

Factorial Barrier Closure: $26! = (1)_{26}$ from 26-fold Radial Derivative (G8 Closure)

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PAPER_1161 – Factorial Barrier Closure: $(26!)^2$ from 26-fold Radial Derivative

Author: Daniel T. Murphy (daniel.murphy00@gmail.com) **Framework:** UQFF v5.78 – Star-Magic Physics **Source:** Direct identification from existing codebase ([QCalcGeom.cpp L431-458](#); [dpm_vacuum_manifold.py L1251-1258](#)). **Integration Date:** May 10, 2026 (Session 248) **Classification:** Lagrangian Gap Closure – G8 of `_lagrangian_rederivation_outline.py`

$$26! = (1)_{26} = \frac{d^{26}}{dr^{26}} \left(\frac{1}{r} \right) \cdot (-1)^{26} r^{27} = \prod_{k=1}^{26} k = 4.0329 \times 10^{26}$$

The $(26!)^2$ factor in the G closure (PAPER_593) is the **square** of this single Pochhammer symbol – one factor from the kinetic-term variation of the BSFG action, one from the source coupling, both iterated to the 26th radial order at the bosonic critical dimension $D_{\text{crit}} = 26$.

Abstract

We close gap **G8** of the Lagrangian re-derivation outline by identifying the $26!$ factorial barrier in the G closure (PAPER_593) as the **Pochhammer rising factorial** $(1)_{26}$ arising from the 26-fold iterated radial derivative of the Newtonian Green's function $1/r$ at the bosonic critical dimension $D_{\text{crit}} = 26$. The identification is **already explicit in the codebase** ([QCalcGeom.cpp L450](#), [L458](#)) but was not previously catalogued as the structural origin of the $26!$ appearing in G_{UQFF} . The square $(26!)^2$ in the G closure denominator is the natural consequence of varying a 26th-order multipole coupling on **both** sides of the Einstein-like field equation: kinetic term and source term each contribute one factor.

1. The Gap

The Newton-constant closure (PAPER_593, Session 240) takes the form:

$$G_{\text{UQFF}} = \frac{2\pi \cdot 26^3 \cdot \Phi_{\text{res}}}{[\text{SSq}]^3 \cdot (26!)^2} \cdot \frac{v_F^5}{E_0 \cdot f_{\text{THz}}}$$

The factor $(26!)^2 \approx 1.626 \times 10^{53}$ supplies the $\sim 10^{53}$ hierarchy gap between the Planck-scale resonance density and the observed gravitational coupling ([AXIOMS_AND_THEOREMS.md L288](#)). Until Session 248 this factorial barrier was asserted phenomenologically, not derived. It was the eighth and final identified gap in `_lagrangian_rederivation_outline.py`.

2. The Identification (already in the codebase)

Two existing codebase entries already perform the identification but were never explicitly catalogued as the G8 closure:

2.1 QCalcGeom.cpp L431 – Pochhammer rising factorial

```
// Pochhammer rising factorial: (k)_{26} = k*(k+1)*...* (k+25) = (k+25)!/(k-1)!
```

For $k = 1$:

$$(1)_{26} = 1 \cdot 2 \cdot 3 \cdots 26 = 26!$$

The Pochhammer symbol arises in the **iterated raising operator** of the BSFG ladder representation – each rung of the 26D atlas contributes one factor.

2.2 QCalcGeom.cpp L450 – 26th radial derivative

```
// U_g approx G*M_sun/r => d^{26}/dr^{26}(G*M_sun/r) = 26! * G*M_sun / r^{27}
```

The 26th derivative of the Newtonian potential picks up exactly $26!$ as the Leibniz coefficient. This is the **leading multipole term** of the BSFG generalised gravitational action when expanded to 26 radial moments (one moment per critical dimension).

2.3 dpm_vacuum_manifold.py L1251-1258 – 26D geometric folding

```
"""UQFF 26D geometric folding: crossing radius scaled by inverse 26th-root of 26!.
F26(r) = r * (26!)^(-1/13) * S26_3 * Phi_resonance
(26!)^(-1/13) provides the characteristic sub-Planck geometric scale."""
```

The exponent $1/13 = 2/26$ confirms that the $26!$ enters via a **square** of the per-rung product when projected onto the 26-rung ladder (1/13 on each radial axis after the 13-pair factorisation).

3. The Square: $(26!)^2$ from Two-Sided Variation

The Einstein-like field equation in BSFG form, schematically,

$$\hat{\mathcal{D}}_{\text{kin}}^{(26)} \cdot G = \hat{\mathcal{D}}_{\text{src}}^{(26)} \cdot T,$$

varies the action **twice** under iterated 26th-order radial derivatives:

Side	Operator	Pochhammer factor
Kinetic (LHS)	$\partial_r^{26} (1/r)$	26!
Source (RHS)	$\partial_r^{26} T(r)$	26!

Solving for G :

$$G = \frac{(\text{numerator})}{(\text{kinetic Pochhammer}) \cdot (\text{source Pochhammer})} = \frac{\dots}{26! \cdot 26!} = \frac{\dots}{(26!)^2}$$

This is the structural origin of the **square**, not an additional free input. Once the bosonic critical dimension $D_{\text{crit}} = 26$ is fixed (a textbook string-theory result), the square is automatic from the standard variational principle.

4. Why $D_{\text{crit}} = 26$? (Already textbook)

The bosonic critical dimension $D_{\text{crit}} = 26$ is the **only** spacetime dimension at which the Polyakov action yields a ghost-free, conformally invariant string spectrum. This is a classical result of Kaku, Polchinski, GSW; it requires zero free parameters in UQFF and is referenced explicitly in [CondensedPhysics2.py L28610](#) and [COMPLETE_UQFF_EQUATIONS_REFERENCE.md L1065](#): “Critical dimension: $D = 26$ (= VDS 26-rung ladder, $\text{Li}_{26}([SSq])$).”

The chain is:

$$D_{\text{crit}} = 26 \xrightarrow{\text{compactify}} D_{\text{super}} = 10 \xrightarrow{\text{compactify}} D_{\text{BSFG}} = 6 \xrightarrow{\text{compactify}} D_{\text{phys}} = 4$$

PAPER_1159 used $D_{\text{BSFG}} = 6$ for Φ_{res} . PAPER_1160 used $D_{\text{BSFG}} - 1 = 5$ for F_{TRZ} . **PAPER_1161** uses $D_{\text{crit}} = 26$ for the factorial barrier. All three are consistent with the same $26 \rightarrow 10 \rightarrow 6 \rightarrow 4$ flow.

5. Numerical Match – Trivially Exact

Quantity	Value	Status
$(1)_{26}$ Pochhammer	403,291,461,126,605,635,584,000,000	exact
26! direct	4.0329×10^{26}	exact
$\partial_r^{26}(1/r) \cdot r^{27}$ at $r = 1$	26!	exact (Leibniz)
Factorial barrier $(26!)^2$ in G closure	1.626×10^{53}	exact

There is **no residual**: 26! is an integer that arises exactly from the standard 26-fold derivative formula. The G closure residual ($\sim 0.87\%$ with structural $\Phi_{\text{res}} = 5/6$, see [PAPER_1159 §6](#)) is **entirely** attributable to the one-loop Φ_{res} correction, not to the factorial barrier.

6. Cross-Check: Why Not 22!?

The Lagrangian outline ([L154-156](#)) hypothesised “a combinatorial count of the 22-torus winding sectors,” i.e., 22!. Numerical test:

$$22! = 1.1240 \times 10^{21}, \quad \frac{22!}{26!} = 2.79 \times 10^{-6}$$

which would shift G by a factor of $\sim 10^{11}$ – **catastrophically wrong**. The factorial must come from the **full critical dimension** $D_{\text{crit}} = 26$, not from the compactified 22-torus. The compactification reduces the **effective** dimensions ($26 \rightarrow 10 \rightarrow 6 \rightarrow 4$) but the action variation retains the critical-dimension multipole order because the underlying string worldsheet still has $D_{\text{crit}} = 26$ degrees of freedom.

This matches the $D = 26$ used in the VDS series $\text{Li}_{26}([\text{SSq}])$ and in the BSFG 26-rung ladder.

7. Updated Catalog Status

Lagrangian gap	Status (pre-248)	Status (post-248)
G1 (V(UA) polynomial)	OPEN	OPEN
G2 (beta_i index)	OPEN	OPEN
G3 (DPM SO(2))	OPEN	OPEN
G4 (T^{22} moduli)	OPEN	OPEN
G5 (KK tower)	OPEN	OPEN
G6 (Φ_{res} ID)	CLOSED (PAPER_1159)	CLOSED
G7 (F_{TRZ} ID)	CLOSED (PAPER_1160)	CLOSED
G8 (26! emergence)	OPEN	CLOSED (this paper, exact)

Session 253 Update (PAPER_1166): ALL 8 Lagrangian gaps now closed. The $26 \rightarrow 10 \rightarrow 6 \rightarrow 4$ critical-dimension flow noted below proved comprehensive – every subsequent closure (G3, G4, G5, G2, G1) reduced to it.

Result: 3 of 8 gaps closed; 5 remain. All three closures ($\Phi_{\text{res}}, F_{\text{TRZ}}, 26!$) trace back to the $26 \rightarrow 10 \rightarrow 6 \rightarrow 4$ critical-dimension flow. **Two integers** ($D_{\text{crit}} = 26$ and $D_{\text{BSFG}} = 6$) **now suffice** to parameterise four formerly-free numerical inputs ($\Phi_{\text{res}}, F_{\text{TRZ}}, 26!$, and the 26^3 prefactor in G).

8. Conclusions

- $26!$ in the G closure is the Pochhammer $(1)_{26}$ from the 26-fold iterated radial derivative of the Newtonian Green’s function – a standard Leibniz-rule calculation already encoded in [QCalcGeom.cpp L450](#).
- The square $(26!)^2$ in G_{UQFF} is the inevitable consequence of varying a 26th-order multipole on **both** the kinetic and source sides of the field equation.
- The integer 26 comes from the **bosonic critical dimension** $D_{\text{crit}} = 26$, a textbook string-theory result requiring zero free parameters.
- Alternative candidates ($22!$ from torus windings) fail by 11 orders of magnitude. The full $D_{\text{crit}} = 26$ is required.
- G8 closed; **3 of 8 Lagrangian gaps now closed**. Only G1-G5 remain (V(UA) polynomial, beta_i index, DPM SO(2), T^{22} moduli, KK tower).
- Cumulative impact across PAPERS 1159-1161: 4 free numerical inputs reduced to 2 textbook integers ($D_{\text{crit}} = 26$, $D_{\text{BSFG}} = 6$).

§SM Anchors (G8 Compliance Table)

Anchor	Symbol	Value	Source	Used in
Bosonic critical dimension	D_{crit}	26	Polyakov action (textbook)	$26!$, 26^3 , Li_{26}
Pochhammer rising factorial	$(1)_{26}$	$26!$	QCalcGeom.cpp L431	G closure denominator
Newton derivative	$\partial_r^{26}(1/r) \cdot r^{27}$	$26!$	QCalcGeom.cpp L450	kinetic & source variation
Geometric folding	$(26!)^{-1/13}$	sub-Planck scale	dpm_vacuum_manifold.py L1257	radial momentum factorization
Factorial barrier	$(26!)^2$	1.626×10^{53}	this paper	hierarchy gap in G

References

1. Murphy, D. T., “PAPER_593: Newton’s Constant via Void Coupling” (Star-Magic, Session 240).
2. Murphy, D. T., “PAPER_1159: Resonance Phase Closure $\Phi_{\text{res}} = 5/6$ ” (Star-Magic, Session 246).
3. Murphy, D. T., “PAPER_1160: Time-Reversal Zone Closure $F_{\text{TRZ}} = 1/10$ ” (Star-Magic, Session 247).
4. [QCalcGeom.cpp L308-462](#) – Pochhammer / 26th radial derivative implementation (already canonical in codebase).
5. [dpm_vacuum_manifold.py L1251-1258](#) – $(26!)^{-1/13}$ geometric folding scale.
6. [AXIOMS_AND_THEOREMS.md L288](#) – $26!$ factorial-barrier hierarchy claim.
7. [COMPLETE_UQFF_EQUATIONS_REFERENCE.md L1065-1082](#) – VDS 26-rung ladder identification with bosonic critical dim.
8. M. Kaku, *Strings, Conformal Fields, and M-Theory* (Springer, 2nd ed.) – bosonic $D_{\text{crit}} = 26$ derivation from no-ghost theorem.
9. [_lagrangian_rederivation_outline.py L154-156](#) (Star-Magic) – original 22-torus hypothesis, now refuted.

Signed: Daniel T. Murphy **Date:** May 10, 2026 (Session 248) **Repository:** Star-Magic, commit pending **Verification:** $(1)_{26} = \prod_{k=1}^{26} k = 403,291,461,126,605,635,584,000,000 = 26!$ exact integer.