CoolProp IF97 Mathcad Add-in DLL Function Verification (IAPWS Units)

This IF97 Mathcad Add-in DLL is based on the CoolProp implementation of the Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam (2007) and supplementary releases for additional transport and physical properties for pure water substance. This implementation of the IF97 equations is:

- Entirely free and open-source
- Written in optimized standard C++ code so it will compile anywhere
- Fast
- Easy-to-use (just a single header)

This Mathcad worksheet provides verification of of the CoolProp IF97 implementation and Mathcad Add-in DLL by comparing calculated values to tabulated values for computer program verification contained in the various IAPWS release documents.

IF97 Mathcad Functions

After copying the `if97_EN.xml` file to the directory

```
C:\Program Files (x86)\Mathcad\Mathcad 15\doc\funcdoc
```

the IF97 functions can be accessed through the Mathcad menus under **Insert | Function...**, or from the ***f(x)*** button on the toolbar. On the function panel, the IF97 functions will be under the if97 category and will all begin with the **if97_** prefix.

CoolProp IF97 Implementation Version

This Mathcad function returns the version number of the CoolProp IF97 implementation:

if97_getvers(0) = "v2.0.0 (IAPWS Units)"

CoolProp IF97 Units

By default, CoolProp IF97 will take all inputs/outputs in straight SI units of Pa, J, kg, m, etc. However, the IAPWS IF97 Release documents use units of MPa and kJ for the basic thermodynamic property equation and verification tables. The CoolProp IF97 implementation will use the IAPWS units if the calling codes sets the `IAPWS_UNITS` compiler flag. If set, the string `" (IAPWS Units)"` will be appended to ***if97_getvers()*** output, as demonstrated above.

This is the IAPWS Units verification file and should ***only*** be used if ***IAPWS_UNITS*** is switched ***ON***

IF97 Thermodynamic Properties Verification for v, h, u, s, Cp, w

These tables below contain the Reference [a] published values for verification of computer programs. Each entry shows the page number and table where the values can be found in the Reference [a] document.

TP :=

T (K)	P (MPa)		
300	3	"Region 1"	"Page 9, Table 5"
300	80	"Region 1"	"Page 9, Table 5"
500	3	"Region 1"	"Page 9, Table 5"
300	0.0035	"Region 2"	"Page 17, Table 15"
700	0.0035	"Region 2"	"Page 17, Table 15"
700	30	"Region 2"	"Page 17, Table 15"
650	25.58370180	"Region 3"	"Page 32, Table 33"
650	22.29306430	"Region 3"	"Page 32, Table 33"
750	78.30956390	"Region 3"	"Page 32, Table 33"
1500	0.5	"Region 5"	"Page 40, Table 42"
1500	30	"Region 5"	"Page 40, Table 42"
2000	30	"Region 5"	"Page 40, Table 42"

$T1 := TP^{\langle 1 \rangle} \cdot K$

$P1 := TP^{\langle 2 \rangle} \cdot MPa$

$i := 1 .. rows(P1)$

$j := 1 .. 6$

$kJ \equiv 1000 \cdot J$

IF97 :=

v (m³/kg)	h (kJ/kg)	u (kJ/kg)	s (kJ/kg-K)	Cp (kJ/kg-K)	w (m/s)
0.001002151680	115.331273	112.324818	0.392294792	4.17301218	1507.739210
0.000971180894	184.142828	106.448356	0.368563852	4.01008987	1634.690540
0.001202418000	975.542239	971.934985	2.580419120	4.65580682	1240.713370
39.4913866	2549.911450	2411.691600	8.522389670	1.91300162	427.920172
92.3015898	3335.683750	3012.628190	10.174999600	2.08141274	644.289068
0.005429466190	2631.494740	2468.610760	5.175402980	10.35050920	480.386523
0.002	1863.430190	1812.262790	4.054272730	13.89357170	502.005554
0.005	2375.124010	2263.658680	4.854387920	44.65793420	383.444594
0.002	2258.688450	2102.069320	4.469719060	6.34165359	760.696041
1.3845509	5219.768550	4527.493100	9.654088750	2.61609445	917.068690
0.0230761299	5167.235140	4474.951240	7.729701330	2.72724317	928.548002
0.0311385219	6571.226040	5637.070380	8.536405230	2.88569882	1067.369480

Verification using Newton-Raphson reverse lookup in Region 3

$$M_{i,1} := \text{if97\_vtp}\left(\frac{T1_i}{K}, \frac{P1_i}{MPa}\right) \qquad M_{i,2} := \text{if97\_htp}\left(\frac{T1_i}{K}, \frac{P1_i}{MPa}\right) \qquad M_{i,3} := \text{if97\_utp}\left(\frac{T1_i}{K}, \frac{P1_i}{MPa}\right)$$

$$M_{i,4} := \text{if97\_stp}\left(\frac{T1_i}{K}, \frac{P1_i}{MPa}\right) \qquad M_{i,5} := \text{if97\_cptp}\left(\frac{T1_i}{K}, \frac{P1_i}{MPa}\right) \qquad M_{i,6} := \text{if97\_wtp}\left(\frac{T1_i}{K}, \frac{P1_i}{MPa}\right)$$

Calculate the relative error in the calculated values from the published values: $ERR_{i,j} := \frac{\left|IF97_{i,j} - M_{i,j}\right|}{IF97_{i,j}}$

$$E := \text{augment}\left(TP^{\langle 1\rangle}, TP^{\langle 2\rangle}, ERR\right)$$

Relative Error Between Calculated and Published Values

(Newton Raphson Reverse Lookup in Region 3)

T [K]	p [Mpa]	ε_v	ε_h	ε_u	ε_s	ε_{Cp}	ε_w
300	3	3.1E-10	1.9E-10	1.6E-10	1.0E-09	9.7E-10	2.2E-10
300	80	2.2E-11	1.4E-09	2.0E-09	1.1E-09	8.8E-11	1.9E-09
500	3	2.8E-09	1.0E-10	9.0E-11	2.0E-11	4.5E-10	2.5E-09
300	0.0035	9.6E-10	3.3E-10	9.9E-10	3.1E-10	5.1E-10	6.1E-10
700	0.0035	1.9E-10	1.1E-09	2.1E-10	2.1E-09	1.8E-09	6.7E-10
700	30	8.5E-10	1.8E-09	4.0E-10	4.4E-10	8.0E-10	3.5E-10
650	25.5837	6.0E-10	9.6E-11	1.9E-09	9.7E-10	6.3E-09	1.7E-09
650	22.29306	1.6E-08	5.9E-09	1.6E-09	3.1E-09	6.9E-08	6.0E-09
750	78.30956	1.4E-10	2.0E-09	1.1E-09	8.1E-10	9.2E-10	3.2E-10
1500	0.5	8.8E-10	2.3E-10	4.0E-10	3.4E-10	1.5E-09	3.3E-10
1500	30	2.0E-09	1.7E-11	3.7E-10	4.9E-10	8.3E-10	2.3E-10
2000	30	9.7E-10	2.1E-10	4.5E-10	1.3E-10	4.2E-10	1.1E-09

The largest errors are in v and Cp at 22.3 MPa and 650 K and are accurate to about 7 significant figures. These are in Region 3 where the reverse lookup occurs for density. The remaining values are accurate to at least 8 significant figures or more. The verification values for Cv are not listed in the IAPWS documents.

NOTE: The Mathcad wrapper is implemented with the REGION3_ITERATE compiler switch turned ON. While this is slightly slower, the results are more accurate in REGION3 as shown below.

Verification using supplemental reverse lookup equations in Region 3

Relative Error Between Calculated and Published Values

(Supplemental Reverse Lookup Functions in Region 3)

T [K]	p [Mpa]	ε_v	ε_h	ε_u	ε_s	ε_{Cp}	ε_w
300	3	3.1E-10	1.9E-10	1.6E-10	1.0E-09	9.7E-10	2.2E-10
300	80	2.2E-11	1.4E-09	2.0E-09	1.1E-09	8.8E-11	1.9E-09
500	3	2.8E-09	1.0E-10	9.0E-11	2.0E-11	4.5E-10	2.5E-09
300	0.0035	9.6E-10	3.3E-10	9.9E-10	3.1E-10	5.1E-10	6.1E-10
700	0.0035	1.9E-10	1.1E-09	2.1E-10	2.1E-09	1.8E-09	6.7E-10
700	30	8.5E-10	1.8E-09	4.0E-10	4.4E-10	8.0E-10	3.5E-10
650	25.5837	4.2E-06	1.3E-06	1.3E-06	1.0E-06	2.2E-05	8.8E-06
650	22.29306	1.1E-07	2.9E-08	2.2E-08	2.1E-08	4.8E-07	4.5E-08
750	78.30956	1.4E-06	3.6E-07	4.5E-07	3.5E-07	1.7E-06	1.7E-06
1500	0.5	8.8E-10	2.3E-10	4.0E-10	3.4E-10	1.5E-09	3.3E-10
1500	30	2.0E-09	1.7E-11	3.7E-10	4.9E-10	8.3E-10	2.3E-10
2000	30	9.7E-10	2.1E-10	4.5E-10	1.3E-10	4.2E-10	1.1E-09

The largest errors are in Region 3 where the reverse lookup occurs for density. These errors are on the order of 1E-6 using just the supplemental reverse lookup equations for density, while the error in the other regions is on the order of 1.0E-9 or lower.

Timing Test

Supplemental Reverse Lookup Functions

$$\mathrm{Time}_1 := \left| \begin{array}{l} \text{for } j \in 1..100 \\ \quad \left| \begin{array}{l} t_0 \leftarrow \mathrm{time}(0) \\ \text{for } i \in 1..10000 \\ \quad \left| \begin{array}{l} a \leftarrow \mathrm{if97\_vtp}\left(\frac{T1_7}{K}, \frac{P1_7}{Pa}\right) \\ b \leftarrow \mathrm{if97\_vtp}\left(\frac{T1_8}{K}, \frac{P1_8}{Pa}\right) \\ c \leftarrow \mathrm{if97\_vtp}\left(\frac{T1_9}{K}, \frac{P1_9}{Pa}\right) \end{array} \right. \\ \Delta t_j \leftarrow \mathrm{time}(0) - t_0 \end{array} \right. \\ \mathrm{mean}(\Delta t) \end{array} \right. = 0.028$$

Timing Test

Newton-Raphson Iterative Reverse Lookup

$$\mathrm{Time}_2 := \left| \begin{array}{l} \text{for } j \in 1..100 \\ \quad \left| \begin{array}{l} t_0 \leftarrow \mathrm{time}(0) \\ \text{for } i \in 1..10000 \\ \quad \left| \begin{array}{l} a \leftarrow \mathrm{if97\_vtp}\left(\frac{T1_7}{K}, \frac{P1_7}{Pa}\right) \\ b \leftarrow \mathrm{if97\_vtp}\left(\frac{T1_8}{K}, \frac{P1_8}{Pa}\right) \\ c \leftarrow \mathrm{if97\_vtp}\left(\frac{T1_9}{K}, \frac{P1_9}{Pa}\right) \end{array} \right. \\ \Delta t_j \leftarrow \mathrm{time}(0) - t_0 \end{array} \right. \\ \mathrm{mean}(\Delta t) \end{array} \right. = 0.072$$

Using the supplemental reverse lookup equations for density to get an accurate initial guess for pressure and then using Newton-Raphson to iterate on the original Region 3 equation increases the time to calculate density (specific volume) values in this region by a factor of about 2.6, but is far better than the factor of 17 indicated in the supplemental release document using an arbitrary initial guess.

Region 3 Supplemental Reverse Functions Verification

The tables below contain the Reference [b] published values for verification of computer programs. These values can be found in Table 5 (page 13) and Table 13 (page 20) of Reference [b].

$$R1 := \begin{pmatrix} 50 & 630 & 1.470853100 \\ 80 & 670 & 1.503831359 \\ 50 & 710 & 2.204728587 \\ 80 & 750 & 1.973692940 \\ 20 & 630 & 1.761696406 \\ 30 & 650 & 1.819560617 \\ 26 & 656 & 2.245587720 \\ 30 & 670 & 2.506897702 \\ 26 & 661 & 2.970225962 \\ 30 & 675 & 3.004627086 \\ 26 & 671 & 5.019029401 \\ 30 & 690 & 4.656470142 \\ 23.6 & 649 & 2.163198378 \\ 24 & 650 & 2.166044161 \\ 23.6 & 652 & 2.651081407 \\ 24 & 654 & 2.967802335 \\ 23.6 & 653 & 3.273916816 \\ 24 & 655 & 3.550329864 \\ 23.5 & 655 & 4.545001142 \\ 24 & 660 & 5.100267704 \end{pmatrix} \quad R2 := \begin{pmatrix} 23 & 660 & 6.109525997 \\ 24 & 670 & 6.427325645 \\ 22.6 & 646 & 2.117860851 \\ 23 & 646 & 2.062374674 \\ 22.6 & 648.6 & 2.533063780 \\ 22.8 & 649.3 & 2.572971781 \\ 22.6 & 649 & 2.923432711 \\ 22.8 & 649.7 & 2.913311494 \\ 22.6 & 649.1 & 3.131208996 \\ 22.8 & 649.9 & 3.221160278 \\ 22.6 & 649.4 & 3.715596186 \\ 22.8 & 650.2 & 3.664754790 \\ 21.1 & 640 & 1.970999272 \\ 21.8 & 643 & 2.043919161 \\ 21.1 & 644 & 5.251009921 \\ 21.8 & 648 & 5.256844741 \\ 19.1 & 635 & 1.932829079 \\ 20 & 638 & 1.985387227 \\ 17 & 626 & 8.483262001 \\ 20 & 640 & 6.227528101 \end{pmatrix}$$

$$R3 := \begin{pmatrix} 21.5 & 644.6 & 2.268366647 \\ 22 & 646.1 & 2.296350553 \\ 22.5 & 648.6 & 2.832373260 \\ 22.3 & 647.9 & 2.811424405 \\ 22.15 & 647.5 & 3.694032281 \\ 22.3 & 648.1 & 3.622226305 \\ 22.11 & 648 & 4.528072649 \\ 22.3 & 649 & 4.556905799 \\ 22 & 646.84 & 2.698354719 \\ 22.064 & 647.05 & 2.717655648 \\ 22 & 646.89 & 3.798732962 \\ 22.064 & 647.15 & 3.701940010 \end{pmatrix}$$

Combine columns and split out ***p***, ***T***, and ***v***.

$$Rev := \text{stack}(R1, R2, R3)$$

$$P_{rev} := Rev^{\langle 1 \rangle} \cdot MPa \qquad T_{rev} := Rev^{\langle 2 \rangle} \cdot K$$

$$v_{rev} := Rev^{\langle 3 \rangle} \cdot 10^{-3}$$

$$j := 1 .. \text{length}(v_{rev})$$

Calculate the specific volumes using the IF97 supplemental reverse functions:

$$v'_{rev_j} := \text{if97\_vtp}\left(\frac{T_{rev_j}}{K}, \frac{P_{rev_j}}{MPa}\right)$$

Relative Error

$$\varepsilon_{rev} := \overrightarrow{\frac{v_{rev} - v'_{rev}}{v_{rev}}}$$

$$RMS_3 := \sqrt{\frac{\left[\sum \overrightarrow{(v_{rev} - v'_{rev})}\right]^2}{\text{length}(v_{rev})}} = 5.257 \times 10^{-13}$$

RMS error is well below the 10 significant digits listed in the tables.

$$Rev^{\langle 4 \rangle} := v'_{rev} \cdot 10^3 \qquad Rev^{\langle 5 \rangle} := \varepsilon_{rev}$$

<== *Tack calculated values and rel. error onto table.*

p [MPa]	*T* [K]	*v* [m³/kg]	*v calc* [m³/kg]	*rel. error* [*unitless*]
50	630	1.4708531	1.4708531	-7.540579442E-11
80	670	1.503831359	1.503831359	3.055710859E-10
50	710	2.204728587	2.204728587	-2.607323165E-11
80	750	1.97369294	1.97369294	-6.136484297E-11
20	630	1.761696406	1.761696406	2.670563350E-10
30	650	1.819560617	1.819560617	2.589454497E-10
26	656	2.24558772	2.24558772	-1.327314827E-11
30	670	2.506897702	2.506897702	1.477602538E-10
26	661	2.970225962	2.970225962	-1.059882133E-12
30	675	3.004627086	3.004627086	-1.191517559E-10
26	671	5.019029401	5.019029401	-8.908076031E-12
30	690	4.656470142	4.656470142	6.751252519E-11
23.6	649	2.163198378	2.163198378	-1.450261830E-10
24	650	2.166044161	2.166044161	2.009126820E-11
23.6	652	2.651081407	2.651081407	1.607426161E-10
24	654	2.967802335	2.967802335	2.029112835E-11
23.6	653	3.273916816	3.273916816	1.958965329E-11
24	655	3.550329864	3.550329864	9.591066802E-11
23.5	655	4.545001142	4.545001142	7.714460305E-11
24	660	5.100267704	5.100267704	8.368087891E-11
23	660	6.109525997	6.109525997	1.854665487E-11
24	670	6.427325645	6.427325645	4.643044848E-11
22.6	646	2.117860851	2.117860851	1.519907637E-10
23	646	2.062374674	2.062374674	-7.114438656E-11
22.6	648.6	2.53306378	2.53306378	-1.663914381E-10
22.8	649.3	2.572971781	2.572971781	3.302213485E-11
22.6	649	2.923432711	2.923432711	6.199398210E-13
22.8	649.7	2.913311494	2.913311494	-1.417239015E-11
22.6	649.1	3.131208996	3.131208996	-2.080171902E-12
22.8	649.9	3.221160278	3.221160278	1.977936043E-11
22.6	649.4	3.715596186	3.715596186	-1.311510140E-10
22.8	650.2	3.66475479	3.66475479	1.040397240E-10
21.1	640	1.970999272	1.970999272	5.481543383E-11
21.8	643	2.043919161	2.043919161	4.214148618E-11
21.1	644	5.251009921	5.251009921	-1.910636882E-11

** These values and the RMS error above were calculated with the REGION3_ITERATE switch turned OFF to verify the reverse lookup equations. With REGION3_ITERATE switched ON, the calculated values will be slightly more accurate (as demonstrated above) but will not match these values past six significant figures.*

Saturation Curve Verification

Set up Pressure and Temperature matrix based on tabulated values from IAPWS-IF97 on page 35, Table 36.

IAPWS Values

$$P_{IAPWS} := \begin{pmatrix} 0.1 \\ 1 \\ 10 \end{pmatrix} \cdot MPa$$

$$T_{IAPWS} := \begin{pmatrix} 372.755919 \\ 453.035632 \\ 584.149488 \end{pmatrix} \cdot K$$

$$ERR := \overrightarrow{\frac{T_{IAPWS} - T2}{T_{IAPWS}}} = \begin{pmatrix} 1.043 \times 10^{-9} \\ -8.641 \times 10^{-10} \\ 2.521 \times 10^{-12} \end{pmatrix}$$

Calculated Values

$$T2 := \overrightarrow{if97\_tsatp\left(\frac{P_{IAPWS}}{MPa}\right) \cdot K} = \begin{pmatrix} 372.756 \\ 453.036 \\ 584.149 \end{pmatrix} K$$

$$P2 := \overrightarrow{if97\_psatt\left(\frac{T_{IAPWS}}{K}\right) \cdot MPa} = \begin{pmatrix} 0.1 \\ 1 \\ 10 \end{pmatrix} \cdot MPa$$

$$ERR2 := \overrightarrow{\frac{P_{IAPWS} - P2}{P_{IAPWS}}} = \begin{pmatrix} -1.39 \times 10^{-8} \\ 9.008 \times 10^{-9} \\ -2.003 \times 10^{-11} \end{pmatrix}$$

The calculated values agree to the published values to about 8 significant figures or more.

Triple Point: $T_t := if97\_ttrip(0) \cdot K = 273.16\,K$ $\quad P_t := if97\_ptrip(0) \cdot MPa = 611.656\,Pa$

Critical Point: $T_c := if97\_tcrit(0) \cdot K = 647.096\,K$ $\quad P_c := if97\_pcrit(0) \cdot MPa = 22.064 \cdot MPa$

Set up Curve Ranges:

$$\Delta P := \frac{P_c - P_t}{1000} = 0.022 \cdot MPa \qquad \Delta T := \frac{T_c - T_t}{600} = 0.623\,K$$

$$P_s := P_t, P_t + \Delta P .. P_c \qquad T_s := T_t, T_t + \Delta T .. T_c$$

Saturation Functions :

$$T_{sat}(P) := round\left(if97\_tsatp\left(\frac{P}{MPa}\right), 7\right) \cdot K$$

$$P_{sat}(T) := round\left(if97\_psatt\left(\frac{T}{K}\right), 9\right) \cdot MPa$$

check :

$$T_{sat}(P_t) = 273.1599775\,K \qquad P_{sat}(T_t) = 611.657\,Pa$$

$$T_{sat}(P_c) = 647.096\,K \qquad P_{sat}(T_c) = 22.064 \cdot MPa$$

Region 2/3 Functions :

$T_{23}(P) := \text{if97\_t23}\left(\frac{P}{Pa}\right)\cdot K$ $\quad P_{23}(T) := \text{if97\_p23}\left(\frac{T}{K}\right)\cdot MPa$

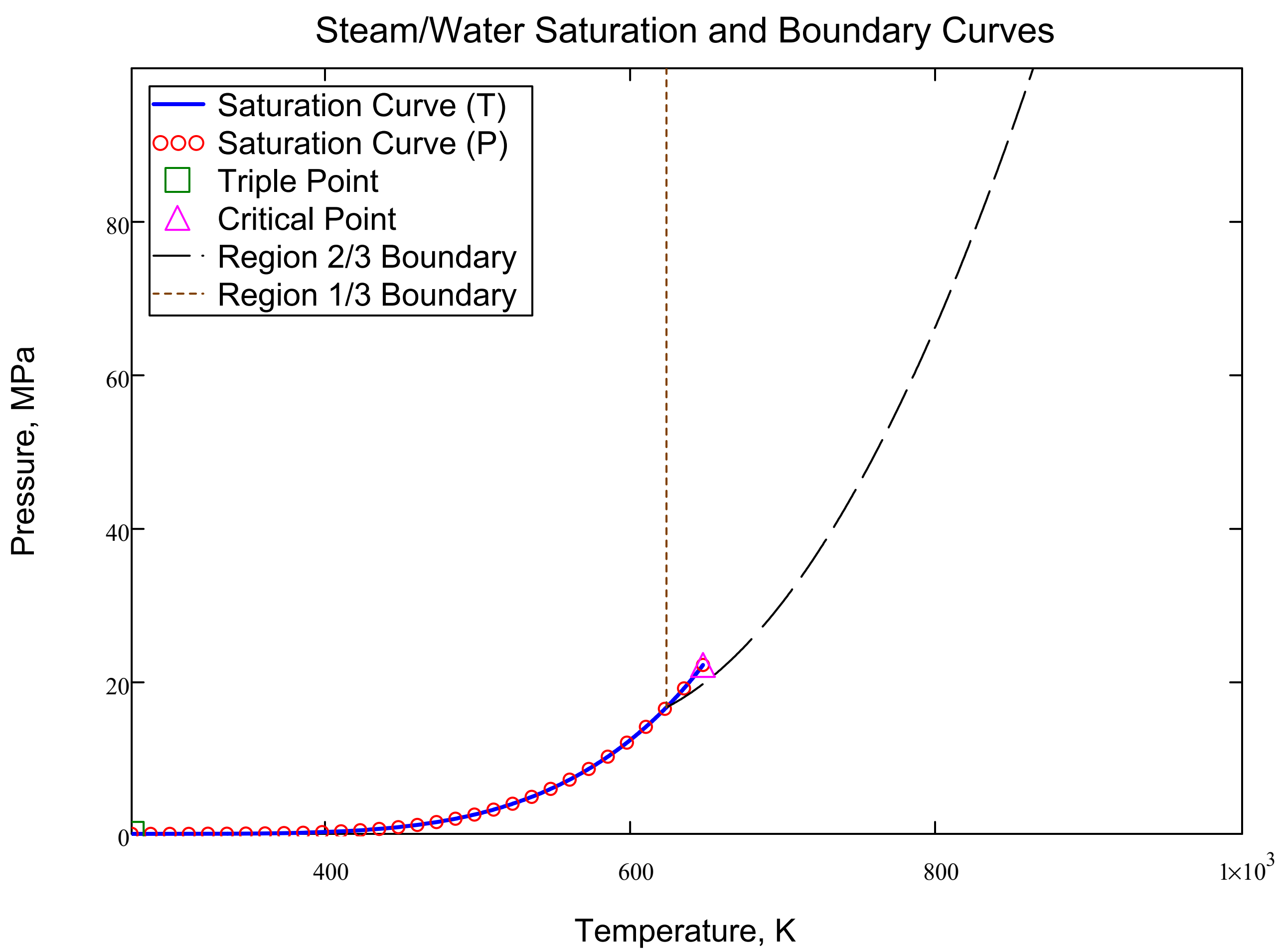
$T_{Bmin} := 623.15\cdot K$ $\quad T_{Bmax} := 863.15\cdot K$

$\Delta T_B := \frac{T_{Bmax} - T_{Bmin}}{300} = 0.8\,K$ $\quad T_B := T_{Bmin}, T_{Bmin} + \Delta T_B .. T_{Bmax}$

Region 1/3 Curve :

$T_{13_1} := T_{Bmin}$ $\quad T_{13_2} := T_{Bmin}$

$P_{13_1} := P_{23}(T_{Bmin}) = 16.529\cdot MPa$ $\quad P_{13_2} := 100\cdot MPa$



Saturation Function Verification and Continuity Check

Discrete Pressure Isobars

$p1 := 0.05 \cdot MPa$ $\quad p4 := P_c$

$p2 := .5 \cdot MPa$ $\quad p5 := 50 \cdot MPa$

$p3 := 5 \cdot MPa$ $\quad p6 := 100 \cdot MPa$

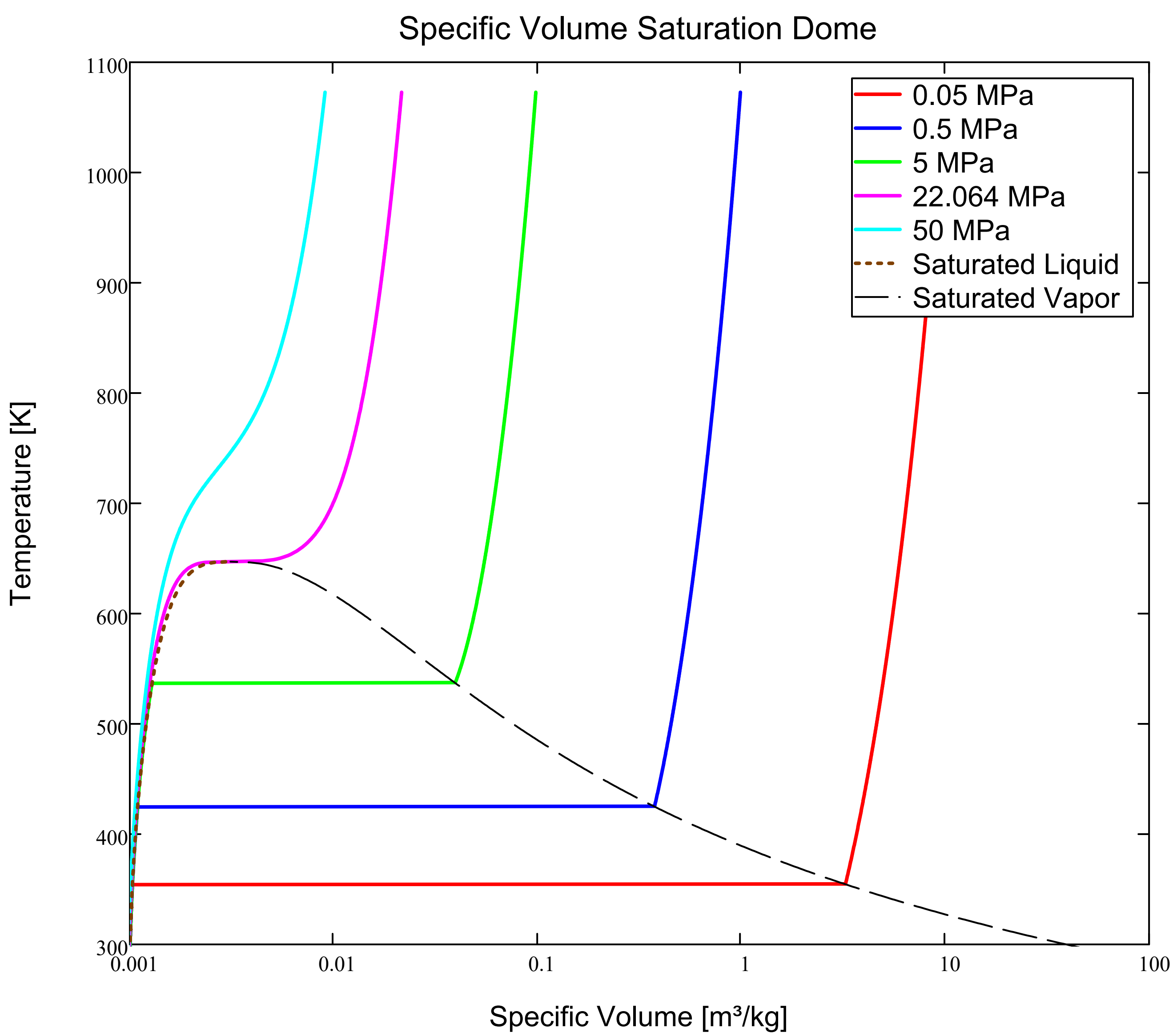
Temperature Range

$TT := T_t, T_t + \Delta T .. 1073.15 \cdot K$

Define Specific Volume Functions with Units

$$v_{tp}(T,P) := if97\_vtp\left(\frac{T}{K}, \frac{P}{MPa}\right) \cdot \frac{m^3}{kg} \qquad v_f(P) := if97\_vf\left(\frac{P}{MPa}\right) \cdot \frac{m^3}{kg}$$

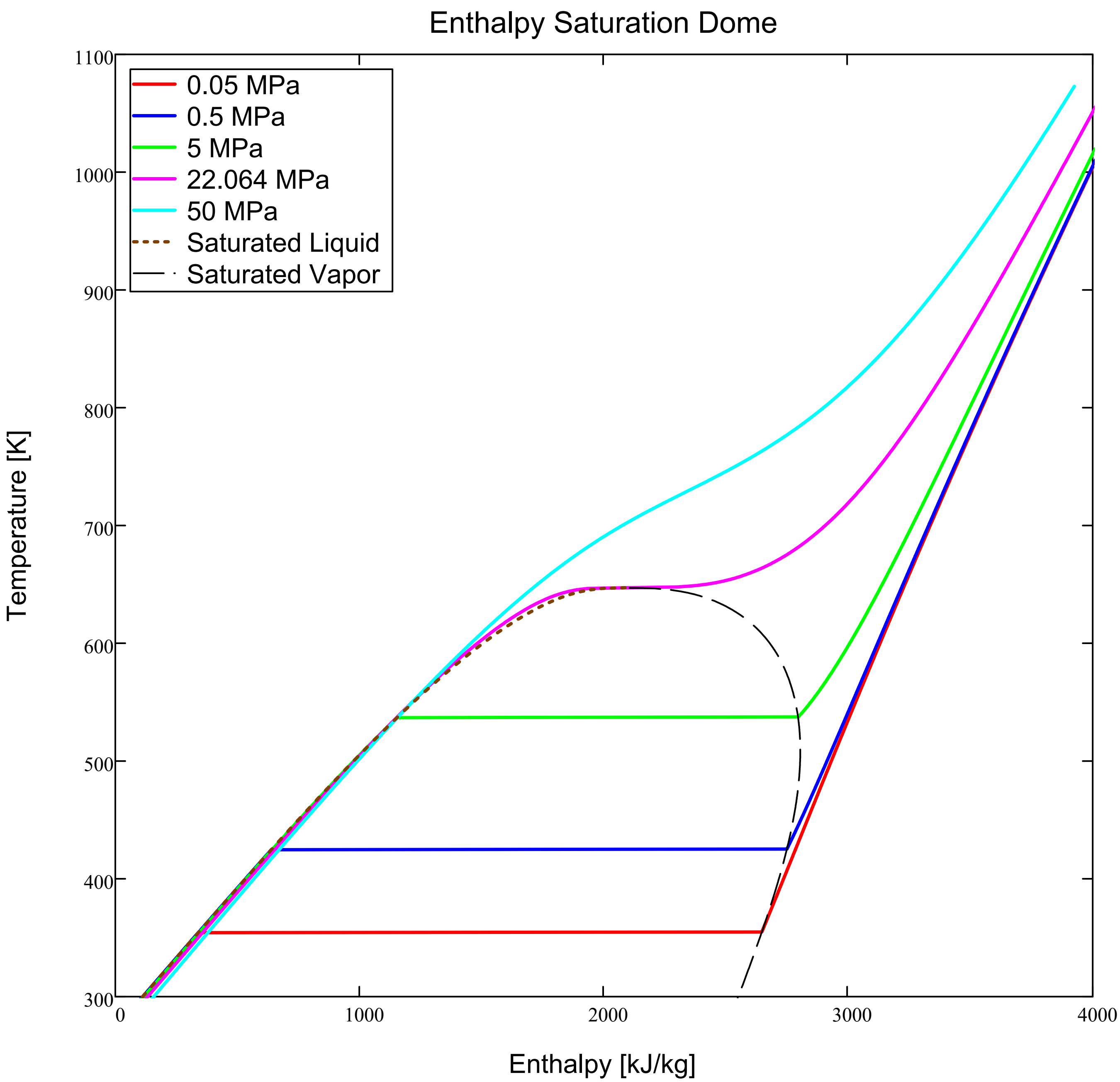
$$v_g(P) := if97\_vg\left(\frac{P}{MPa}\right) \cdot \frac{m^3}{kg}$$



Define Enthalpy Unit Functions:

$$h_{tp}(T,P) := \text{if97\_htp}\left(\frac{T}{K}, \frac{P}{MPa}\right)\cdot\frac{kJ}{kg} \qquad h_f(P) := \text{if97\_hf}\left(\frac{P}{MPa}\right)\cdot\frac{kJ}{kg}$$

$$h_g(P) := \text{if97\_hg}\left(\frac{P}{MPa}\right)\cdot\frac{kJ}{kg}$$



Reverse Function Verification

The ***Pressure-Enthalpy*** tables below contain the published *T*(*p,h*) values for verification of computer programs from Table 7 (p. 11), and Table 24 (p. 25) of Reference [b], as well as Table 5 (p. 8) of Reference [c].

Define the Backward unit function: $T_{ph}(p,h) := \text{if97\_tph}\left(\frac{p}{MPa}, \frac{h}{kJ \cdot kg^{-1}}\right) \cdot K$

Validation Points		
p [Mpa]	h [kJ/kg]	T [K]
3	500	3.91798509E+02
80	500	3.78108626E+02
80	1500	6.11041229E+02
0.001	3000	5.34433241E+02
3	3000	5.75373370E+02
3	4000	1.01077577E+03
5	3500	8.01299102E+02
5	4000	1.01531583E+03
25	3500	8.75279054E+02
40	2700	7.43056411E+02
60	2700	7.91137067E+02
60	3200	8.82756860E+02
20	1700	6.29308389E+02
50	2000	6.90571834E+02
100	2100	7.33616301E+02
20	2500	6.41841805E+02
50	2400	7.35184862E+02
100	2700	8.42046088E+02

Extract Table Values:

$P_{val} := H^{\langle 1 \rangle} \cdot MPa$ $\qquad h_{val} := H^{\langle 2 \rangle} \cdot \frac{kJ}{kg}$

$T_{val} := H^{\langle 3 \rangle} \cdot K$

Calculate Values and Error from IF97:

$T_{calc} := \overrightarrow{T_{ph}(P_{val}, h_{val})}$ $\qquad \varepsilon_{calc} := \overrightarrow{\frac{T_{calc} - T_{val}}{T_{val}}}$

Add columns to table to show below:

$H^{\langle 4 \rangle} := T_{calc} \cdot K^{-1}$

$H^{\langle 5 \rangle} := \varepsilon_{calc}$

p [Mpa]	h [kJ/kg]	T_{val} [K]	T_{ph} [K]	Error [%]
3	500	3.91798509E+02	3.91798509E+02	-6.06368815E-10
80	500	3.78108626E+02	3.78108626E+02	-3.18951355E-10
80	1500	6.11041229E+02	6.11041229E+02	6.58978409E-10
0.001	3000	5.34433241E+02	5.34433241E+02	7.14274394E-10
3	3000	5.75373370E+02	5.75373370E+02	4.14487923E-10
3	4000	1.01077577E+03	1.01077577E+03	-4.29270363E-09
5	3500	8.01299102E+02	8.01299102E+02	-1.79453874E-10
5	4000	1.01531583E+03	1.01531583E+03	-4.60337915E-09
25	3500	8.75279054E+02	8.75279054E+02	-3.18181654E-10
40	2700	7.43056411E+02	7.43056411E+02	-5.09931672E-11
60	2700	7.91137067E+02	7.91137067E+02	-5.91197171E-10
60	3200	8.82756860E+02	8.82756860E+02	-3.97333666E-10
20	1700	6.29308389E+02	6.29308389E+02	-6.45404924E-11
50	2000	6.90571834E+02	6.90571834E+02	-1.83830850E-11
100	2100	7.33616301E+02	7.33616301E+02	6.21502647E-11
20	2500	6.41841805E+02	6.41841805E+02	3.62164110E-11
50	2400	7.35184862E+02	7.35184862E+02	-1.05677432E-11
100	2700	8.42046088E+02	8.42046088E+02	3.95011786E-11

The ***Pressure-Entropy*** tables below contain the published *T*(*p,s*) values for verification of computer programs from Table 9 (p. 12) and Table 29 (p. 29) of Reference [b], as well as Table 12 (p. 13) of Reference [c].

Define the Backward unit function:

$$T_{ps}(p,s) := \text{if97\_tps}\left(\frac{p}{MPa}, \frac{s}{kJ\cdot kg^{-1}\cdot K^{-1}}\right)\cdot K$$

Validation Points		
p [Mpa]	s [kJ/kg-K]	T [K]
3	0.5	3.07842258E+02
80	0.5	3.09979785E+02
80	3	5.65899909E+02
0.1	7.5	3.99517097E+02
0.1	8	5.14127081E+02
2.5	8	1.03984917E+03
8	6	6.00484040E+02
8	7.5	1.06495556E+03
90	6	1.03801126E+03
20	5.75	6.97992849E+02
80	5.25	8.54011484E+02
80	5.75	9.49017998E+02
20	3.8	6.28295987E+02
50	3.6	6.29715873E+02
100	4	7.05688024E+02
20	5	6.40117644E+02
50	4.5	7.16368752E+02
100	5	8.47433283E+02

Extract Table Values:

$$P_{val} := S^{\langle 1\rangle}\cdot MPa \qquad s_{val} := S^{\langle 2\rangle}\cdot\frac{kJ}{kg\cdot K}$$

$$T_{val} := S^{\langle 3\rangle}\cdot K$$

Calculate Values and Error from IF97:

$$T_{calc} := \overrightarrow{T_{ps}\left(P_{val}, s_{val}\right)} \qquad \varepsilon_{calc} := \overrightarrow{\frac{T_{calc} - T_{val}}{T_{val}}}$$

Add columns to table to show below:

$$S^{\langle 4\rangle} := T_{calc}\cdot K^{-1}$$

$$S^{\langle 5\rangle} := \varepsilon_{calc}$$

p [Mpa]	s [kJ/kg-K]	T_{val} [K]	T_{ps} [K]	Error [%]
3	0.5	3.07842258E+02	3.07842258E+02	-1.16593447E-09
80	0.5	3.09979785E+02	3.09979785E+02	-5.85349353E-10
80	3	5.65899909E+02	5.65899909E+02	-4.26752613E-10
0.1	7.5	3.99517097E+02	3.99517097E+02	-8.62513236E-10
0.1	8	5.14127081E+02	5.14127081E+02	8.67720550E-10
2.5	8	1.03984917E+03	1.03984917E+03	3.11125061E-09
8	6	6.00484040E+02	6.00484040E+02	-3.11288704E-10
8	7.5	1.06495556E+03	1.06495556E+03	4.02534045E-09
90	6	1.03801126E+03	1.03801126E+03	-7.77092364E-10
20	5.75	6.97992849E+02	6.97992849E+02	6.64551491E-10
80	5.25	8.54011484E+02	8.54011484E+02	4.69806536E-11
80	5.75	9.49017998E+02	9.49017998E+02	-4.52629750E-10
20	3.8	6.28295987E+02	6.28295987E+02	-4.85905316E-11
50	3.6	6.29715873E+02	6.29715873E+02	4.47509004E-11
100	4	7.05688024E+02	7.05688024E+02	-5.88468653E-11
20	5	6.40117644E+02	6.40117644E+02	5.59035752E-11
50	4.5	7.16368752E+02	7.16368752E+02	3.73543571E-11
100	5	8.47433283E+02	8.47433282E+02	-4.07984759E-11

The ***Enthalpy-Entropy*** tables below contain the published *p*(*h,s*) values for verification of computer programs from Table 3 (p. 6) and Table 9 (p. 10) of Reference [d], as well as Table 5 (p. 10) of Reference [e].

Define the Backward unit function:

$$P_{hs}(h,s) := \text{if97\_phs}\left(\frac{h}{kJ\cdot kg^{-1}}, \frac{s}{kJ\cdot kg^{-1}\cdot K^{-1}}\right)\cdot MPa$$

Validation Points		
h [kJ/kg]	s [kJ/kg-K]	P [MPa]
0.001	0	9.80098061E-04
90	0	9.19295473E+01
1500	3.4	5.86829442E+01
2800	6.5	1.37101277E+00
2800	9.5	1.87974384E-03
4100	9.5	1.02478900E-01
2800	6	4.79391144E+00
3600	6	8.39551921E+01
3600	7	7.52716144E+00
2800	5.1	9.43920206E+01
2800	5.8	8.41457412E+00
3400	5.8	8.37690388E+01
1700	3.8	2.55570325E+01
2000	4.2	4.54087347E+01
2100	4.3	6.07812334E+01
2600	5.1	3.43499926E+01
2400	4.7	6.36392489E+01
2700	5	8.83904328E+01
1800	5.3	3.46847550E+02
2400	6	4.25137331E+02
2500	5.5	5.22557901E+02

Extract Table Values:

$$h_{val} := HS^{\langle 1\rangle}\cdot\frac{kJ}{kg} \qquad s_{val} := HS^{\langle 2\rangle}\cdot\frac{kJ}{kg\cdot K}$$

$$P_{val} := HS^{\langle 3\rangle}\cdot MPa$$

Calculate Values and Error from IF97:

$$P_{calc} := \overrightarrow{P_{hs}(h_{val}, s_{val})} \qquad \varepsilon_{calc} := \overrightarrow{\frac{P_{calc} - P_{val}}{P_{val}}}$$

Add columns to table to show below:

$$HS^{\langle 4\rangle} := P_{calc}\cdot MPa^{-1}$$

$$HS^{\langle 5\rangle} := \varepsilon_{calc}$$

h [kJ/kg]	s [kJ/kg-K]	p [Mpa]	P_{hs} [MPa]	Error [%]
0.001	0	9.80098061E-04	9.80098061E-04	-1.91728168E-10
90	0	9.19295473E+01	9.19295473E+01	-3.63058134E-11
1500	3.4	5.86829442E+01	5.86829442E+01	-6.31604358E-12
2800	6.5	1.37101277E+00	1.37101277E+00	-1.46793174E-10
2800	9.5	1.87974384E-03	1.87974384E-03	-1.22487408E-10
4100	9.5	1.02478900E-01	1.02478900E-01	-4.45720804E-10
2800	6	4.79391144E+00	4.79391144E+00	6.85274914E-12
3600	6	8.39551921E+01	8.39551921E+01	5.73789681E-11
3600	7	7.52716144E+00	7.52716144E+00	-4.70439583E-11
2800	5.1	9.43920206E+01	9.43920206E+01	-2.23157438E-12
2800	5.8	8.41457412E+00	8.41457412E+00	-5.07377900E-12
3400	5.8	8.37690388E+01	8.37690388E+01	8.32158554E-12
1700	3.8	2.55570325E+01	2.55570325E+01	1.13319989E-10
2000	4.2	4.54087347E+01	4.54087347E+01	-4.95967189E-11
2100	4.3	6.07812334E+01	6.07812334E+01	7.66201523E-11
2600	5.1	3.43499926E+01	3.43499926E+01	-6.69435353E-11
2400	4.7	6.36392489E+01	6.36392489E+01	5.56526539E-11
2700	5	8.83904328E+01	8.83904328E+01	-1.36014787E-11

Water Viscosity Property Verification

Set up Pressure and Temperature matrix based on tabulated values from IAPWS Reference [f], page 8, Table 4. First define: $\mu Pa \equiv 10^{-6} Pa$

Define Viscosity Unit Functions:

$$\mu_{tp}(T,P) := \text{if97\_mutp}\left(\frac{T}{K}, \frac{P}{MPa}\right) \cdot Pa \cdot s$$

$$\mu_{t\rho}(T,\rho) := \text{if97\_mutrho}\left(\frac{T}{K}, \frac{\rho}{kg \cdot m^{-3}}\right) \cdot Pa \cdot s$$

Saturated Liquid

$$\mu_f(p) := \text{if97\_muf}\left(\frac{p}{MPa}\right) \cdot Pa \cdot s$$

Function of T and ρ only set up for verification purposes

Saturated Vapor

$$\mu_g(p) := \text{if97\_mug}\left(\frac{p}{MPa}\right) \cdot Pa \cdot s$$

IAPWS Values

$$T_\mu := \begin{pmatrix} 298.15 \\ 298.15 \\ 373.15 \\ 433.15 \\ 433.15 \\ 873.15 \\ 873.15 \\ 873.15 \\ 1173.15 \\ 1173.15 \\ 1173.15 \end{pmatrix} \cdot K \qquad \rho_\mu := \begin{pmatrix} 998 \\ 1200 \\ 1000 \\ 1 \\ 1000 \\ 1 \\ 100 \\ 600 \\ 1 \\ 100 \\ 400 \end{pmatrix} \cdot \frac{kg}{m^3} \qquad \mu^* := \begin{pmatrix} 889.7351 \\ 1437.649467 \\ 307.883622 \\ 14.538324 \\ 217.685358 \\ 32.619287 \\ 35.802262 \\ 77.430195 \\ 44.217245 \\ 47.640433 \\ 64.154608 \end{pmatrix} \cdot \mu Pa \cdot s$$

Calculated Values

$$\overrightarrow{\mu_{t\rho}(T_\mu, \rho_\mu)} = \begin{pmatrix} 889.735100 \\ 1437.649467 \\ 307.883622 \\ 14.538324 \\ 217.685358 \\ 32.619287 \\ 35.802262 \\ 77.430195 \\ 44.217245 \\ 47.640433 \\ 64.154608 \end{pmatrix} \cdot \mu Pa \cdot s$$

Relative Error from Published Values

$$Err_{IAPWS} := \overrightarrow{\left(\frac{\mu^* - \overrightarrow{\mu_{t\rho}(T_\mu, \rho_\mu)}}{\mu^*}\right)} =$$

	1
1	$-1.684 \cdot 10^{-10}$
2	$2.168 \cdot 10^{-10}$
3	$-1.109 \cdot 10^{-9}$
4	$-3.341 \cdot 10^{-8}$
5	$-1.218 \cdot 10^{-9}$
6	$7.977 \cdot 10^{-10}$
7	$7.768 \cdot 10^{-9}$
8	$-2.935 \cdot 10^{-9}$
9	$1.097 \cdot 10^{-8}$
10	$-1.702 \cdot 10^{-9}$
11	$2.364 \cdot 10^{-9}$

Root Mean Square

$$RMS(M) := \sqrt{\frac{\sum_{i=1}^{rows(M)} \sum_{j=1}^{cols(M)} \left(M_{i,j}\right)^2}{rows(M) \cdot cols(M)}}$$

$$RMS(Err_{IAPWS}) = 1.095 \times 10^{-8}$$

The next two tables represent the experimental data from Reference [g] (Appendix A) for water viscosity measurements [µPa-s] taken over the full IF97 range of temperatures and pressures. The data is presented here in two halves, but will be combined and extracted into matrices for comparison with the calculated values. Note that there is a typographical error in Reference [g] that printed the temperature ir column fourteen in the Appendix A table as 500°C, when it should have been 450°C in the temperature series.

Appendix A-1: *Table of Critically-Evaluated Experimental Data*

Pressure, Mpa	Temperature, °C										
	0	**25**	**50**	**75**	**100**	**150**	**200**	**250**	**300**	**350**	**375**
0.1	1791	890.9	547.1	377.3	12.42	14.29	16.26	18.3	20.36	22.43	23.45
0.5	1790	891.2	546.7	378	281.7	182.3	16.05	18.16	20.25	22.32	23.43
1	1789	891.1	546.8	378.2	281.9	182.4	15.92	18.09	20.21	22.29	23.4
2.5	1786	890.8	547.1	378.5	282.3	182.8	134.6	17.85	20.07	22.22	23.37
5	1780	890.3	547.7	379.2	283.1	183.4	135.2	106.5	19.88	22.15	23.33
7.5	1774	889.8	548.3	379.8	283.8	184.1	135.9	107.2	19.75	22.12	23.34
10	1768	889.4	548.7	380.4	284.7	184.7	136.4	107.8	87.1	22.16	23.39
12.5	1762	889.1	549.1	381	285.3	185.3	137	108.5	88	22.35	23.57
15	1756	888.7	549.5	381.6	286	186	137.6	109.1	89	22.84	23.88
17.5	1750	888.5	550	382.3	286.7	186.6	138.2	109.8	89.9	67.3	24.49
20	1744	888.2	550.4	382.9	287.4	187.3	138.8	110.4	90.8	69.5	25.85
22.5	1738	887.9	550.9	383.5	288	187.9	139.4	111.1	91.6	71.4	48.2
25	1733	887.6	551.3	384.2	288.7	188.5	140	111.7	92.4	73	58.8
27.5	1728	887.4	551.8	384.8	289.4	189.1	140.6	112.3	93.1	74.4	62.4
30	1723	887.2	552.3	385.5	290	189.8	141.2	112.9	93.9	75.7	64.9
35	1713	886.8	553.3	386.7	291.4	191	142.3	114.1	95.3	78	68.6
40	1705	886.6	554.3	388	292.7	192.2	143.5	115.3	96.5	79.9	71.3
45	1697	886.5	555.3	389.3	294	193.4	144.6	116.4	97.8	81.7	73.7
50	1690	886.4	556.3	390.6	295.4	194.6	145.8	117.6	99	83.4	75.9
55	1684	886.5	557.4	392	296.7	195.8	146.9	118.7	100.2	84.9	77.8
60	1679	886.7	558.5	393.3	298	197	148	119.7	101.3	86.3	79.5
65	1674	886.9	559.7	394.6	299.4	198.2	149	120.8	102.5	87.7	81.1
70	1670	887.3	560.9	395.9	300.7	199.4	150.1	121.9	103.6	89	82.5
75	1666	887.7	562	397.3	302	200.6	151.2	122.9	104.6	90.3	83.9
80	1662	888.3	563.3	398.6	303.4	201.8	152.3	123.9	105.6	91.4	85.2
85	1659	888.8	564.5	400	304.6	203	153.3	124.9	106.6	92.6	86.4
90	1656	889.5	565.8	401.4	305.9	204.2	154.3	125.9	107.6	93.7	87.5
95	1653	890.3	567.1	402.8	307.3	205.4	155.4	126.9	108.6	94.7	88.7
100	1651	891.1	568.4	404.2	308.6	206.5	156.4	127.9	109.6	95.8	89.8

Appendix A-2: *Table of Critically-Evaluated Experimental Data*

Pressure, Mpa \ Temperature, °C	400	425	450	475	500	550	600	650	700	750	800
0.1	24.47	25.49	26.5	27.51	28.52	30.53	32.55	34.6	36.6	38.6	40.5
0.5	24.44	25.49	26.53	27.57	28.64	30.67	32.77	34.7	36.7	38.5	40.3
1	24.43	25.49	26.53	27.58	28.65	30.68	32.79	34.8	36.8	38.5	40.4
2.5	24.41	25.49	26.54	27.59	28.66	30.72	32.84	34.8	36.8	38.6	40.4
5	24.42	25.52	26.6	27.66	28.73	30.82	32.77	34.9	36.9	38.7	40.6
7.5	24.46	25.58	26.68	27.76	28.81	30.94	32.87	34.9	37	38.8	40.7
10	24.52	25.65	26.75	27.82	28.95	31.08	33.02	35.1	37.2	39	40.9
12.5	24.69	25.81	26.91	27.98	29.09	31.19	33.2	35.2	37.4	39.2	41.1
15	24.98	26.06	27.13	28.18	29.3	31.44	33.4	35.5	37.6	39.4	41.2
17.5	25.37	26.38	27.42	28.42	29.49	31.7	33.7	35.7	37.8	39.6	41.4
20	26.03	26.83	27.8	28.76	29.81	31.98	33.9	35.9	38	39.8	41.6
22.5	27.11	27.5	28.31	29.17	30.17	32.38	34.2	36.2	38.2	39.8	41.9
25	29.1	28.43	28.99	29.7	30.56	32.73	34.6	36.5	38.5	40.2	41.9
27.5	33.88	29.81	29.84	30.33	31.08	33.11	34.9	36.8	38.7	40.4	42.2
30	43.97	31.84	30.97	31.06	31.68	33.6	35.3	37.2	39	40.7	42.5
35	56.4	39.47	34.19	33.17	33.1	34.6	36.1	37.9	39.8	41.3	43
40	62.1	49.26	39.16	36.06	35.2	35.7	37.5	38.8	40.4	42	43.7
45	65.8	55.6	44.87	39.9	37.6	37.4	38.6	40	41.2	43.1	44.4
50	68.2	60.1	50.5	44	40.5	39.1	40	40.6	42.2	43.7	45.3
55	70.9	63.6	55.3	48.4	43.9	41	41.4	41.8	42.5	44.6	45.9
60	73.1	66.1	59.2	52.3	47.6	43.1	41.7	42.9	43.2	44.8	46.6
65	75.2	68.1	62.3	55.5	50.8	45.1	43.2	43.9	44.2	45.4	46.8
70	76.9	70.5	64.9	58.8	53.7	47.5	44.8	44.3	44.4	46.2	47.4
75	78.5	72.2	66.9	61.3	56.2	49.7	45.7	45.5	45.6	46.8	48.1
80	79.9	74	68.3	63.6	58.7	52.1	47.4	47	46.6	47.3	48.6
85	81.4	75.8	70.2	65.5	60.8	54	49.9	47.6	47.6	48.1	49
90	82.7	77.2	72.3	67.3	62.8	55.8	51.4	48.9	49.1	48.9	49.7
95	83.6	78.6	73.8	69.1	64.6	57.7	53.6	50.9	49.5	49.8	50.3
100	85	79.8	74.6	69.8	66.1	59.3	55.1	52.1	50.5	51.1	51

$M := \mathrm{augment}(M1, M2)$

$P_{IAPS} := \mathrm{submatrix}(M, 2, \mathrm{rows}(M), 1, 1)\cdot \mathrm{MPa}$

$i := 1 .. \mathrm{rows}(P_{IAPS})$

$T_{IAPS} := \mathrm{submatrix}(M, 1, 1, 2, \mathrm{cols}(M))^{T}\ °C$

$j := 1 .. \mathrm{rows}(T_{IAPS})$

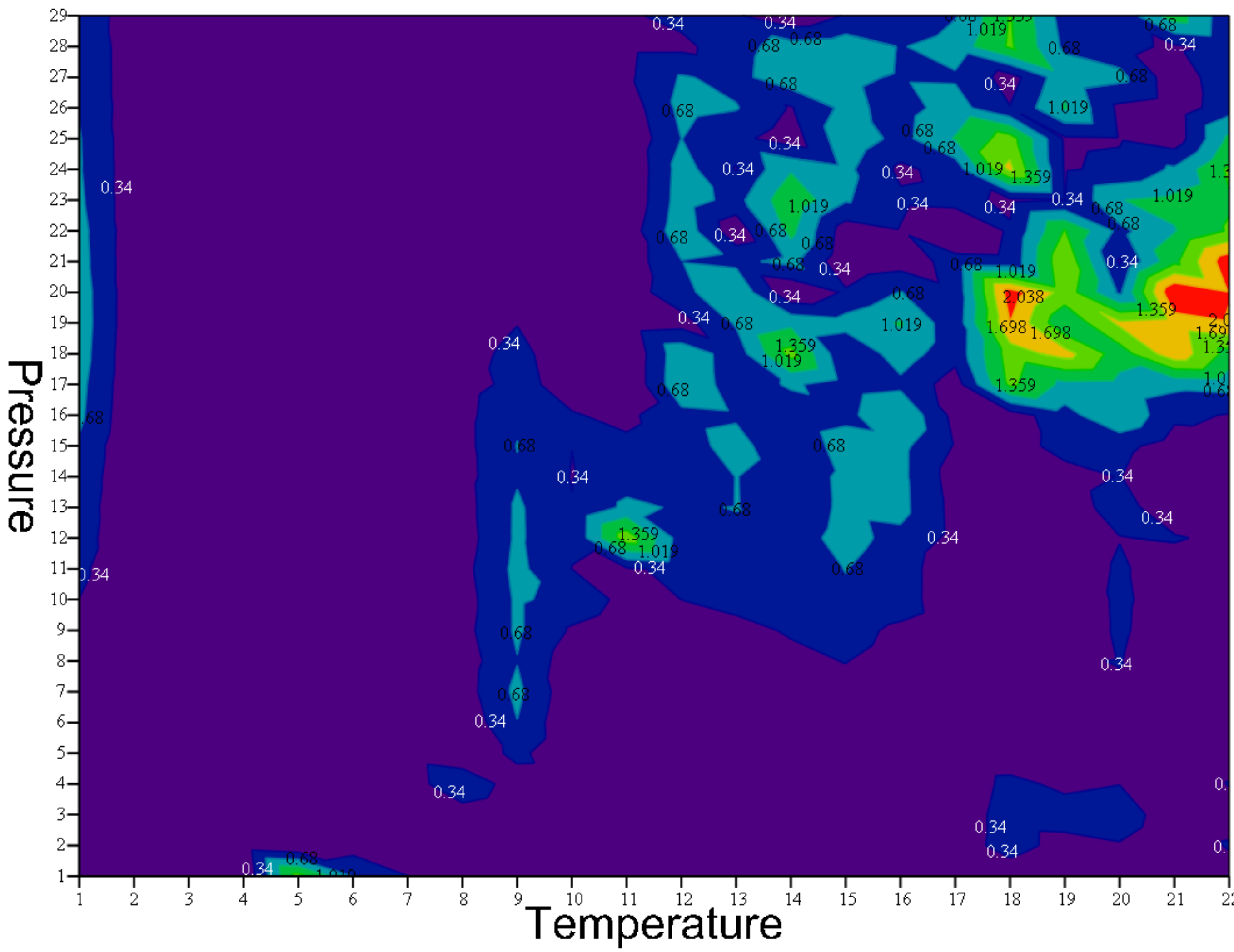
$\mu_{IAPS} := \mathrm{submatrix}(M, 2, \mathrm{rows}(M), 2, \mathrm{cols}(M))\cdot(\mu \mathrm{Pa}\cdot \mathrm{s})$

Calculate IF97 values: $\mu_{calc_{i,j}} := \mu_{tp}\left(T_{IAPS_j}, P_{IAPS_i}\right)$

Calculate IF97 relative error: $err_{IAPS} := \overrightarrow{\left(\frac{\left|\mu_{IAPS} - \mu_{calc}\right|}{\mu_{IAPS}\cdot\%}\right)}$

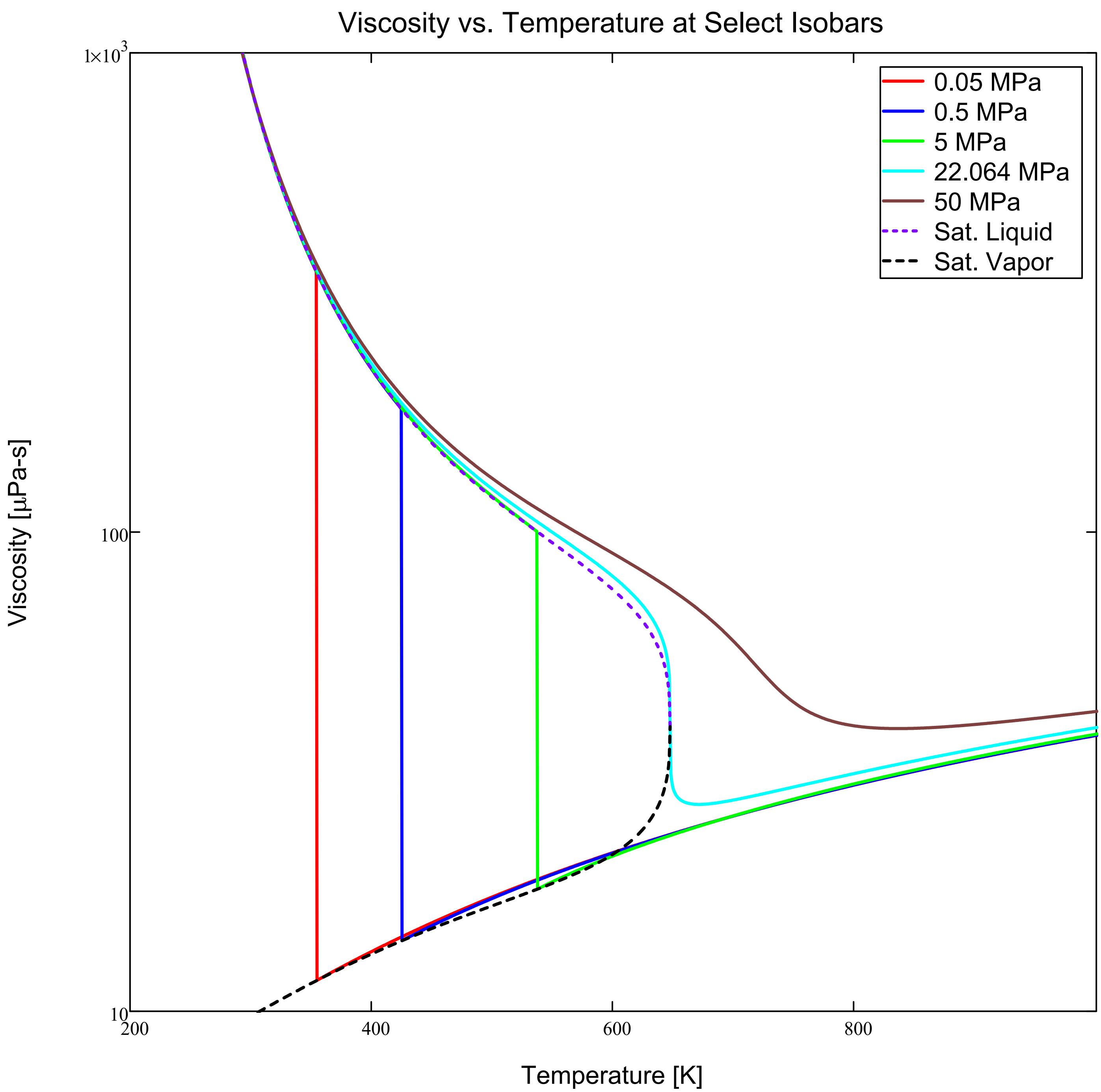
$RMS\left(err_{IAPS}\right) = 0.526771\ \%$ $\qquad$ $max\left(err_{IAPS}\right) = 2.377\ \%$

Relative IF97 Viscosity Error (%) from IAPS Experimental Data



The RMS error is around 0.5% and the maximum relative error is about 2.4% in the high temperature, moderate pressure vapor region. This maximum occurs in an area where the measurement uncertainty is between 2.6 and 3.0%. Relative error everywhere else is predominantly well below the relative measurement uncertainty of 1.4 to 2.6%.

The plot below ensures confirms that viscosity as a function of temperature and pressure are accurate and continuous, including saturation curve functions.



Water Thermal Conductivity Property Verification

Set up Pressure and Temperature matrix based on tabulated values from IAPWS Reference [h], pages 13-14, Table 7, 8, and 9.

Define Viscosity Unit Functions:

$$k_{tp}(T,P) := \text{if97\_ktp}\left(\frac{T}{K},\frac{P}{MPa}\right)\cdot\frac{W}{m\cdot K}$$

Saturated Liquid

$$k_f(p) := \text{if97\_kf}\left(\frac{p}{MPa}\right)\cdot\frac{W}{m\cdot K}$$

Saturated Vapor

$$k_g(p) := \text{if97\_kg}\left(\frac{p}{MPa}\right)\cdot\frac{W}{m\cdot K}$$

IAPWS Values

T [K]	P [Mpa]	rho [kg/m³]	k [W/m-K]
620	20	613.2277774	0.481485195
620	50	699.2260433	0.54503894
650	0.3	1.004521407	0.052231102
800	50	218.0300122	0.177709914
647.35	21.98406271	222	0.366879411
647.35	22.13216002	322	1.241824148
298.15	0.1	997.0474354	0.6065

$$T_k := M^{\langle 1\rangle}\cdot K \qquad P_k := M^{\langle 2\rangle}\cdot MPa \qquad k_{IAPWS} := M^{\langle 4\rangle}\cdot\frac{W}{m\cdot K}$$

Calculated Values

$$\overrightarrow{k_{tp}\left(M^{\langle 1\rangle}\cdot K, M^{\langle 2\rangle}\cdot MPa\right)} = \begin{pmatrix} 0.481 \\ 0.545 \\ 0.052 \\ 0.178 \\ 0.367 \\ 1.242 \\ 0.607 \end{pmatrix}\cdot\frac{W}{m\cdot K}$$

Relative Error from Published Values

$$Err_k := \overrightarrow{\frac{k_{IAPWS} - k_{tp}(T_k, P_k)}{k_{IAPWS}}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.875\times 10^{-15} \\ 7.742\times 10^{-14} \\ -2.61\times 10^{-5} \end{pmatrix}$$

Root Mean Square

$$RMS(Err_k) = 9.863\times 10^{-6}$$

The next two tables represent the experimental data from Reference [i] (Appendix A) for water thermal conductivity measurements [W/mK] taken over the full IF97 range of temperatures and pressures. The data is presented here in two halves for display purposes, but will be combined and extracted into matrices for comparison with the calculated values.

Table A.1 Critically Evaluated Experimental Data Reduced to a Uniform Grid

	Temperature [°C]											
Pressure [MPa]	0	25	50	75	100	150	200	250	300	350	375	400
0.1	563	610	643	664	25	28.9	33.3	38.1	43.3	49.0	52.0	54.9
0.5	563	610	643	664	680	688	34.1	38.7	43.7	49.1	52.6	55.5
1	564	611	643	666	681	689	35.9	39.5	44.3	49.5	53.0	56.0
2.5	566	611	644	666	682	690	668	43.8	46.5	50.9	54.7	56.9
5	567	613	645	668	683	691	671	625	52.7	54.1	56.5	58.6
7.5	570	614	647	669	685	694	673	628	63.6	59.6	60.5	62.7
10	571	615	648	669	686	695	675	631	557	68.2	65.3	66.9
12.5	571	616	649	672	687	697	678	634	562	81.2	73.6	72.4
15	573	617	650	673	689	700	680	638	566	107.5	84.8	79.9
17.5	573	618	651	674	691	701	682	639	571	452	104.2	90.0
20	574	619	653	676	691	703	684	641	576	465	144	104.9
22.5	574	620	654	678	692	705	686	646	581	476	478	124.1
25	577	621	655	679	694	707	689	648	588	482	400	166.4
27.5	578	622	656	680	696	708	690	651	589	490	413	240.8
30	578	623	658	681	697	710	692	653	593	498	426	337
35	580	625	660	684	700	714	696	660	601	511	453	384
40	583	626	662	686	702	717	700	664	608	526	471	399
45	584	629	664	690	705	721	704	670	615	537	486	425
50	586	630	666	692	708	724	708	673	621	547	498	444
55	589	633	667	694	710	726	712	678	629	558	510	461
60	590	635	670	697	713	729	715	682	634	566	525	476
65	592	638	673	699	715	733	718	688	639	574	535	489
70	597	639	674	702	718	735	721	691	645	582	546	499
75	599	641	675	705	720	738	725	696	648	589	554	511
80	599	645	677	707	723	739	729	699	653	598	564	521
85	601	646	680	706	726	742	732	702	659	604	571	532
90	604	648	681	710	728	745	735	707	665	611	578	544
95	608	650	685	713	731	748	739	711	669	615	586	553
100	609	650	686	716	735	749	742	715	672	624	594	561

Table A.1 Critically Evaluated Experimental Data Reduced to a Uniform Grid

	Temperature [°C]									
Pressure [MPa]	425	450	475	500	550	600	650	700	750	800
0.1	57.9	60.6	63.8	67.1	73.1	79.9	86.4	93.4	100.5	107.5
0.5	58.5	61.4	64.5	67.7	74	80.5	87.2	93.8	100.9	108.0
1	58.6	61.7	64.7	68	74.3	81	87.7	94.3	101.4	108.6
2.5	59.6	62.6	65.6	68.7	75.1	81.5	88.8	95.3	102.4	109.5
5	60.9	64	66.4	69.3	75.4	81.5	91.4	95.7	103.6	109.6
7.5	64	66.7	69.5	73.3	80	87.3	96.4	101	108.1	112.4
10	67.4	69.4	72.1	75.6	82.5	89.4	97.5	102.9	111.2	118.1
12.5	72	74.1	76.1	79.4	85	90.7	97.9	102.9	109.9	116.3
15	77.8	78.4	79.3	82.4	87.5	93.4	100.3	105.6	112.7	118
17.5	84.8	84	84.2	85.7	90.2	96.2	102.5	106	114.4	119.7
20	93.7	90.8	90.1	91.6	94.9	98.6	105.5	109.3	116.8	122.7
22.5	105.9	98.6	95.9	96	98.1	102.6	107.6	112.1	119.2	123.7
25	120.6	108.3	102.8	101.5	102.3	105.7	110.7	114.5	121.5	126.2
27.5	139.2	120.3	111.1	107.3	106.1	108.7	113	118	123.4	127.8
30	175	133.8	119.4	114.1	110.6	112.3	116.2	119.9	125.7	130.2
35	260.5	176.3	144.3	129.7	121.1	119.8	122.7	125.1	130	134.6
40	331	233.2	178.9	152.9	133.9	129.2	129.5	131.8	135.8	139.3
45	365	287	219	180.1	148.2	138.5	136.4	137.7	141.1	144.5
50	381	325	263	211	164	150	145	145	146	149
55	401	354	297	244	184	162	154	152	153	155
60	423	366	322	277	207	176	164	159	159	161
65	438	387	332	299	228	191	175	168	166	167
70	453	406	355	322	253	205	186	178	173	173
75	467	421	376	327	269	218	198	186	180	178
80	480	435	393	346	298	235	209	196	190	185
85	488	448	410	366	312	246	222	206	196	194
90	500	460	424	385	308	259	233	215	205	201
95	510	473	434	396	322	273	243	226	214	207
100	519	484	445	412	338	288	255	236	221	215

$T_k := \mathrm{stack}\left(T1^T, T2^T\right)\,°C$ $\qquad$ $j := 1 .. \mathrm{rows}(T_k)$

$P_k := P1 \cdot MPa$ $\qquad$ $i := 1 .. \mathrm{rows}(P_k)$

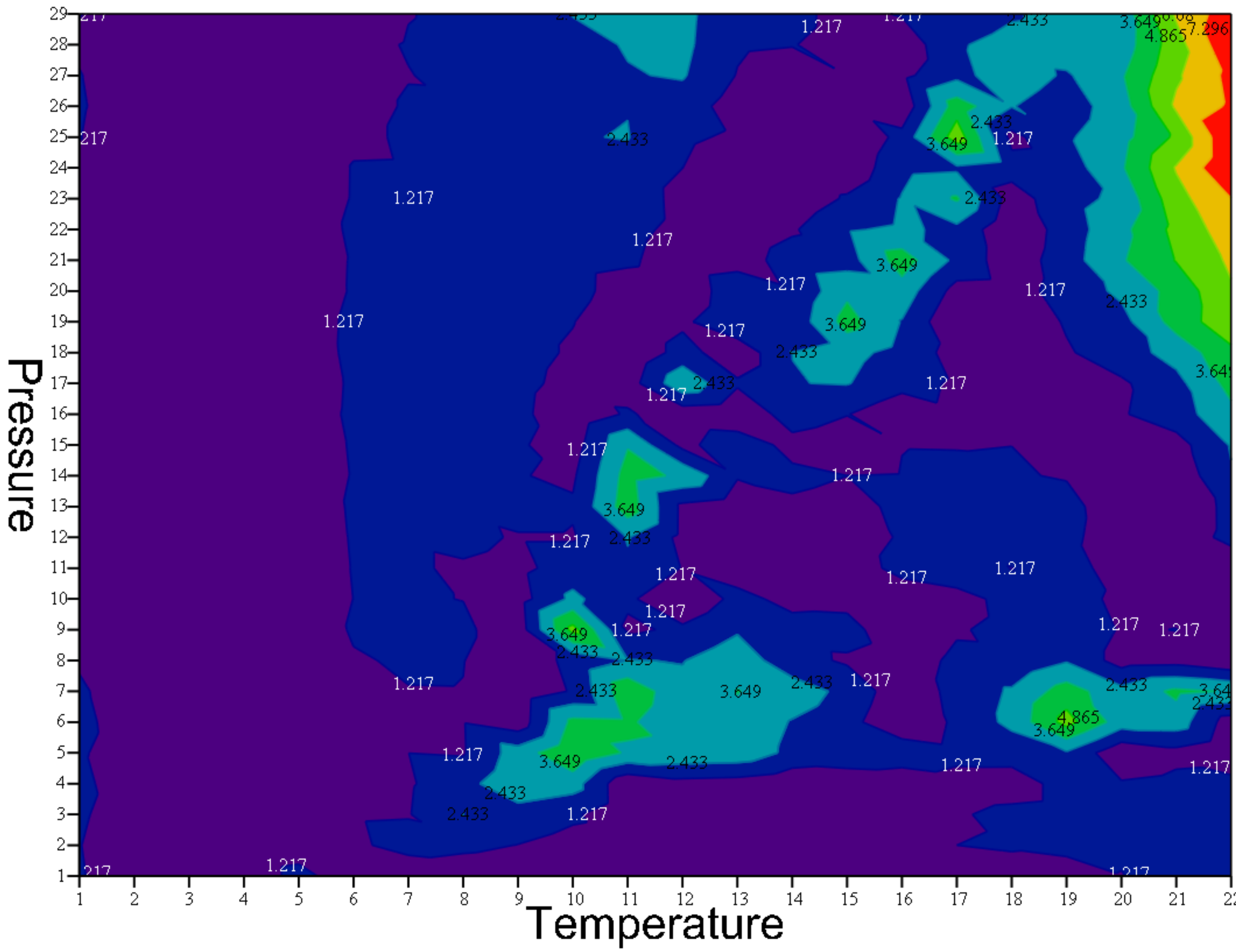
$k_{exp} := \mathrm{augment}(K1, K2) \cdot \dfrac{mW}{m \cdot K}$

Calculate IF97 values: $k_{calc_{i,j}} := k_{tp}\left(T_{k_j}, P_{k_i}\right)$

Calculate IF97 relative error: $errk_{IAPS} := \overrightarrow{\left(\frac{\left|k_{exp} - k_{calc}\right|}{k_{exp} \cdot \%}\right)}$

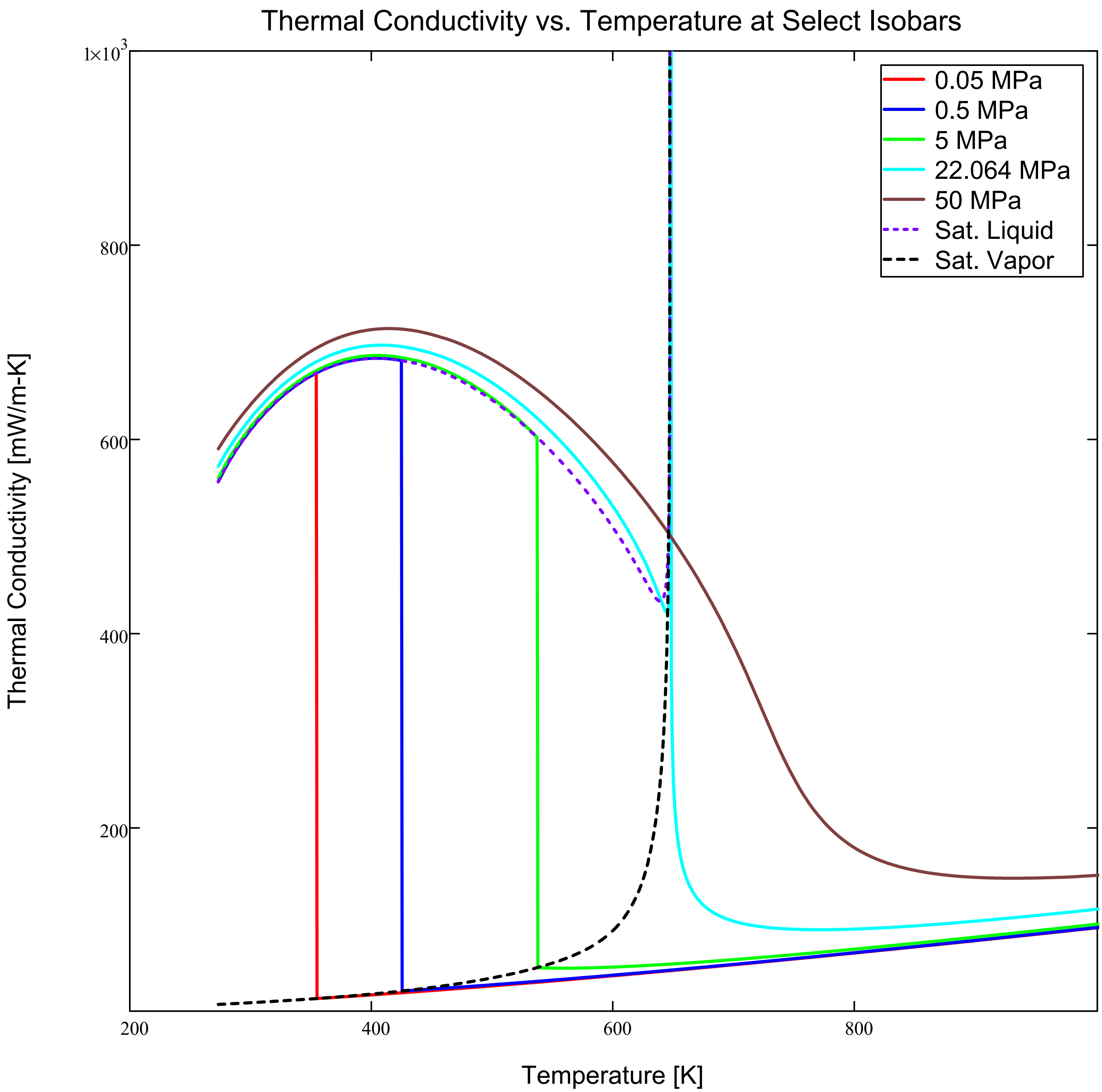
$RMS\left(errk_{IAPS}\right) = 1.995181$ % $\max\left(errk_{IAPS}\right) = 8.512$ %

Relative IF97 Thermal Conductivity Deviation (%) from 1985/2008 IAPS Experimental Data



The fairly consistent deviation from the 2008 data of around 1.217% is consistent with the shift in the 2011 correlations based on the addition of newer experimental data to the database; still within the minimum data uncertainty of 1.5%. The larger error at high temperature and pressure is relative to values that were extrapolated beyond the validity of the 1985/2008 data. High pressure data was incorporated into the 2011 formulation.

The plot below ensures confirms that thermal conductivity as a function of temperature and pressure are accurate and continuous, including saturation curve functions.



Water Surface Tension Verification

Set up matrix based on tabulated values from IAPWS Reference [j], pages 4-5, Table 1.

First define: $mN \equiv 0.001 \cdot N$ $\qquad mN/m \equiv \frac{mN}{m}$

$M_\sigma :=$

Temperature	Experimental σ	Uncertainty Δσ	Calculated σ_{calc}	Error $\sigma_{calc} - \sigma$
°C	mN/m	mN/m	mN/m	mN/m
0.01	75.64	0.38	75.65	0.01
5	74.94	0.37	74.94	0.00
10	74.23	0.37	74.22	-0.01
15	73.49	0.37	73.49	0.00
20	72.74	0.36	72.74	0.00
25	71.98	0.36	71.97	-0.01
30	71.19	0.36	71.19	0.00
35	70.41	0.35	70.40	-0.01
40	69.59	0.35	69.60	0.01
45	68.78	0.34	68.78	0.00
50	67.93	0.34	67.94	0.01
55	67.09	0.34	67.10	0.01
60	66.24	0.33	66.24	0.00
65	65.36	0.33	65.37	0.01
70	64.47	0.32	64.48	0.01
75	63.57	0.32	63.58	0.01
80	62.68	0.31	62.67	-0.01
85	61.76	0.31	61.75	-0.01
90	60.82	0.30	60.82	0.00
95	59.88	0.30	59.87	-0.01
100	58.92	0.29	58.91	-0.01
105	57.95	0.29	57.94	-0.01
110	56.97	0.28	56.96	-0.01
115	55.98	0.28	55.97	-0.01
120	54.97	0.27	54.97	0.00

$T_\sigma := M_\sigma^{\langle 1 \rangle} \, °C$ $\qquad \sigma_{exp} := M_\sigma^{\langle 2 \rangle} \cdot mN/m$ $\qquad \sigma_u := M_\sigma^{\langle 3 \rangle} \cdot mN/m$ $\qquad \sigma_{calc} := M_\sigma^{\langle 4 \rangle} \cdot mN/m$

Set up a unit handling function for surface tension, σ(T):

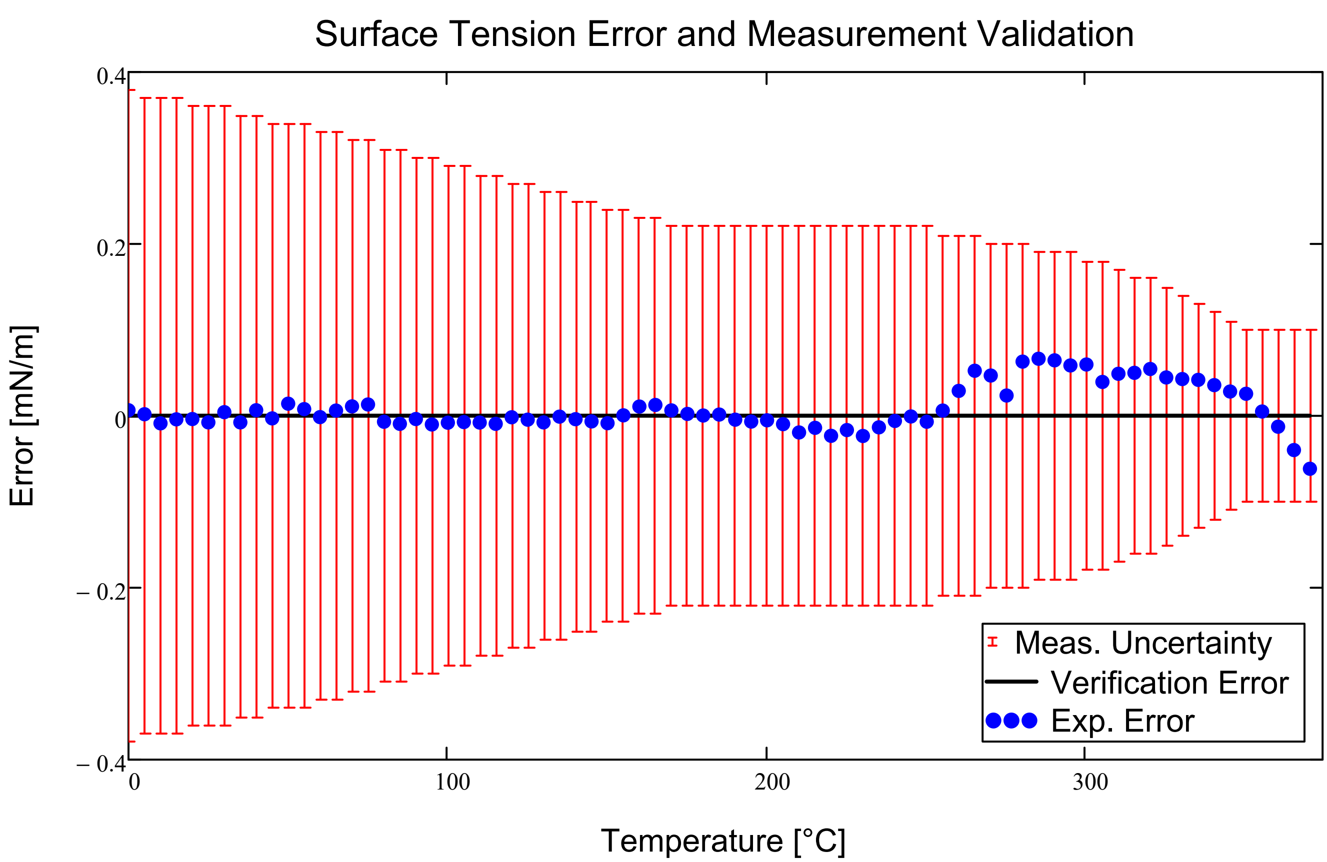
$$\sigma(T) := \text{if97\_sigma}\left(\frac{T}{K}\right)\cdot\frac{mN}{m}$$

Calculate error from the published verification values.

$$\varepsilon_{97} := \overrightarrow{\frac{\text{Round}\left(\sigma\left(T_{\sigma}\right), 0.01mN/m\right) - \sigma_{calc}}{mN/m}} \qquad \max\left(\varepsilon_{97}\right) = 1.388 \times 10^{-14}$$

Calculate the error from the experimental values:

$$\varepsilon_{\sigma} := \overrightarrow{\left(\sigma\left(T_{\sigma}\right) - \sigma_{exp}\right)}$$



The calculated values match the published values exactly to the published precision. The difference between the calculated values and the experimental measurements from IAPS data is within the measurement uncertainty of the data.

Water Prandtl Number Verification

Plot Prandtl Number vs. Temperature for continuity and reasonableness check.

Define Prandtl Unit Functions:

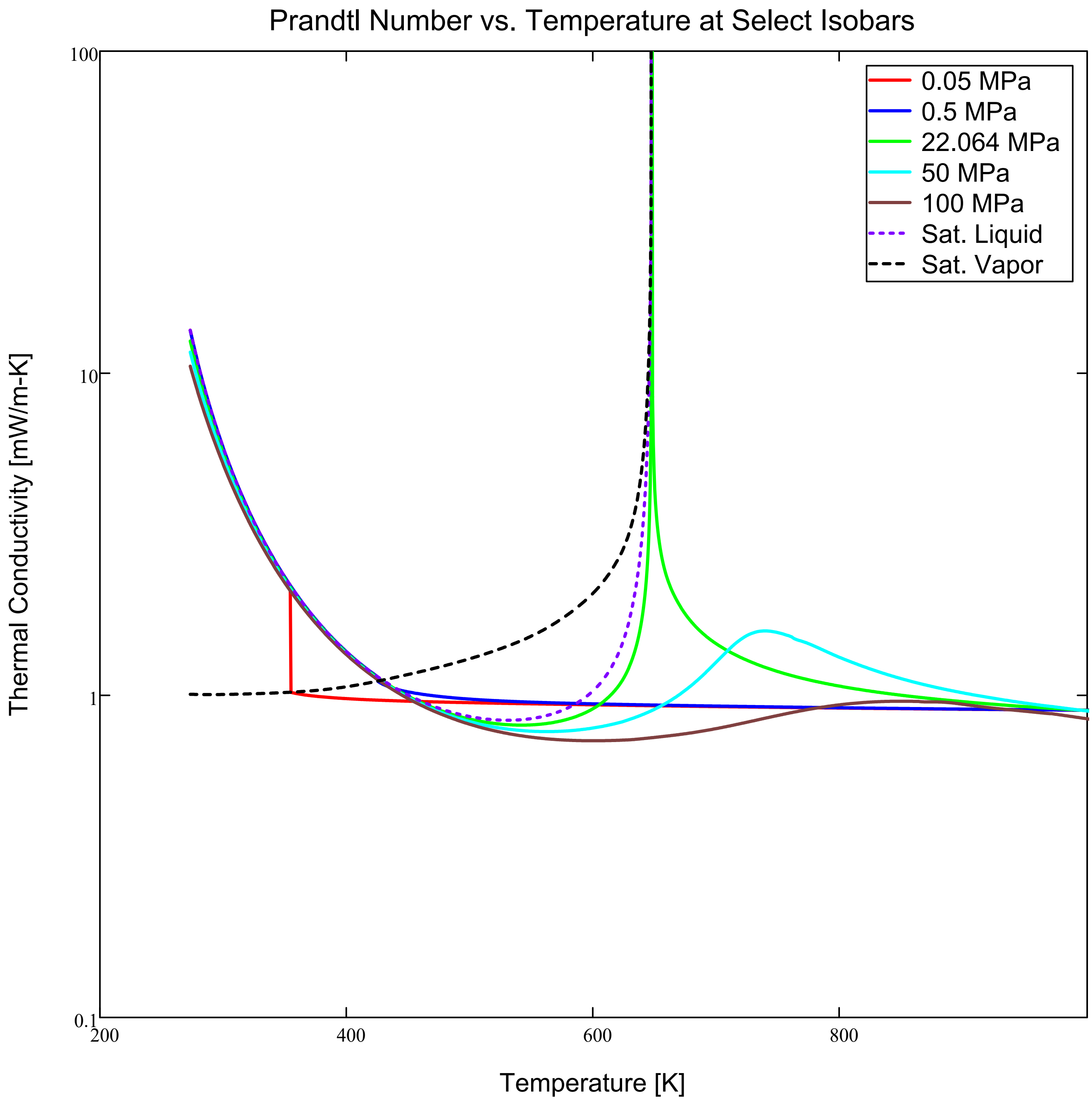
$Pr_{tp}(T, P) := \text{if97\_prtp}\left(\frac{T}{K}, \frac{P}{MPa}\right)$

Saturated Liquid

$Pr_f(p) := \text{if97\_prf}\left(\frac{p}{MPa}\right)$

Saturated Vapor

$Pr_g(p) := \text{if97\_prg}\left(\frac{p}{MPa}\right)$



References

[a] IAPWS, *Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, Lucerne, Switzerland, August 2007. [*IAPWS R7-97(2012)*]

[b] IAPWS, *Revised Supplementary Release on Backward Equations for Specific Volume as a Function of Pressure and Temperature v(p,T) for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, Moscow, Russia, 2014. [*IAPWS SR5-05(2016)*]

[c] IAPWS, *Revised Supplementary Release on Backward Equations for the Functions T(p,h), v(p,h) and T(p,s), v(p,s) for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, Moscow, Russia, June 2014. [IAPWS SR3-03]

[d] IAPWS, *Revised Supplementary Release on Backward Equations for Pressure as a Function of Enthalpy and Entropy p(h,s) for Regions 1 and 2 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, Moscow, Russia, June 2014. [IAPWS SR2-01(2014)]

[e] IAPWS, *Revised Supplementary Release on Backward Equations p(h,s) for Region 3, Equations as a Function of h and s for the Region Boundaries, and an Equation Tsat(h,s) for Region 4 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, Moscow, Russia, June 2014. [IAPWS SR4-04(2014)]

[f] IAPWS, *Release on the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance*, Berlin, Germany, September 2008. [*IAPWS R12-08*].

[g] IAPWS, *Revised Release on the IAPS Formulation 1985 for the Viscosity of Ordinary Water Substance*, Vejle, Denmark, August 2003.

[h] IAPWS, Release on the IAPWS Formulation 2011 for the Thermal Conductivity of Ordinary Water Substance, Plzeň, Czech Republic, September 2011. [*IAPWS R15-11*]

[i] IAPWS, *Revised Release on the IAPS Formulation 1985 for the Thermal Conductivity of Ordinary Water Substance*, London, England, August 1998.

[j] IAPWS, *Revised Release on Surface Tension of Ordinary Water Substance*, Moscow, Russia, June 2014. [*IAPWS R1-76(2014)*]