

Low Cycle Fatigue: Physics and Equations for Review

David Fieser

Hugh Shortt

lcf-strain-life. Both authors contributed equally.

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Abstract

This is a self-contained statement of the fatigue physics implemented in the toolkit, written for review by a materials or fatigue specialist. It contains the equations, the symbols and units, the conventions, and the source for each relation. It contains no software detail. A short sign-off table is at the end.

1 Conventions

All analysis uses true stress and true strain. Stress and modulus are in MPa. Strain is a dimensionless fraction. Life is expressed in reversals $2N_f$, where one cycle is two reversals. The fatigue strength exponent b and the fatigue ductility exponent c are negative.

Symbol	Meaning	Unit
σ, ε	true stress, true strain	MPa, -
$\Delta\sigma/2, \Delta\varepsilon/2$	stress, total strain amplitude	MPa, -
$\Delta\varepsilon_e/2, \Delta\varepsilon_p/2$	elastic, plastic strain amplitude	-
σ_m, σ_{max}	mean stress, maximum stress	MPa
E	Young's modulus	MPa
σ'_f, b	fatigue strength coefficient, exponent	MPa, -
ε'_f, c	fatigue ductility coefficient, exponent	-
K', n'	cyclic strength coefficient, strain-hardening exponent	MPa, -
$2N_f$	reversals to failure	-
K_t, K_f, q	stress concentration, fatigue notch factor, notch sensitivity	-

2 True stress and strain

Engineering values e and $\sigma_{\text{eng}} = F/A_0$ convert to true values by

$$\varepsilon = \ln(1 + e), \quad \sigma = \sigma_{\text{eng}}(1 + e). \quad (1)$$

Valid up to necking. Reference: Dowling, Mechanical Behavior of Materials, 4th ed.

3 Cyclic stress-strain, Ramberg-Osgood

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}, \quad \Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}. \quad (2)$$

The second form is the doubled hysteresis branch. Reference: Ramberg and Osgood 1943, NACA TN 902. Dowling 4th ed., Eq. 14.12.

4 Strain-life

Elastic, Basquin:

$$\frac{\Delta\sigma}{2} = \sigma'_f(2N_f)^b, \quad \frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_f}{E}(2N_f)^b. \quad (3)$$

Plastic, Coffin-Manson:

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f(2N_f)^c. \quad (4)$$

Total strain-life and the elastic-plastic transition life:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c, \quad 2N_t = \left(\frac{\varepsilon'_f E}{\sigma'_f} \right)^{1/(b-c)}. \quad (5)$$

Compatibility, checked and reported but not forced:

$$n' = \frac{b}{c}, \quad K' = \frac{\sigma'_f}{(\varepsilon'_f)^{b/c}}. \quad (6)$$

The plastic line is fit over the low cycle regime, excluding near-runout points whose plastic strain is at measurement noise level. References: Basquin 1910, Coffin 1954, Manson 1953, Dowling 4th ed. Eq. 14.3 to 14.6.

5 Mean-stress corrections

Morrow, elastic term shifted by the mean stress:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c. \quad (7)$$

Modified Morrow, both terms shifted:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + \varepsilon'_f \left(\frac{\sigma'_f - \sigma_m}{\sigma'_f} \right)^{c/b} (2N_f)^c. \quad (8)$$

Smith-Watson-Topper:

$$\sigma_{max} \frac{\Delta\varepsilon}{2} = \frac{(\sigma'_f)^2}{E}(2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{b+c}, \quad \sigma_{ar} = \sqrt{\sigma_{max} \sigma_a}. \quad (9)$$

Walker, with the equivalent fully-reversed amplitude and the steel estimate of the exponent:

$$\sigma_{ar} = \sigma_{max}^{1-\gamma} \sigma_a^\gamma, \quad \gamma_{steel} = 0.8818 - 2.00 \times 10^{-4} \sigma_u. \quad (10)$$

Morrow equivalent fully-reversed amplitude:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\sigma'_f}. \quad (11)$$

With $\gamma = 0.5$ Walker reduces to SWT. References: Morrow 1968. Smith, Watson, Topper 1970, J. Materials 5(4):767. Walker 1970. Dowling, Calhoun, Arcari 2009, Fatigue Fract. Eng. Mater. Struct. (steel γ). Dowling 4th ed. Eq. 9.18 to 9.21.

6 Hysteresis energy and cyclic response

The plastic strain energy density per cycle is the closed loop area

$$W = \oint \sigma d\varepsilon. \quad (12)$$

The tension-compression asymmetry of a cycle is $R_{TC} = |\sigma_{max}|/|\sigma_{min}|$. Peak and valley stress versus cycle give the cyclic hardening and softening response.

7 Variable amplitude, cycle counting

Irregular histories are reduced to closed cycles by the rainflow method, which pairs reversals into hysteresis loops while preserving their order in time. Reference: ASTM E1049-85(2017), Downing and Socie 1982, Matsuishi and Endo 1968.

Level-crossing counting records positive-slope crossings at and above a reference level and negative-slope crossings below it. Peak counting records peaks at and above the reference and valleys below it. Both follow ASTM E1049-85(2017), sections 5.2 and 5.3.

The racetrack (gate) filter condenses a history before counting by removing swings smaller than a gate while keeping the order of the large reversals. Reference: Fuchs, Nelson, Burke, and Toomay, SAE paper 730565, 1973.

8 Cumulative damage

Palmgren-Miner linear damage, failure at a critical sum, the default being one:

$$D = \sum_i \frac{n_i}{N_{f,i}}. \quad (13)$$

Double Linear Damage Rule, Manson-Halford, with the Phase I life fraction referenced to the longest life in the spectrum:

$$f_I = 0.35 \left(\frac{N_f}{N_{long}} \right)^{0.25}, \quad N_I = N_f f_I, \quad N_{II} = N_f (1 - f_I). \quad (14)$$

Phase I accumulates to one, then Phase II accumulates to one. Corten-Dolan, where σ_1 is the maximum stress in the block and α_i the cycle fractions:

$$N = \frac{N_{f,1}}{\sum_i \alpha_i (\sigma_i / \sigma_1)^d}. \quad (15)$$

References: Palmgren 1924, Miner 1945, Manson and Halford 1981 (Int. J. Fracture 17:169), Corten and Dolan 1956.

For stress-based collectives the allowable life comes from a one-slope Woehler line with a knee at (S_D, N_D) :

$$N = N_D \left(\frac{S_a}{S_D} \right)^{-k} \quad (S_a \geq S_D), \quad (16)$$

and below the knee one of three treatments: infinite life (Miner original), the same slope k continued (Miner elementary), or the flatter fictitious slope $2k - 1$ (Haibach):

$$N = N_D \left(\frac{S_a}{S_D} \right)^{-(2k-1)} \quad (S_a < S_D). \quad (17)$$

References: Miner 1945, Haibach 1970, described in Haibach, Betriebsfestigkeit, 3rd ed., Springer, 2006.

9 Notch local-strain approach

Neuber, combined with the cyclic curve, and its range form:

$$\frac{(K_t S)^2}{E} = \sigma \varepsilon, \quad \frac{(K_t \Delta S)^2}{E} = \Delta \sigma \Delta \varepsilon. \quad (18)$$

Glinka equivalent strain energy density:

$$\frac{(K_t S)^2}{2E} = \frac{\sigma^2}{2E} + \frac{\sigma}{n' + 1} \left(\frac{\sigma}{K'} \right)^{1/n'}. \quad (19)$$

Fatigue notch factor and notch sensitivity:

$$K_f^{\text{Peterson}} = 1 + \frac{K_t - 1}{1 + a/r}, \quad K_f^{\text{Neuber}} = 1 + \frac{K_t - 1}{1 + \sqrt{\beta/r}}, \quad q = \frac{K_f - 1}{K_t - 1}. \quad (20)$$

Neuber tends to overestimate and Glinka to underestimate the local strain. The measured value usually lies between them. References: Neuber 1961, Molski and Glinka 1981, Peterson 1974.

10 Statistics and design curves

Life is the dependent variable in the linearized regression, $\log_{10} N = A + B \log_{10}(\Delta \varepsilon/2)$. Confidence and prediction intervals use the residual standard error and the Student t quantile. A reliability and confidence design curve is the mean reduced by $k s$, where k is the one-sided Owen tolerance factor. Right-censored runouts are handled by maximum likelihood rather than deletion. References: ASTM E739, withdrawn 2024 and used as the de facto reference, Owen 1963, Williams, Lee and Rilly 2003 (Int. J. Fatigue 25:427).

Outlier screening operates on the residuals of the log-life regression. Runouts are censored observations, not outliers, and are excluded from the screen. A single suspect point uses the two-sided Grubbs test, several suspect points use the generalized extreme studentized deviate test, whose approximation is intended for $n \geq 15$. Influence is reported through leverage, externally studentized residuals against a Bonferroni-corrected t critical value, and Cook's distance against the $4/n$ screening threshold. Flagged points are reported, never deleted automatically. References: Grubbs 1969 (Technometrics 11:1), Rosner 1983 (Technometrics 25:165), Cook 1977 (Technometrics 19:15), NIST/SEMATECH e-Handbook of Statistical Methods, sections 1.3.5.17.1 and 1.3.5.17.3.

11 Estimation of strain-life constants

When no strain-controlled test data exists, the four constants are estimated from monotonic properties. S_u is the ultimate tensile strength in MPa, HB the Brinell hardness, RA the reduction in area as a fraction, and $\tilde{\varepsilon}_f = \ln[1/(1 - RA)]$ the true fracture ductility. Estimates are screening values, not substitutes for test data.

Medians method, steels (recommended default) and aluminum alloys:

$$\begin{aligned} \text{steel: } & \sigma'_f = 1.5 S_u, \quad b = -0.09, \quad \varepsilon'_f = 0.45, \quad c = -0.59, \\ \text{aluminum: } & \sigma'_f = 1.9 S_u, \quad b = -0.11, \quad \varepsilon'_f = 0.28, \quad c = -0.66. \end{aligned} \quad (21)$$

Uniform Material Law, steels, with the ductility correction $\psi = 1$ for $S_u/E \leq 0.003$, else $\psi = 1.375 - 125 S_u/E$:

$$\sigma'_f = 1.50 S_u, \quad b = -0.087, \quad \varepsilon'_f = 0.59 \psi, \quad c = -0.58, \quad K' = 1.65 S_u, \quad n' = 0.15, \quad (22)$$

and for aluminum and titanium alloys $\sigma'_f = 1.67 S_u$, $b = -0.095$, $\varepsilon'_f = 0.35$, $c = -0.69$. The law loses validity as S_u approaches 2.2 GPa, where ψ reaches zero.

Universal slopes, any metal:

$$\sigma'_f = 1.9 S_u, \quad b = -0.12, \quad \varepsilon'_f = 0.76 \tilde{\varepsilon}_f^{0.6}, \quad c = -0.6. \quad (23)$$

Modified universal slopes, steels:

$$\sigma'_f = 0.623 E \left(\frac{S_u}{E} \right)^{0.832}, \quad b = -0.09, \quad \varepsilon'_f = 0.0196 \left(\frac{S_u}{E} \right)^{-0.53} \tilde{\varepsilon}_f^{0.155}, \quad c = -0.56. \quad (24)$$

Hardness method, steels with roughly 150 to 700 HB:

$$\sigma'_f = 4.25 HB + 225, \quad b = -0.09, \quad \varepsilon'_f = \frac{0.32 HB^2 - 487 HB + 191000}{E}, \quad c = -0.56. \quad (25)$$

References: Meggiolaro and Castro 2004 (Int. J. Fatigue 26:463, also the comparative evaluation over 845 metals), Baeumel and Seeger 1990, Manson 1965 (Exp. Mech. 5:193), Muralidharan and Manson 1988 (J. Eng. Mater. Technol. 110:55), Roessle and Fatemi 2000 (Int. J. Fatigue 22:495).

12 Elevated temperature

Frequency-modified Coffin-Manson, coefficient form:

$$C_f = C_o \left(\frac{f}{f_{ref}} \right)^{k-1}, \quad \frac{\Delta \varepsilon_p}{2} = C_f (2N_f)^c. \quad (26)$$

Linear time-fraction creep-fatigue damage, fatigue plus creep:

$$D = \sum_i \frac{n_i}{N_{f,i}} + \sum_j \frac{t_j}{t_{r,j}}, \quad (27)$$

checked against a bilinear creep-fatigue interaction envelope. References: Coffin 1971, Robinson 1952, ASTM E2714-13(2020).

13 Multiaxial, survey only

Critical-plane parameters, provided for evaluation. The full plane search is not yet implemented.

$$P_{FS} = \frac{\Delta \gamma_{max}}{2} \left(1 + k \frac{\sigma_{n,max}}{\sigma_y} \right), \quad P_{BM} = \frac{\Delta \gamma_{max}}{2} + S \Delta \varepsilon_n, \quad P_{SWT} = \sigma_{n,max} \frac{\Delta \varepsilon_1}{2}. \quad (28)$$

References: Fatemi and Socie 1988 (Fatigue Fract. Eng. Mater. Struct. 11(3):149), Brown and Miller 1973, Smith, Watson, Topper 1970.

14 Conventions and assumptions to confirm

- True stress-strain conversion assumed valid up to necking.
- Plastic strain amplitude taken as $\Delta\varepsilon_t/2 - \Delta\sigma/(2E)$.
- Failure criterion is a percent load drop from the stabilized half-life peak load, default 30 percent.
- Default mean-stress model for variable amplitude is SWT, notch default is Neuber, damage default is Miner.
- The plastic strain-life line is fit over the low cycle regime only.

15 Reviewer sign-off

Reviewer name and date, then a verdict per section. Suggested verdicts: OK, OK with note, or needs change.

Section	Verdict	Note
True stress and strain		
Ramberg-Osgood		
Strain-life		
Mean-stress corrections		
Energy and cyclic response		
Cycle counting and filtering		
Cumulative damage and knee variants		
Notch local-strain		
Statistics, design curves, outlier screen		
Estimation of strain-life constants		
Elevated temperature		
Multiaxial survey		
Conventions and assumptions		