

A FORMULA FOR SCHUBERT ELEMENTS VIA ANCHORED PATHS

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Definition 1. For a permutation w , let $\mathbf{c}^*(w)$ be the Lehmer code of w^{-1} .

Let μ be a dominant permutation with $\mathbf{c}^*(\mu) = (\lambda_1, \dots, \lambda_n)$ and let u be a permutation such that $\ell(u\mu) = \ell(\mu) - \ell(u)$.

For a positive integer p , we say that there is a *p-anchored Bruhat path* between u and w if there exists a sequence of pairwise distinct positive integers $a_1, a_2, \dots, a_k$ with $a_i \neq p$ for any i such that $w = ut_{pa_1} \cdots t_{pa_k}$ and $\ell(ut_{pa_1} \cdots t_{pa_i}) = \ell(u) + i$ for all i .

We have

$$\mathfrak{S}_{u\mu}(x) = \sum_P (-1)^{\text{sign}(P)} e_{\lambda_1 - \ell(u_0, u_1), \lambda_1}(x) e_{\lambda_2 - \ell(u_1, u_2), \lambda_2}(x) \cdots e_{\lambda_n - \ell(u_{n-1}, u_n), \lambda_n}(x)$$

where the sum is over all sequences $P = (u_0, \dots, u_n)$ of permutations with $u_0 = 1$ and $u_n = u$ such that there is an i -anchored Bruhat path between u_{i-1} and u_i for all i , and $\text{sign}(P)$ is the sum over all i of the number of $j < i$ such that $u_{i-1}(j) \neq u_i(j)$.