

Phantom terms, quantum annihilating pairs, and positivity

`full_CEM` for a permutation u produces a dictionary $\{R \mapsto \text{cem_dict}\}$ where each R is an RC graph for u , and `cem_dict` is its Grassmannian factorization in $\mathcal{E}$. However, some tensors in the CEM dict, when squashed via $\mathcal{R}$, land on an RC graph for a *different* permutation $u' \neq u$. These are the **phantoms**—they live in $\ker(\mathcal{R} - (\text{expected perm}))$.

When two CEM elements are multiplied, a phantom tensor can pair with a legitimate tensor and produce:

- A **bad** RC graph (wrong permutation or wrong weight) that gets killed, and simultaneously
- A **good** RC graph (correct permutation, correct weight) that fills a gap.

These come in pairs—a **quantum annihilating pair**—where the phantom contribution simultaneously subtracts a term that should not be present and adds one that should.

This is why the crystal filter works despite the apparent signs: the phantoms are not creating cancellation in the coefficient c_{uv}^w . Instead, they perform a **crystal-theoretic correction**, swapping contributions that landed in the wrong crystal component for ones in the right component. The net effect on c_{uv}^w is always non-negative because:

1. Without phantoms, the total count would be correct but distributed incorrectly across crystal components.
2. The phantoms redistribute: -1 from a forbidden component, $+1$ to a missing one.
3. After filtering by `is_highest_weight` and `extremal_weight == trimcode`, each surviving component has been corrected to its true (non-negative) multiplicity.

A proof would therefore need to show that phantom pairs always act as **transpositions on the set of crystal components**: each phantom involvement swaps exactly one bad contribution for one good one, preserving the total and maintaining non-negativity after the crystal filter.

Formally, the phantoms should generate an action on the fiber $\mathcal{R}^{-1}(w)$ that is compatible with the crystal structure, and the quantum annihilating pairs are precisely the orbits of this action that cross crystal component boundaries.