Tools to Calculate Adiabatic Invariants from Dynamic Simulations of Earth's Magnetosphere

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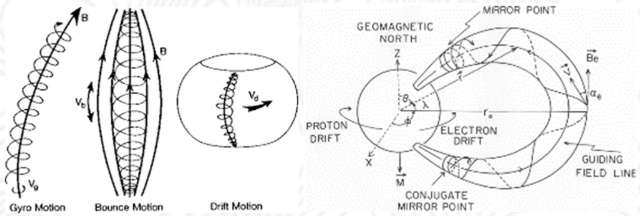
Introduction: Adiabatic Invariants

Trapped Particles in Magnetospheres undergo three periodic motions, each with their own characterizing invariant parameter (variable in parenthesis).

1. **Gyration** around a field line (μ)
2. **Bounce motion** along a field line (K)
3. **Drift azimuthally** around the magnetized body (L^*)



- Phase space density $f(\vec{v},\vec{x})$ can recast in terms $f(\mu, K, L^*, \phi_\mu, \phi_K, \phi_{L^*})$
- When put in this form f will remain constant during slow-changing (slower than drift period time scale) reconfigurations of the global magnetic field when no work is done
- This property is essential for studying the dynamics of trapped particles during geomagnetic storms

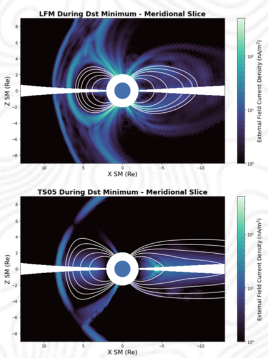
Calculation from Dynamic Simulations

The calculation requires a global magnetic filed model. Existing practices use empirical magnetic models:

- Tsyganenko (T96-TS05), Olson & Pfitzer (Quiet / Dynamic), Alexeev, Ostapenko & Maltsev, Mead & Fairfield, and more

We argue there is an advantage to being able to use magnetic fields from simulations:

1. Empirical models don't capture fine current structures like MHD models do (see right plots: LFM-RCM at top is MHD and TS05 below is empirical)
2. Studies which guide test particles through simulation fields should always use the simulation fields for maximal self-consistency



Algorithm for Gyration Invariant μ

No special algorithm is required for calculation of μ, because it does not require global magnetic field knowledge. The relativistic equation is given by:

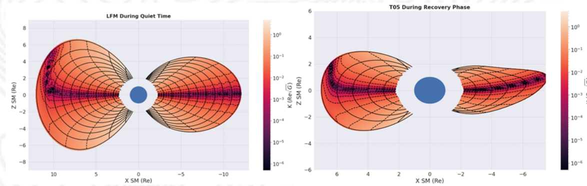
$$\mu = \frac{p^2}{2m_0 B}.$$

Algorithm for Bounce Motion Invariant K

- Algorithm uses native simulation grid
- Traces field along bounce path using Runge-Kutta 45
- Takes subset of field line trace between mirroring magnetic field intensities (B_m). This is the bounce path.
- Once bounce path is determined, numerically integrate:

$$K = \int_{s_1}^{s_2} \sqrt{B_m - B(s)}\, ds$$

Plot below shows K for particles mirroring at varying magnetic latitudes in the LFM and TS05 models (SM coordinates) ↓

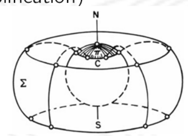


Algorithm for Drift Shell Invariant L^*

- Iterate over N_{MLT} equally spaced local times
  1. Use a linear search for field line at increasing/decreasing radii to find field line conserves $K(B_m)$
  2. First advance in large steps, then backtrack and take small steps if gone too far
- Once drift shell is determined, use numerical integration with spline smoothing over polar cap (Stokes simplification)

Basic Equation:

$$L^* = \frac{2\pi B_E R_E^2}{\iint \vec{B} \cdot \hat{n}\, ds}$$



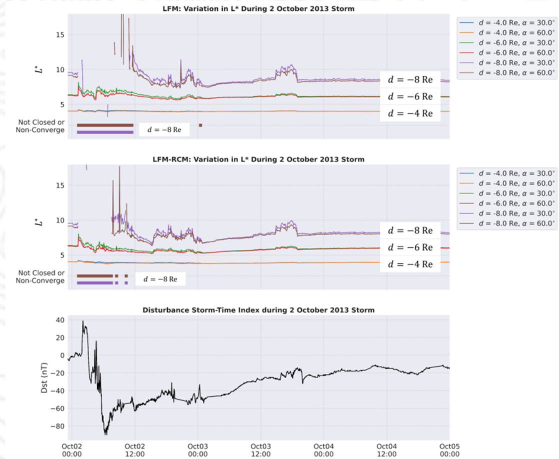
Stokes Simplification (at Model Inner Boundary):

$$L^* = \left(\frac{R_{in}}{R_E}\right) \frac{2\pi}{\int_0^{2\pi} \sin^2(\theta(\phi))\, d\phi}$$

Comparison of Results from LFM Simulations during Geomagnetic Storm

Calculated adiabatic invariants during 2 October 2013 geomagnetic storm

- Calculated at fixed points with fixed local pitch angles
- Calculation can be parallelized over time
- Biggest deviations in L^* from dipole L^* ($L = L^*$) occur farther into the magnetosphere where the external field holds greater influence
- Differences between models in the duration of non-closed / non-convergent drift shells during main phase of storm
- Different in structure of L^* during early recovery phase



Analysis of Phase Space Density with RBSP Data

Previously established observational techniques tracks time evolution of $f(L^*)$ at fixed μ, K to investigate energization processes (Green et al., 2004)

- $f(L^*)$ reflects a truer "state variable" during storms than $f(L)$
- Different processes will distort the $f(L^*)$ curve over time; such as radial diffusion (top left →) and internal acceleration (top right →)
- Curve be calculated from instruments measuring flux $j(\alpha, E)$ such as RBSP

Method:

- Interpolate flux distribution at α/E corresponding to fixed μ/K
- Compute L^* corresponding to K and ephemeris location

Example (bottom right →) shows combination of radial diffusion and precipitation loss