VIVYD – A CODE FOR SIMULATING THE GENERALIZED VIV MODEL

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Abstract. - it is a generic model. It can be degenerated into various known generalized VIV models. The code is provided open so that many other users can use it

Contents

1. Introduction

This report presents a numerical implementation for the solution of a system of differential equations that model 2-D Vortex-Induced Vibrations (VIV) with added stochastic forcing in the wake equation. This model extends the recent model proposed by Rigo et al. [REF], and might be useful to simulate a direct response of the model in view of identifying its parameters from experimental data.

The system features:

- a generalized cubic expression for the nonlinearity in the wake equation, responsible for the limit cycle behavior
- a coupling between structural displacement and fluid forces, through the position of the wake
- a coupling between fluid forces and structural displacement, through position, velocity and acceleration of the cylinder
- additional stochastic loading terms in the wake equation representing the near-wall turbulence in the wake of the cylinder

In the direct integration approach, the governing equations are integrated using the `odeint` function from `SciPy`.

In the forward identification problem, XXX [to de developed]

2. Mathematical Description of the Model

2.1. **Governing Equations.** The system of equations solved is based on the following set of dimensionless differential equations:

$$\ddot{y} + 2\xi_s \dot{y} + y = M_0 q + \eta_1, \tag{2.1}$$

$$\ddot{q} + \Omega\left(aq^2 + b\dot{q}^2 - \varepsilon\right)\dot{q} + \Omega^2 q = A_2 \ddot{y} + A_1 \dot{y} + A_0 y + \eta_2, \tag{2.2}$$

$$\dot{\eta}_1 = -\nu_1 \eta_1 + \sigma_1 \, w_1(t), \tag{2.3}$$

$$\dot{\eta}_2 = -\nu_2 \eta_2 + \sigma_2 \, w_2(t), \tag{2.4}$$

where:

- $y\left(\tau\right)$ and $\dot{y}\left(\tau\right)$ represent the displacement and velocity of the structure, respectively,
- $q\left(\tau\right)$ and $\dot{q}\left(\tau\right)$ represent the displacement and velocity of the fluid-induced motion,
- $\eta_1\left(\tau\right)$ and $\eta_2\left(\tau\right)$ represent the additive stochastic forcing, obtained by coloring white noise processes $w_1\left(\tau\right)$ and $w_2\left(\tau\right)$,
- ξ_s, and Ω are system parameters related to the structural damping and the scaled upstream wind velocity
- ε, a, and b are used to parameterize the nonlinear restoring force in the wake equation
- $\nu_{1,2}$ and $\sigma_{1,2}$ represent the characteristic frequency and intensity of the noise used in both the structural and wake equations,
- A_0, A_1, A_2 and M_0 are system parameters related to the fluid-structure interaction

The terms $w_1(\tau)$ and $w_2\left(\tau\right)$ represent two independent unit-intensity white noises, used to create the Ornstein-Uhlenbeck processes $\eta_1\left(\tau\right)$ and $\eta_2\left(\tau\right)$.

ρ	: air density (usually 1.25 kg/m^3)
St	: Strouhal number
f_0	: natural frequency (in Hz)
ω_0	: natural circular frequency (in rad/s)
D	: crossflow diameter (in m)
V	: upstream wind velocity (in m/s)
f_s	: shedding frequency (in Hz)

TABLE 1. Symbols

2.2. **Scaling and non-dimensionalization.** Equations (2.1-2.4) are written in a dimensionless version, as a function of the dimensionless time

$$\tau = \omega_0 t$$

where t is the physical time and ω_0 is the natural circular frequency of the structural system. Parameter Ω is used to represent the mistuning between the structure and the fluid system. It is related to the upstream wind velocity.

The structural and fluid variables $y(\tau)$ and $q(\tau)$ represent the dimensionless motion and inclination of the wake lamina. Usually the structural displacement is expressed as a function of the crossflow diameter D,

$$y(\tau) = \frac{\mathrm{y}(t(\tau))}{D}$$

where y(t) is the transverse displacement. The fluid coordinate $q(\tau)$ may have a different meaning depending on which model is considered, but typically it is a scaled version of the inclination of the lamina, whenever interpreted as such, or alternatively the fluctuating lift coefficient.

2.3. **Relation to existing wake-oscillator models.** In this section, we review some existing models of the literature and indicate how they can be expressed in the form of the general dimensionless version given above. Table 1 summarizes a few symbols which are common to all models.

2.3.1. *Hartlen-Currie (1970).* Hartlen and Currie's model of VIV is usually recognized as a major step in the derivation of models able to reproduce the synchronization features of the phenomenon [1]. Their model reads

$$\ddot{x}_r + 2\xi_s \dot{x}_r + x_r = a\Omega^2 c_L \tag{2.5}$$

$$\ddot{c}_L + \left(\frac{\gamma}{\Omega}\dot{c}_L^2 - \alpha\Omega\right)\dot{c}_L + \Omega^2 c_L = b\dot{x}_r \tag{2.6}$$

where $x_r = x/D$ is the transverse displacement expressed in diameters, and

$$a = \frac{\rho D^2}{8\pi^2 \mathrm{St}^2 \left(M/L\right)} \quad ; \quad \Omega = \frac{f_s}{f_0} = \mathrm{St}\frac{V}{f_0 D}$$

where M/L is the mass per unit length of the body and other symbols are reported in Table 1. Parameters α, γ and b are tuned on experimental data. They notice that the limit cycle amplitude, for small $\alpha\Omega$, is

$$C_{L_0} = \left(\frac{4\alpha}{3\gamma}\right)^{1/2}$$

which defines the ratio α/γ when C_{L_0} is determined experimentally. To compare meaningful information with experimental data, they use the following values for their parameters

$$\mathrm{St} = 0.2, \quad a = 0.00218, \quad \alpha = 0.02, \quad \gamma = 0.667, \quad b = 0.4.$$

The chosen values for α and γ results in $C_{L0} = 0.2$, which, together with $\mathrm{St} = 0.2$, are assumed to represent the subcritical range of Reynolds number.

It is observed that equations (2.5)-(2.6) can be recast in the format of (2.1)-(2.4). By comparing the coefficients in the different terms, the equivalence summary in Table 2 (p. 6) can be obtained.

2.3.2. *Tamura (1979, 1980, 1983).* Tamura's wake oscillator [2] is described by the following equations

$$Y'' + \left(2\eta + n(f + C_D)\frac{\Omega}{S^\star}\right) Y' + Y = -\frac{fn\Omega^2}{S^{\star 2}}\alpha \tag{2.7}$$

$$\alpha'' - 2\zeta\Omega\left(1 - \frac{4f^2}{C_{L0}^2}\alpha^2\right)\alpha' + \Omega^2\alpha = -m^\star Y'' - \Omega S^\star Y' \tag{2.8}$$

$$C_L = -f\left(\alpha + \frac{S^\star}{\Omega}\dot{Y}\right)$$

where $Y = y/D$, α and C_L are the three dimensionless variables in the model. The last equation is a memoryless transformation of the angular position of the wake α and the transverse oscillating frequency $\dot{Y}$ which can be used to determine a physical representation for the lift coefficient.

This model was analytically derived from a Birkhoff oscillator, assuming a wake lamina is oscillating behind the cylindrical body. It is based on the earlier works of Funakawa who derived a similar model but with a linear equation for the wake, which is therefore unable to model all features related to the limit cycle, in particular synchronization and phase entrainment. All parameters of Tamura's model have a physical meaning since they are derived from a model: $S^\star = 2\pi\mathrm{St}$, f chosen as $f = 1.16$ is related to a geometric quantity used to express the lift on a fixed cylinder, as a function of the inclination of the lamina, $C_D = 1.2$ is the drag coefficient, $n = \rho D^2/2\,(M_c/s)$ is the mass ratio (M_c/s is the mass per unit length of the cylinder), $\zeta = 0.038$ represents the negative damping associated with the effect of the discharged vortex, and $m^\star = 0.625$ is another geometric parameter representing the position of the centroid of the lamina. Finally C_{L_0} is the amplitude of the lift coefficient for a fixed cylinder. In the original paper it is tentatively proposed to use $C_{L_0} = 0.4$ since several authors had observed RMS values of C_{L_0} in the range $[0.2; 0; 6]$.

In his paper [2] illustrates the results of his model by comparison with various experiments. The mass ratio n varies in the different simulations to match experimental tests:

- Feng (19xx): $n = 0.00257$, i.e. $M_0 = -\frac{fn\Omega^2}{(2\pi \mathrm{St})^2} = -0.0019\Omega^2$
- Ferguson (19xx): $n = 0.00330$, i.e. $M_0 = -\frac{fn\Omega^2}{(2\pi \mathrm{St})^2} = -0.00242\Omega^2$
- Yamaguchi (19xx): $n = 0.00178$, i.e. $M_0 = -\frac{fn\Omega^2}{(2\pi \mathrm{St})^2} = -0.00131\Omega^2$

It is observed that equations (2.7)-(2.8) can be recast in the format of (2.1)-(2.4). By comparing the coefficients in the different terms, the equivalence summary in Table 2 (p. 6) can be obtained.

2.3.3. *Fachinetti, de Langre, Biolley.*

2.3.4. *Rigo, Andrianne, Denoël (2022).* The Rigo et al. model is a wake model only. It is already written in a dimensionless form, very similar to the generic format discussed and implemented here:

$$\ddot{q} - \left(\alpha q^2 + \beta q\dot{q} + \gamma \dot{q}^2 + \delta\right)\dot{q} + q = \eta.$$

Comparison with the general form shows that $\Omega = 1$,

$$\ddot{q} + \Omega\left(a q^2 + b\dot{q}^2 - \varepsilon\right)\dot{q} + \Omega^2 q = A_2\ddot{y} + A_1\dot{y} + A_0 y + \eta_2.$$

2.3.5. *Denoël.*

2.3.6. *Krenk.*

2.3.7. *Ogink-Metrikine.*

2.3.8. *Summary.*

2.4. **Relation to existing spectral models.**

	HC	T	FDLB	RAD	D	KX	OM
$y(\tau)$	x_r	Y		$-$			
$q(\tau)$	c_L	α		$q(\tau)$			
ξ_s	ξ_s	$\xi_s + \frac{\Omega n(f+C_D)}{4\pi \mathrm{St}}$		$-$			
Ω	Ω	Ω		1			
ε	α	2ζ		δ			
a	$-$	$8\Omega\zeta\frac{f^2}{C_{L_0}^2}$		$-\alpha$			
b	$\frac{\gamma}{\Omega^2}$	$-$		$-\gamma$			
A_0	$-$	$-$		$-$			
A_1	b	$-2\pi \mathrm{St}\Omega$		$-$			
A_2	$-$	$-m^\star$		$-$			
M_0	$a\Omega^2$	$-\frac{fn\Omega^2}{(2\pi \mathrm{St})^2}$		$-$			
$\eta_1(\tau)$	$-$	$-$		$-$			
$\eta_2(\tau)$	$-$	$-$		$\eta(\tau)$			

TABLE 2. Summary of the recasting of existing VIV models in the general dimensionless format (2.1)-(2.4). Legend: Hartlen-Currie (HC), Tamura (T), Fachinetti et al. (FDLB), Rigo et al. (RAD), Denoël (D), Krenk-XXX (KX), Ogink-Metrikine (OM).

	HC	T	FDLB	RAD	D	KX	OM
ξ_s	ξ_s	$\xi_s + 0.94\Omega\, n$		$-$			
ε	0.02	0.076		0.063			
a	$-$	2.56		0.09			
b	$\frac{2}{3\Omega^2} \sim 0.667$	$-$		-0.009			
A_0	$-$	$-$		$-$			
A_1	0.4	-1.26Ω		$-$			
A_2	$-$	-0.625		$-$			
M_0	$0.002\Omega^2 \sim 0.002$	$-0.73n\Omega^2$		$-$			
$\eta_1(\tau)$	$-$	$-$		$-$			
$\eta_2(\tau)$	$-$	$-$		η			

TABLE 3. Numerical values proposed by some authors to model VIV in the subcritical Reynolds regime.

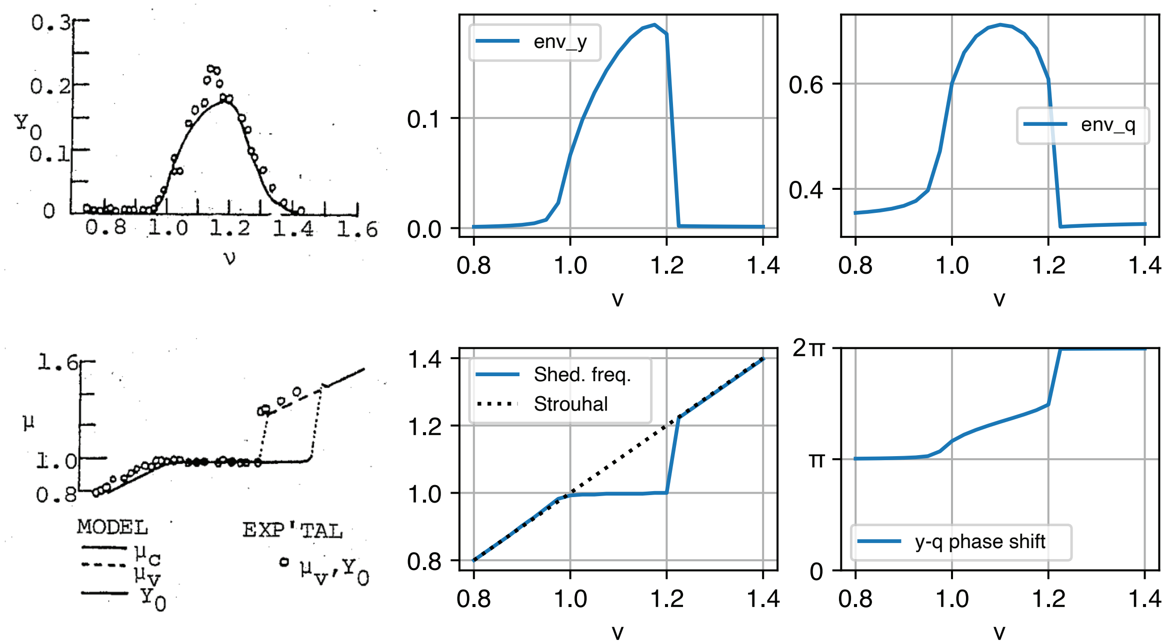

FIGURE 5.1.

3. Numerical Method for the Direct Integration

The system of equations is solved numerically using the `odeint` function from the `SciPy` library, which applies a variable-coefficient adaptive step-size integration method. The time series t spans a duration of 2500 units, discretized into 5000 steps.

To simulate realistic VIV dynamics under stochastic forcing, we generate a white noise process $w(t)$, which drives the stochastic component of the system through the equation for $\dot{n}_0$. This noise is interpolated to match the time steps of the solution.

4. Statistical Analysis

After solving the system of differential equations, we analyze the solution by computing:

- The mean and standard deviation of the displacement y_0,
- The envelope of the solution using a simple maxima-based approach,
- Histograms of the solution and its envelope to study the distribution of the response.

5. Validations

5.1. **Tamura 1983 - OK.** Tam81 wrong ??

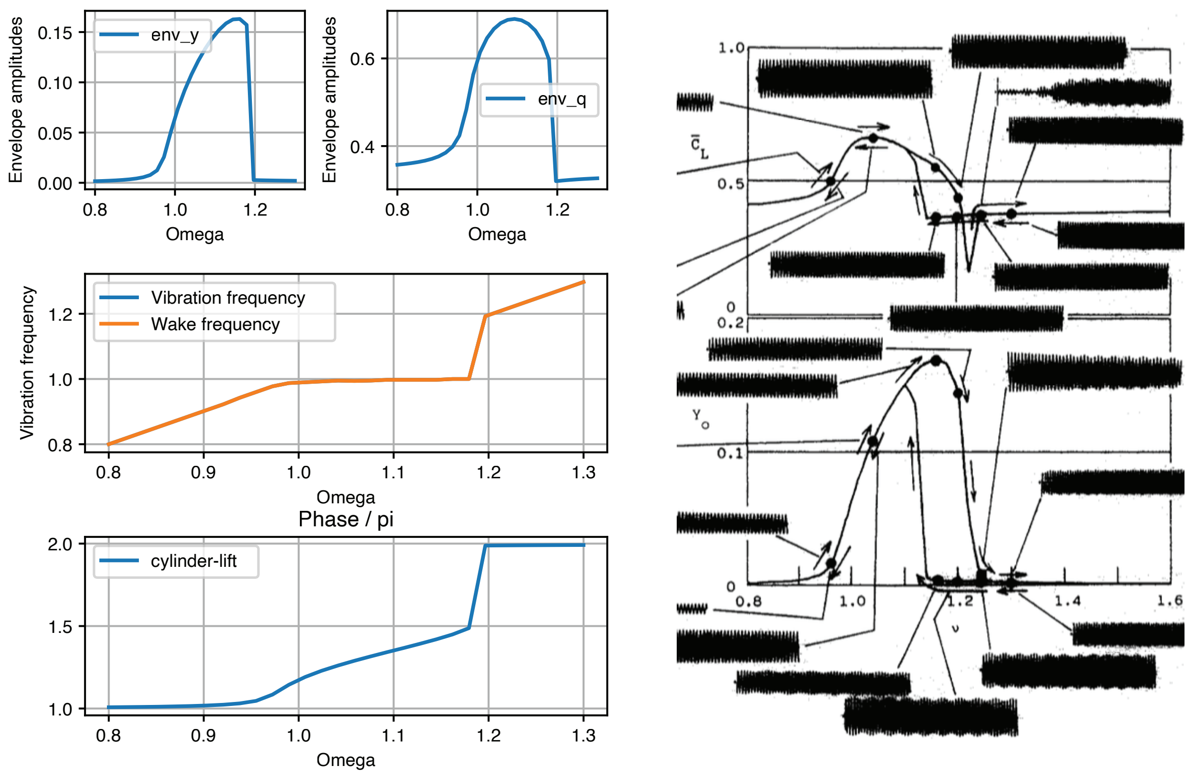

Figure 5.2.

5.2. **Hartlen-Currie 1970.**

6. Results

The resulting time series is analyzed for different values of the system parameters ξ_s and η. The output includes both the statistical properties of the structural displacement and the fluid-induced motion, as well as the statistical properties of the envelope of these motions. The results are saved for further post-processing.

7. Numerical Method for the Inverse Identification Problem

to be developed

8. Conclusion

This numerical approach provides a way to simulate and statistically analyze Vortex-Induced Vibrations in a structure under stochastic forcing. The solution includes both

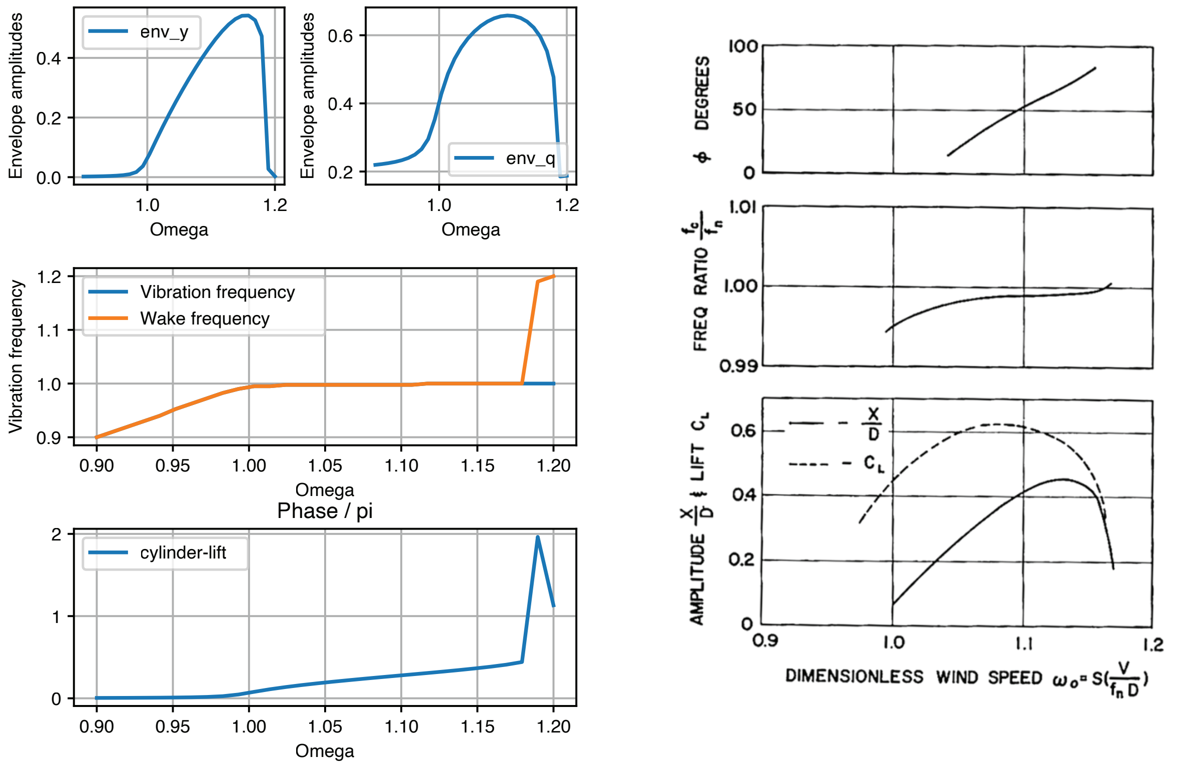

FIGURE 5.3.

the time evolution of the system and statistical properties, which are crucial for understanding the long-term behavior of VIV in stochastic environments.

REFERENCES

[1] Ronald T Hartlen and Iain G Currie. Lift-oscillator model of vortex-induced vibration. *Journal of the Engineering Mechanics Division*, 96(5):577–591, 1970.

[2] Y Tamura and G Matsui. Wake-oscillator model of vortex-induced oscillation of circular cylinder. In *Wind Engineering*, pages 1085–1094. Elsevier, 1980.