GLOBAL NAVIGATION SATELLITE SYSTEM

GLONASS

INTERFACE CONTROL DOCUMENT

General Description of Code Division Multiple Access Signal System

Edition 1.0

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Definitions and acronyms

AFS – Atomic Frequency Standard

CDMA – Code Division Multiple Access

CS – Central Synchronizer

ECEF – Earth-Centered Earth-Fixed (coordinate system)

FDMA – Frequency Division Multiple Access

GLONASS – Global Navigation Satellite System

GMST – Greenwich Mean Sidereal Time

GPS – Global Positioning System

GSO – Geosynchronous Orbit

GST – Greenwich Sidereal Time

HEO – Highly Elliptical Orbit

ICD – Interface Control Document

IERS – International Earth Rotation Service

JD0 – current Julian date at 00:00 in Moscow Time

L1OC – CDMA Open Service Navigation Signal in L1 frequency band

L1OF – FDMA Open Service Navigation Signal in L1 frequency band

L1SC – CDMA Secured Service Navigation Signal in L1 frequency band

L1SF – FDMA Secured Service Navigation Signal in L1 frequency band

L2OCp – CDMA Open Service Navigation Signal in L2 frequency band (pilot signal)

L3OC – CDMA Open Service Navigation Signal in L3 frequency band

LDMP – Long-time Dynamic Model Parameters

MEO – Medium Earth Orbit

MT – Moscow Time

PS – Pseudoframe Size

PZ-90 – system of geodetic parameters “Parametry Zemli 1990” and designation of the Earth’s reference ellipsoid used in this system of geodetic parameters

RMS – root mean square (error)

RST – Received Signal Time

RT – Receiver Time

SSE – Sun-SV-Earth (plane and angle)

SV – Space Vehicle

TAI – International Atomic Time

TEC – Total Electron Content

UE – User Equipment

UT1 – Universal Time – mean solar time at 0° longitude accounted for the effect of polar motion on position of meridians

UTC – Coordinated Universal Time

UTC(SU) – Coordinated Universal Time of Russia

$\langle x \rangle$ – extracting the integer part of x

$\langle\langle x \rangle\rangle$ – computing the integer nearest to x

1 Introduction

1.1 The Global Navigation Satellite System (GLONASS) is used to provide positioning, velocity and precise time for marine, air, terrestrial, and space users, as well as to accomplish additional information functions.

1.2 The GLONASS system consists of three segments:

- Space Segment;
- Control Segment;
- User Segment.

The GLONASS Space Segment consists nominally of 24 to 30 operational Medium Earth Orbit (MEO) space vehicles (SV) in circular orbits with the nominal altitude of 19,100 km, the inclination of 64.8º, and the orbital period of 11 h 15 min 44 s. The orbital planes were selected to ensure continuous global signal coverage of the Earth's surface and the near-Earth space up to 2,000 km.

The number of the reserved ranging codes enables expanding the orbital constellation to include up to 64 SVs. The SV ID number transmitted in the service part of CDMA signal navigation message strings equals to the ranging code number listed in the table of the related ICD. The orbital constellation can be expanded both by adding a SV within an orbital plane or between orbital planes, and by building MEO, geosynchronous (GSO) and highly elliptical (HEO) orbital extensions.

The baseline constellation described in the GLONASS ICD for Navigation Radiosignal in Bands L1, L2 Frequency will be sustained until L1 and L2 FDMA signals are supported. The SVs of this constellation will transmit L1 and L2 FDMA signals.

SVs of orbital extensions can only transmit either CDMA signals or CDMA and FDMA signals together (provided spare carrier frequencies are available).

In the new GLONASS CDMA signals a SV ID number can be inconsistent with the number of its orbital slot.

The Control Segment consists of a System Control Center and a network of Telemetry, Control and Command Stations. The Control Segment is responsible for monitoring the Space Segment operation, continuous SV orbit determination and time synchronization, and upload of software and non-recurrent control commands as well as of navigation message data or raw data used to generate navigation message.

The User Segment is comprised of a wide variety of user receivers employed to receive navigation signals, measure navigation parameters and provide solution for positioning, velocity determination and timing based on the processed measurements.

1.3 To determine position and velocity GLONASS user receivers receive signals from at least 4 (3) SVs and perform one-way measurements of pseudorange and radial range-rate with respect to each SV, as well as receive and process navigation messages contained within signals. A navigation message describes position of SVs in space and time. Processing the measurements and navigation message data results in 3 (2) coordinates of a user location and 3 (2) components of the user velocity vector. Also the user receiver time scale (RT) is synchronized with one of the following time scales: GLONASS time, Moscow Time (MT), the Coordinated Universal Time of Russia (UTC(SU)), or International Atomic Time (TAI).

For a timing solution (provided the user location is known), the GLONASS user receiver should receive signals from at least one SV.

The information used for planning positioning calculation sessions, selecting the effective constellation and acquiring the broadcast signals is relayed in the form of almanac within a navigation message (see Appendices M and N).

1.4 Advanced GLONASS-capable user receiver hardware shall be designed to receive and process signals from the baseline constellation described in the GLONASS ICD for Navigation Radiosignal in Bands L1, L2 as well as from its orbital extensions and local ground-based navigation signal sources (pseudollites, local area augmentation systems).

Expandable GLONASS functional capabilities shall be supported by updating user receiver software without modifying its hardware.

1.5 GLONASS CDMA signals use string structure of navigation message data. Strings contain service and data sections.

Service sections of strings are similar for each signal and include a SV ID number, type of a string, time stamp, signal parameters as well as CRC check bits.

The structure of a data section is defined by its type. Each string type contains the complete individual block of data. In order to process this data one doesn't need to know the data contained in other string types. This does not refer to the ephemeris and clock data broadcast in the three consecutive strings.

Arrangement of data in navigation message strings is provided in the Interface Control Documents for the corresponding signals.

Future GLONASS evolutions may involve the inclusion into their navigation message of additional new string types which are either modifications of the old string types defined in an earlier version of the GLONASS ICD or containing new types of data. The old string types are expected to be supported for the long-term transitional period to provide backward compatibility with the existing user equipment. The day of the discontinuation of support for the old string types will be specified in the next version of the GLONASS ICD. To mitigate the impact of the new string types on user equipment designed in accordance with the old versions of the GLONASS ICD, user receivers are expected to be able to ignore all unknown string types. An earlier manufactured user receiver is expected to be able to support new string types and discontinue use of specific old string types through updating its software.

The sequence of the received navigation message strings is not predetermined. The conventional repeating fragment of a string packet is pseudoframe.

A pseudoframe is a string packet containing immediate and non-immediate data starting with the three strings of ephemeris and clock data. The remaining strings of a pseudoframe contain non-immediate data. The composition of non-immediate data strings in adjacent pseudoframes can vary. The composition of non-immediate data strings transmitted at the same time intervals can also vary between various SVs. It results in a considerably higher non-immediate data rate to multichannel user receivers.

Example

Provided L1OC signal pseudoframe includes 6 strings, at an interval of a pseudoframe length various SVs may transmit the following:

SV1

- strings of Types 10-12 with their immediate data;
- string of Type 20 with SV-1 almanac;
- string of Type 20 with SV-2 almanac;
- string of Type 20 with SV-3 almanac;

SV2

- strings of Types 10-12 with their immediate data;
- string of Type 25 with Earth Rotation Parameters (relevant for all SVs);
- string of Type 20 with SV-4 almanac;
- string of Type 20 with SV-5 almanac;

SV3

- strings of Types 10-12 with their immediate data;
- string of Type 16 including SV orientation parameters for noon/midnight turn maneuver (relevant only for the particular SV for the limited time interval);
- string of Type 20 with SV-6 almanac;
- string of Type 20 with SV-7 almanac;

SV4

- strings of Types 10-12 with their immediate data;
- string of Type 20 with SV-8 almanac;
- string of Type 20 with SV-9 almanac;
- string of Type 20 with SV-10 almanac;

The next pseudoframe may comprise the following information:

SV1

- strings of Types 10-12 with their immediate data;
- string of Type 20 with SV-4 almanac;
- string of Type 20 with SV-5 almanac;
- string of Type 20 with SV-6 almanac;

SV2

- strings of Types 10-12 with their immediate data;
- string of Type 20 with SV-6 almanac;
- string of Type 20 with SV-7 almanac;
- string of Type 20 with SV-8 almanac;

SV3

- strings of Types 10-12 with their immediate data;
- string of Type 16 with SV orientation parameters for noon/midnight turn maneuver (relevant only for the particular SV for the limited time interval);
- string of Type 20 with SV-8 almanac;
- string of Type 20 with SV-9 almanac;

SV4

- strings of Types 10-12 with their immediate data;
- string of Type 25 with Earth Rotation Parameters (relevant for all SVs);
- string of Type 20 with SV-11 almanac;
- string of Type 20 with SV-12 almanac.

The pseudoframe size and the rule for its generation are determined by the orbital constellation structure and urge for its effective use.

These settings may vary both for different SVs and for L1 and L3 CDMA signals of the same SV. Thus changes in the non-immediate data strings sequence in L1 and L2 signals of the same SV leads to even higher non-immediate data group receipt rate in multichannel and multifrequency receivers.

The size of the current pseudoframe for each signal is transmitted in the data section of Type 10 string as a value of PS parameter (Pseudoframe Size).

Some string types are transmitted only when necessary:

- Strings of Type 16 containing noon/midnight turn maneuver parameters are transmitted just preceding the turn start. They stop being broadcast on completion of the maneuver (see Appendix R);
- Strings of Type 60 with textual information and strings of Type 50 containing Cospas-Sarsat notices of receipt are transmitted when necessary;
- Strings of Type 31 and 32 containing long-term dynamic model parameters (LDMP, see J.3) may be transmitted both regularly and at certain time intervals depending on GLONASS signal use;
- individual SVs may not transmit almanac data (strings of Type 20), substituting them with strings of other types.

1.6 GLONASS CDMA signals allocate substantially more space for ephemeris parameters allowing transmission of ephemeris and clock data for GEO and HEO SVs.

Ephemeris and clock data in CDMA signals is referenced to a SV’s center of mass in contrast to that for FDMA signals. Ephemeris numerical values tied to the same instant are equal for L1OC and L3OC.

User receiver calculates the position of the SV’s antenna phase center to which the measurements are referenced in the PZ-90 Geodetic Coordinate System. For this purpose navigation message strings of Types 10-12 contain coordinates of transmitting antenna phase centers in a SV-fixed reference system. The origin of this system coincides with the SV’s center of mass. The position of the SV's center of mass may shift as the SV's attitude control system depletes propellant. In this regard the phase center coordinates of the signal transmitting antennas in a SV's coordinate system may also shift.

User receiver recalculates the SV's center of mass coordinates into the corresponding coordinates of its antenna phase center taking into account rotations the SV performs for solar panels Sun pointing during nominal operations. During noon/midnight turn maneuvers user receiver shall take into account Type 16 string parameters (see Appendix R). Attribute P2 = 1 in the service part of a navigation message indicates that a SV performs a noon/midnight turn maneuver. Any SV may remain in this regime for not more than 15 min.

Provided the user does not employ Type 16 string parameters to calculate the antenna phase center in the PZ-90 Geodetic Coordinate System during the noon/midnight turn maneuver, the maximum approximation error ΔR_{max} of the estimated range may be evaluated as

$$\Delta R_{max} = 0{,}5 * \sqrt{\left(\Delta Ypc^2 + \Delta Zpc^2\right)},$$

where ΔYpc, ΔZpc are Y and Z coordinates of the signal transmitting antenna phase center. The coordinates belong to a reference system originating from the SV's center of mass.

1.7 FDMA signals of SV number j contain navigation message with polynomial model parameters. These parameters allow receiver to calculate the offset between time of L1SF signal (referred to as VT signal in previous L1 and L2 FDMA Signal ICD) and GLONASS time. They also allow calculating offset $\Delta\tau^j$ between L1 signal and L2 signal time. Thus for GLONASS FDMA signals each SV uses L1SF as a basic signal, because it transmits $\Delta\tau^j$ offset for transformation to L2 signal time.

The navigation message of each GLONASS CDMA signal contains polynomial model parameters allowing receiver to calculate the offset between time of the signal and GLONASS time. Consequently GLONASS CDMA signals do not use basic signal concept – time of each signal is independent. L2OCp signal is an exception; it is a pilot signal which does not contain navigation message. That is why string of Type 12 of the L1OC signal navigation message includes $\Delta\tau^j_{L2}$ offset between L2OCp time and L1OC time.

When generating clock data in rebroadcasting and multiplication modes corrections to signal time are generated as relative to GLONASS time (see 3.2).

When generating clock data using intersatellite measurements the above mentioned corrections are generated as relative to the orbital system timescale based on signal time scales of all the SVs included in the orbital constellation. Accuracy of the orbital system time scale referencing to other time scales (MT and others) may decline to some extent. However it does not have any influence on user positioning accuracy.

The clock data generation regime is indicated in the navigation message by attributing specific values to R_T^j parameter in Type 10-12 strings.

1.8 In contrast to FDMA signals, ephemeris and clock data update period in GLONASS CDMA signals is not strictly determined. Ephemeris and clock data may be referenced to any instant within 24 hours with 90 s increment and may change at intervals divisible by 90 seconds.

1.9 CDMA signals navigation messages contain almanac parameters only for the SV transmitting these signals. SV almanac parameters transmitted within the navigation messages of all its CDMA signals are the same.

In CDMA signals SV nominal orbit inclinations and nodical period values are assumed as 64.8º and 40,544 s, respectively. These values differ from those transmitted by FDMA signals (63º and 43,200 s, respectively). These changes of nominal values in CDMA signals were made to approximate them to the real orbit parameters.

CDMA signals navigation messages contain almanacs for all SVs transmitting CDMA including those temporarily not usable for navigation. The number of SVs in almanac changes as orbital constellation composition changes. The number of SVs included into the almanac is indicated by N_S parameter of string Type 20. It is supposed that SVs temporarily unusable for navigation are used for information purposes of GLONASS.

Individual SVs may not transmit the full set of signals while still being usable for navigation. A set of signals transmitted by a given SV is indicated by a SR_A parameter in string Type 20.

In the process of GLONASS development orbital augmentations at other orbits (GSO, HEO, etc.) may be established. In this case the composition, capacity and nominal values of almanac parameters will probably be different for new orbits. The composition of almanac parameters is defined by «TO» (Type of Orbit) attribute in the beginning of the data section of Type 20 string. This attribute is in fact a subtype of Type 20 string which defines the composition and the structure of almanac parameters for the given types of orbits. TO = «00» states that almanac string structure refers to the existing GLONASS circular orbits with nominal height of 19,100 km, inclination of 64.8º, and orbital period of 11 h 15 min 44 s. Almanac parameters for this type are described in the Interface Control Documents for the corresponding signals.

Introduction of orbital augmentations at different orbits will be supported by publication of the corresponding Interface Control Document follow-ups (revisions). Almanac parameters for new orbit types, as well as for the existing ones, will be transmitted in Type 20 string and defined by «TO» attribute distinct from «00».

User receivers receiving GLONASS CDMA signals should:

- ensure software upgrade capability to process orbital augmentations almanac.
- ignore the Type 20 strings of the almanac with the unknown «TO» values.

1.10 Service part of the GLONASS CDMA signals navigation message strings contains «H^j» (health) flag for the navigation signal of SV number j and «l^j» data validity flag for a given string.

H^j=1 means the signal is not applicable for positioning and timing solutions. At the same time the overall system data and augmentation data transmitted within the navigation message are valid and can be used.

For example, if H^j=1 the following data may be used:

- almanac parameters in Type 20 strings;
- Earth rotation parameters and other parameters from Type 25 string;
- augmentation data in the form of Cospas-Sarsat notices of receipt from Type 50 strings;
- augmentation data in the form of text messages from Type 60 strings.

l^j=1 indicates non-applicability of the data transmitted in the string. If at the same time the navigation signal health flag is indicated as H^j=0, the ranging code transmitted by the SV may be used by a receiver to generate measurements.

2 Scope

2.1 This Interface Control Document (ICD) defines general characteristics of the GLONASS system, types of navigation radio signals being transmitted and navigation message general processing algorithms.

GLONASS SVs may transmit open service and secured service navigation signals which are either FDMA only, or both FDMA and CDMA, or CDMA only, depending on a SV modification.

Interface specifications for open service FDMA signals in L1 and L2 radio frequency bands are defined in the document "GLONASS. INTERFACE CONTROL DOCUMENT. Navigational radiosignal in Bands L1, L2." (http://russianspacesystems.ru).

Interface specifications for open service CDMA signals in L1, L2 and L3 frequency bands are defined by the individual dedicated documents.

2.2 Russian Rocket and Space Engineering and Information Systems Corporation, Joint Stock Company (Russian Space Systems, JSC) – the designer of GLONASS mission payload – is assigned as the developer of ICD and is responsible for its drafting, coordination, revision and maintenance.

The current Document comes into force provided that it is signed by the following persons/entities:

- GLONASS Chief Designer;
- Russian Rocket and Space Engineering and Information Systems Corporation, Joint Stock Company (Russian Space Systems, JSC) of ROSCOSMOS State Space Corporation which is the leading organization on the GLONASS payload, service radiofrequency and telemetry systems, ground control and command facilities, and a set of user equipment for different user groups;
- Academician M.F. Reshetnev Information Satellite Systems (ISS, JSC) of ROSCOSMOS State Space Corporation – prime for development and integration of GLONASS satellites, including system integration of space, launch, and ground control complexes, on-board mission software used to generate navigation message and SV control data;
- Research and Development Center (Korolev) of the Central Research Institute of the Russian Federation Space Forces – leading research and development organization of the Russian Ministry of Defense on the GLONASS system;

- Russian Institute of Radionavigation and Time (RIRT, OJSC) of Ministry of Industry and Trade of Russian Federation responsible for developing timing facilities of special and dual use, facilities for generating space segment time scale; synchronization of GLONASS timing facilities and developing user equipment for different user groups;
- Central Research Institute of Machine Building, Federal State Unitary Enterprise (TSNIIMASH, FSUE) – the head research institute of the ROSCOSMOS State Space Corporation.

ICD is approved by the authorized representatives of ROSCOMOS State Space Corporation and Space Forces. ICD comes into force on approval of the Commanding General of the Space Forces and the Director General of the ROSCOSMOS State Space Corporation.

In the course of GLONASS evolution its individual parameters may change. The author of ICD bears responsibility for coordinating any suggested modifications among all responsible parties and, if necessary, for drafting new edition of the Document containing such modifications.

Modifications and new editions of ICD come into force on approval of the Commanding General of the Space Forces and the Director General of the ROSCOSMOS State Space Corporation.

The Russian Space Systems, JSC is responsible for official distribution of the GLONASS ICD.

3 Time scales used in GLONASS

3.1 Determination of position and time in GLONASS are carried out using GLONASS time. Time of each navigation signal (signal time) broadcast by an SV and a receiver time (RT) are synchronized with GLONASS time. User can also use available information (including navigation message) to do transformation to the following time scales:

- Moscow Time (MT);
- Coordinated Universal Time of Russia (UTC(SU));
- Universal Time (UT1);
- International Atomic Time (TAI);
- GPS time.

Depending on receiver architecture, RT may undergo regular discrete or seamless synchronization based on processed measurements.

3.2 GLONASS time (T_{GL}) is a mathematical timescale established based on timing signals of several highly stable hydrogen frequency standards combined within the Central Synchronizer (CS) of GLONASS. T_{GL} is corrected simultaneously with the planned correction for ±1 s of $T_{UTC(SU)}$ and T_{MT}. As a result there is no offset for an integer number of seconds between GLONASS time and MT. In future transition to the continuous GLONASS time may be implemented. To accomplish this, navigation data structure for CDMA signals provides for the related capacity.

3.3 Signal Time – time scale determined by phase of a signal at the phase center of an SV antenna. Signal time is generated and maintained by an on-board clock based on atomic frequency standards, and is synchronized with GLONASS time.

Navigation data for GLONASS FDMA signals contains the estimated parameters of the polynomial model for transformation from L1 signal time to GLONASS time, as well as offset for transformation from L1 signal time to L2 signal time.

Navigation data for any GLONASS CDMA signal contains the estimated parameters of the polynomial model for transformation from that signal time to GLONASS time (see Appendix D), as well as for transformation from time of the pilot component of this signal to time of its data component.

When GLONASS time is corrected for ±1 s during scheduled leap second corrections of $T_{UTC(SU)}$, simultaneous correction of signal time for all SVs is carried out through changing second pulse sequence time stamps of all SVs'. Navigation data provides advance notifications to users of the day and the sign of the correction.

General recommendations on carrying out computations in receivers at the time of the scheduled leap correction are stated in Appendix E.

3.4 UT1 is mean solar time at 0° longitude accounted for the effect of polar motion on position of meridians. GLONASS navigation data contains parameters of polynomials used to determine the position of the instantaneous Earth's pole (Appendix L) and an offset between UT1 and UTC(SU) estimated accounting for polar motion (Appendix B).

3.5 TAI is a continuous time scale calculated by the International Bureau of Weights and Measures (Bureau International des Poids et Mesures, BIPM, France). GLONASS navigation message contains the requisite data for transformation from UTC(SU) to TAI (also see Appendix H).

3.6 UTC(SU) is the time scale referenced to UTC as maintained by the Russian Federation Primary Time Standard. UTC(SU) is a stepped uniform time scale. It is corrected periodically with ±1 s, as per the decision of BIPM, when UT1 – UTC modulo offset reach 0.75 to 0.9 s. UTC(SU) is corrected, as a rule, once per year (once per 1.5 year) at the end of a quarter (at 00 h: 00 min: 00 s): on the night of December 31/morning of January 1 , night of March 31/morning of April 1, night of June 30/morning of July 1, night of September 30/morning of October 1 in UTC(SU) (at 3 h 00 min 00 s in MT). The correction is carried out simultaneously by all users reproducing or employing UTC(SU).

3.7 MT is generated as UTC(SU) plus 3 h (10800 s). MT is corrected simultaneously with the planned correction of UTC(SU). GLONASS navigation message contains the requisite data for transformation from GLONASS time to MT (see Appendix D).

3.8 GPS time is conventional composite time established by the GPS Control Segment. GPS time may differ from UTC because GPS time shall be a continuous time scale, while UTC is corrected periodically with an integer number of leap seconds. Since July 2015 GPS time is 17 s ahead of UTC.

4 GLONASS geodetic reference

4.1 Ephemeris broadcast within immediate data are used to calculate position and velocity vector components for the SV's center of mass in the "Parametri Zemli 1990" (PZ-90) geodetic reference system of the latest version (algorithm for calculating position and velocity vector components for the SV's center of mass based on broadcast ephemeris is described in Appendix J).

Algorithm for recalculating position of a SV's center of mass into a position (in PZ-90 reference system) of its antenna phase center to be used in position fixes are described in Appendix R.

In "Parametri Zemli 1990" (PZ-90) the location of a point in space is unambiguously defined by:

- geocentric spatial Cartesian coordinates (X, Y, Z);
- geodetic coordinates: latitude (B), longitude (L), and height (H).

4.2 In geocentric spatial Cartesian reference system the location of a point in space is defined by the projection of the point on coordinate axes. The position of axes is determined as follows (Figure 4.1):

- the origin O is aligned with the center of mass of the Earth including ocean and atmosphere masses, which is specified in the planetary model of the Earth's gravitational field and is also aligned with the center of the PZ-90 Earth's ellipsoid;
- OZ axis is directed to the Conventional Reference Pole as defined by the International Earth Rotation and Reference System Service (IERS). Positive values count starts at O origin and directs to the Conventional Reference Pole as recommended by IERS. Negative values count starts at O origin and directs to the opposite of the Conventional Reference Pole;
- OX axis stems from O origin and coincides with the line formed by the intersection of the Equatorial plane and the Zero Meridian plane as set by IERS and Bureau International de l'Heure (BIH). Positive values count starts at O origin and directs to the intersection point of the Equator with the Zero Meridian plane (Xa point). Negative values count starts at O origin and directs to the opposite of the intersection point of the Equator with the Zero Meridian plane (Xb point);
- OY axis completes a right-handed system. Positive values count starts at O origin and directs to Ya point. Negative values count starts at O origin and directs to Yb point.

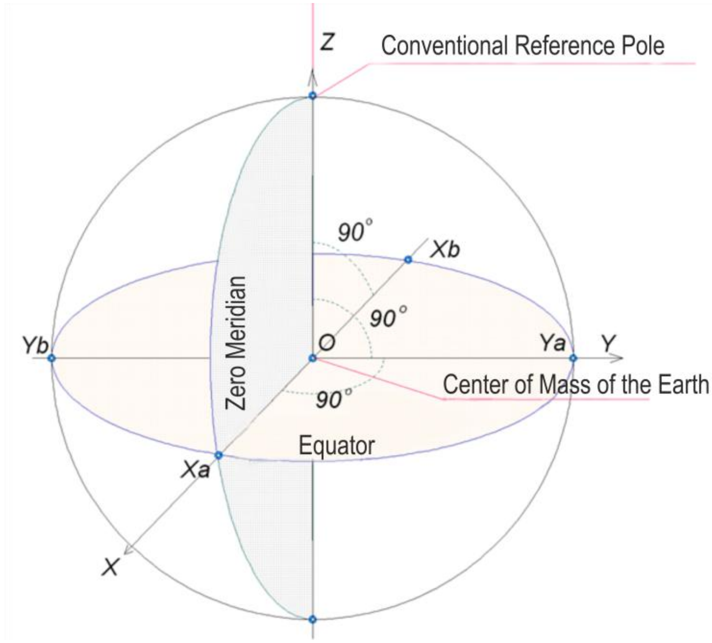


Figure 4.1 – Axes position in the geocentric spatial Cartesian reference system (X, Y и Z)

4.3 Geodetic coordinate system (*B*, *L*, and *H*) includes:

- the PZ-90 Earth's ellipsoid and its orientation parameters in relation to the Earth and OX, OY and OZ axes of the geocentric spatial Cartesian reference system;
- the geodetic coordinate system (*B*, *L* и *H*).

The main parameters of the PZ-90 Earth's ellipsoid are provided in Table 4.1.

The geodetic coordinate system (*B*, *L*, and *H*) is defined as follows (Figure 4.2):

- geodetic latitude *B* of M point is defined as a flat acute angle between the Equatorial plane and the normal line to the PZ-90 Earth's ellipsoid at M point. The geodetic latitude goes for 0° at the equator to +90° N (positive latitude) at the North Pole or -90° S (negative latitude) at the South Pole;
- geodetic longitude *L* of M point is defined as a dihedral angle between the Zero Meridian plane and the plane of the meridian passing through M point. The geodetic longitude goes for 0° from the Zero Meridian to 360° at the Zero Meridian;
- geodetic height *H* of M point is defined as a distance alone the normal line from the surface of the PZ-90 Earth's ellipsoid to M point. The geodetic height of M point, situated over the surface of the ellipsoid as related to its center is positive, while that situated under – is negative.

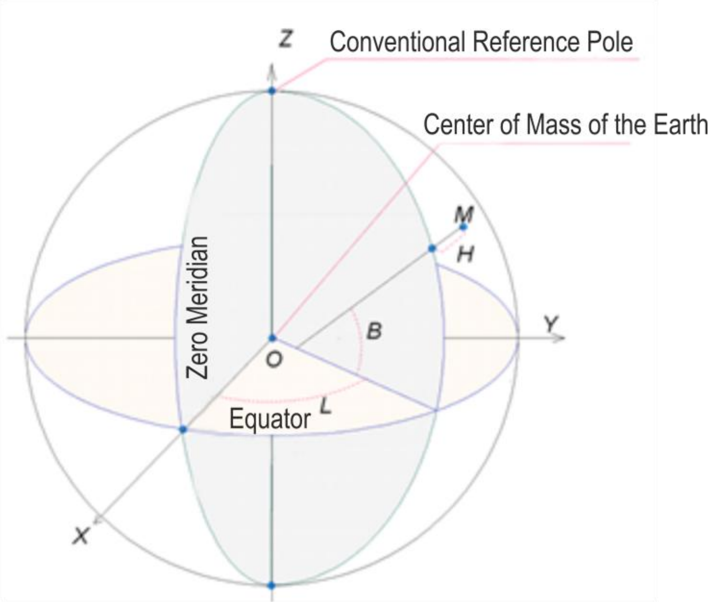


Figure 4.2 – Geodetic coordinate system (B, L, and H)

The geodetic coordinate system ensures unambiguous positioning in relation to the surface of the PZ-90 Earth's ellipsoid.

The poles in the geodetic coordinate system are specific points at which all meridians converge and which lack one coordinate – the geodetic longitude *L*.

The fundamental geodetic constants and the main parameters of the PZ-90 Earth's ellipsoid, employed in "Parametri Zemli 1990" are specified in Table 4.1.

Table 4.1 – Fundamental geodetic constants and main parameters of the PZ-90 Earth's ellipsoid

Angular velocity of the Earth (ω_3) wrt vernal equinox	$7.292115 \cdot 10^{-5}$ rad/s
Geocentric gravitational constant (mass of the Earth's atmosphere included) (fM)	$398600.4418 \cdot 10^9$ m^3 / s^2
Geocentric gravitational constant of the Earth's atmosphere (fM_a)	$0.35 \cdot 10^9$ m^3 / s^2
Light velocity in vacuum (c)	299792458 m/s
Semi-major axis of the PZ-90 Earth's ellipsoid (a_e)	6378136 m
PZ-90 Earth's ellipsoid flattening factor (α)	1 / 298.25784
Normal gravity at the ellipsoid equator (γ_a)	978032.84 mGal
Correction in acceleration of normal gravity due to attraction of atmosphere at sea level ($\delta\gamma_a$)	−0.87 mGal
Second degree zonal coefficient of normal potential (J_2^0)	$1082625.75 \cdot 10^{-9}$
Fourth degree zonal coefficient of normal potential (J_4^0)	$-2370.89 \cdot 10^{-9}$
Sixth degree zonal coefficient of normal potential (J_6^0)	$6.08 \cdot 10^{-9}$
Eighth degree zonal coefficient of normal potential (J_8^0)	$1.40 \cdot 10^{-11}$
Normal potential on the PZ-90 Earth's ellipsoid surface (U_0)	62636861.4 m^2 / s^2

Note: In ballistic measurements the normalized coefficients of normal potential are employed:

$$\overline{C}_{20}^{0} = -484164.954 \cdot 10^{-9};$$

$$\overline{C}_{40}^{0} = 790.296 \cdot 10^{-9}.$$

There is a relationship between zonal coefficient of normal potential J_n^0 and zonal harmonics of geopotential $\overline{C}_{n,0}^{0}$ of the same degree *n*:

$$J_n^0 = -\left|\sqrt{2n+1}\right| \cdot \overline{C}_{n,0}^{0},$$

which leads to:

$$J_2^0 = -\left|\sqrt{5}\right| \cdot \overline{C}_{20}^{0};$$

$$J_4^0 = -3 \cdot \overline{C}_{40}^{0};$$

$$J_6^0 = -\left|\sqrt{13}\right| \cdot \overline{C}_{60}^{0};$$

$$J_8^0 = -\left|\sqrt{17}\right| \cdot \overline{C}_{80}^{0}.$$

Besides, when transferring from the Earth's normal to abnormal gravitational field, one shall take into account that:

- normal potential of the Earth's gravity is described by the series containing only fully normalized coefficients of an even degree *n* and zero order for gravity potential spherical function series expansion $\overline{C}^{0}_{n,0}$;
- the Earth's gravity perturbing potential is described by a set of zonal harmonics $\Delta\overline{C}_{n,m}$, starting with *n = 2*, where $\Delta\overline{C}_{n,m} = \overline{C}_{n,m}$, except for zonal harmonic coefficients of the degrees *n = 2, 4, 6*, and *8*, and zero order, used to describe the Earth's gravity potential, for which $\Delta\overline{C}_{n,0} = \overline{C}_{n,0} - \overline{C}^{0}_{n,0}$.

5 General signal properties

5.1 General characteristics of radionavigation signals

5.1.1 Coherence of carrier oscillations and modulating sequences

Carrier frequencies of all transmitted signals and clock rates of modulating sequences are coherently derived from the on-board atomic frequency standard (AFS). The nominal value of the pulse rate at AFS output is set in the way it is 5.0 MHz as seen by a user on the ground and signal phase secular drifts due to relativistic effects are minimized. In particular for GLONASS circular orbits of 19,100 km the nominal pulse rate at AFS output as seen on the ground is 4.99999999782 MHz (see Appendix T).

5.1.2 Signal polarization

Transmitted signals shall be right-handed circularly polarized (RHCP). For the angular range of $\pm 19°$ from nadir, ellipticity should be no worse than 0.7.

5.1.3 Carrier phase noise

The phase noise spectral density of the unmodulated carrier shall be such that a phase locked loop of 10 Hz one-sided noise bandwidth shall be able to track the carrier to accuracy no worse than 0.01 radians rms.

5.1.4 Spurious transmissions

In-band spurious transmissions, from the SV, which are defined as transmissions within the bands specified below, which are not expressly components of L1, L2 and L3 signals,

L1 (1592.9 – 1610) MHz

L2 (1237.8 – 1256.8) MHz

L3 (1190.35 – 1212.23) MHz

shall be at or bellow -40 dBc over the respective band specified above.

SVs are equipped with filters minimizing spurious transmissions in the bands specified bellow:

(1610.6 – 1613.8) MHz,

(1660.0 – 1670.0) MHz,

to the level set by ITU-RRA.769.

5.1.5 Multiple access interference

For CDMA signals multiple access interference is defined by the intercorrelation properties of ranging codes and depends on the number of elementary symbols N in the periods of these codes. Multiple access interference power in relation to the power of the desired signal shall be at the level of $10 \cdot \log_{10}(1/\mathrm{N})$. For example, if $\mathrm{N} = 1023$ the average power shall be -30 dB.

5.1.6 User-received signal level

The minimum received power measured at the output of a 3 dBi linearly polarized user receiving antenna (located near ground) when the SV is above 5° elevation angle, shall be no less than −158.5 dBW (see Appendix A).

5.2 General message data description

5.2.1 The GLONASS CDMA signal-in-space data are transmitted as a variable sequence of strings.

Strings are conventionally divided into service and data sections. The structure of a service section is the same for each signal and includes satellite ID number, type of the string, time of the beginning of the string, signal parameters and CRC bits to check the integrity of the string data.

The structure of a data section depends on its type. Each string type contains the complete individual block of data. In order to process this data one doesn't need to know the data contained in other string types. Orbit and clock data is the exception, because that data occupies three types of strings and is transmitted as continuous packet.

The structure of string data is provided in ICDs for the respective signals.

The message design may evolve together with future evolutions of GLONASS. This evolution may involve the inclusion of additional new string types which can either contain new data types or modify the existing string types.

5.2.2 The GLONASS navigation message contains immediate (ephemeris and clock corrections), non-immediate (almanac, the Earth’s rotation parameters, ionosphere model parameters, etc.), as well as service and other data used to broaden GLONASS information capabilities. Service and other data description is provided in separate documents.

5.2.3 The conventional repeating fragment of a string sequence is a pseudoframe. A pseudoframe is the set of strings of immediate and non-immediate data starting with the three strings of ephemeris and clock data. The remaining strings of a pseudoframe contain non-immediate data.

5.2.4 Immediate data (ephemeris) includes parameters of precise SV orbit models and coefficients of the precise polynomial models, describing transformation from signal time to GLONASS time. The maximum interval of updating immediate data (ephemeris) is 30 min. In individual cases this interval can be increased. Any update of immediate data is accompanied with the essential change of field t_b in the message.

The maximum ephemeris repetition interval is 45 s.

Each SV broadcasts its own immediate data only.

5.2.5 Non-immediate data includes the parameters of coarse SV orbit models, the coefficients of coarse polynomial models, describing an offset between signal time and GLONASS time, as well as the Earth’s rotation parameters, ionosphere model parameters and other non-immediate data. Various types of non-immediate data are updated at various intervals. The maximum update interval for all non-immediate data is 48 h.

The maximum almanac repetition interval is 5 min.

To increase the delivery rate of non-immediate data to a receiver, in the new CDMA signals this data is transmitted by various SVs with various time shifts. Different signals of the same SV can contain non-immediate data with various time shifts.

6 Monitoring GLONASS signal-in-space performance

Monitoring GLONASS signal-in-space performance means monitoring the quality of both signals and message data transmitted by SVs. In GLONASS signal-in-space performance monitoring results are delivered to users using the following means:

Each SV transmits in its message data signal health and data validity attributes: H^j and l^j.

H^j attribute is transmitted in each string of a message. The value of $H^j = 0$ means the signal can be used for position fixes in receivers.

l^j attribute is transmitted in each string of a message. The value of $l^j = 0$ means this particular string can be used for positioning by users.

Accuracy factors F_E^j and F_T^j are also transmitted. These factors relate to ephemeris (E) and timing (T) errors. They contain signal-in-space range error (SISRE, σ) to SV number j.

SR_A attribute transmitted in non-immediate data strings indicates which CDMA signals are transmitted by SV j_A.

When deciding upon applicability of each SV for positioning GLONASS users shall take into analysis each of the three attributes: H^j, l^j and SR_A. Apart from H^j, l^j and SR_A analysis, GLONASS capable receivers shall use RAIM.

Users are also recommended to monitor ephemeris and clock data for consistency at any new reference instant t_b versus previous reference instant t_b. For example, one shall calculate ephemeris and clock corrections at the midpoint of interval between those two instants, relying on both old and new ephemeris and clock data. If a significant discrepancy is observed, for example of more than 30 m (100 ns) as for both ephemeris and clock data, one shall be using the data of the previous reference instant t_b up to the next reference instant t_b.

Appendix A
User-received signal level

The guaranteed user-received minimum RF signal levels are specified in 5.1.6, taking into account the following assumptions:

- the minimum received power is measured at the output of a 3 dBi linearly polarized user receiving antenna;
- the SV is at or above 5° elevation angle;
- the SV angular orientation error is of 1° (towards signal level decrease).

The SV angular orientation error shall not be more than ±1°, after the SV is stabilized with respect to the reference coordinate system.

The maximum user-received RF signal level, taking into account the above assumptions, is expected not to be greater than -155.2 dBW. This value is estimated, assuming the user receiving antenna has the characteristics provided above, atmospheric loss is 0.5 dB, and the SV angular orientation error is of 1° (towards signal level increase).

Appendix B
UT1 calculation algorithm

To calculate T_{UT1} in UT1 time scale non-immediate message data contains the number of the day N_B within a four-year interval and B_0, B_1, B_2 parameters defined for the beginning of the day with number N_B. This information enables calculating $\tau_{UT1} = T_{UT1} - T_{UTC(SU)}$ offset expressed in terms of seconds of UT1 and UTC(SU) time scales, which corresponds to arbtraty instant t_{MT} expressed in terms of seconds of MT falling in the day with the calendar number N within the four-year interval N_4:

$$\tau_{UT1} = B_0 + B_1 \cdot \frac{\Delta t_{UT1}}{86400} + \frac{B_2}{2} \cdot \left(\frac{\Delta t_{UT1}}{86400} \right)^2 ,$$

where $\Delta t_{UT1} = \Delta N_{UT1} \cdot 86400 + t_{MT}$;

$$\Delta N_{UT1} = \begin{cases} N - N_B - \left\langle\left\langle \frac{N - N_B}{1461} \right\rangle\right\rangle \cdot 1461 \text{ if } N_4 \neq 27, \\ N - N_B - \left\langle\left\langle \frac{N - N_B}{1460} \right\rangle\right\rangle \cdot 1460 \text{ if } N_4 = 27. \end{cases}$$

$T_{UT1}(t_{MT})$ in UT1 time scale which corresponds to the instant t_{MT} expressed in terms of seconds in MT falling in the day with the calendar number N within the four-year interval is calculated as per the following formula:

$$T_{UT1}(t_{MT}) = \mathrm{mod}_{86400}(t_{MT} - 10800 \text{ c} + \tau_{UT1}) = \mathrm{mod}_{86400}(t_{UTC(SU)} + \tau_{UT1}).$$

UTC(SU) leap second corrections which lead to jump changes of $\tau_{UT1} = T_{UT1} - T_{UTC(SU)}$ offset are conducted in the beginning of the UTC(SU) day. Whereas the MT day is 10800 s (3 h) ahead of that of UTC(SU). That is why, if the UTC(SU) correction is conducted on the N_B day of MT, the value of the transmitted $\tau_{UT1} = T_{UT1} - T_{UTC(SU)}$ parameter in the beginning of day N_B of MT, shall differ by 1 second from the true value of $\tau_{UT1} = T_{UT1} - T_{UTC(SU)}$ 3 hours after the beginning of this day.

To avoid a mistake the user should use the combination of attribute KP (scheduled correction) and attribute A (anomaly of the next string), transmitted within service sections of all message strings.

Appendix C
Transformation from GLONASS time to GPS time

To calculate GPS time T_{GPS} in terms of seconds of GPS time, the immediate message data contains the fractional part of seconds $\tau_{GPS}(t_b) = T_{GPS}(t_b) - T_{GL}(t_b) + 10800 - \Delta T(t_b)$ and the integer part of seconds $\Delta T(t_b)$ in the offset of $T_{GPS}(t_b)$ in relation to $T_{GL}(t_b) - 10800$ (GLONASS time minus 3 h) at the instant t_b of MT. It results in the estimation of $\hat{T}_{GPS}(t_b)$ of GPS time for the instant t_b:

$$\hat{T}_{GPS}(t_b) = \tau_{GPS}(t_b) + T_{GL}(t_b) - 10800 + \Delta T(t_b).$$

The difference between t_b and any instant t of GLONASS time shall not be less than a half of interval between two neighboring values t_b, hence the transformation to the GPS time shall be carried out using the following formula

$$\hat{T}_{GPS}(t) = \mathrm{mod}_{604800}\left[\hat{T}_{GPS}(t_b) + \frac{\dot{\tau}_c(t_b)}{A_1}(t - t_b)\right],$$

where A_1 is the parameter transmitted by GPS SVs in their navigation data.

The integer number of seconds $\Delta T(t_b)$, included in the formula used to estimate $\hat{T}_{GPS}(t_b)$, shall be calculated in accordance with the following algorithm:

1) In accordance with the algorithm, provided in Appendix K, the number of the weekday Day_{MT} at 00:00 in MT shall be computed. $Day_{MT} = 0$ corresponds to Monday, $Day_{MT} = 1$ – to Tuesday, etc.

2) $T_{UTC(SU)}$ and $Day_{UTC(SU)}$ in UTC(SU) shall be computed, corresponding to T_{GL} and Day_{MT}:

$$x(t_b) = \mathrm{mod}_{86400}[T_{GL} + \tau_c(t_b) + \dot{\tau}_c(t_b) \cdot \Delta t_b - 10800],$$

where calculation of the sum of the first three terms in the square brackets, is described in Appendix D.

If $x(t_b) \geq 0$, then

$$T_{UTC(SU)}(t_b) = x(t_b), \quad Day_{UTC(SU)} = Day_{MT},$$

otherwise

$$T_{UTC(SU)}(t_b) = mod_{86400}(x(t_b)), \quad Day_{UTC(SU)} = Day_{MT} - 1.$$

3) Assuming $T_{UTC(USNO)}(t_b) = T_{UTC(SU)}(t_b)$ and $Day_{UTC(USNO)} = Day_{UTC(SU)}$, whose accuracy is quite acceptable for computing an integer number of seconds ΔT, $T^{week}_{UTC(USNO)}$ in UTC(USNO) time scale for a week interval shall be computed according to the following formula, provided a week, as in GPS, starts at Sunday night

$$T^{week}_{UTC(USNO)}(t_b) = mod_{604800}\left[T_{UTC(SU)}(t_b) + 86400 \cdot \left(Day_{UTC(SU)} + 1\right)\right].$$

4) The coarse estimation of $\hat{T}^{coarse}_{GPS}(t_b)$ shall be made based on the assumption that $T_{UTC(USNO)}(t_b) = T_{UTC(SU)}(t_b)$.

If $T_{GL}(t_b)$ value is at the left of the instant of correction carried out at the end of a quarter provided the leap second scheduled correction attribute KP=01 or KP=11, then

$$\hat{T}^{coarse}_{GPS}(t_b) = mod_{604800}\left[T^{week}_{UTC(USNO)}(t_b) + \Delta t_{LS} + A_0\right],$$

otherwise

$$\hat{T}^{coarse}_{GPS}(t_b) = mod_{604800}\left[T^{week}_{UTC(USNO)}(t_b) + \Delta t_{LSF} + A_0\right],$$

where Δt_{LS}, Δt_{LSF} and A_0 are the parameters transmitted by GPS SVs in their navigation data.

5) The integer number $\Delta T(t_b)$ of seconds in the offset of $T_{GPS}(t_b)$ relative to GLONASS time $T_{GL}(t_b)$ at the instant t_b of MT shall be computed using:

$$\Delta T(t_b) = \left\langle\left\langle \hat{T}_{GPS}^{coarse} - T_{GL}(t_b) + 10800 - \tau_{GPS}(t_b) \right\rangle\right\rangle .$$

Appendix D
Transformation from received signal time to GLONASS time and MT using immediate navigation data

The transformation from the timescale of the received signal $T^j_{\{signal\}}$ of SV j to GLONASS time T_{GL} at a arbitrary instant t_{MT} of MT is conducted according to the formula

$$T_{GL} = \mathrm{mod}_{86400}\left[T^j_{\{signal\}} + \tau^j(t_b) - \Delta t_b \cdot \gamma^j(t_b) - \Delta t_b^2 \cdot \beta^j(t_b)\right],$$

where T_{GL} is expressed in seconds;

$\Delta t_b = t_{MT} - t_b - \left\langle\left\langle \dfrac{t_{MT} - t_b}{86400} \right\rangle\right\rangle \cdot 86400$;

$\tau^j(t_b)$, $\gamma^j(t_b)$, $\beta^j(t_b)$ are the parameters transmitted within the immediate navigation data at the instant t_b in MT.

For practical applications value Δt_b (in seconds) can be computed according to the formula

$$\Delta t_b = \frac{T^j_{\{signal\}} + \tau^j(t_b) + \tau_c(t_b) - t_b - \left\langle\left\langle \dfrac{T^j_{\{signal\}} + \tau^j(t_b) + \tau_c(t_b) - t_b}{86400} \right\rangle\right\rangle \cdot 86400}{1 + \gamma^j(t_b) - \dot{\tau}_c(t_b)}.$$

The transformation from GLONASS time to MT shall be performed according to the formula

$$t_{MT} = \mathrm{mod}_{86400}\left[T_{GL} + \tau_c(t_b) + \Delta t_b \cdot \dot{\tau}_c(t_b)\right],$$

where $\Delta t_b = \dfrac{T_{GL} + \tau_c(t_b) - t_b - \left\langle\left\langle \dfrac{T_{GL} + \tau_c(t_b) - t_b}{86400} \right\rangle\right\rangle \cdot 86400}{1 - \dot{\tau}_c(t_b)}$.

Appendix E
Receiver operation during GLONASS time and MT leap second corrections

One shall take into account the specific aspects of receiver operation during scheduled corrections of GLONASS time and MT in the following situations:

- acquisition using message strings;
- pseudorange measurements;
- using ephemeris and clock data following the scheduled leap second correction of GLONASS time and MT.

The specific aspects of receiver operation during scheduled leap second corrections of GLONASS time and MT in the specified situations are listed below.

E.1 Receiver synchronization issues associated with scheduled leap second corrections of GLONASS time and MT

At any time each channel of a multichannel receiver may operate in either of the two modes: acquisition of a signal from a specific SV or tracking a signal from this SV. Acquisition in each channel either may be supported by other channels that have already acquired the signal (assisted acquisition) or may not (non-assisted acquisition).

A channel acquires a signal by estimating firstly a ranging code phase. As code periods in all GLONASS signals are chosen to be divisible by the value of the second correction, the correction itself shall not affect the code phase acquisition.

Following the code phase acquisition, symbol synchronization that is determination of navigation bit boundaries, detection and convolution decoding of the stream of these bits shall be performed. As a result the stream of data bits emerges. As data and code bits in all GLONASS signals constitute integer number within a second interval, scheduled leap second correction shall not affect the determination of these bits boundaries.

The subsequent decoding of the data bit stream requires determination of message string boundaries. The algorithm for determination of message string boundaries is based on extracting regularly repeating preambles having a fixed structure from the data bit stream. During the scheduled correction the regularity of preambles is impaired. That is why the nominal operation of the string boundaries determination algorithm can be maintained provided the algorithm is notified in advance of both the anomaly of the string following the extracted preamble and the anomaly type, that is either positive second (KP = 01) or negative second (KP = 11) change of the string length. This cannot be accomplished during non-assisted acquisition. That is why the string boundaries determination algorithm shall fail during non-assisted acquisition. As a result the anomalous string boundaries can be determined at later stages of the algorithm operation following the recovery of preambles regularity. This situation shall occur only in the receiver channel which is first to perform the non-assisted acquisition during the scheduled leap second correction of GLONASS time (normally once a year).

In assisted acquisition the information on the anomalous string time and its type can be extracted in advance from the strings decoded in the receiver channels that have already performed acquisition. The difference between the times of the strings in various channels shall not exceed 100 ms. That is why the error of the latest anomalous string time delivery into the acquiring channel shall not exceed 100 ms. This enables the preamble followed by the anomalous string to be extracted. Thus GLONASS time leap second correction induced algorithm failures can be avoided.

When tracking a signal from a specific SV each receiver channel is able to extract the preamble followed by the anomalous string. To this end KP and A service fields shall be analyzed in each string. If A = 0 in the current string, then the next string shall be nominal, that is of the length specified in the ICD of the respective signal. If A = 1, then the next string shall be anomalous. The anomaly type is defined by KP attribute. If KP = 11, the string is 1 second longer. If KP = 01, the string is 1 second shorter.

E.2 Pseudorange measurement generation issues associated with scheduled leap second corrections of GLONASS time and MT

Pseudorange measurements in a receiver shall be generated using signal parameters tied to the SVs’ signal timescales all being in the similar state: either pre-correction state or post-correction state. The accomplishment of this condition depends on whether RT is synchronized with GLONASS time at the instant of generating pseudorange measurements in the receiver channels.

At the initial phase of a receiver operation before the first fix pseudorange measurements are generated generally based on the arbitrary shifted RT that is not synchronized with GLONASS time. It shall be accounted for that pseudorange measurement algorithms in a receiver use preamble time stamps transmitted in message strings. Extracting preamble time stamps from message strings shall be preceded by the acquisition using these strings and extraction of the content of their service fields. Thus pseudorange measurements in a receiver shall be generated following the extraction of preamble time stamps and service data carrying the anomaly attribute of the next string (A=0 – nominal, A=1 – anomalous) as well as KP (scheduled correction) attribute.

Based on the analysis of A and KP attributes in a receiver the exact instant of the scheduled leap second correction of MT and GLONASS time can be defined within ± 10 ms accuracy in the timescale of the received signal (RST) in the channel first to acquire the anomalous string. This correction shall be about 75 ms ahead of the rear boundary of the anomalous string in the channel first to acquire the string. This assumed instant of the correction is depicted in Figure E.1 with the dotted vertical line for the both possible cases: of the longer (KP=01) and shorter (KP=11) strings.

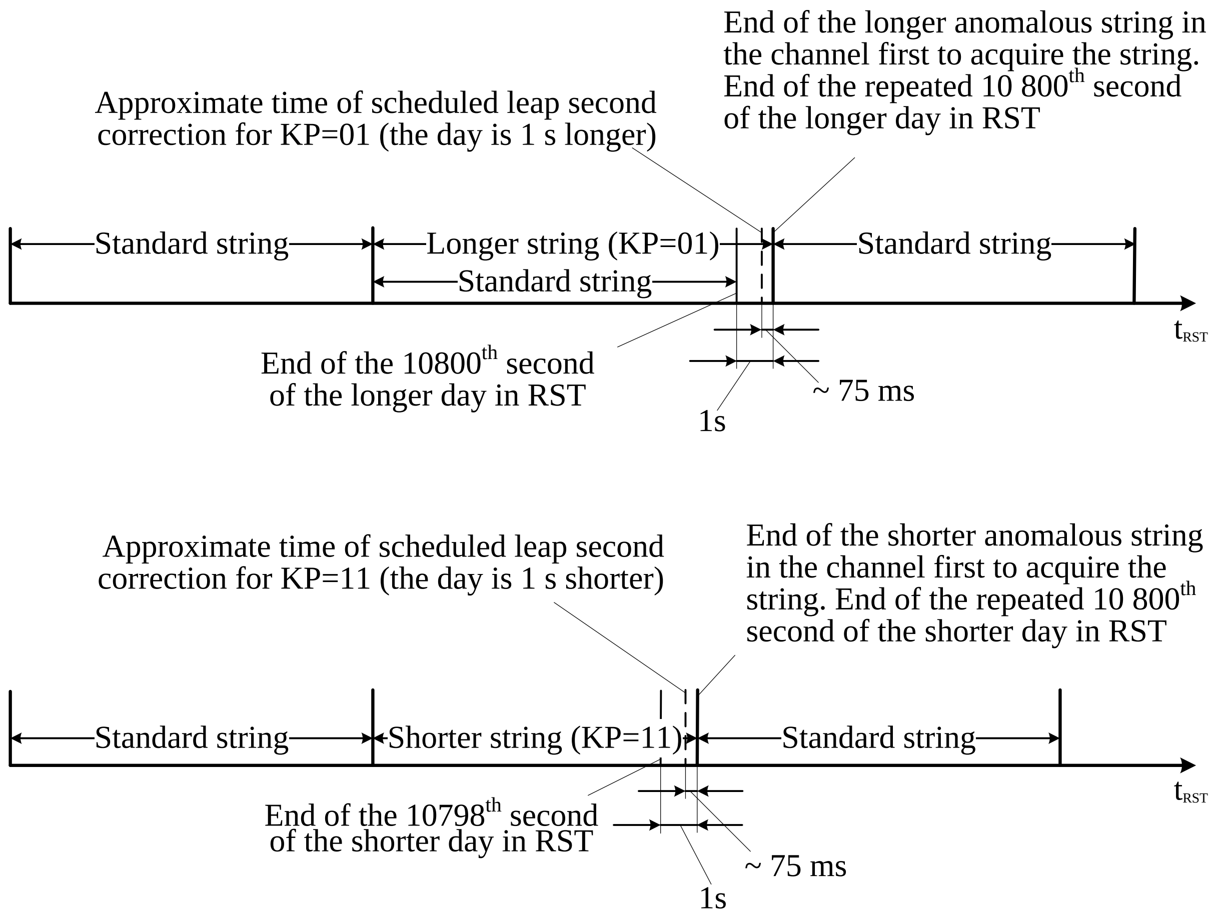


Figure E.1 – Approximate location of the MT and GLONASS time scheduled leap second correction in relation to the rear boundary of the anomalous string of the message in the channel first to acquire the string

Pseudorange measurement generation depends on the location of measurement instant in RST in the channel first to acquire the string provided RT is not synchronized with MT and GLONASS time (see Figure E.1):

- if the measurement instant is on the left of the approximate time of scheduled correction, the pseudorange measurements shall be tied to signal times in the state prior to the scheduled correction. To this end pseudorange measurements shall be generated assuming the standard string length in all receiver channels; that is the strings in receiver channels end after the 10800th second of the beginning of the day; and at the instants of these endings the string number counters in all channels increases by one.
- if the measurement instant is on the right of the approximate time of scheduled correction, the pseudorange measurements shall be tied to signal times in the state subsequent to the scheduled correction. To this end pseudorange measurements shall also be generated assuming the standard string length in all receiver channels. However the continuation of time count in the longer anomalous strings shall be delayed for 1 s with respect to the end of the previous string; and ranging code periods count shall be started 1 s later following the ending of that string. The continuation of time count in the shorter anomalous strings shall be started 1 s prior to the end of the previous string; and ranging code periods count shall also be started 1 s prior to the ending of the previous string.

Processing pseudorange measurements generated as per the recommendations above enables estimating the offset of RT relative to GLONASS and MT time as well as correcting RT. In doing so, RT shall be synchronized with GLONASS time and MT to an accuracy determined by the accuracy of positioning solution.

Provided RT is synchronized with GLONASS and MT time, RT correction for ± 1 s (during any scheduled leap second correction) can be accomplished without pseudorange measurements. The indication for this type of RT correction is appearance of the string carrying attribute of anomaly A=1, following the current string. If the scheduled correction attribute is KP=01, then the second with the number 10800 shall be followed by the second with the same number (10800). If the scheduled correction attribute is KP=11, then the second with the number 10798 shall be followed by the second with the number 10800.

Provided RT is synchronized with GLONASS time and MT pseudorange measurements shall be generated relative to the position of the measurement generation instant in the corrected RT:

- If the position of the measurement generation instant in the corrected RT is to the left of the instant of the RT correction (if KP=01 it is the termination instant of the repeated second with the number 10800 in RT, if KP=11 it is the termination instant of the second with the number 10800), pseudorange measurements shall be referenced to signal times in a state prior to the MT and GLONASS time scheduled leap second correction. To accomplish this, pseudorange measurements shall be generated assuming standard length of strings in all channels, that is, strings in receiver channels shall terminate after the 10800th second from the beginning of the day in timescales of the received signals (RST) and string number counters shall add a unit of time in the instants of these terminations;
- If the position of the measurement generation instant in the corrected RT is to the right of the instant of the RT correction, pseudorange measurements shall be referenced to signal times in a state following MT and GLONASS time scheduled leap second correction. Pseudorange measurements in this case can also be generated assuming standard length of strings in all channels. However, continuation of time count in the timescales of the received signals (RST) in long anomalous strings shall be delayed for 1 second relative to the termination of the previous string. The count of ranging code periods in these strings shall be initiated 1 second after the termination of that string. Continuation of time count in the timescales of received signals in short anomalous strings shall be initiated 1 second prior to the termination of the previous string. The count of ranging code periods in these strings shall also be initiated 1 second prior to the termination of the previous string.

E.3 Ephemeris data use associated with GLONASS time and MT scheduled leap second corrections

GLONASS time and MT leap second correction accompanied with RT correction results in one second difference of the clock readings for the corresponding timescales before and after the leap second correction. This means that when using after correction ephemeris and clock data that have been received before the correction, one shall use MT clock readings in which the correction has not been introduced. This situation occurs immediately after the scheduled leap second correction when the receiver hasn't been able to retrieve new ephemeris and clock data in time. MT clock readings after the correction can only be used as applied to new ephemeris and clock data retrieved only at a certain interval after the correction.

E.4 Non-immediate data use associated with GLONASS time and MT scheduled leap second corrections

The following parameters of non-immediate data received prior to the leap second correction shall be corrected after the leap second correction:

- parameters of orbit almanac (string Type 20);
- parameters of quadratic polynomial for determining an offset between τ_{UT1} and $UTC - TAI$ (string Type 25).

These string types can still be transmitted for some time containing non-immediate data referenced to MT in a state prior to the scheduled leap second correction. User receivers are able to differentiate between non-immediate data referenced to MT before and after its scheduled leap second correction either by modification KP attribute in service parts of the corresponding strings.

User receivers can implement the following way of using non-immediate data referenced to MT before the scheduled leap second correction:

- in almanac data (string Type 20), t_{λ_A} (time of the ascending node passage) shall be changed for ± 1 s;
- in the Earth's rotation parameters, τ_{UT1} and $UTC - TAI$ (string Type 25) shall be changed for ± 1 s.

Appendix F

Using number of four-year interval N_4 in MT

Number of a four-year interval N_4 transmitted in immediate data shall be determined in MT and shall always be associated with the instant t_b in MT, to which immediate data (ephemeris and clock data) relates. In other words, N_4 (hereinafter ephemeris N_4), transmitted within immediate data shall always represent the number of a four-year interval in MT comprising the MT day containing the instant t_b.

To calculate the current Gregorian date and time (see Appendix K), it is necessary to determine N_4^{cur} of a current four-year interval in MT. The first year of the first current four-year interval corresponds to 1996, that is, $N_4^{cur} = 1$ on interval 1996-1999 (MT). Current four-year intervals in user receivers shall be counted in RT. The instant of adding a unit to N_4^{cur} shall be determined in a user receiver as the termination of the 1461th day of RT (for all four-year intervals, excluding the 27th four-year interval that shall contain 1460 days).

At four-year interval boundaries ephemeris N_4 can differ from N_4^{cur} of the current four-year interval. That is why ephemeris N_4, associated with the instant t_b, to which immediate data (ephemeris and clock data) relates, shall be used in user receivers to calculate corrections to signal time and to determine SV position. At the same time N_4^{cur} (number of the current four-year interval) shall be used to calculate the current Gregorian date and time (see Appendix K).

Appendix G

Using number of day N_T in a four-year interval in MT

N_T day number in a four-year interval, transmitted within immediate data, shall be determined in MT and shall always be associated with the instant t_b in MT, to which immediate data (ephemeris and clock data) relates. In other words, N_T (hereinafter ephemeris N_T), transmitted within immediate data, shall always be the number of a day, containing the instant t_b.

To calculate a current date and time in the Gregorian Calendar (see Appendix K) it is necessary to determine N_T^{cur} of a current day in a four-year interval of MT. The first day in each four-year interval corresponds to the first day in MT of the next leap year, excluding 2100, which is not a leap year according to the Gregorian Calendar. RT shall be used to count current days in user receivers. The instant of adding a unit to N_T^{cur} shall be determined in a user receiver as the termination of the 86399th second. A number of a current second in a user receiver shall be calculated in RT taking into account scheduled leap second corrections of UTC(SU), MT and GLONASS time. Since MT is 3 h (10800 s) ahead of UTC(SU), RT correction shall involve the following steps: when UTC(SU) is corrected for +1 s, the RT second following the 10800th second shall have the same number (10800). All the subsequent seconds of RT shall add one unit relative to the previous second. When UTC(SU) is corrected for -1 s, the RT second following the 10798th second shall have the number 10800. All the subsequent seconds of RT shall add one unit relative to the previous second. GLONASS navigation message shall carry specific attributes indicating the time and the correction sign (positivity/negativity) of UTC(SU), MT, GLONASS time, and timescales of all navigation signals transmitted by GLONASS SVs (also see Appendix E).

At day boundaries an ephemeris N_T can differ from N_T^{cur} of a current day. That is why to calculate corrections to signal time and determine a SV's position in a user receiver, one shall use the value of ephemeris N_T, associated with t_b, to which immediate data (ephemeris and clock data) relates. To calculate a current date and time in the Gregorian Calendar (see Appendix K), it is necessary to use N_T^{cur} value – the number of the current day.

Appendix H
Transformation from UTC(SU) to TAI

GLONASS navigation message contains an offset between UTC(SU) and TAI at the beginning of day N_B in MT:

$$UTC(SU) - TAI = T_{UTC(SU)} - T_{TAI}\ .$$

UTC(SU) leap second corrections, which lead to uneven changes of $UTC(SU) - TAI$ offset, normally occur at the beginning of the UTC(SU) day. In MT a day inception is 10800 s (3 h) ahead of that in UTC(SU). That is why if the leap second correction is conducted on the N_B day of MT, the value of the transmitted $UTC(SU) - TAI$ parameter in the beginning of day N_B of MT, shall differ by 1 second from the true value of $UTC(SU) - TAI$ 3 hours after the beginning of this day.

To mitigate the error the user must account for the combination of KP and A scheduled leap second correction attributes of the anomaly of the following string, transmitted in every string of a message.

Appendix J
Algorithms for determination of SV center of mass position and velocity vector components using ephemeris data

This appendix includes three example algorithms for calculating position and velocity vector components for the SV's center of mass for a given instant t_i in MT using ephemeris data:

- precise algorithm (precise calculation for a 30-minute interval of an orbit);
- simplified algorithm (simplified calculation for a 30-minute interval of an orbit);
- long-term algorithm, employing parameters of the long-term dynamic model (precise calculation for a 4-hour interval of an orbit).

Table J.1 provides the SV's center of mass position prediction errors obtained through the numerical integration using the Runge-Kutta forth-order method with a 1 minute increment for the identical initial data for the precise, simplified, and long-term algorithms.

Table J.1 – Position prediction errors at various intervals (m)

Algorithm example	Integration interval			
	5 min	10 min	15 min	4 h
Precise	0.13	0.18	0.25	>30
Simplified	0.03 – 0.42	0.04 – 0.56	0.05 – 0.77	>100
Long-term	0.03 – 0.42	0.04 – 0.56	0.05 – 0.77	0.25 – 1

The error range corresponding to the simplified and long-term algorithms in Table J.1 is SV modification related. Depending on GLONASS SV modification, the string types 10, 11, 12 can contain either reference or coordinated ephemerides (velocities and accelerations). In ground control segment a SV's motion trajectories are calculated using complex models involving a great number of disturbances. The reference ephemerides are the certain coordinates of position within the trajectory at the instant t_b. So the reference ephemerides provide the maximum practical accuracy of a SV's trajectory retrieval at the instant t_b. However user receivers employ much simpler models to predict a SV position leading to increasing systematic prediction errors when deviating of t_b. The coordinated ephemerides at the same instant t_b are calculated based on the reference ephemerides to minimize average systematic prediction errors on the prediction interval using simpler SV dynamic models inherent to user receivers.

In Table J.1 the lower boundary of the values corresponds to the coordinated ephemerides broadcast in string types 10, 11, and 13, and the upper – to the reference ephemerides. In future only coordinated ephemerides will be broadcast by all blocks of SVs.

When broadcasting coordinated ephemerides, the prediction error using precise algorithm changes insignificantly.

On a short interval, for example at t_b ±15 min it is recommended to use the simplified algorithm, while on a long interval, for example from +15 min to +4 h, it is recommended to use the long-term algorithm, employing parameters of the long-term dynamic model.

IGS (International GNSS Service, http://igs-ip.net/home) precise orbits were used to calculate the errors. According to the IGS website, the maximum orbit accuracy does not exceed ±2.5 cm.

J.1 Precise algorithm for determination of position and velocity vector components for the SV's center of mass for the given instant of MT

J.1.1 Calculation algorithm

A user shall recalculate the ephemerides at the instant t_b in MT into those at the given instant t_i in MT ($|\tau_i| = |t_i - t_b|$ < 15 min) using numerical integration of differential equations for the motion of a SV's center of mass. The terms on the right side of the equations shall include accelerations determined by the geocentric gravitational constant GM (mass of the Earth's atmosphere included), the second degree zonal coefficient of normal potential J_2^0, which describes the polar flattening, and also by the accelerations induced by gravitational perturbations from the Moon and the Sun. These motion equations shall be defined as the following system

$$
\begin{aligned}
&\frac{dx_0}{dt} = Vx_0, \\
&\frac{dy_0}{dt} = Vy_0, \\
&\frac{dz_0}{dt} = Vz_0, \\
&\frac{dVx_0}{dt} = -\hat{GM}\cdot\hat{x}_0 - \frac{3}{2}J_2^0\,\hat{GM}\cdot\hat{x}_0\rho^2(1-5\hat{z}_0{}^2) + j_{x0s} + j_{x0m}, \\
&\frac{dVy_0}{dt} = -\hat{GM}\cdot\hat{y}_0 - \frac{3}{2}J_2^0\,\hat{GM}\cdot\hat{y}_0\rho^2(1-5\hat{z}_0{}^2) + j_{y0s} + j_{y0m}, \\
&\frac{dVz_0}{dt} = -\hat{GM}\cdot\hat{z}_0 - \frac{3}{2}J_2^0\,\hat{GM}\cdot\hat{z}_0\rho^2(3-5\hat{z}_0{}^2) + j_{z0s} + j_{z0m},
\end{aligned}
\tag{J.1}
$$

where $\hat{GM} = \frac{GM}{r^2}$; $\hat{x}_0 = \frac{x_0}{r_0}$; $\hat{y}_0 = \frac{y_0}{r_0}$; $\hat{z}_0 = \frac{z_0}{r_0}$; $\rho = \frac{a_e}{r_0}$;

$r_0 = \sqrt{x_0{}^2 + y_0{}^2 + z_0{}^2}$;

$j_{x0s}, j_{y0s}, j_{z0s}$ – accelerations induced by gravitational perturbations of the Sun;

$j_{x0m}, j_{y0m}, j_{z0m}$ – accelerations induced by gravitational perturbations of the Moon;

a_e = 6378136 m – semi-major axis of the PZ-90 Earth's ellipsoid;

GM = (398600441.8 ± 0.8) $\cdot 10^6$ m^3/s^2 – geocentric gravitational constant (mass of the Earth's included);

$J_2^0 = 1082625.75 \cdot 10^{-9}$ – second degree coefficient of normal potential, which describes the Earth's polar flattening.

The reference assumptions needed to determine ephemerides in accordance with the precise algorithms include: N_4 – the number of an ephemeris four-year interval; N_T – the number of an ephemeris day in an ephemeris four-year interval; t_b – the instant in MT from the GLONASS immediate data; the position and velocity vector components of a SV center of mass for the instant t_b from the GLONASS immediate data; t_i – the instant in MT for which the position and velocity vector components of a SV center of mass shall be determined.

The motion differential equations (J.1) shall be integrated in the orthogonal inertial geocentric coordinate system $OX_0Y_0Z_0$. Its origin is aligned with the PZ-90 geodetic system origin, OX_0 axis is directed to the vernal equinox point, OZ_0 axis is directed to the North Pole and coincides with OZ axis of the PZ-90 geodetic system; and OY_0 axis completes a right-handed system.

Integration is executed using a numerical method, for example the Runge-Kutta forth-order method.

Accelerations induced by gravitational perturbations of the Sun and the Moon in the orthogonal inertial geocentric coordinate system $OX_0Y_0Z_0$ shall be calculated using the formulae:

$$
\begin{aligned}
j_{x0k} &= \hat{G}_k\left[\frac{\xi_k - \hat{x}_k}{\Delta_k} - \xi_k\right],\\
j_{y0k} &= \hat{G}_k\left[\frac{\eta_k - \hat{y}_k}{\Delta_k} - \eta_k\right],\\
j_{z0k} &= \hat{G}_k\left[\frac{\Im_k - \hat{z}_k}{\Delta_k} - \Im_k\right],
\end{aligned}
\tag{J.2}
$$

where $\hat{x}_k = \frac{x_0}{r_k}$; $\hat{y}_k = \frac{y_0}{r_k}$; $\hat{z}_k = \frac{z_0}{r_k}$; $\hat{G}_k = \frac{G_k}{r_k^2}$;

$\Delta_k = \left[(\xi_k - \hat{x}_k)^2 + (\eta_k - \hat{y}_k)^2 + (\Im_k - \hat{z}_k)^2\right]^{3/2}$;

k – index of perturbing body, k = “m” for the Moon и k = “s” for the Sun;

G_m – the Moon’s gravitational field constant which is $4902.799 \cdot 10^9\ m^3/s^2$;

G_s – the Sun’s gravitational field constant which is $13271244.0 \cdot 10^{13}\ m^3/s^2$.

The direction cosines ξ_k, η_k, $\Im_k$ and distance r_k of perturbing body in the $OX_0Y_0Z_0$ coordinates are calculated one time (at the instant t_b) for the whole interval of reproduction (± 15 min).

The direction cosines ξ_s, η_s, $\Im_s$ and distance r_s to the Sun included in (J.2) are calculated using the algorithm provided in the Appendix S; the direction cosines and the distance to the Moon ξ_m, η_m, $\Im_m$, r_m are calculated using the following formulae[1)]:

---

[1] G.Duboshin, Celestial Mechanics: Fundamental Tasks and Methods; Moscow: Nauka, 1975; V.Abalakin, Fundamentals of Ephemeris Astronomy; Moscow: Nauka, 1979

$$\begin{aligned}
\xi_m &= (\sin\vartheta_m \cos\Gamma' + \cos\vartheta_m \sin\Gamma')\,\xi_{11} + (\cos\vartheta_m \cos\Gamma' - \sin\vartheta_m \sin\Gamma')\,\xi_{12}\,,\\
\eta_m &= (\sin\vartheta_m \cos\Gamma' + \cos\vartheta_m \sin\Gamma')\,\eta_{11} + (\cos\vartheta_m \cos\Gamma' - \sin\vartheta_m \sin\Gamma')\,\eta_{12}\,,\\
\Im_m &= (\sin\vartheta_m \cos\Gamma' + \cos\vartheta_m \sin\Gamma')\,\Im_{11} + (\cos\vartheta_m \cos\Gamma' - \sin\vartheta_m \sin\Gamma')\Im_{12}\,,\\
r_m &= a_m \cdot (1 - e_m \cos E_m)\,,
\end{aligned} \tag{J.3}$$

where $\sin\vartheta_m = \dfrac{\sqrt{1-e_m^2}\,\sin E_m}{1 - e_m \cos E_m}$;

$\cos\vartheta_m = \dfrac{\cos E_m - e_m}{1 - e_m \cos E_m}$;

$E_m = q_m + e_m \sin E_m$ – Kepler's equation for eccentric anomaly shall be solved by iteration; initial approximation $E_m = q_m$, until $|E_m - E_{m\,(\text{prev. iteration})}|$ is less than 10^{-8};

$\xi_{11} = \sin\Omega_m \cdot \cos\Omega_m (1 - \cos i_m)$;

$\xi_{12} = 1 - \sin^2\Omega_m (1 - \cos i_m)$;

$\eta_{11} = \xi^* \cdot \cos\varepsilon - \Im^* \cdot \sin\varepsilon$;

$\eta_{12} = \xi_{11} \cdot \cos\varepsilon + \eta^* \cdot \sin\varepsilon$;

$\Im_{11} = \xi^* \cdot \sin\varepsilon + \Im^* \cdot \cos\varepsilon$;

$\Im_{12} = \xi_{11} \cdot \sin\varepsilon - \eta^* \cdot \cos\varepsilon$;

$\xi^* = 1 - \cos^2\Omega_m (1 - \cos i_m)$;

$\eta^* = \sin\Omega_m \cdot \sin i_m$;

$\Im^* = \cos\Omega_m \cdot \sin i_m$;

$a_m = 3.84385243 \cdot 10^5$ km – semi-major axis of the Moon's orbit;

$e_m = 0.054900489$ – the Moon's orbit eccentricity;

$i_m = 0.0898041080$ rad – mean inclination of the Moon's orbit to the ecliptic.

The Moon and the Sun nutation parameters at the instant t_b shall be calculated according to the following formulae:

- mean anomaly of the Moon, rad:

$$q_m = 2.3555557435 + 8328.6914257190 \cdot T + 0.0001545547 \cdot T^2\,;$$

- mean longitude of the ascending node of the Moon, rad:

$$\Omega_m = 2.1824391966 - 33.7570459536 \cdot T + 0.0000362262 \cdot T^2\,;$$

- mean longitude of the Moon's orbit perigee, rad:

$$\Gamma' = 1.4547885346 + 71.0176852437 \cdot T - 0.0001801481 \cdot T^2;$$

- mean inclination of Earth equator to ecliptic, rad:

$$\varepsilon = 0.4090926006 - 0.0002270711 \cdot T;$$

- time form epoch 2000, 1st January, 12:00 (UTC(SU)) to the instant t_b of GLONASS ephemeris broadcast (in MT) (in Julian centuries of 36525 ephemeris day each):

$$T = (JD0 + (t_b - 10800)/86400 - 2451545.0)/36525,$$

where JD0 – the current Julian date at 00:00 in MT. Its calculation algorithm using GLONASS navigation message data is provided in the Appendix K;

t_b – the instant in MT, to which GLONASS ephemerides are tied, in seconds;

10800 – difference between MT and (UTC(SU)), in seconds;

2451545.0 – Julian date for 12 hours, 1st January, 2000 (UTC(SU)).

The initial conditions for integration of the system (J.1) are the position for the SV's center of mass $x_0(t_b)$, $y_0(t_b)$, $z_0(t_b)$ and its velocity vector components $\dot{x}_0(t_b)$, $\dot{y}_0(t_b)$, $\dot{z}_0(t_b)$ in the inertial geocentric coordinate system $OX_0Y_0Z_0$ for the instant t_b in MT. These initial conditions are derived by reproduction of the coordinates $x(t_b)$, $y(t_b)$, $z(t_b)$ of the SV's center of mass and the velocity vector components $\dot{x}(t_b)$, $\dot{y}(t_b)$, $\dot{z}(t_b)$ realized in the ECEF PZ-90 geodetic coordinate system. The reproduction shall be carried out according to the formulae:

$$
\begin{aligned}
&x_0(t_b) = x(t_b)\cdot\cos\bigl(S(t_b)\bigr) - y(t_b)\cdot\sin\bigl(S(t_b)\bigr),\\
&y_0(t_b) = x(t_b)\cdot\sin\bigl(S(t_b)\bigr) + y(t_b)\cdot\cos\bigl(S(t_b)\bigr),\\
&z_0(t_b) = z(t_b),\\
&\dot{x}_0(t_b) = \dot{x}(t_b)\cdot\cos\bigl(S(t_b)\bigr) - \dot{y}(t_b)\cdot\sin\bigl(S(t_b)\bigr) - \omega_E\cdot y_0(t_b),\\
&\dot{y}_0(t_b) = \dot{x}(t_b)\cdot\sin\bigl(S(t_b)\bigr) + \dot{y}(t_b)\cdot\cos\bigl(S(t_b)\bigr) + \omega_E\cdot x_0(t_b),\\
&\dot{z}_0(t_b) = \dot{z}(t_b),\\
&S(t_b) = GST + \omega_E\cdot(t_b - 10800),
\end{aligned}
\tag{J.4}
$$

where $\omega_E = 7.2921151467\cdot10^{-5}$ rad/s – is the mean angular velocity of the Earth relative to vernal equinox;

GST is the Greenwich Sidereal Time, in radians.

Upon integration the coordinates of the SV center of mass $x_0(t_i)$, $y_0(t_i)$, $z_0(t_i)$ and its velocity vector components $\dot{x}_0(t_i)$, $\dot{y}_0(t_i)$, $\dot{z}_0(t_i)$ obtained in the inertial coordinate system $OX_0Y_0Z_0$, can be recalculated into those realized in the ECEF PZ-90 (Oxyz) using the formulae

$$
\begin{aligned}
&x(t_i) = x_0(t_i)\cdot\cos\bigl(S(t_i)\bigr) + y_0(t_i)\cdot\sin\bigl(S(t_i)\bigr),\\
&y(t_i) = -x_0(t_i)\cdot\sin\bigl(S(t_i)\bigr) + y_0(t_i)\cdot\cos\bigl(S(t_i)\bigr),\\
&z(t_i) = z_0(t_i),\\
&\dot{x}(t_i) = \dot{x}_0(t_i)\cdot\cos\bigl(S(t_i)\bigr) + \dot{y}_0(t_i)\cdot\sin\bigl(S(t_i)\bigr) + \omega_E\cdot y(t_i),\\
&\dot{y}(t_i) = -\dot{x}_0(t_i)\cdot\sin\bigl(S(t_i)\bigr) + \dot{y}_0(t_i)\cdot\cos\bigl(S(t_i)\bigr) - \omega_E\cdot x(t_i),\\
&\dot{z}(t_i) = \dot{z}_0(t_i),\\
&S(t_i) = GST + \omega_E\cdot(t_i - 10800).
\end{aligned}
\tag{J.5}
$$

Coordinates and velocity vector components (J.5) for the center of mass of SV j for the given instant t_i in MT are determined for the position of the ECEF PZ-90 coordinate system it takes at the same instant t_i. However coordinates and velocity vector components of a user shall be determined for the position of the ECEF PZ-90 coordinate system it shall take after signal propagation. These features shall be taken into account while processing in UE.

Notes

1 Accelerations j_{x0m}, j_{y0m}, j_{z0m}, j_{x0s}, j_{y0s}, j_{z0s} in equation (J.1) can either be assumed as constants and computed once for the instant t_b using formulae (J.2) or excluded from (J.1) provided integration results are corrected at a later stage:

$$\Delta x = (j_{x0m} + j_{x0s}) \cdot \tau^2 / 2,\ \Delta y = (j_{y0m} + j_{y0s}) \cdot \tau^2 / 2,\ \Delta z = (j_{z0m} + j_{z0s}) \cdot \tau^2 / 2,$$

$$\Delta \dot{x} = (j_{x0m} + j_{x0s}) \cdot \tau,\ \Delta \dot{y} = (j_{y0m} + j_{y0s}) \cdot \tau,\ \Delta \dot{z} = (j_{z0m} + j_{z0s}) \cdot \tau,$$

$$\tau = t_i - t_b .$$

Ephemeris prediction error shall not grow by more than 10% in this case.

2 Direction cosines ξ_{0k}, η_{0k}, $\Im_{0k}$ can be computed using formulae (J.3).

3 The origin of the Greenwich (right-handed) coordinate system is the center of mass of the Earth, z axis is directed along the Earth's rotational axis to the Mean North Pole of the epoch 1900-1905, and x axis – to the intersection point of the Greenwich meridian and the equatorial plane.

4 To compute the coordinates of position for the SV center of mass for the given instant t_i in MT the user can utilize projections of accelerations $\ddot{x}(t_b)$, $\ddot{y}(t_b)$, $\ddot{z}(t_b)$ on the axes of the right-handed coordinate system, transmitted within navigation message data. Before integrating differential equation system (J.1) these accelerations should be transformed into the orthogonal inertial geocentric coordinate system $OX_0Y_0Z_0$ using the formulae:

$$(J_{x0m} + J_{x0s}) = \ddot{x}(t_b) \cdot \cos(S) - \ddot{y}(t_b) \cdot \sin(S),$$

$$(J_{y0m} + J_{y0s}) = \ddot{x}(t_b) \cdot \sin(S) + \ddot{y}(t_b) \cdot \cos(S),$$

$$(J_{z0m} + J_{z0s}) = \ddot{z}(t_b) .$$

Ephemeris prediction error in this case shall not grow more than by 25 % if compared to computations involving accelerations induced by lunar and solar perturbations (J.2).

5 Instead of GST, formulae (J.4) and (J.5) can utilize GMST (Greenwich Mean Sidereal Time), computed using the method provided in Appendix K.

J.1.2 Example for calculating coordinates of position for SV's center of mass for the given instant t_i (MT) using precise algorithm

The ephemeris parameters for the GLONASS SV set in the PZ-90 coordinates for t_b = 11700 of 07.09.2012 (N_T = 251, N_4 = 5) of MT are provided in the Table below:

$x(t_b)$ = 7003.008789 km	$y(t_b)$= – 12206.626953 km	$z(t_b)$= 21280.765625 km
$\dot{x}(t_b)$ = 0.7835417 km/s	$\dot{y}(t_b)$ = 2.8042530 km/s	$\dot{z}(t_b)$ = 1.3525150 km/s

The task is to compute the SV position in PZ-90 for t_i = 12300 of 07.09.2012 (N_T = 251, N_4 = 5) of MT.

The result of recalculating the SV position in PZ-90 from t_b to t_i (MT) is the following:

$x(t_i)$ = 7523.174819 km	$y(t_i)$= – 10506.961965 km	$z(t_i)$= 21999.239413 km
$\dot{x}(t_i)$ = 0.950126007 km/s	$\dot{y}(t_i)$ = 2.855687825 km/s	$\dot{z}(t_i)$= 1.040679862 km/s

The values of accelerations induced by the Sun's and the Moon's perturbations computed at the instant t_b in the PZ-90 were used on the interval of recalculation:

$J_{x0m}(t_i) = -5.035590\cdot10^{-10}$ km/s^2	$J_{y0m}(t_i) = 7.379024\cdot10^{-10}$ km/s^2	$J_{z0m}(t_i) = -1.648033\cdot10^{-9}$ km/s^2
$J_{x0sc}(t_i) = 4.428827\cdot10^{-10}$ km/s^2	$J_{y0s}(t_i) = 3.541631\cdot10^{-10}$ km/s^2	$J_{z0s}(t_i) = -8.911601\cdot10^{-10}$ km/s^2

The computation of the current Julian Date JD0 and the Greenwich Mean Sidereal Time (GMST) using N_T^{cur} of the current day and N_4^{cur} of the current four-year interval included in the algorithm is provided in Appendix K.

For N_T^{cur} = 251, N_4^{cur} = 5: JD0 = 2456177.5, GMST = 29191.442830.

J.2 Simplified algorithm for determination of position and velocity vector components for the SV’s center of mass for the given instant in MT

J.2.1 Calculation algorithm

A user shall recalculate the PZ-90 coordinates of position and velocity vector components for the SV’s center of mass at the instant t_b in MT into those at the given instant t_i in MT using numerical integration of the following differential equations:

$$
\begin{aligned}
\frac{dx}{dt} &= V_x, \\
\frac{dy}{dt} &= V_y, \\
\frac{dz}{dt} &= V_z, \\
\frac{dV_x}{dt} &= -\frac{GM}{r^3}x - \frac{3}{2}J_2^0\frac{GM\cdot a_e^2}{r^5}x(1-5\frac{z^2}{r^2}) + \omega_E^2 x + 2\omega_E V_y + \ddot{x}, \\
\frac{dV_y}{dt} &= -\frac{GM}{r^3}y - \frac{3}{2}J_2^0\frac{GM\cdot a_e^2}{r^5}y(1-5\frac{z^2}{r^2}) + \omega_E^2 y - 2\omega_E V_x + \ddot{y}, \\
\frac{dV_z}{dt} &= -\frac{GM}{r^3}z - \frac{3}{2}J_2^0\frac{GM\cdot a_e^2}{r^5}z(3-5\frac{z^2}{r^2}) + \ddot{z},
\end{aligned}
$$

where $r = \sqrt{x^2 + y^2 + z^2}$;

a_e = 6378136 m – semi-major (equatorial) axis of the PZ-90 Earth’s ellipsoid;

GM = $(398600441.8\pm0.8)\cdot10^6$ m^3/s^2 – gravitational constant (mass of the Earth’s atmosphere included);

$J_2^0 = 1082625.75\cdot10^{-9}$ – second degree zonal coefficient of normal potential;

ω_E = $7.2921151467\cdot10^{-5}$ rad/s – mean angular velocity of the Earth relative to the vernal equinox.

The initial conditions for simplified algorithm integration are the coordinates $x = x(t_b)$, $y = y(t_b)$, $z = z(t_b)$, velocity vector components $V_x = \dot{x}(t_b)$, $V_y = \dot{y}(t_b)$, $V_z = \dot{z}(t_b)$ and perturbing accelerations $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ for the SV’s center of mass at the instant t_b in MT transmitted within navigation message data. Perturbing accelerations $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ are considered constants on an interval of t_b ±15 min.

Numeric integration can be executed using for example the Runge-Kutta forth-order method.

To mitigate the effect of t_i parameter jumping during date boundaries, the integration shall be initiated at the instant t_b and completed at the instant t_i', that can be determined according to the formula

$$t_i' = t_i - \left\langle \frac{t_i - t_b}{86400} \right\rangle \cdot 86400,$$

where operation $\langle x \rangle$ involves computing an integer nearest to x;

t_i, t_b, and t_i' are expressed in seconds.

J.2.2 Example for calculating coordinates of position for SV's center of mass at the given instant t_i (MT) using simplified algorithm

The coordinates, velocity vector components and perturbing accelerations for the SV's center of mass at the instant t_b = 11700 of 07.09.2012 (MT) in the PZ-90 coordinate system are set and provided in the Table below:

$x(t_b)$ = 7003.008789 km	$y(t_b)$= – 12206.626953 km	$z(t_b)$= 21280.765625 km
$\dot{x}(t_b)$ = 0.7835417 km/s	$\dot{y}(t_b)$ = 2.8042530 km/s	$\dot{z}(t_b)$ = 1.3525150 km/s
$\ddot{x}(t_b)$ = 0 km/s^2	$\ddot{y}(t_b)$ = $1.7 \cdot 10^{-9}$ km/s^2	$\ddot{z}(t_b)$ = $-5.41 \cdot 10^{-9}$ km/s^2

The task is to compute coordinates and velocity vector components for the SV's center of mass at the instant t_i = 12300 of 07.09.2012 (MT) in the PZ-90 coordinate system using the simplified algorithm.

Result:

$x(t_i)$ = 7523.174853 km	$y(t_i)$ = – 10506.962176 km	$z(t_i)$ = 21999.239866 km
$\dot{x}(t_i)$ = 0.95012609 km/s	$\dot{y}(t_i)$ = 2.85568710 km/s	$\dot{z}(t_i)$ = 1.04068137 km/s

J.3 Long-term algorithm for determination of position and velocity vector components for the SV's center of mass for the given instant in MT employing long-term dynamic model parameters

J.3.1 Calculation algorithm

A user shall recalculate the PZ-90 coordinates of position and velocity vector components for the SV's center of mass at the instant t_b of MT into those at the given instant t_i of the same timescale taking into account the long-term dynamic model parameters using numerical integration of the following differential equations (further referred to as "long-term algorithm"):

$$
\begin{aligned}
&\frac{dx}{dt} = V_x, \\
&\frac{dy}{dt} = V_y, \\
&\frac{dz}{dt} = V_z, \\
&\frac{dV_x}{dt} = -\frac{GM}{r^3}x - \frac{3}{2}J_2^0\frac{GM\cdot a_e^2}{r^5}x(1-5\frac{z^2}{r^2}) + \omega_E^2 x + 2\omega_E V_y + \ddot{x} + a_x, \\
&\frac{dV_y}{dt} = -\frac{GM}{r^3}y - \frac{3}{2}J_2^0\frac{GM\cdot a_e^2}{r^5}y(1-5\frac{z^2}{r^2}) + \omega_E^2 y - 2\omega_E V_x + \ddot{y} + a_y, \\
&\frac{dV_z}{dt} = -\frac{GM}{r^3}z - \frac{3}{2}J_2^0\frac{GM\cdot a_e^2}{r^5}z(3-5\frac{z^2}{r^2}) + \ddot{z} + a_z,
\end{aligned}
$$

where $r = \sqrt{x^2 + y^2 + z^2}$;

$$
\begin{aligned}
a_x &= \Delta a_{x0} + a_{x1}(t-t_b) + a_{x2}(t-t_b)^2 + a_{x3}(t-t_b)^3 + a_{x4}(t-t_b)^4; \\
a_y &= \Delta a_{y0} + a_{y1}(t-t_b) + a_{y2}(t-t_b)^2 + a_{y3}(t-t_b)^3 + a_{y4}(t-t_b)^4; \\
a_z &= \Delta a_{z0} + a_{z1}(t-t_b) + a_{z2}(t-t_b)^2 + a_{z3}(t-t_b)^3 + a_{z4}(t-t_b)^4;
\end{aligned}
$$

a_e = 6378136 m – semi-major (equatorial) axis of the PZ-90 Earth's ellipsoid;

GM = (398600441.8±0.8)·106 m^3/s^2 – gravitational constant (mass of the Earth's atmosphere included);

$J_2^0 = 1082625.75\cdot 10^{-9}$ – second degree zonal coefficient of normal potential;

ω_E=7.2921151467·10^{-5} rad/s – mean angular velocity of the Earth relative to the vernal equinox.

Initial data needed to compute the coordinates and velocity vector components for the SV's center of mass based on the long-term algorithm are coordinates $x = x(t_b)$, $y = y(t_b)$, $z = z(t_b)$, velocity vector components $V_x = \dot{x}(t_b)$ $V_y = \dot{y}(t_b)$, $V_z = \dot{z}(t_b)$ for the SV's center of mass at the instant t_b, and perturbing accelerations $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, as well as long-term dynamic model parameters. These parameters are coefficients of four-degree polynomials which allow to compute additional accelerations $a_x(t,t_b)$, $a_y(t,t_b)$, $a_z(t,t_b)$. Coordinates, velocity vector components at the instant t_b, and perturbing accelerations are transmitted within string of Types 10, 11, 12. Long-term dynamic model parameters are transmitted within string of Types 31 and 32. Perturbing accelerations $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ and long-term dynamic model parameters are constants on an interval of t_b +4 hours.

The Runge-Kutta forth-order method can be used for numeric integration.

To mitigate the effect of t_i parameter jumping during date boundaries, the integration shall be initiated at the instant t_b and completed at the instant t_i':

$$t_i' = t_i - \left\langle \frac{t_i - t_b}{86400} \right\rangle \cdot 86400,$$

where operation $\langle x \rangle$ means calculating an integer nearest to x;

t_i, t_b, and t_i' values are expressed in seconds.

J.3.2 Example for calculating coordinates of position and velocity vector components for SV's center of mass at the given instant t_i (MT) using the long-term algorithm (involving long-term dynamic model parameters)

The PZ-90 coordinates, velocity vector components and perturbing accelerations for the SV's center of mass, as well as the long-term dynamic model parameters at the instant t_b = 306000 of 12.01.2013 (MT) are set and provided in the Table below:

$x(t_b)$ =	2290.0216875	km
$\dot{x}(t_b)$ =	-0.43945587147	km/s
$\ddot{x}(t_b)$ =	$-2.2591848392 \cdot 10^{-9}$	km/s^2
$\Delta a_{x0}(t_b)$=	$-1.3642421 \cdot 10^{-12}$	km/s^2
$a_{x1}(t_b)$ =	$-1.6237011735 \cdot 10^{-13}$	km/s^3
$a_{x2}(t_b)$ =	$1.7485470537 \cdot 10^{-16}$	km/s^4
$a_{x3}(t_b)$ =	$-1.0455562943 \cdot 10^{-20}$	km/s^5
$a_{x4}(t_b)$ =	$5.3011452831 \cdot 10^{-26}$	km/s^6
$y(t_b)$ =	19879.8775810	km
$\dot{y}(t_b)$ =	2.12254652940	km/s
$\ddot{y}(t_b)$ =	$2.4629116524 \cdot 10^{-9}$	km/s^2
$\Delta a_{y0}(t_b)$=	$1.1368684 \cdot 10^{-12}$	km/s^2
$a_{y1}(t_b)$ =	$1.2870815524 \cdot 10^{-12}$	km/s^3
$a_{y2}(t_b)$ =	$2.6054733458 \cdot 10^{-17}$	km/s^4
$a_{y3}(t_b)$ =	$-2.2786344334 \cdot 10^{-20}$	km/s^5
$a_{y4}(t_b)$ =	$1.0112818152 \cdot 10^{-24}$	km/s^6
$z(t_b)$ =	15820.0775420	km
$\dot{z}(t_b)$ =	-2.61032191480	km/s
$\ddot{z}(t_b)$ =	$-3.3505784813 \cdot 10^{-9}$	km/s^2
$\Delta a_{z0}(t_b)$ =	$-1.5916158 \cdot 10^{-12}$	km/s^2
$a_{z1}(t_b)$ =	$-1.3594680937 \cdot 10^{-13}$	km/s^3
$a_{z2}(t_b)$ =	$-1.5930995672 \cdot 10^{-17}$	km/s^4
$a_{z3}(t_b)$ =	$1.1662419456 \cdot 10^{-20}$	km/s^5
$a_{z4}(t_b)$ =	$-5.5518137243 \cdot 10^{-25}$	km/s^6

The task is to compute the PZ-90 coordinates and velocity vector components for the SV's center of mass at the instant t_i = 45000 of 12.01.2013 (MT) using the long-term algorithm.

Result:

$x(t_b)$ = -5994.7163090 km
$\dot{x}(t_b)$ = -2.21922119660 km/s

$y(t_b)$ = 9242.4696773 km
$\dot{y}(t_b)$ = -2.24157215710 km/s

$z(t_b)$ = -22981.9999270 km
$\dot{z}(t_b)$ = -0.32535557997 km/s

J.4 Accounting for PZ-90 rotation

Numerical integration of differential equations using precise, simplified, or long-term algorithms results in the coordinates of position and velocity vector components for the SV's center of mass at the given instant t_i in MT corresponding to the state of the rotating PZ-90 coordinate system at that instant. However the coordinates of position and velocity vector components for the user shall be determined for the PZ-90 state, it takes after signal propagation. That is why it is necessary to recalculate the coordinates and velocity vector components from the state of the rotating PZ-90 at the instant t_i, to the state it takes after signal propagation.

Recalculating the velocity vector components for the SV's center of mass can be neglected when objects moving with velocities of less than 5 km/s.

Appendix K

Algorithm for calculation of the current Julian date JD0, Gregorian date, and GMST

Initial data for calculation of JD0, Gregorian date (GD) and GMST include the number N_4^{cur} of the current four-year interval and the number N_T^{cur} of the current day. Their count in UE shall be carried out in RT (see Appendices F and G).

The algorithm for recalculating N_4^{cur} and N_T^{cur} into JD0, generally accepted Gregorian date, and GMST is provided below.

1) Compute JD0 for 00:00 in MT:

$$JD0 = 1461 \cdot \left(N_4^{cur} - 1\right) + N_T^{cur} + 2450082.5 - (N_T^{cur} - 3)/25 + C_1 + C_2,$$

where C_1 = 44195 is a constant to be used following the end of the year 2119 of MT (following the first reset of N_4 parameter to “1” in navigation message data”);

C_2 = 45290 is a constant to be used following the end of the year 2239 of MT (following the second reset of N_4 parameter to “1”). The above formula for calculation of $JD0$ is valid till year 2364.

2) Compute the number of Julian day for the current date:

JDN = JD0 + 0.5.

3) Compute the interim coefficients:

a = JDN + 32044,
b = (4a + 3) / 146097,
c = a – (146097b) / 4,
d = (4c + 3) / 1461,
e = c – (1461d) / 4,
m = (5e + 2) / 153.

4) Compute the Gregorian day, month, and year:

Day = e – (153m + 2) / 5 + 1;

Month = m + 3 – 12 (m / 10);

Year = 100b + d – 4800 + (m / 10).

All division operations shall be integer, fractional part shall be discarded. That is why, for example, 12 · (m / 10) in the formula for calculating month shall not be computed as (12 · m) / 10.

5) Compute the week day as a remainder of division of JDN by 7. “0” stand for Monday, “1” – for Tuesday, etc.

6) Compute GMST (in rad) using the formula

$$GMST = ERA + 0.0000000703270726 + 0.0223603658710194 \cdot T_\Delta + \\ + 0.0000067465784654 \cdot T_\Delta^2 - 0.0000000000021332 \cdot T_\Delta^3 - \\ - 0.0000000001452308 \cdot T_\Delta^4 - 0.0000000000001784 \cdot T_\Delta^5 ,$$

where ERA is the Earth’s rotation angle in radians, computed according to the formula

$ERA = 2\pi \cdot (0.7790572732640 + 1.00273781191135448 \cdot (JD0 - 2451545.0))$;

T_Δ – time from Epoch 2000, 1st January, 00:00 (UTC(SU)) till the current Epoch in Julian centuries, containing 36525 ephemeris days each;

$T_\Delta = (JD0 - 2451545.0) / 36525$.

The example for recalculating N_T^{cur} of the current day and N_4^{cur} of the current four-year interval determined in RT, into JD0, Gregorian date, and GMST:

It is stated that:

N_T^{cur} = 251 (number of the current day in a four-year interval);

N_4^{cur} = 5 (number of the current four-year interval).

Result:

JD0 = 2456177.5

JDN = 2456178

Day = 7

Month = 9

Year = 2012

GMST = 29191.442830

Appendix L

Algorithm for determination of the Conventional Reference Pole (Conventional International Origin) coordinates (X_p, Y_p) on the surface of the PZ-90 reference ellipsoid

Refer to Directive Document *РД50-25645.325-89* for description of the Earth Rotation Parameters use.

Appendix M
Algorithm for determination of position and velocity vector components for the SV's center of mass using almanac data

The user shall compute the coordinates of position and velocity vector components for the SV's center of mass using almanac in order to determine ID numbers of visible SVs, to predict SV entry into visibility zone, and to enable targeting to ensure acquisition of the signal from the selected SV. Methodological errors of computation supported by almanac data are provided in Table M.1.

Table M.1. – Prediction errors for the SV's position and velocity

Prediction interval	1 day	2 days	5 days	15 days	30 days	2 months	3 months
Maximum magnitude of position error vector, km	2.5	4	7	16	25	60	120
rms projection of position error vector on SV-to-receiver line of sight, km	0.5	0.8	2.5	6	11	25	51
Maximum magnitude of velocity error vector, m/s	0.5	0.8	1	2	3	8	25
rms projection of velocity error vector on SV-to-receiver line of sight, m/s	0.1	0.2	0.6	1.3	2.5	5	10

Input data used to compute the coordinates of position and velocity vector components for the SV's center of mass include the instant t_i (MT) and almanac data transmitted in the navigation message.

M.1 Algorithm for determination of position and velocity vector components for the SV's center of mass for the specified instant t_i (MT) of day N within a four-year interval using almanac data

Index j indicating SV ID number is truncated for simplicity in the algorithm below.

1) The interval of prediction Δt_{pr} shall be determined in seconds:

$$\Delta t_{pr} = \Delta N_A \cdot 86400 + (t_i - t_{\lambda_A}),$$

where $\Delta N_A = \begin{cases} N - N_A - \left\langle\left\langle \dfrac{N - N_A}{1461} \right\rangle\right\rangle \cdot 1461 \text{ if } N_4 \neq 27, \\ N - N_A - \left\langle\left\langle \dfrac{N - N_A}{1460} \right\rangle\right\rangle \cdot 1460 \text{ if } N_4 = 27; \end{cases}$

N is the calendar number of a day within a four-year interval starting with a leap year, containing the specified instant t_i in seconds MT;

N_A is the calendar number of a day (MT) within a four-year interval, transmitted in non-immediate data;

$\langle\langle x \rangle\rangle$ is operation of computing the integer nearest to x.

2) The number of whole orbits W on a prediction interval shall be determined:

$$W = \left\langle \frac{\Delta t_{pr}}{T_{av} + \Delta T_A} \right\rangle,$$

where $\langle x \rangle$ means extraction of integer;

T_{av} is the averaged value of a SV draconic orbital period, in seconds (T_{av} is defined in ICD for the corresponding signal).

ΔT_A is the correction to the averaged value of a SV draconic orbital period, in seconds.

3) The current inclination shall be determined:

$$i = \left(\frac{i_{av}}{180^{o}} + \Delta i_A \right) \cdot \pi \text{ rad},$$

where i_{av} is defined in ICD for the corresponding signal;

Δi_A is the correction to the mean inclination of orbit.

4) The mean draconic period in orbit W+1 and mean motion shall be determined:

$$T_{dr} = T_{av} + \Delta T_A + (2W+1) \cdot \Delta \dot{T}_A,$$
$$n = 2\pi / T_{dr},$$

where ΔT_A is the correction to the mean draconic orbital period;

$\Delta \dot{T}_A$ is the half rate of draconic period change.

5) The successive approximation m = 0, 1, 2… shall be used to determine the semi-major axis *a* of the SV orbit:

$$a^{(m+1)} = \sqrt[3]{\left(\frac{T_{osc}^{(m)}}{2\pi}\right)^2 \cdot GM},$$

$$p^{(m+1)} = a^{(m+1)}\left(1-(\varepsilon_A)^2\right),$$

$$T_{osc}^{(m+1)} = \frac{T_{dr}}{1-\frac{3}{2}\cdot J_2^0\left(\frac{a_e}{p^{(m+1)}}\right)^2\left[\left(2-\frac{5}{2}\cdot\sin^2 i\right)\cdot\frac{\left(1-(\varepsilon_A)^2\right)^{3/2}}{\left(1+\varepsilon_A\cdot\cos(\omega_A\pi)\right)^2}+\frac{\left(1+\varepsilon_A\cdot\cos(\omega_A\pi)\right)^3}{1-(\varepsilon_A)^2}\right]},$$

where GM = (398600441.8 ± 0.8)×10^6 m^3/s^2 is the gravitational constant (mass of Earth's atmosphere included) (as according to Table 4.1);

ε_A is the eccentricity of the SV orbit at the instant t_{λ_A};

a_e = 6378136 m is the semi-major axis of the PZ-90 reference ellipsoid (as according to Table 4.1);

J_2^0 = 1082.62575·10^{-6} is the second degree zonal coefficient of normal potential (as according to Table 4.1);

ω_A is the argument of perigee at the instant t_{λ_A}.

$T_{osc}^{(0)} = T_{dr}$ shall be taken as an initial approximation. Iteration shall terminate when $\left|a^{(m+1)} - a^{(m)}\right| \le 1\,\text{cm}$.

6) The current longitude of the ascending node and argument of perigee for the SV's orbit shall be determined allowing for their secular variation induced by the Earth's flattening:

$$\lambda = \lambda_A \cdot \pi - \left\{ \omega_3 + \frac{3}{2} J_2^0 \cdot n \cdot \left(\frac{a_e}{p} \right)^2 \cos i \right\} \Delta t_{pr},$$

$$\omega = \omega_A \cdot \pi - \frac{3}{4} J_2^0 n \left(\frac{a_e}{p} \right)^2 (1 - 5\cos^2 i) \cdot \Delta t_{pr},$$

where $\omega_E = 7.2921150 \cdot 10^{-5}$ rad/s is the angular velocity of the Earth (see Table 4.1).

7) The mean longitude at the instant the SV passes the current ascending node shall be determined:

$$L_1 = \omega + E_0 - \varepsilon_A \sin E_0,$$

where $E_0 = -2 \cdot \operatorname{arctan}\left(\sqrt{\frac{1-\varepsilon_A}{1+\varepsilon_A}} \cdot \tan\frac{\omega}{2} \right).$

8) The current mean longitude of the SV shall be determined:

$$L = L_1 + n(\Delta t_{pr} - (T_{av} + \Delta T_A)W - \Delta \dot{T}_A W^2).$$

9) Parameters a, ε_A, i, λ, ω, L shall be corrected allowing for periodic perturbations induced by the Earth's flattening according to the following formulae:

$$a' = a + \delta a_2 - \delta a_1$$
$$\varepsilon' = \sqrt{h'^2 + l'^2}$$
$$i' = i + \delta i_2 - \delta i_1$$
$$\lambda' = \lambda + \delta\lambda_2 - \delta\lambda_1$$
$$\omega' = \operatorname{arctan}\frac{h'}{l'}$$
$$L' = L + \delta L_2 - \delta L_1$$

where $h' = h + \delta h_2 - \delta h_1;$

$l' = l + \delta l_2 - \delta l_1;$

$h = \varepsilon_A \sin\omega;$

$l = \varepsilon_A \cos\omega.$

$\delta a_k, \delta h_k, \delta l_k, \delta i_k, \delta\lambda_k, \delta L_k$, for $k = 1, 2$ shall be computed according to the formulae

$$B = \frac{3}{2} J_2^0 \left(\frac{a_e}{a}\right)^2,$$

$$\frac{\delta a_k}{a} = 2B\left(1 - \frac{3}{2}\sin^2 i\right)(l\cdot\cos L_k + h\cdot\sin L_k) +$$
$$+ B\sin^2 i\left(\frac{1}{2}h\cdot\sin L_k - \frac{1}{2}l\cdot\cos L_k + \cos 2L_k + \frac{7}{2}l\cdot\cos 3L_k + \frac{7}{2}h\cdot\sin 3L_k\right),$$

$$\delta h_k = B\left(1 - \frac{3}{2}\sin^2 i\right)\left[\sin L_k + \frac{3}{2}l\cdot\sin 2L_k - \frac{3}{2}h\cdot\cos 2L_k\right] - \frac{1}{4}B\sin^2 i \times$$
$$\times\left[\sin L_k - \frac{7}{3}\sin 3L_k + 5l\cdot\sin 2L_k - \frac{17}{2}l\cdot\sin 4L_k + \frac{17}{2}h\cdot\cos 4L_k + h\cdot\cos 2L_k\right] +$$
$$+\left(-\frac{1}{2}B\cos^2 i\cdot l\cdot\sin 2L_k\right),$$

$$\delta l_k = B\left(1 - \frac{3}{2}\sin^2 i\right)\left[\cos L_k + \frac{3}{2}l\cdot\cos 2L_k + \frac{3}{2}h\cdot\sin 2L_k\right] - \frac{1}{4}B\sin^2 i \times$$
$$\times\left[-\cos L_k - \frac{7}{3}\cos 3L_k - 5h\cdot\sin 2L_k - \frac{17}{2}l\cdot\cos 4L_k - \frac{17}{2}h\cdot\sin 4L_k + l\cdot\cos 2L_k\right] +$$
$$+\frac{1}{2}B\cos^2 i\cdot h\cdot\sin 2L_k,$$

$$\delta\lambda_k = -B\cos i\left(\frac{7}{2}l\cdot\sin L_k - \frac{5}{2}h\cdot\cos L_k - \frac{1}{2}\sin 2L_k - \frac{7}{6}l\cdot\sin 3L_k + \frac{7}{6}h\cdot\cos 3L_k\right),$$

$$\delta i_k = \frac{1}{2}B\sin i\cdot\cos i\left(-l\cdot\cos L_k + h\cdot\sin L_k + \cos 2L_k + \frac{7}{3}l\cdot\cos 3L_k + \frac{7}{3}h\cdot\sin 3L_k\right),$$

$$\delta L_k = 2B\left(1 - \frac{3}{2}\sin^2 i\right)\left(\frac{7}{4}l\cdot\sin L_k - \frac{7}{4}h\cdot\cos L_k\right) + 3B\sin^2 i \times$$
$$\times\left(-\frac{7}{24}h\cdot\cos L_k - \frac{7}{24}l\cdot\sin L_k - \frac{49}{72}h\cdot\cos 3L_k + \frac{49}{72}l\cdot\sin 3L_k + \frac{1}{4}\sin 2L_k\right) +$$
$$+ B\cos^2 i\cdot\left(\frac{7}{2}l\cdot\sin L_k - \frac{5}{2}h\cdot\cos L_k - \frac{1}{2}\sin 2L_k - \frac{7}{6}l\cdot\sin 3L_k + \frac{7}{6}h\cdot\cos 3L_k\right).$$

The values of argument L_k, k = 1.2, where L_2 = L shall be substituted consecutively in the above formulae.

Note – At the current stage of the algorithm short-period disturbances of the SV's orbit elements, caused by effects of the second zonal geopotential harmonics shall be accounted for. The amplitude of short-period disturbances for GLONASS SV orbits is within 1.5-2 km in the Greenwich coordinates. As from the existing practice, such almanac errors are insignificant for most GLONASS users. For these users stage 9 of the algorithm can be skipped. Stage 9 enables nearly 2-3 times lesser systematic errors of Greenwich coordinates (for the first day of prediction).

Stages 10–12 describe computation of position and velocity vector components for the SV's center of mass using parameters a, ε_A, i, λ, ω, L. If stage 9 is not skipped the corrected values $a', \varepsilon', i', \lambda', \omega', L'$ shall be taken.

10) The eccentric anomaly shall be determined by Kepler's equation

$$L - \omega = E - \varepsilon \cdot \sin E\,.$$

As a rule, the scheme of successive approximations m = 0, 1, 2, etc. shall be used:

$$E^{(m+1)} = L - \omega + \varepsilon \cdot \sin E^{(m)}\,,$$

where $E^{(0)} = L - \omega$ shall be taken as initial approximation. Iteration shall be terminated, when $\left|E^{(m+1)} - E^{(m)}\right| \le 10^{-9}$.

11) True anomaly υ and argument of latitude u for the SV shall be determined:

$$\upsilon = 2\arctan\left(\sqrt{\frac{1+\varepsilon_A}{1-\varepsilon_A}}\tan\frac{E}{2}\right),$$

$$u = \upsilon + \omega.$$

12) The coordinates of position for the SV's center of mass in the geocentric orthogonal coordinate system shall be computed:

$$p = a(1 - (\varepsilon_A)^2)\,,$$

$$r = \frac{p}{1 + \varepsilon_A \cos\upsilon},$$

$$x(t_i) = r(\cos\lambda\cos u - \sin\lambda\sin u\cos i),$$

$$y(t_i) = r(\sin\lambda\cos u + \cos\lambda\sin u\cos i),$$

$$z(t_i) = r\sin u\sin i.$$

13) The velocity vector components for the SV's center of mass in the geocentric orthogonal coordinate system shall be computed:

$$v_r = \sqrt{\frac{\mu}{p}}\ \varepsilon_A \sin\upsilon,$$

$$v_u = \sqrt{\frac{\mu}{p}}(1 + \varepsilon_A \cos\upsilon),$$

$$\dot{x}(t_i) = v_r(\cos\lambda\cos u - \sin\lambda\sin u\cos i) - v_u(\cos\lambda\sin u + \sin\lambda\cos u\cos i) + \omega_3 y(t_i),$$

$$\dot{y}(t_i) = v_r(\sin\lambda\cos u + \cos\lambda\sin u\cos i) - v_u(\sin\lambda\sin u - \cos\lambda\cos u\cos i) - \omega_3 x(t_i),$$

$$\dot{z}(t_i) = v_r \sin u\sin i + v_u \cos u\sin i.$$

M.2 Example for calculating coordinates of position and velocity vector components for SV's center of mass using almanac data

A SV's almanac is set as follows:

N_A = 1452

t_{λ_A} = 33571.625

ΔT_A = 0.01953124999975

$\Delta \dot{T}_A$ = 6.103515625E-05

λ_A = –0.293967247009277

ω_A = 0.57867431640625

ε_A = 0.000432968139648438

Δi_A = -0.00012947082519531

The task is to compute the OXYZ orthogonal system coordinates of position and velocity vector components for the SV's center of mass at the instant t_i = 51300 s within day N = 1453 of a four-year interval.

Computation:

1) Prediction interval in days determined:

Δt_{pr} = 104128.375

2) The number of orbits on the prediction interval determined:

W = 2

3) The current inclination determined:

i = 1.1305666106990377

4) The mean draconic period and mean motion determined:

T_{dr} = 40544.019836425781
n = 0.00015497193747756143

5) The semi-major axis determined through successive approximation:

m	a	p	T_{osc}
0	25508.047485485004	25508.042703710456	40547.946040115967
1	25509.694225238574	25509.689443155326	40547.945533182959
2	25509.694012622691	25509.689230539483	40547.945533248407
3	25509.694012650143	25509.689230566935	40547.945533248399

6) The current longitude of the ascending node and the argument of perigee determined allowing for their secular variation induced by the Earth's flattening:

$\lambda = -8.5173843140309469$

$\omega = 1.8178836298808301$

7) The mean longitude at the instant the SV passes the current ascending node determined:

$E_0 = -1.8174637892065451$

$L_1 = 0.00083970352771615942$

8) The mean longitude of the SV determined:

$L = 3.5714451660610322$

9) Parameters a, ε_A, i, λ, ω, L are corrected allowing for periodic perturbations induced by the Earth's flattening:

$a' = 25508{,}9554310860556;$

$\varepsilon' = 0{,}00042419917873569115;$

$i' = 1{,}13055979412986857;$

$\lambda' = -8{,}51736802279423523;$

$\omega' = 1{,}96580151879618214;$

$L' = 3{,}57148542469343516;$

$h' = 0{,}00039153353705544239;$

$l' = -0{,}00016323735050805419;$

$h = 0{,}00041981843111904164;$

$l = -0{,}00010589567905904274;$

$B = 0{,}00010151884398503961;$

k	1	2
L_k	0,00083970352771615942	3,5714451660610322
$\frac{\delta a_k}{a}$	8,30615563565482404e-005	5,41085801381285305e-005
δh_k	1,69203051231874733e-008	-2,82679737584760494e-005
δl_k	4,61175901454555875e-005	-1,12240813035558812e-005
$\delta \lambda_k$	6,05435071360537479e-008	1,63517802188070858e-005
δi_k	1,95658219192184023e-005	1,27492527500887458e-005
δL_k	1,10334212545863083e-008	4,02696658242106401e-005

10) The eccentric anomaly determined through the Kepler's equation:

m	$E^{(m)}$
0	1,60568390589725301
1	1.60610784694660425
2	1.60610784063575716
3	1.60610784063585167
4	1.60610784063585167

11) The SV's true anomaly and the argument of latitude determined:

$\upsilon = 1.60653177221277219$;

u = 3.57233329100895433.

12) The geocentric orthogonal coordinate system coordinates for the SV's center of mass computed:

p = 25508,9508408785184;

r = 25509,3374532650379;

$x(t_i)$ = 10697,1164874360654;

$y(t_i)$ = 21058,2924241863210;

$z(t_i)$ = –9635,67933963303405.

13) The velocity vector components for the SV's center of mass in the SV's center of mass computed:

v_r = 0,00167577247194655653;

v_u = 3,95290163460661152;

$\dot{x}(t_i) = -0{,}686100809921691084;$

$\dot{y}(t_i) = -1{,}13654864124521881;$

$\dot{z}(t_i) = -3{,}24998587740305799.$

Appendix N

Algorithm for calculation of ranging code phase and pseudo-Doppler shift of the received carrier frequency for SV j at an arbitrary time of MT using almanac data

Almanac based estimated ranging code phase of data and pilot components and the pseudo-Doppler shift of the received carrier frequency for SV j shall enable targeting to ensure signal acquisition from this SV. The estimation shall be conducted using the following algorithm.

N.1 The coordinates $x^j(t_{MT})$, $y^j(t_{MT})$, $z^j(t_{MT})$ of position and velocity vector components $\dot{x}^j(t_{MT})$, $\dot{y}^j(t_{MT})$, $\dot{z}^j(t_{MT})$ for SV j at the instant t_{MT} in MT shall be calculated according to the algorithm described in Appendix M.

N.2 Using the known coordinates of UE $x_{UE}(t_{MT})$, $y_{UE}(t_{MT})$, $z_{UE}(t_{MT})$, range $\hat{R}^j(t_{MT})$ to SV j shall be computed according to the formula

$$\hat{R}^j(t_{MT}) = \sqrt{\left(x^j(t_{MT}) - x_{UE}(t_{MT})\right)^2 + \left(y^j(t_{MT}) - y_{UE}(t_{MT})\right)^2 + \left(z^j(t_{MT}) - z_{UE}(t_{МДВ})\right)^2}\ .$$

$\hat{R}^j(t_{MT})$ divided by the speed of light yields $\hat{\tau}^j = \hat{R}^j(t_{MT})/c$ that is signal traveling time from SV j to UE.

N.3 Time $\hat{t}_{pr}$ of MT preceding time t_{MT} for the signal traveling time shall be computed in seconds using he formula

$$\hat{t}_{pr} = t_{MT} - \hat{\tau}^j .$$

N.4 The predicted value of ranging code phase of data and pilot components $\hat{\varphi}^j$ for the signal received at the instant t_{MT}, in cycles, shall be computed using the formula

$$\hat{\varphi}^j = \frac{\mathrm{mod}_{T_{PRN}}\left[\hat{t}_{pr} - \tau_c(t_b) - (t_{MT} - t_b)\cdot\dot{\tau}_c(t_b) - \tau_A^j\right]}{T_{PRN}},$$

where T_{PRN} is the ranging code period of data or pilot component.

N.5 Using the known velocity vector components of UE $\dot{x}_{UE}(t_{MT})$, $\dot{y}_{UE}(t_{MT})$, $\dot{z}_{UE}(t_{MT})$, the predicted value of radial velocity (range rate) $\hat{\dot{R}}^j(t_{UE})$ of SV j relative to the UE at the instant t_{MT} shall be computed with the formula

$$\hat{\dot{R}}^j(t_{MT}) = h_x^j\left(\dot{x}^j(t_{MT}) - \dot{x}_{UE}(t_{MT})\right) + h_y^j\left(\dot{y}^j(t_{MT}) - \dot{y}_{UE}(t_{MT})\right) + h_z^j\left(\dot{z}^j(t_{MT}) - \dot{z}_{UE}(t_{MT})\right),$$

where $h_x^j = \dfrac{x^j(t_{MT}) - x_{UE}(t_{MT})}{\hat{R}^j(t_{MT})}$, $h_y^j = \dfrac{y^j(t_{MT}) - y_{UE}(t_{MT})}{\hat{R}^j(t_{MT})}$, $h_z^j = \dfrac{z^j(t_{MT}) - z_{UE}(t_{MT})}{\hat{R}^j(t_{MT})}$.

N.6 Using the known UE oscillator's frequency shift $\Delta\hat{f}_{oscil}(t_{MT})$ relative to its nominal value f_{oscnom}, pseudo-Doppler shift $\hat{f}_{pd}(t_{MT})$ of the received carrier frequency for SV j at the instant t_{MT} shall be computed:

$$\hat{f}_{pd}(t_{MT}) = -\frac{\hat{\dot{R}}^j(t_{MT})}{\lambda_{nom}^j} - k^j \cdot \Delta\hat{f}_{oscil}(t_{MT}),$$

where λ_{nom}^j is the nominal wavelength of the signal carrier frequency;

$k^j = f_{nom}/f_{oscnom}$ is the ratio of nominal carrier frequency f_{nom} to nominal frequency f_{oscnom} of UE oscillator.

Appendix P

Recommendations on use of accuracy factors F_E^j, F_T^j

If an operative attribute H^j transmitted in a signal is “0” (which means the signal is usable), accuracy factors F_E^j, F_T^j can be used by the user to calculate weights $1/(\sigma^j)^2$, with which pseudorange measurements of SV j shall be processed, where:

$$(\sigma^j)^2 = (\sigma_{FE}^j)^2 + (\sigma_{FT}^j)^2$$

σ_{FE}^j, σ_{FT}^j are rms user equivalent range errors for SV j, expressed in meters and introduced by ephemeris and clock errors depending on accuracy factors F_E^j и F_T^j, transmitted by the SV in its immediate data;

If an operative attribute H^j is “1” (not usable signal), accuracy factors F_E^j, F_T^j can be inconsistent with the real status of the system.

Appendix Q
Algorithm of using ionosphere model parameters

Three parameters of the adaptive ionosphere model are transmitted within GLONASS CDMA signal data to be used as input data for computation of ionospheric corrections. These are:

- c_A is the numerical factor for the peak TEC (Total Electron Content) of F2 ionosphere layer;
- $c\_F_{10.7}$ is the corrected value of solar activity index;
- c_A_p is the daily geomagnetic activity index (further on referred to as A_p).

These parameters enable better accuracy of the computed ionosphere model based corrections by compensating geo- and heliophysical disturbances in atmosphere.

Q.1 Ionospheric correction algorithm

To compensate ionosphere effect in single-frequency UE the following correction shall be introduced:

- for pseudoranges, m:

$$\Delta S_{ion} = 0.40364 \cdot \frac{I_e}{f^2}; \tag{Q.1}$$

- for velocities, m/s:

$$\Delta V_{ion} = 0.40364 \cdot \frac{\dot{I}_e}{f^2}, \tag{Q.2}$$

where f is the signal carrier frequency, in GHz;

I_e is the total electron content (TEC) integrated along the signal propagation path, $1\times10^{16}\ m^{-2}$;

$\dot{I}_e$ is the rate of change of TEC integrated along the signal propagation path, $1\cdot10^{16}\ m^{-2}\cdot s^{-1}$.

This section describes algorithms for computing TEC integrated along the signal propagation path:

- uniform for terrestrial and space users;
- for terrestrial users.

The first uniform algorithm is more complex and enables better accuracy of computing TEC as well as wider application. The second algorithm is easier to implement, but it has limited applications (used only by terrestrial users) and results in lager errors of TEC at less than 30° elevation angles. These errors however do not exceed 1 m provided the signal frequency of 1.6 GHz.

The input data for computation are:

- estimates of UE position in geocentric orthogonal coordinates;
- estimates of the SV's transmitter position in geocentric orthogonal coordinates;
- signal carrier frequency;
- time of observation in UTC(SU);
- corrected value of solar activity index $c\_F_{10.7}$ for the observation date;
- daily geomagnetic activity index A_p for the date of observation;
- numerical factor for the maximum electron content of F2 ionosphere layer N_{max};
- adaptation coefficient c_A.

Q.2 Evaluating TEC values for terrestrial and space users

In a general way TEC value I_e along the propagation path of signal l shall be determined using the formula:

$$I_e = \int_l N_e dl,$$

where N_e is the electron content in the given points of the signal path.

A user can choose between different integration methods. However it is recommended the integration increment shall be chosen with respect to a height rather than a distance along the signal path. It relates to the fact, that N_e varies more significantly with the height for the fixed latitude and longitude than it does with the latitude or longitude for the fixed height.

To obtain N_e for a given point in space the model for TEC distribution in ionosphere shall be employed.

Below is the algorithm for calculation of N_e for a given point in space.

Q.2.1 Computation of local time slt in radians:

$$\mathrm{slt} = (\mathrm{ut} + \mathrm{glong}/15) \cdot \pi/12,$$

where ut is UTC(SU) time in hours;

glong is the longitude of the given point in degrees.

Q.2.2 Computation of normalized value of relative Sun Spot Number (SSN) r:

$$r = 0.01 \cdot W,$$

where $W = \sqrt{167273 + 1123.6 \cdot (c\_F_{10.7} - 63.7)} - 408.99$ is the Wolf number.

Q.2.3 Computation of solar declination dec (in rad):

$$\mathrm{dec} = \arcsin\{0.39795 \cdot \sin[\pi \cdot (\mathrm{month} - 3.167)/6]\},$$

where month is the number of a month.

Q.2.4 Computation of geomagnetic latitude mlat (in rad):

$$\mathrm{mlat} = \arcsin[0.98 \cdot \sin(\mathrm{glat}) + 0.2 \cdot \cos(\mathrm{glat}) \cdot \cos(\mathrm{glong} + 1.2)],$$

where glat is the latitude of the point, rad;

glong is the longitude of the point, rad.

Q.2.5 Computation of geomagnetic longitude *mlong* (in rad):

$$\mathrm{mlong}' = \mathrm{arctg}\left(\frac{\mathrm{smlon}}{\mathrm{cmlon}}\right),$$

where mlong' is the arc tangent of the geomagnetic longitude;

$\mathrm{smlon} = 0.2 \cdot \cos(\mathrm{glat}) \cdot \sin(\mathrm{glong} + 1.2)$ is the sine of the geomagnetic longitude;

$\mathrm{cmlon} = 0.98 \cdot \sin(\mathrm{mlat}) - \sin(\mathrm{glat})$ is the cosine of the geomagnetic longitude.

In this subsection glat and glong are expressed in radians.

As arc tangent function yields in the interval from minus π/2 to π/2, and the geomagnetic longitude is in the interval from 0 to 2π, one shall take into account positivity/negativity of geomagnetic longitude sine *smlon* and cosine *cmlon* to determine the geomagnetic longitude *mlong* correctly:

$$\mathrm{mlong} = \begin{cases} \mathrm{mlong}', \text{ if } \mathrm{cmlon} > 0 \text{ and } \mathrm{smlon} \geq 0 \ (1^{st} \text{ quarter of circumference}) \\ \mathrm{mlong}' + 2\pi, \text{ if } \mathrm{cmlon} > 0 \text{ and } \mathrm{smlon} < 0 \ (4^{th} \text{ quarter of circumference}) \\ \mathrm{mlong}' + \pi, \text{ if } \mathrm{cmlon} < 0 \ (2^{nd} \text{ and } 3^{rd} \text{ quarters of circumference}) \\ \pi / 2, \text{ if } \mathrm{cmlon} = 0 \text{ and } \mathrm{smlon} > 0 \\ 3\pi / 2, \text{ if } \mathrm{cmlon} = 0 \text{ and } \mathrm{smlon} < 0 \end{cases}$$

Q.2.6 Computation of the magnetic inclination *dip* (in radians):

$$\mathrm{dip} = \mathrm{arctg}\left[2 \cdot \mathrm{tg}(\mathrm{mlat})\right].$$

Q.2.7 Computation of F2-layer peak height *h max*:

$$\mathrm{h\,max} = 240 + \mathrm{A} + \mathrm{B} + \mathrm{C},$$

where A, B, and C are the values computed using the formulae

$\mathrm{A} = 10 \cdot \cos(\mathrm{mlat}) \cdot \cos\left[\pi \cdot (\mathrm{month}/3 - 1.5)\right],$

$\mathrm{B} = \mathrm{r} \cdot \left[75 + 83 \cdot \cos(\mathrm{mlat}) \cdot \sin(\mathrm{mlat}) \cdot \sin(\mathrm{dec})\right],$

$\mathrm{C} = 30 \cdot \cos\left(\mathrm{slt} - 4.5 \cdot |\mathrm{mlat}| - \pi\right).$

Q.2.8 Computation of m3000 parameter:

$$\mathrm{m3000} = \frac{1490}{\mathrm{h\,max} + 176}.$$

Q.2.9 Computation of F2-layer peak concentration Nmax, $1 \cdot 10^{11}\ \mathrm{m}^{-3}$:

$$\mathrm{N max} = 0.66 \cdot \mathrm{D} \cdot \mathrm{E} \cdot \mathrm{F} \cdot \mathrm{G} \cdot \mathrm{H} \cdot \mathrm{I} \cdot \mathrm{J},$$

where D, E, F, G, H, I, J are the values determined according to the formulae

$$\mathrm{D} = \mathrm{K} \cdot \mathrm{L} \cdot \exp\{-1.1 \cdot [\cos(\mathrm{slt} - 0.873) + 1]\},$$

where $\mathrm{K} = 0.9 + 0.32 \cdot \sin(\mathrm{dec}) \cdot \sin(\mathrm{mlat});$

$$\mathrm{L} = 1 + \sin(\mathrm{mlat}) \cdot \left[\cos\left(\mathrm{slt} + \frac{\pi}{4}\right)\right]^2 \cdot \sin(\mathrm{dec});$$

$$\mathrm{E} = \{1 - 0.4 \cdot [\cos(\mathrm{mlat})]^{10}\} \cdot \left[1 + 0.6 \cdot [\cos(\mathrm{mlat})]^{10} \cdot \left[\cos\left(\mathrm{slt} + \frac{\pi}{4}\right)\right]^2\right] \cdot \mathrm{M} \cdot \mathrm{N},$$

where M and N are the interim calculation values determined as per the formulae

$$\begin{aligned}\mathrm{M} = 1 + [\cos(\mathrm{mlat})]^8 \cdot [\cos(|\mathrm{mlat}| - 0.2618)]^{12} \times \\ \times \left(1 - 0.2 \cdot \mathrm{r} + 0.6 \cdot \sqrt{\mathrm{r}}\right) \cdot \exp\{0.25 \cdot [1 - \cos(\mathrm{slt} - 0.873)]\}\end{aligned},$$

$$\mathrm{N} = 1 + 0.05 \cdot \left[0.5 - \cos\left(2 \cdot \mathrm{month} \cdot \frac{\pi}{6}\right) + \cos\left(\mathrm{month} \cdot \frac{\pi}{6}\right)\right],$$

$$\mathrm{F} = \{1.2 - 0.5 \cdot [\cos(\mathrm{mlat})]^2\} \cdot \mathrm{O} \cdot \mathrm{P} \cdot \exp\left\{3 \cdot \cos\left[\mathrm{mlat} \cdot \frac{\sin(\mathrm{slt}) - 1}{2}\right]\right\},$$

where O and P are the interim values determined as per the formulae

$$\mathrm{O} = 1 + 0.05 \cdot \mathrm{r} \cdot \cos\left(\mathrm{month} \cdot \frac{\pi}{6}\right) \cdot [\sin(\mathrm{mlat})]^3,$$

$$\mathrm{P} = 1 - 0.15 \cdot \exp\left[-\sqrt{\left(12 \cdot \mathrm{mlat} + 4 \cdot \frac{\pi}{3}\right)^2 + \left(\frac{\mathrm{month}}{2} - 3\right)^2}\right],$$

$$G = \begin{cases} 1 + r + (0.204 + 0.03 \cdot r) \cdot r^2, \text{ if } r < 1.1, \\ 2.39 + 1.53 \cdot [\sin(\text{mlat})]^2 \cdot \{[1 + r + (0.204 + 0.03 \cdot r) \cdot r^2)] - 2.39\}, \text{ if } r \geq 1.1, \end{cases}$$

$$H = 1 + 0.1 \cdot [\cos(\text{mlat})]^3 \cdot \cos\left[2 \cdot \left(\text{mlong} - 7 \cdot \frac{\pi}{18}\right)\right],$$

$$I = R \cdot \left\{1 + 0.03 \cdot \left[0.5 - \cos\left(2 \cdot \text{month} \cdot \frac{\pi}{6}\right) + \cos\left(\text{month} \cdot \frac{\pi}{6}\right)\right]\right\},$$

where

$$R = 1 + \{0.15 - 0.5 \cdot (1 + r) \cdot [1 - \cos(\text{mlat})] \cdot \exp[-0.33 \cdot (\text{month} - 6)^2]\} \cdot \exp\left[-18 \cdot \left(|\text{dip}| - 4 \cdot \frac{\pi}{18}\right)^2\right],$$

$$J = \frac{(S + T)}{G} + U,$$

where $S = 0.2 \cdot [1 - \sin(|\text{mlat}| - 0.5236)] \cdot \left\{1 + 0.6 \cdot \cos\left[\frac{\pi}{3} \cdot (\text{month} - 4)\right]\right\} \cdot \cos\left[\frac{\pi}{6} \cdot (\text{month} - 1)\right],$

$$T = \left[0.13 - 0.06 \cdot \sin\left(\left|\,|\text{mlat}| - \frac{\pi}{9}\right|\right)\right] \cdot \cos\left[\frac{\pi}{3} \cdot (\text{month} - 4.5)\right] - V,$$

where $V = \begin{cases} 0, \text{ if } 1 - \cos(\text{slt}) \leq 0.0001 \\ [\cos(\text{mlat} + \text{dec})]^3 \cdot [0.15 + 0.3 \cdot \sin(|\text{mlat}|)] \times \\ \times \exp\{0.25 \cdot \ln[1 - \cos(\text{slt})]\}, \text{ if } 1 - \cos(\text{slt}) > 0.0001 \end{cases}$

$$U = 0.7 \cdot \left\{X + 0.1778 \cdot \frac{r^2}{G} \cdot \cos\left[\frac{\pi}{3} \cdot (\text{month} - 4.3)\right]\right\} \cdot \exp(-1 \cdot Y \cdot Z),$$

where

$$X = 1 + 0.085 \cdot \left\{\left[\cos\left(\frac{\pi}{6} \cdot (0.5 \cdot \text{month} - 1)\right)\right]^3 \cdot \cos\left(\text{mlat} - \frac{\pi}{6}\right) + \right.$$
$$\left. + \cos\left(\text{mlat} + \frac{\pi}{4}\right) \cdot \left[\cos\left(\frac{\pi}{6} \cdot [0.5 \cdot \text{month} - 4]\right)\right]^2\right\},$$

$$Y = 1.3 + \left\{0.139 \cdot \left[1 + \cos\left(\text{mlat} - \frac{\pi}{4}\right)\right] + 0.0517 \cdot r\right\} \cdot r^2,$$

$$Z = \cos[\text{mlat} + \text{dec} \cdot \cos(\text{slt})] - \cos(\text{mlat}).$$

Q.2.10 Computation of F2-layer critical frequency fof2:

$$\mathrm{fof}\,2 = \sqrt{\frac{\mathrm{N\,max}}{0.124}}\ .$$

Q.2.11 Computation of the height scale (thickness parameter) B_{bot} (below F2-layer peak):

$$B_{bot} = 0.385 \cdot \frac{\mathrm{N\,max}}{0.01 \cdot \exp\left[-3.467 + 0.857 \cdot \ln\left(\mathrm{fof}\,2^2\right) + 2.02 \cdot \ln(\mathrm{m3000})\right]}.$$

Q.2.12 Computation of the height scale (thickness parameter) B_{top} (above F2-layer peak):

$$B_{top} = \frac{B_{bot} \cdot b_{ok}}{\left(0.041163 \cdot \frac{B_{bot} \cdot b_{ok} - 150}{100} - 0.183981\right) \cdot \frac{B_{bot} \cdot b_{ok} - 150}{100} + 1.424472},$$

where $b_{ok} = \begin{cases} 6.705 - 0.01 \cdot W - 0.008 \cdot \mathrm{h\,max}, & \text{April through Sepember,} \\ -7.77 + 0.097 \cdot \left(\frac{\mathrm{h\,max}}{B_{bot}}\right)^2 + 0.153 \cdot \mathrm{N\,max}, & \text{other months,} \end{cases}$

and if b_{ok} < 2, then b_{ok} = 2, if b_{ok} > 8, then b_{ok} = 8.

Q.2.13 Computation of correction coefficients c_N and c_h:

In this subsection glat is expressed in radians and glong – in degrees.

The correction coefficients c_N and c_h shall be computed provided the condition Ap > 27 nT is met. The following expressions shall be used:

- calculation of the atmospheric temperature T_{ns} for Ap = 0:

$$T_{ns} = a_{ns} \cdot (1 + b_{ns} + c_{ns}) \cdot 0.001,$$

where $a_{ns} = 3.3 \cdot W + 705$;

$b_{ns} = 0.2 \cdot \sin(2 \cdot \pi \cdot z_{ns}) \cdot \sin(glat)$;

$z_{ns} = (month \cdot 30.5 - 96)/365$;

$c_{ns} = 0{,}12 \cdot \cos(glat) \cdot C_{slt}$;

$C_{slt} = \cos[15 \cdot (dLt - 15) \cdot \pi/180]$;

$dLt' = ut + \frac{glong}{15}$ is the local time (in hours).

If $dLt' < 0$, then $dLt = dLt' + 24$. If $dLt' \geq 24$, then $dLt = dLt' - 24$. Otherwise $dLt = dLt'$;

- calculation of the atmospheric temperature T_{nd} under geomagnetic disturbances A_p:

$$T_{nd} = T_{ns} + (4.5 \cdot A_p - 100) \cdot [\sin(mlat)]^2 \cdot 0.001;$$

- calculation of c_N coefficient:

$$c_N = e^{dExp},$$

where $dExp = 1.3 \cdot (dLog + dSlag1 + dSlag2)$;

$dLog = \ln\left(\frac{2 - 1.8 \cdot T_{ns} + 0.8 \cdot T_{ns}^2}{2 - 1.8 \cdot T_{nd} + 0.8 \cdot T_{nd}^2}\right)$;

$dSlag1 = -0.0022 \cdot (1 - 0.3 \cdot C_{slt}) \cdot (4.5 \cdot A_p - 100) \cdot [\sin(mlat)]^2$;

$dSlag2 = 0.0007 \cdot (1 - 0.3 \cdot C_{slt}) \cdot A_p \cdot [\cos(mlat)]^4$;

- calculation of c_h coefficient:

$$c_h = 0.2 \cdot A_p \cdot (1 - 0.001 \cdot A_p) \cdot (1 + 0.01 \cdot mlat).$$

Q.2.14 Correction of altitude parameters of TEC distribution

- F2-layer peak height:

$$h'_{max} = h\max + c_h,$$

where h'_{max}, $h\max$ are the corrected and the initial (see Q.2.7) values of F2-layer peak height in km.

- F2-layer peak concentration:

$$N'_{max} = N\max \cdot c_N ,$$

where N'_{max}, $N\max$ are the corrected and the initial (see Q.2.9) values of F2-layer peak concentration, in $1 \cdot 10^{16}$ m^{-2};

- The height scale (thickness) above F2-layer peak:

$$B'_{top} = B_{top} \cdot [1 - \ln(c_N)], \qquad (Q.3)$$

where B'_{top}, B_{top} are the corrected and the initial values (see Q.2.12).

- The height scale (thickness) below F2-layer peak:

$$B'_{bot} = B_{bot} \cdot \left[1 - \frac{\ln(c_N)}{2}\right], \qquad (Q.4)$$

where B'_{bot}, B_{bot} are the corrected and the initial values (see Q.2.11).

According to Q.2.13, coefficients c_N and c_h may not be calculated provided the condition $Ap > 27$ nT is not met. In this case the correction of altitude parameters for TEC distribution shall be carried out, i.e.:

$$h'_{max} = h\max ,$$

$$N'_{max} = N\max ,$$

$$B'_{top} = B_{top} ,$$

$$B'_{bot} = B_{bot} .$$

Q.2.15 Correction of F2-layer peak concentration Nmax allowed for c_A adaptation coefficient:

$$\tilde{N}_{max} = N'_{max} \cdot c\_A \,.$$

Q.2.16 Computation of TEC in a given point in space, $1 \cdot 10^{16}$ m^{-2}:

$$N_e = A \cdot \frac{e^y}{(1+e^y)^2} \,,$$

where A shall be determined as per the formula

$$A = 4 \cdot \tilde{N}_{max} \,; \qquad \text{(Q.5)}$$

$$y = \begin{cases} (h - h'_{max}) / B'_{bot}, & h < h'_{max}, \\ y' \Big/ \left(1 + \dfrac{12.5 \cdot y'}{100 + 0.1 \cdot y'}\right), \quad y' = (h - h'_{max}) / B'_{top}, & h > h'_{max}; \end{cases}$$

h is the height of the point expressed in km.

Q.3 Evaluating TEC values for terrestrial users

To employ the algorithm it is necessary to obtain TEC along the slant – TEC_vert for the location of UE.

Q.3.1 Obtain TEC along the vertical:

$$TEC\_vert = A \cdot (0.5 \cdot B'_{bot} + 0.9 \cdot B'_{top}) \cdot 0.01,$$

where A is determined using the formula (Q.5);

B'_{bot} and B'_{top} are the height scales (thicknesses below and above the F-2-layer peak, obtained though (Q.3) and (Q.4), respectively.

Q.3.2 Evaluate TEC

To evaluate I_e the following formula shall be used

$$I_e = TEC\_vert / \left[1 - \left[\sin(zet) / (1 + 400 / Re)\right]^2\right]^{1/2},$$

where $zet = \mathrm{arctg}\left[sdel/\left(cdel - Re/r_2\right)\right]$ is the slant's zenith angle;

$sdel = \left(1 - cdel^2\right)^{1/2}$;

$cdel = \sin(\varphi_1)\cdot\sin(\varphi_2) + \cos(\varphi_1)\cdot\cos(\varphi_2)\cdot\cos(\lambda_2 - \lambda_1)$;

r2=Re+h2;

Re is the Earth's radius;

$\varphi 1$ is the UE location latitude, degrees;

$\lambda 1$ is the UE location longitude, degrees;

$h2$ is the signal transmitter height, km;

$\varphi 2$ is the signal transmitter latitude, degrees;

$\lambda 2$ is he signal transmitter longitude, degrees.

To evaluate $\dot{I}_e$ for the given instant t, one shall use the formulae

$$\dot{I}_e = \left[I_e(t) - I_e(t + \Delta t)\right]/\Delta t,$$

where Δt is the increment of 1 s.

Under the described model, ionospheric correction error ΔS_{ion} features residual errors not exceeding 4 m (0.95 probability).

Q.4 Example for evaluating TEC for the given location

Input data:

ut = 14 is the time of evaluation in UTC(SU), h;

month = 3 is the number of the month in the year;

h = 700 is the height of the given location, km;

glat = 60 is the latitude of the given location, degrees;

glong = 30 is the longitude of the given location, degrees;

c_A = 0.8 is the numerical factor for electron content of F2-layer peak;

$c\_F_{10.7}$ = 70 is the corrected index of solar activity in solar flux units (sfu), where 1 sfu=$1\cdot 10^{-22}$ $W/(m^2 \cdot Hz)$;

c_Ap = Ap =30 is the daily index of geomagnetic activity, nT.

Constants:

π = 3.14159265358979.

The obtained evaluation results are:

Q.4.1 Computation of the local time slt in radians:

$$slt = (ut + glong/15) \cdot \pi/12 = (14 + 30/15) \cdot \pi/12 = 4.18879020478639$$

Q.4.2 Computation of the normalized value of relative Sun Spot Number (SSN):

$$W = \sqrt{167273 + 1123.6 \cdot (c\_F_{10.7} - 63{,}7)} - 408{,}99 = 8.56440364101058,$$

$$r = 0.01 \cdot W = 0.0856440364101058.$$

Q.4.3 Computation of the solar declination dec, rad:

$$dec = \arcsin\{0.39795 \cdot \sin[\pi \cdot (3 - 3.167)/6]\} = -0.0347598176827493.$$

Q.4.4 Computation of the geomagnetic latitude, rad:

In this subsection latitude glat and longitude glong of the given location shall be brought to radians.

$$\begin{aligned}
mlat &= \arcsin[0.98 \cdot \sin(glat) + 0.2 \cdot \cos(glat) \cdot \cos(glong + 1.2)] = \\
&= \arcsin[0.98 \cdot \sin(1.0471975511966) + \\
&+ 0.2 \cdot \cos(1.0471975511966) \cdot \cos(0.523598775598299 + 1.2)] = \\
&= 0.985383484299204.
\end{aligned}$$

Q.4.5 Computation of the geomagnetic longitude, rad:

In this subsection latitude glat and longitude glong of the given location shall be brought to radians.

$$smlon = 0.2 \cdot \cos(glat) \cdot \sin(glong + 1.2) = 0.098834840300598;$$

$$cmlon = 0.98 \cdot \sin(mlat) - \sin(glat) = -0.049211041180020;$$

$$\mathrm{mlong}' = \mathrm{arctg}\left(\frac{\mathrm{smlon}}{\mathrm{cmlon}}\right) = -1.108820608385621.$$

Provided cmlon < 0, the geomagnetic longitude shall be computed using the formula:

$$\mathrm{mlong} = \mathrm{mlong}' + \pi = 2.032772045204172.$$

Q.4.6 Computation of the magnetic inclination, rad:

$$\mathrm{dip} = \mathrm{arctg}[2 \cdot \mathrm{tg}(\mathrm{mlat})] = \mathrm{arctg}[2 \cdot \mathrm{tg}(0.985383484299204)] = 1.25072715977841$$

Q.4.7 Computation of the F2-layer peak height:

$$\begin{aligned} &\mathrm{h\,max} = 240 + \mathrm{A} + \mathrm{B} + \mathrm{C} = \\ &= 240 - 8.88568934667369\mathrm{E}{-16} + 6.3095323959819 - 29.1009481763898 = \\ &= 217.208584219592 \end{aligned}$$

where A, B, C are values computed using the formulae:

$$\begin{aligned} &\mathrm{A} = 10 \cdot \cos(\mathrm{mlat}) \cdot \cos[\pi \cdot (\mathrm{month}/3 - 1.5)] = \\ &= 10 \cdot \cos(0.985383484299204) \cdot \cos[\pi \cdot (3/3 - 1.5)] = \\ &= -8.88568934667369\mathrm{E}{-16}, \end{aligned}$$

$$\begin{aligned} &\mathrm{B} = \mathrm{r} \cdot (75 + 83 \cdot \cos(\mathrm{mlat}) \cdot \sin(\mathrm{mlat}) \cdot \sin(\mathrm{dec})) = \\ &= 0.0856440364101058 \cdot [75 + 83 \cdot \cos(0.985383484299204) \times \\ &\times \sin(0.985383484299204) \cdot \sin(-0.0347598176827493)] = \\ &= 6.3095323959819, \end{aligned}$$

$$\begin{aligned} &\mathrm{C} = 30 \cdot \cos(\mathrm{slt} - 4.5 \cdot |\mathrm{mlat}| - \pi) = \\ &= 30 \cdot \cos(4.18879020478639 - 4.5 \cdot |0.985383484299204| - \pi) = \\ &= -29.1009481763898. \end{aligned}$$

Q.4.8 Computation of m3000 parameter:

$$\mathrm{m3000} = \frac{1490}{\mathrm{h\,max} + 176} = \frac{1490}{217.208584219592 + 176} = 3.78933741479024$$

Q.4.9 Computation of the F2-layer peak concentration, $1 \cdot 10^{11}\ \mathrm{m}^{-3}$:

$$N\max = 0.66 \cdot D \cdot E \cdot F \cdot G \cdot H \cdot I \cdot J = 3.40176218779566,$$

where D, E, F, G, H, I, J are the values computed using the formulae

$$D = K \cdot L \cdot \exp\{-1.1 \cdot [\cos(slt - 0.873) + 1]\} = 0.874325439422186,$$

where $K = 0.9 + 0.32 \cdot \sin(dec) \cdot \sin(mlat) = 0.890730905733522,$

$$L = 1 + \sin(mlat) \cdot \left[\cos\left(slt + \frac{\pi}{4}\right)\right]^2 \cdot \sin(dec) = 0.998059651309945,$$

$$E = \left\{1 - 0.4 \cdot [\cos(mlat)]^{10}\right\} \times$$
$$\times \left\{1 + 0.6 \cdot [\cos(mlat)]^{10} \cdot \left[\cos\left(slt + \frac{\pi}{4}\right)\right]^2\right\} \cdot M \cdot N = 1.07453125208824,$$

where M and N are the interim calculation values determined as per the formulae

$$M = 1 + [\cos(mlat)]^8 \cdot [\cos(|mlat| - 0.2618)]^{12} \cdot \left(1 - 0.2 \cdot r + 0.6 \cdot \sqrt{r}\right) \times$$
$$\times \exp\{0.25 \cdot [1 - \cos(slt - 0.873)]\} = 1.00051897465433$$

$$N = 1 + 0.05 \cdot \left[0.5 - \cos\left(2 \cdot month \cdot \frac{\pi}{6}\right) + \cos\left(month \cdot \frac{\pi}{6}\right)\right] = 1.075,$$

$$F = \left\{1.2 - 0.5 \cdot [\cos(mlat)]^2\right\} \cdot O \cdot P \cdot \exp\left(3 \cdot \cos\left(mlat \cdot \frac{\sin(slt) - 1}{2}\right)\right) =$$
$$= 6.45730603201738,$$

where O and P are the interim values determined as per the formulae

$$O = 1 + 0.05 \cdot r \cdot \cos\left(month \cdot \frac{\pi}{6}\right) \cdot [\sin(mlat)]^3 = 1,$$

$$P = 1 - 0.15 \cdot \exp\left[-\sqrt{\left(12 \cdot mlat + 4 \cdot \frac{\pi}{3}\right)^2 + \left(\frac{month}{2} - 3\right)^2}\right] =$$
$$= 0.999999984471864,$$

$$G = \begin{cases} 1 + r + (0.204 + 0.03 \cdot r) \cdot r^2 \quad \text{if } r < 1.1 \\ 2.39 + 1.53 \cdot [\sin(\text{mlat})]^2 \cdot \{[1 + r + (0.204 + 0.03 \cdot r) \cdot r^2] - 2.39\}, \ \text{if} \quad r \geq 1.1 \end{cases} =$$

$$= 1.0871592019243,$$

$$H = 1 + 0.1 \cdot [\cos(\text{mlat})]^3 \cdot \cos\left[2 \cdot \left(\text{mlong} - 7 \cdot \frac{\pi}{18}\right)\right] =$$

$$= 0.999135201805969,$$

$$I = R \cdot \left\{1 + 0.03 \cdot \left[0.5 - \cos\left(2 \cdot \text{month} \cdot \frac{\pi}{6}\right) + \cos\left(\text{month} \cdot \frac{\pi}{6}\right)\right]\right\} =$$

$$= 1.04558943719772,$$

where

$$R = 1 + \{0.15 - 0.5 \cdot (1 + r) \cdot (1 - \cos(\text{mlat})) \cdot \exp[-0.33 \cdot (\text{month} - 6)^2]\} \times$$

$$\times \exp\left[-18 \cdot \left(|\text{dip}| - 4 \cdot \frac{\pi}{18}\right)^2\right] = 1.00056405473466,$$

$$J = \frac{(S + T)}{G} + U = 0.748062500429652,$$

where

$$S = 0.2 \cdot [1 - \sin(|\text{mlat}| - 0.5236)] \times$$

$$\times \left\{1 + 0.6 \cdot \cos\left[\frac{\pi}{3} \cdot (\text{month} - 4)\right]\right\} \cdot \cos\left[\frac{\pi}{6} \cdot (\text{month} - 1)\right] = 0.072079085569345$$

$$T = \left[0.13 - 0.06 \cdot \sin\left(\left||\text{mlat}| - \frac{\pi}{9}\right|\right)\right] \cdot \cos\left[\frac{\pi}{3} \cdot (\text{month} - 4.5)\right] - V =$$

$$= -0.086906884576849,$$

$$\text{where } V = \begin{cases} 0, \ \text{if} \ 1 - \cos(\text{slt}) \leq 0.0001 \\ [\cos(\text{mlat} + \text{dec})]^3 \cdot [0.15 + 0.3 \cdot \sin(|\text{mlat}|)] \times \\ \times \exp\{0.25 \cdot \ln[1 - \cos(\text{slt})]\}, \ \text{если} \ 1 - \cos(\text{slt}) > 0.0001 \end{cases} = 0.086906884576849,$$

$$U = 0.7 \cdot \left\{X + 0.1778 \cdot \frac{r^2}{G} \cdot \cos\left[\frac{\pi}{3} \cdot (\text{month} - 4.3)\right]\right\} \cdot \exp(-1 \cdot Y \cdot Z) =$$

$$= 0.761701532303971,$$

where

$$X = 1 + 0.085 \cdot \left\{ \left[\cos\left(\frac{\pi}{6} \cdot (0.5 \cdot \mathrm{month} - 1) \right) \right]^3 \cdot \cos\left(\mathrm{mlat} - \frac{\pi}{6} \right) + \right.$$

$$\left. + \cos\left(\mathrm{mlat} + \frac{\pi}{4} \right) \cdot \left[\cos\left(\frac{\pi}{6} \cdot [0.5 \cdot \mathrm{month} - 4] \right) \right]^2 \right\} = 1.06744908854236,$$

$$Y = 1.3 + \left\{ 0.139 \cdot \left[1 + \cos\left(\mathrm{mlat} - \frac{\pi}{4} \right) \right] + 0.0517 \cdot r \right\} \cdot r^2 = 1.30205125974859,$$

$$Z = \cos[\mathrm{mlat} + \mathrm{dec} \cdot \cos(\mathrm{slt})] - \cos(\mathrm{mlat}) = -0.0145685963255228,$$

Q.4.10 Computation of the F2-layer critical frequency:

$$\mathrm{fof}\,2 = \sqrt{\frac{N\max}{0.124}} = 5.23770618024819.$$

Q.4.11 Computation of the height scale (thickness parameter) below the F2-layer peak:

$$B_{bot} = 0.385 \cdot \frac{N\max}{0.01 \cdot \exp\left[-3.467 + 0.857 \cdot \ln\left(\mathrm{fof}\,2^2 \right) + 2.02 \cdot \ln(\mathrm{m3000}) \right]} =$$

$$= 16.6555096230286.$$

Q.4.12 Computation of the height scale (thickness parameter) above the F2-layer peak:

$$B_{top} = \frac{B_{bot} \cdot b_{ok}}{\left(0.041163 \cdot \frac{B_{bot} \cdot b_{ok} - 150}{100} - 0.183981 \right) \cdot \frac{B_{bot} \cdot b_{ok} - 150}{100} + 1.424472} =$$

$$= 91.4851741931585,$$

where $b_{ok} = 8$.

As the month number is 3 (month=3), then in consistency with the algorithm one shall use the formula

$$b_{ok} = -7.77 + 0.097 \cdot \left(\frac{h\max}{B_{bot}} \right)^2 + 0.153 \cdot N\max = 9.24765491515162$$

Then one shall check parameter b_{ok} for falling into the given interval:

if $b_{ok} < 2$, then $b_{ok} = 2$, if $b_{ok} > 8$, then $b_{ok} = 8$.

Q.4.13 Computation of correction coefficients c_N and c_h:

In this subsection glat is expressed in radians and glong is expressed in degrees.

Allowing for Ap = 30 > 27 nT one shall compute correction coefficients c_N and c_h:

- Calculation of the atmospheric temperature T_{ns} for Ap = 0:

$a_{ns} = 3.3 \cdot W + 705$ = 733.262532015335;

$z_{ns} = (\text{month} \cdot 30.5 - 96)/365$ = – 0.0123287671232877;

$b_{ns} = 0.2 \cdot \sin(2 \cdot \pi \cdot z_{ns}) \cdot \sin(\text{glat})$ = – 0.0134037313851232;

$dLt' = ut + \frac{\text{glong}}{15}$ = 16;

$C_{slt} = \cos[15 \cdot (dLt - 15) \cdot \pi / 180]$ = 0.965925826289068;

$c_{ns} = 0.12 \cdot \cos(\text{glat}) \cdot C_{slt}$ = 0.0579555495773441;

$T_{ns} = a_{ns} \cdot (1 + b_{ns} + c_{ns}) \cdot 0.001$ = 0.76593071102885.

- Calculation of the atmospheric temperature T_{nd} under geomagnetic disturbances A_p:

$$T_{nd} = T_{ns} + (4.5 \cdot A_p - 100) \cdot [\sin(\text{mlat})]^2 \cdot 0.001 = 0.790245058804246.$$

- Calculation of coefficient c_N:

$$dLog = \ln\left(\frac{2 - 1.8 \cdot T_{ns} + 0.8 \cdot T_{ns}^2}{2 - 1.8 \cdot T_{nd} + 0.8 \cdot T_{nd}^2}\right) = 0.0124514466089894;$$

$$dSlag1 = -0.0022 \cdot (1 - 0.3 \cdot C_{slt}) \cdot (4.5 \cdot A_p - 100) \cdot [\sin(mlat)]^2 = -0.0379908998385558;$$

$$dSlag2 = 0.0007 \cdot (1 - 0.3 \cdot C_{slt}) \cdot A_p \cdot [\cos(mlat)]^4 = 0.00139020726016566;$$

$$dExp = 1.3 \cdot (dLog + dSlag1 + dSlag2) = -0.031394019760221;$$

$$c_N = e^{dExp} = 0.969093655789602;$$

- Calculation of coefficient c_h:

$$c_h = 0.2 \cdot A_p \cdot (1 - 0.001 \cdot A_p) \cdot (1 + 0.01 \cdot mlat) = 5.87734931878621.$$

Q.4.14 Correction of the altitude parameters of TEC distribution - Nmax, hmax, B_{top}, B_{bot} – using the obtained c_N and c_h coefficients

- F2-layer peak height:

$$h'_{max} = h\max + c_h = 223.085933538378;$$

- F2-layer peak concentration:

$$N'_{max} = N\max \cdot c_N = 3.29662615469773;$$

- The height scale (thickness) above F2-layer peak

$$B'_{top} = B_{top} \cdot [1 - \ln(c_N)] = 94.3572615595458;$$

- The height scale (thickness) below F2-layer peak

$$B'_{bot} = B_{bot} \cdot \left[1 - \frac{\ln(c_N)}{2}\right] = 16.9169513221396.$$

Q.4.15 Correction of F2-layer peak concentration Nmax:

$$\tilde{N}_{max} = N'_{max} \cdot c\_A = 2.63730092375818$$

Q.4.16 Computation of TEC in the given location in space

Provides the condition $h > h'_{max}$ is met, the following computations shall be carried out.

$y' = (h - h'_{max}) / B'_{top} = 5.05434408098688;$

$y = y' \Big/ \left(1 + \dfrac{12.5 \cdot y'}{100 + 0.1 \cdot y'}\right) = 3.10346011314242;$

$A = 4 \cdot \tilde{N}_{max} = 10.5492036950327;$

$N_e = A \cdot \dfrac{e^y}{(1+e^y)^2} = 0.433770428050415.$

Appendix R
Algorithm for determination of SV's antenna phase center position in PZ-90 geodetic coordinate system

The algorithm in Appendix J, is utilized to compute the coordinates of position for the SV's center of mass. However when UE processes pseudorange measurements, the position of the SV shall be identified with the position of the transmitting antenna phase center. The difference between the SV's center of mass and the antenna phase center can reach as much as several meters. That is why high-precision measurements require that the coordinates of position for the SV's antenna phase center be transformed from the SV-fixed reference system to the PZ-90 geodetic coordinate system. This transformation depends on the attitude of the SV-fixed reference system which in its turn is governed by the need for continuous solar array pointing.

Definitions

Sun-SV-Earth (SSE) plane and angle. Used only for clarifications, never computed in UE.

T_{in} is the time from the beginning of the current day in signal time at the instant of the SV's transition from sun-pointing to noon/midnight turn maneuver (broadcast within the message).

α_s is the angle between the SV's orbital plane and the Earth-Sun vector (used only for clarifications, never computed in UE).

sn is the sign of the noon/midnight turn maneuver (broadcast within the message).

τ_1 is either the interval between T_{in} and the termination instant of angular rate increment with the constant angular acceleration $\dot{\omega}$, or the duration of angular rate decrement with the constant acceleration $\dot{\omega}$ up to the value ω_{out} at the time the SV exits the noon/midnight turn maneuver (broadcast within the message).

τ_2 is the time the SV takes to perform the turn maneuver with a given maximum angular rate ω_{max} (broadcast within the message).

ψ is the yaw angle, i.e. the angle between the SV's linear velocity vector and axis -b (minus b) of the SV-fixed reference system (see Figure R.1, computed in UE).

Ψ_{in} is the yaw angle at time T_{in} of the SV's transition from sun-pointing to turn maneuver (broadcast within the message).

Ψ_t is the current angle of the turning SV (computed in UE).

Ψ_{ft} is the full turn angle (computed in UE).

ω_{max} is the given maximum angular rate of the turning SV (broadcast within the message).

ω_{in} is the angular rate of the SV, entering the turn maneuver (broadcast within the message).

ω_{out} is the angular rate with which the SV is to exit the turn maneuver (computed in UE).

$\dot{\omega}$ is the constant angular acceleration or deceleration of the SV's turn maneuver along r axis of the SV-fixed reference system (broadcast within the message).

R.1 Regimes, preconditioning attitude of Glonass-K2-fixed reference system

Let us introduce a SV-fixed reference system and a SV's displaced reference system. Directions of axes r, b, n of the Glonass-K-fixed reference system are shown on Figure R.1 and determined by the need for continuous solar array pointing.

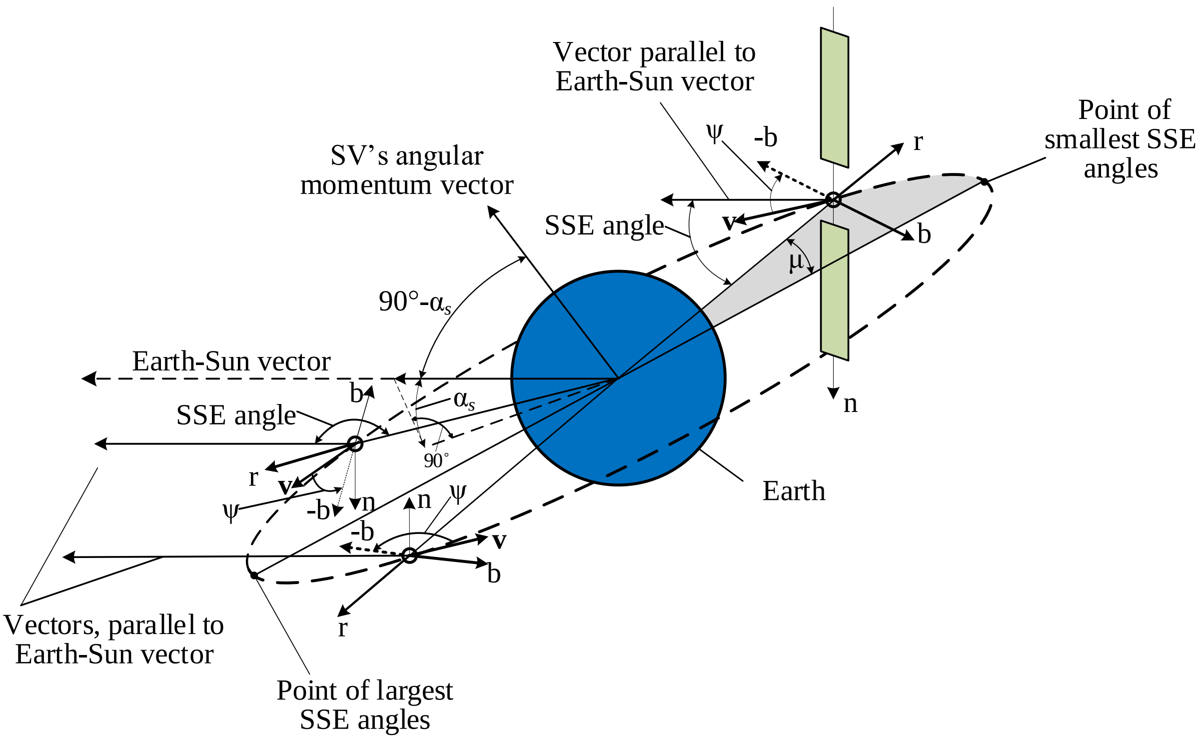


Figure R.1 – SV's passing of points of largest and smallest SSE angles

Unit vector $\mathbf{e}^r$ of axis r is oriented along the straight line connecting the Earth's center and a SV and directed off-Earth. Provided the module (magnitude) of angle α_s between the orbital plane and the direction to the Sun is large ($|\alpha_s| \geq 2^\circ$), the SSE plane rotates quite slowly. In this case unit vector $\mathbf{e}^b$ of b axis is maintained in the SSE plane by the SV's attitude control system as perpendicular to axis r in the direction off the Sun. Unit vector $\mathbf{e}^n$ of n axis completes a right-handed system. The Glonass-K2 solar arrays are oriented along axis n and rotate around this axis in the direction to the Sun (see Figure R.1).

Axes of the SV's displaced reference system match those of the SV-fixed reference system however its origin is moved into the SV's center of mass. The position of the SV's antenna phase center remains unchanged in the SV-fixed reference system while the position of the SV's center of mass changes over the SV's lifetime. This means that the position of the SV's antenna phase center in the SV's displaced reference system also changes over the SV's lifetime. Therefore coordinates Δx_{pc}, Δy_{pc}, Δz_{pc} of the antenna phase center in the SV's displaced reference system are broadcast within the navigation message.

Transformation of coordinates Δx_{pc}, Δy_{pc}, Δz_{pc} of the SV's antenna phase center from the SV's displaced reference system into those of the geocentric orthogonal coordinate system shall be performed using the formula

$$\begin{bmatrix} x_{pc} \\ y_{pc} \\ z_{pc} \end{bmatrix} = \begin{bmatrix} x_{cm} \\ y_{cm} \\ z_{cm} \end{bmatrix} - \mathbf{E} \begin{bmatrix} \Delta x_{pc} \\ \Delta y_{pc} \\ \Delta z_{pc} \end{bmatrix}, \qquad \text{(R.1)}$$

where x_{cm}, y_{cm}, z_{cm} are the coordinates of the SV's center of mass estimated using ephemeris data (see Appendix J for the calculation algorithms);

$\mathbf{E}$ is (3×3)-transformation matrix determined by the formula

$$\mathbf{E} = \begin{bmatrix} \mathbf{e}^r & \mathbf{e}^b & \mathbf{e}^n \end{bmatrix} = \begin{bmatrix} e_x^r & e_x^b & e_x^n \\ e_y^r & e_y^b & e_y^n \\ e_z^r & e_z^b & e_z^n \end{bmatrix}, \qquad \text{(R.2)}$$

$\mathbf{e}^{r} = \begin{bmatrix} e_x^r & e_y^r & e_z^r \end{bmatrix}^T$, $\mathbf{e}^{b} = \begin{bmatrix} e_x^b & e_y^b & e_z^b \end{bmatrix}^T$, $\mathbf{e}^{n} = \begin{bmatrix} e_x^n & e_y^n & e_z^n \end{bmatrix}^T$ are unit vectors of axes r, b, n of the SV-fixed reference determined in the PZ-90 coordinate system at the time of transformation.

Thus, the transformation of coordinates Δx_{pc}, Δy_{pc}, Δz_{pc} of the SV's antenna phase center into those of the PZ-90 coordinate system comes to the computation of transformation matrix **E** or, equally, to the computation of unit vectors $\mathbf{e}^r$, $\mathbf{e}^b$, $\mathbf{e}^n$ of the axes of the SV-fixed reference system realized in the PZ-90 at the instant of the transformation.

The algorithm for computation of transformation matrix **E** relates to the SV-fixed reference system axes directions, the SV's position in orbit determined by angle μ, and the position of the SV's orbital plane with respect to the direction to the Sun, determined by angle α_S (Figure R.1).

An SSE plane constantly rotates as a SV orbits around the Earth. This results in the continuous rotation of the SV-fixed reference system around its r axis. The angular position of the SV-fixed reference system is used to be determined by yaw angle ψ, measured from the direction of the SV's velocity vector **v** to the direction of axis –b (minus b). The positive direction of such measurement is counter-clockwise, if observing the plane formed by **v** and axis –b as from the positive direction of axis r. Examples for yaw angle ψ variation and its rate for various angles α_S between the orbital plane and the direction to the Sun are shown in Figures R.2 и R.3.

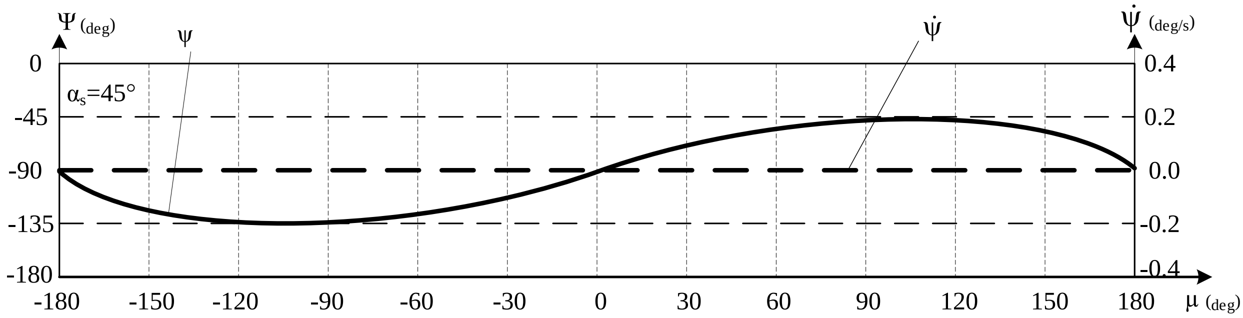


Figure R.2 – The typical pattern of yaw angle ψ variation and its derivative $\dot{\psi}$ as a function of angle μ for large modulo angle α_S (α_S =45°)

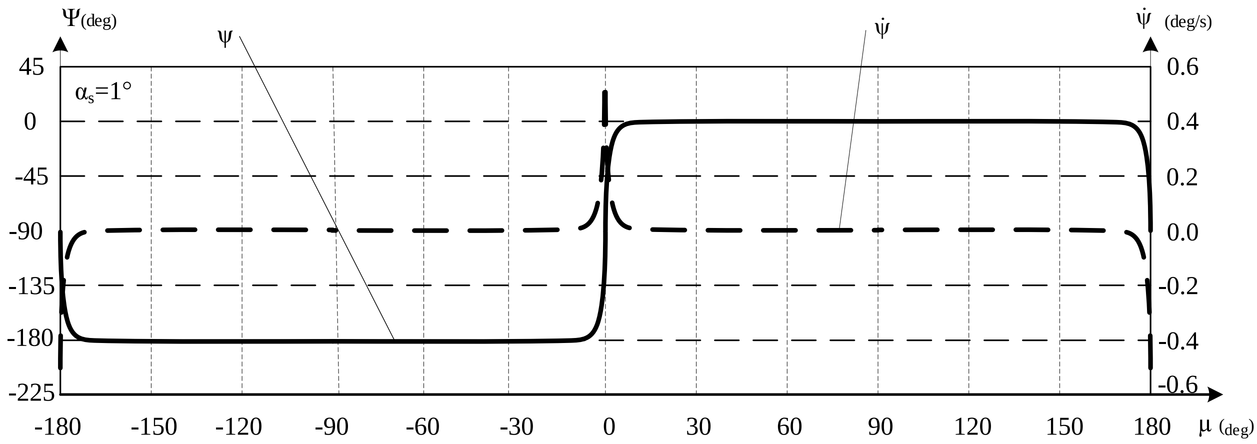
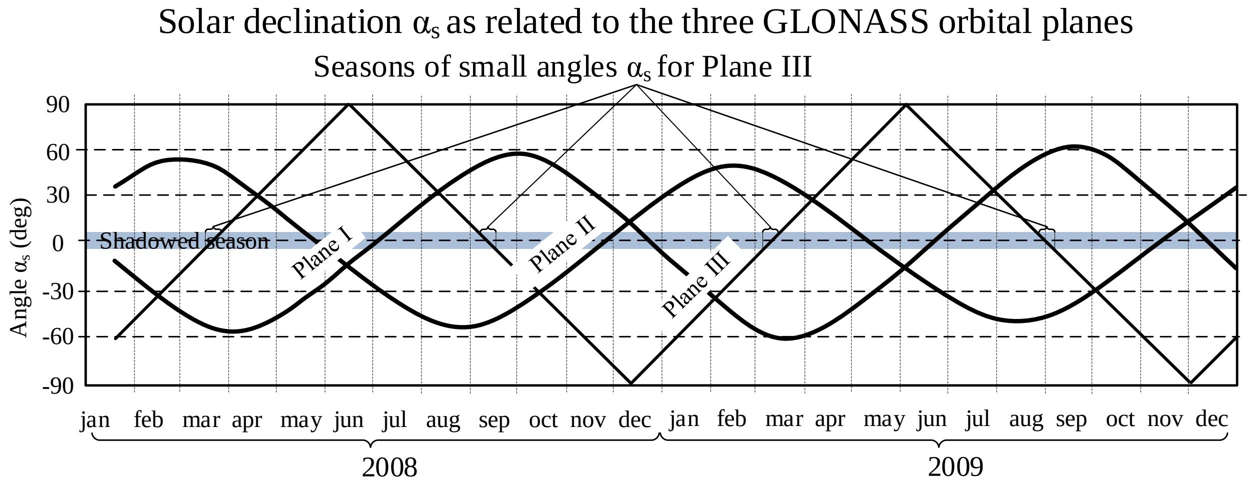


Figure R.3 – The typical pattern of yaw angle ψ variation and its derivative $\dot{\psi}$ as a function of angle μ for small modulo angle α_s (α_s=1°)

Figure R.2 demonstrates, that large modulo α_s values, characteristic of the most part of the orientation process repeatability time, result in a very small rotation rate $\dot{\psi}$ for -b axis that can be implemented through the on-board attitude control capabilities. In this case the orientation of the SV-fixed reference system axes changes very slowly and is only determined at any time by the direction to the Sun (i.e. axes b and -b are continuously maintained by the SV's attitude control system within the SSE plane). This slow rotation of the SV-fixed reference system will further be referred to as the sun-pointing regime.

However, as Figure R.3 demonsrates, the small modulo α_s values ($|\alpha_s|$<2˚), which are observed twice within the orientation process repeatability time in sections adjacent to the points of small ($\mu \approx 0$) and large ($\mu \approx 180$) SSE angles, result in a drastic increase of rotation rate for b axis that is far beyond the SV's attitude control capabilities. In this case the turn maneuver regime shall be employed, under which the SV starts rotating about axis r prior to the instants when the required rotation rate outperforms the SV's attitude control capabilities. Thus, during the turn maneuver sections of orbit the orientation of the SV-fixed reference system axes will be determined by the SV's attitude control algorithms, responsible for performing the turn.

Figure R.4 demonstrates the diagram for variation of angle α_s between the orbital plane and the direction to the Sun for the three GLONASS orbital planes. The diagram reveals that appearance of time intervals of small angle α_s is of seasonal nature.



- Angle α_s varies as the Earth moves around the Sun
- Seasons of small angles α_s repeat every 6 months for Plane III and about every 176 day for Plans I and II
- In a season of small angels α_s during each orbital period (~ 11 h 15 min) a satellite passes once through the small angle zone and once – through the wide angle zone
- Earth shadowed area lasts for not more than 53 min

Figure R.4 – Season repeatability of small angles α_c for GLONASS orbital planes

As an example, seasons for Plane III are marked with curly braces on Figure R.4. A season of small angles α_s comes every 6 months for Plane III and about every 176 days for Planes I and II. These seasons for Planes I and II lasts about 30 days, and for Plane III – for about 40 days.

Within the season of small angles α_s a satellite passes once through the small SSE angle zone and once – through the large SSE angle zone during each orbital period (~ 11 h 15 min). In the beginning of the season the rotation rate of b axis for small and large SSE angles is minor and grows gradually until it reaches its maximum in the middle of the season. Then it is gradually decreased until it reaches value needed for Sun orientation.

The turn maneuver shall be performed when the angle between the orbital plane and the direction to the Sun is smaller than 2°. At the starting instant of the turn the SSE angle is more than 175° when passing large SSE angles and less than 5° when passing small SSE angles. The maximum duration of the turn maneuver is 15 min. As a result during the season each GLONASS satellite shall enter the turn maneuver regime 4 times a day. The aggregate number of turn maneuvers in the season is less than or equal to 22. The rapid calculation shows that the turn maneuver regime takes about 0.1% of a SV’s orbital lifetime. Thus the most of the time the orientation of a SV-fixed reference system is determined by the direction to the Sun.

There is a special attribute P2 for a SV's orientation within the navigation message. P2 = 1 indicates the turn maneuver. In this case the user can either reject using measurements coming from this SV for precise positioning or use parameters of String Type 16 to recalculate the coordinates of the SV's center of mass into the coordinates of the SV's antenna phase center.

It can be concluded that depending on the SV's orientation (Sun-pointing or turn maneuver), transformation matrix elements **E** (R.2) shall be computed differently.

R.2 Computation of transformation matrix for sun-pointing regime as consistent with SV-fixed reference system axes' directions adopted in GLONASS

The input data for computation of transformation matrix **E** (R.2) for sun-pointing regime shall be the SV's center of mass radius vector $\mathbf{r} = (x_{cm} \quad y_{cm} \quad z_{cm})^{T}$ in the PZ-90 at the time of calculation and the unit vector $\mathbf{e}^{c} = [e_x^c \quad e_y^c \quad e_z^c]^{T}$ in the PZ-90 originating from the Earth's center and directed to the center of the apparent Sun. Calculation algorithms for vectors $\mathbf{r}$ and $\mathbf{e}^{c}$ are provided in Appendices J and S, correspondingly.

Computation of unit vector $\mathbf{e}^{r}$ of axis r of a SV-fixed reference system:

$$\mathbf{e}^{r} = [e_x^r \quad e_y^r \quad e_z^r]^{T} = \frac{\mathbf{r}}{|\mathbf{r}|}, \tag{R.3}$$

where $\mathbf{r} = (x_{cm} \quad y_{cm} \quad z_{cm})^{T}$ is the radius vector of the SV's center of mass in the PZ-90, computed using the ephemeris data at the time of calculation.

Computation of unit vector $\mathbf{e}^{n}$ of axis n for a satellite coordinate system:

$$\mathbf{e}^{n} = [e_x^n \quad e_y^n \quad e_z^n]^{T} = -\frac{\mathbf{e}^{r} \times \mathbf{e}^{s}}{|\mathbf{e}^{r} \times \mathbf{e}^{s}|} = -\frac{\mathbf{e}^{r} \times \mathbf{e}^{s}}{\sqrt{1-\left((\mathbf{e}^{r})^{T}\mathbf{e}^{s}\right)^{2}}}, \tag{R.4}$$

where $\times$ is the operation of cross-product; $\mathbf{e}^{s} = [e_x^s \quad e_y^s \quad e_z^s]^{T}$ is the unit vector in the PZ-90 originating from the center of Earth and directed to the center of the apparent Sun.

Computation of unit vector $\mathbf{e}^{\mathrm{b}}$ of axis b for a SV-fixed reference system:

$$\mathbf{e}^{\mathrm{b}} = \begin{bmatrix} \mathrm{e}_{\mathrm{x}}^{\mathrm{b}} & \mathrm{e}_{\mathrm{y}}^{\mathrm{b}} & \mathrm{e}_{\mathrm{z}}^{\mathrm{b}} \end{bmatrix}^{\mathrm{T}} = \mathbf{e}^{\mathrm{r}} \times \mathbf{e}^{\mathrm{n}} . \qquad \text{(R.5)}$$

Unit vectors $\mathbf{e}^{\mathrm{r}}$, $\mathbf{e}^{\mathrm{b}}$, $\mathbf{e}^{\mathrm{n}}$ form vector columns of transformation matrix $\mathbf{E}$ (see (R.2)) for sun-pointing regime.

R.3 Computation of transformation matrix for turn maneuver regime as consistent with SV-fixed reference system axes' directions adopted in GLONASS

The input data for computation of transformation matrix $\mathbf{E}$ (see (R.2)) for turn maneuver regime shall be the SV's center of mass radius vector $\mathbf{r} = \begin{pmatrix} \mathrm{x}_{\mathrm{cm}} & \mathrm{y}_{\mathrm{cm}} & \mathrm{z}_{\mathrm{cm}} \end{pmatrix}^{\mathrm{T}}$, its velocity vector $\mathbf{e}^{\mathrm{v}} = \begin{bmatrix} \mathrm{e}_{\mathrm{x}}^{\mathrm{v}} & \mathrm{e}_{\mathrm{y}}^{\mathrm{v}} & \mathrm{e}_{\mathrm{z}}^{\mathrm{v}} \end{bmatrix}^{\mathrm{T}}$ in the PZ-90 and yaw angle parameter ψ at the time of calculation. Calculation algorithm for vectors $\mathbf{r}$ and $\mathbf{e}^{\mathrm{v}}$ are provided in Appendix J. Calculation algorithm for yaw angle ψ at the instant of calculation is described in section R.4. If yaw angle ψ has been determined, then the columns of transformation matrix $\mathbf{E}$ (see (R.2)) shall be computed following the way below:

Unit vector $\mathbf{e}^{\mathrm{r}}$ of axis r of a SV-fixed reference system shall be computed, as before, according the formula (R.3).

Unit vector $\mathbf{e}^{\mathrm{b}}$ of axis b shall meet the following three conditions:

1) $\left(\mathbf{e}^{\mathrm{r}}\right)^{\mathrm{T}} \mathbf{e}^{\mathrm{b}} = 0$ – orthogonality of unit vector $\mathbf{e}^{\mathrm{b}}$ of axis b to unit vector $\mathbf{e}^{\mathrm{r}}$ of axis r;

2) $\left(\mathbf{e}^{\mathrm{v}}\right)^{\mathrm{T}} \mathbf{e}^{\mathrm{b}} = -\cos\psi$ – unit vector $-\mathbf{e}^{\mathrm{b}}$ of axis -b of a SV-fixed reference system for the turn maneuver is directed to unit vector $\mathbf{e}^{\mathrm{v}} = \begin{bmatrix} \mathrm{e}_{\mathrm{x}}^{\mathrm{v}} & \mathrm{e}_{\mathrm{y}}^{\mathrm{v}} & \mathrm{e}_{\mathrm{z}}^{\mathrm{v}} \end{bmatrix}^{\mathrm{T}}$ of a SV's velocity vector at yaw angle ψ. Unit vector $\mathbf{e}^{\mathrm{v}}$ in the geocentric orthogonal coordinate system shall be computed using the formula

$$\mathbf{e}^{\mathrm{v}} = \mathbf{v} / |\mathbf{v}| , \qquad \text{(R.6)}$$

where velocity vector $\mathbf{v}$ in the geocentric orthogonal coordinate system shall be computed using the ephemeris data at the time of calculation;

3) $\left(\mathbf{e}^{\mathrm{b}}\right)^{\mathrm{T}}\mathbf{e}^{\mathrm{b}}=1$ – normability of unit vector $\mathbf{e}^{\mathrm{b}}$ of axis b.

Taking the above condition in scalar form yields the following system of equations as related to components e_x^b, e_y^b, e_z^b of vector $\mathbf{e}^b=\left[e_x^b \quad e_y^b \quad e_z^b\right]^T$:

$$\begin{aligned}
&e_x^r e_x^b + e_y^r e_y^b + e_z^r e_z^b = 0\\
&e_x^v e_x^b + e_y^v e_y^b + e_z^v e_z^b = -\cos\psi\\
&\left(e_x^b\right)^2 + \left(e_y^b\right)^2 + \left(e_z^b\right)^2 = 1
\end{aligned} \tag{R.7}$$

Expressing e_x^b and e_y^b in terms of $e_z^r e_z^b$, $e_z^v e_z^b$, that are present in the first two equations of the system (R.7), yields:

$$\begin{bmatrix} e_x^b \\ e_y^b \end{bmatrix} = \begin{bmatrix} \dfrac{e_y^r\left(\cos\psi + e_z^b e_z^v\right)}{e_x^r e_y^v - e_y^r e_x^v} - \dfrac{e_z^b e_z^r e_y^v}{e_x^r e_y^v - e_y^r e_x^v} \\ \dfrac{e_z^b e_z^r e_x^v}{e_x^r e_y^v - e_y^r e_x^v} - \dfrac{e_x^r\left(\cos\psi + e_z^b e_z^v\right)}{e_x^r e_y^v - e_y^r e_x^v} \end{bmatrix} \tag{R.8}$$

Substituting expressions for e_x^b and e_y^b, resulting from (R.8), into the third equation of the system (R.7), yields quadratic equation for e_z^b:

$$\left(\frac{e_y^r\left(\cos\psi + e_z^b e_z^v\right)}{e_x^r e_y^v - e_y^r e_x^v} - \frac{e_z^b e_z^r e_y^v}{e_x^r e_y^v - e_y^r e_x^v}\right)^2 + \left(\frac{e_z^b e_z^r e_x^v}{e_x^r e_y^v - e_y^r e_x^v} - \frac{e_x^r\left(\cos\psi + e_z^b e_z^v\right)}{e_x^r e_y^v - e_y^r e_x^v}\right)^2 + \left(e_z^b\right)^2 = 1 \tag{R.9}$$

Solving quadratic equation (R.9) yields two roots e_{z1}^b, e_{z2}^b, which, when substituted into (R.8), result in two vector solutions $\left[e_{x1}^b \quad e_{y1}^b\right]^T$ and $\left[e_{x2}^b \quad e_{y2}^b\right]^T$, respectively. Combining these two solutions with e_{z1}^b and e_{z2}^b yields two complete full vector solutions $\mathbf{e}^{b1}=\left[e_{x1}^b \quad e_{y1}^b \quad e_{z1}^b\right]^T$, $\mathbf{e}^{b2}=\left[e_{x2}^b \quad e_{y2}^b \quad e_{z3}^b\right]^T$ of the system (R.7). Taking into account the normability of unit vectors $\mathbf{e}^r$, $\mathbf{e}^v$, vector solutions $\mathbf{e}^{b1,\,b2}$ can be represented as

$$
\begin{aligned}
\mathbf{e}^{b1,b2} &= \frac{1}{1-\left(\left(\mathbf{e}^{r}\right)^{T}\mathbf{e}^{v}\right)^{2}}\left(\left(\left(\left(\mathbf{e}^{r}\right)^{T}\mathbf{e}^{v}\right)\mathbf{e}^{r}-\mathbf{e}_{v}\right)\cos\psi \pm \mathbf{e}^{r}\times\mathbf{e}^{v}\sqrt{1-\left(\left(\mathbf{e}^{r}\right)^{T}\mathbf{e}^{v}\right)^{2}-\cos^{2}\psi}\right)= \\
&= \frac{1}{\sin^{2}\phi}\left(\left(\cos\phi\cdot\mathbf{e}^{r}-\mathbf{e}^{v}\right)\cos\psi \pm \mathbf{e}^{r}\times\mathbf{e}^{v}\sqrt{\sin^{2}\phi-\cos^{2}\psi}\right)= \\
&= \frac{1}{\sin^{2}\phi}\left(\mathbf{a}\cdot\cos\psi \pm \mathbf{b}\sqrt{\sin^{2}\phi-\cos^{2}\psi}\right),
\end{aligned}
\quad \text{(R.10)}
$$

where $\cos\phi = \left(\mathbf{e}^{r}\right)^{T}\mathbf{e}^{v}$ is the cosine of the angle between axis r and vector $\mathbf{v}$ of the SV’s linear velocity, $\sin^{2}\phi = 1-\left(\left(\mathbf{e}^{r}\right)^{T}\mathbf{e}^{v}\right)^{2}$, $\mathrm{a} = \cos\phi - \mathbf{e}^{v}$, $\mathbf{b} = \mathbf{e}^{r}\times\mathbf{e}^{v}$.

When choosing between two possible values $\mathbf{e}^{b1,b2}$ (R.10) one shall opt for the value $\mathbf{e}^{b}$, which satisfies the condition of negativity for b axis component of the direction to the Sun

$$
\left(\mathbf{e}^{c}\right)^{T}\mathbf{e}^{b} < 0. \quad \text{(R.11)}
$$

Unit vector $\mathbf{e}^{n}$ of axis n completes a right-handed system:

$$
\mathbf{e}^{n} = \mathbf{e}^{r}\times\mathbf{e}^{b} \quad \text{(R.12)}
$$

where $\mathbf{e}^{b}$ is the unit vector of axis b, previously opted for, consistent with the rule (R.11).

After opting under the condition (R.11) for one of the possible values of ± sign in formula (R.10), the value of unit vector $\mathbf{e}^{n}$ can also be computed using the formula

$$
\mathbf{e}^{n} = \frac{1}{\sin^{2}\phi}\left(\mathbf{b}\cdot\cos\psi \pm \mathbf{a}\cdot\sqrt{\sin^{2}\phi-\cos^{2}\psi}\right) \quad \text{(R.13)}
$$

Unit vectors $\mathbf{e}^{r}$, $\mathbf{e}^{b}$, $\mathbf{e}^{n}$ form vector columns of the transformation matrix $\mathbf{E}$ (see (R.2)) for the turn maneuver regime.

R.4 Computation of yaw angle ψ at a given time for turn maneuver regime as consistent with SV-fixed reference system axes' directions adopted in GLONASS

Yaw angle ψ for the given instant T in signal time shall be computed using the formula:

$$\psi = \psi_{in} + sn \cdot \psi_t , \qquad \text{(R.14)}$$

where ψ_{in} is the yaw angle at the time of transition from sun-pointing to turn maneuver, sn is the sign of the turn maneuver transmitted in the NAV message; ψ_t is the absolute value (module) of the turn angle at the given instant T (in signal time). The maximum value of angle ψ_t at the time the SV exits the turn maneuver will further on be referred to as full turn angle ψ_{ft}.

Angles ψ, ψ_{in}, ψ_t in an orbit-fixed reference system $OX_0Y_0Z_0$ are shown in Figures R.5 and R.6 for small and large SSE angles, accordingly, provided angle α_s is positive, and in Figures R.7 and R.8 for small and large SSE angles, provided angle α_s is negative (see Figure R.1). During turn maneuvers when passing small and large SSE angles and provided angle α_s (between the orbital plane and the direction to the Sun) is of the same sign, the SV turns in the opposite direction.

The origin of the orbit-fixed reference system is aligned with the SV's center of mass, axis OX_0 is directed along the straight line connecting the SV's center of mass and the Earth's center in the direction off the Earth. Axis OY_0 coincides with the SV's linear velocity vector and axis OZ_0 completes a right-handed system.

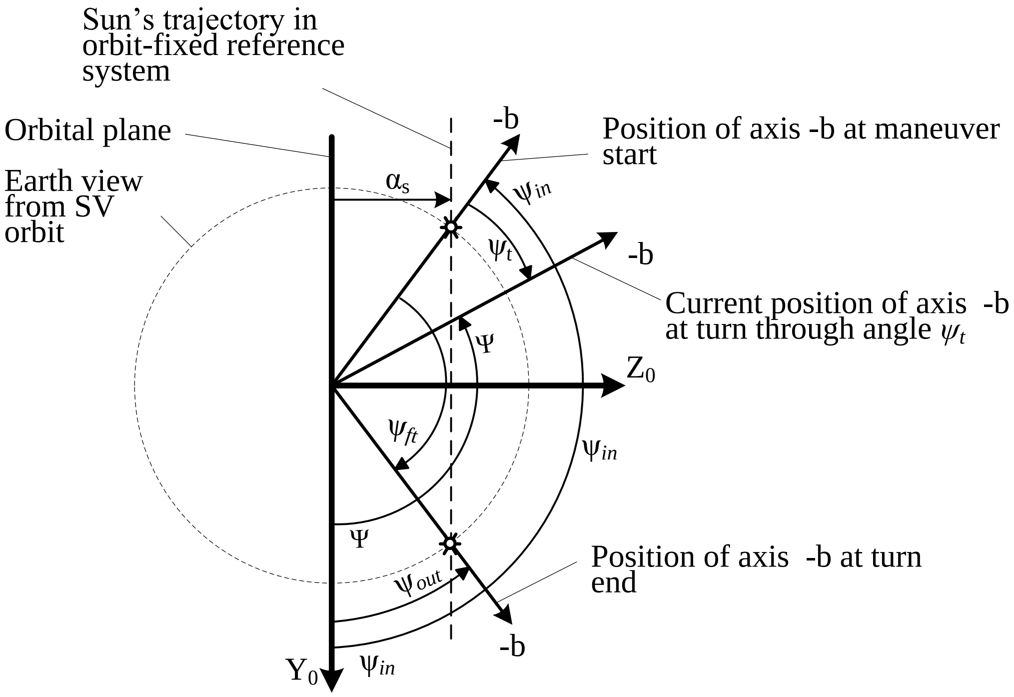


Figure R.5 – Rotation of axis -b during turn maneuver for small SSE angles provided angle α_s is positive

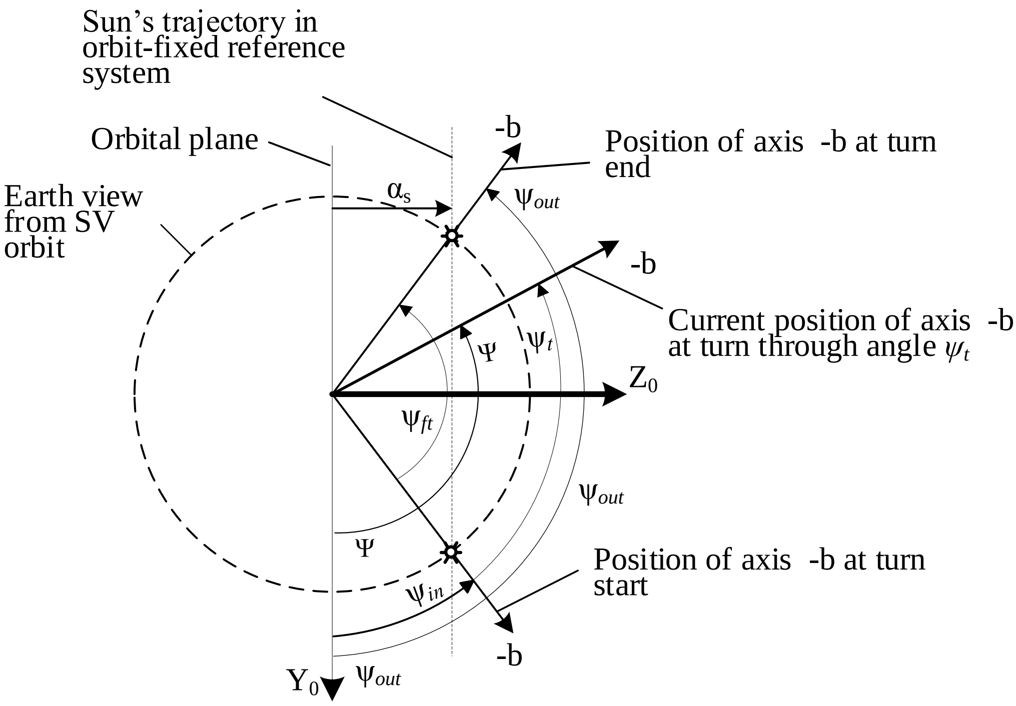


Figure R.6 – Rotation of axis -b during turn maneuver for large SSE angles provided angle α_s is positive

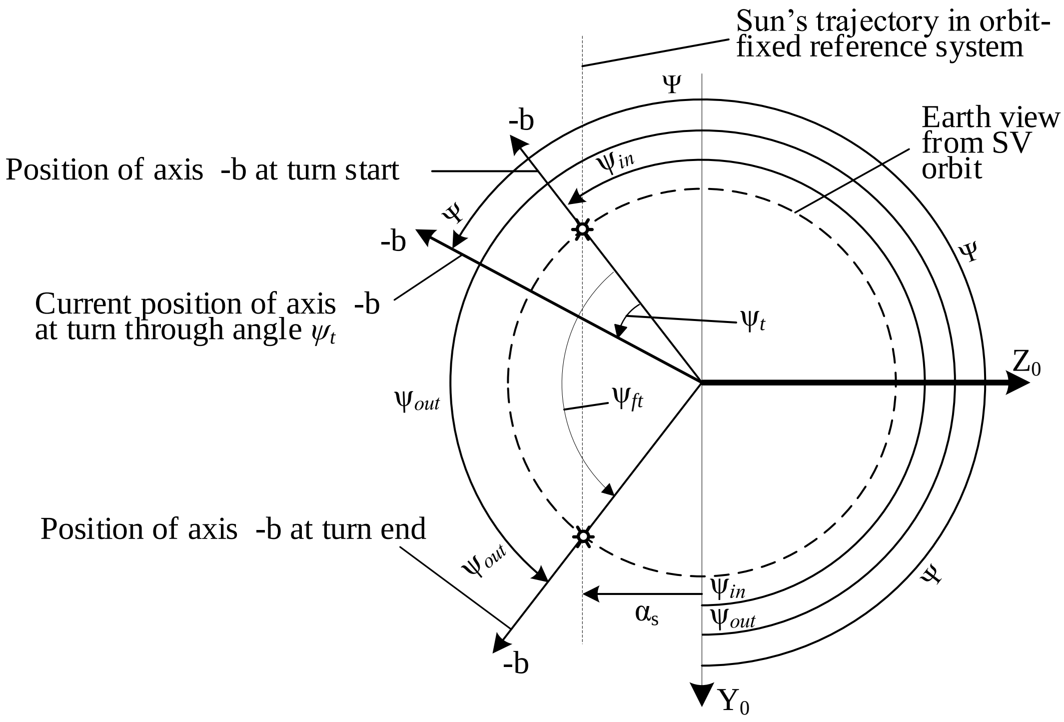


Figure R.7 – Rotation of axis -b during turn maneuver for small SSE angles provided angle α_s is negative

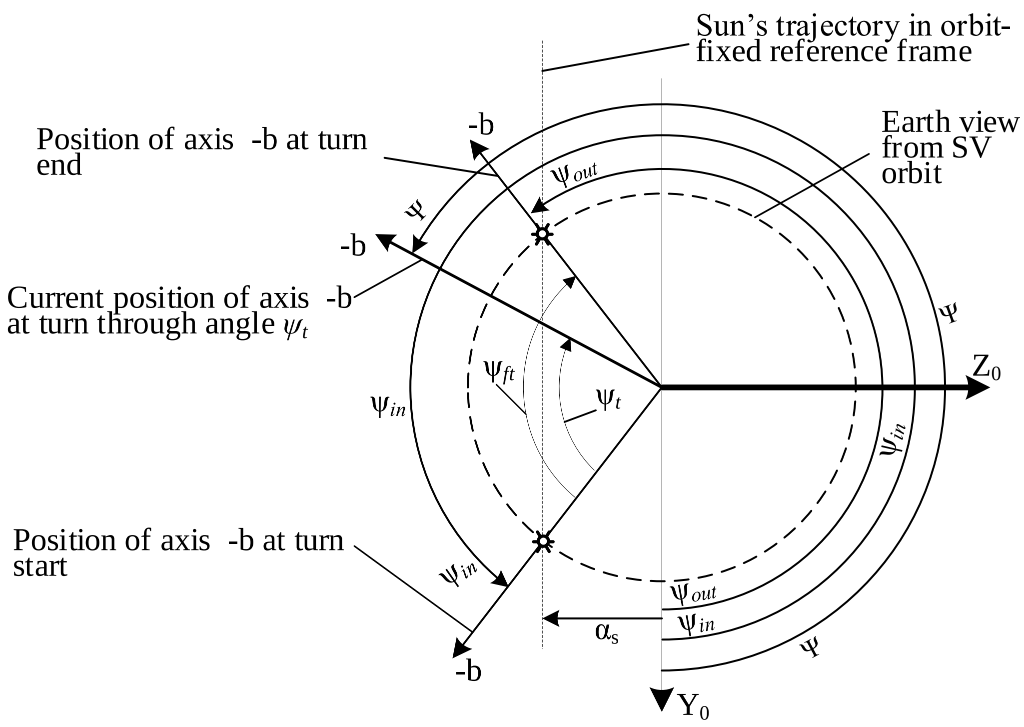


Figure R.8 – Rotation of axis -b during turn maneuver for large SSE angles provided angle α_s is negative

The positive count of all angles is counter-clockwise. The angle ranges from 0 to 2π.

To determine yaw angle ψ during turn maneuver string Type 16 includes specific data to be used to determine turn angle ψ_t at the current instant T (in signal time). A SV enters the turn at T_{in}. At time T_{in} the SV starts increasing angular rate ω_{in} with the constant angular acceleration $\dot{\omega}$. The angular rate increment is limited by either of the two factors:

- For small angles of the full turn ψ_{ft}, the angular rate increment is limited by the necessity to ensure the specified angle of the full turn ψ_{ft} and the specified angular rate ω_{out} at the turn end. The corresponding change in angular rate of the SV is shown in Figure R.7. In this Figure τ_1 is the interval between T_{in} (turn start) to the end of angular rate increment with the constant angular acceleration $\dot{\omega}$. This interval is equal to interval as the angular rate decreases with the constant angular deacceleration $\dot{\omega}$ and reaches $\omega_{out} = \omega_{in}$ at the turn end.
- For large angles of the full turn ψ_{ft}, the angular rate increment is limited by the specified value ω_{max}. In this case, to ensure the specified angle of the full turn ψ_{ft} and the specified angular rate $\omega_{out} = \omega_{in}$ at the turn end there shall be the horizontal leg of the SV's turn with the constant angular rate ω_{max}. Figure R.10 shows this leg. In this Figure τ_2 is the duration of the SV's turn with the constant maximum angular rate ω_{max}. Other designations correspond to those introduced in Figure R.9.

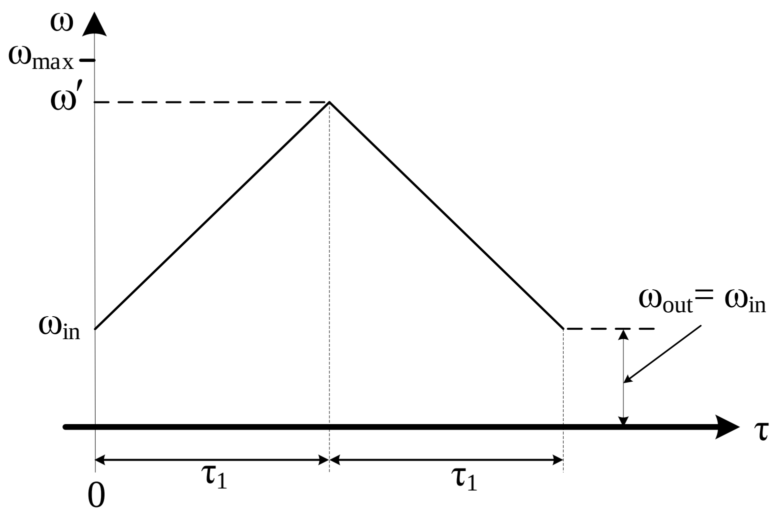


Figure R.9 – Change in angular rate for small angles of full turn ψ_{ft}

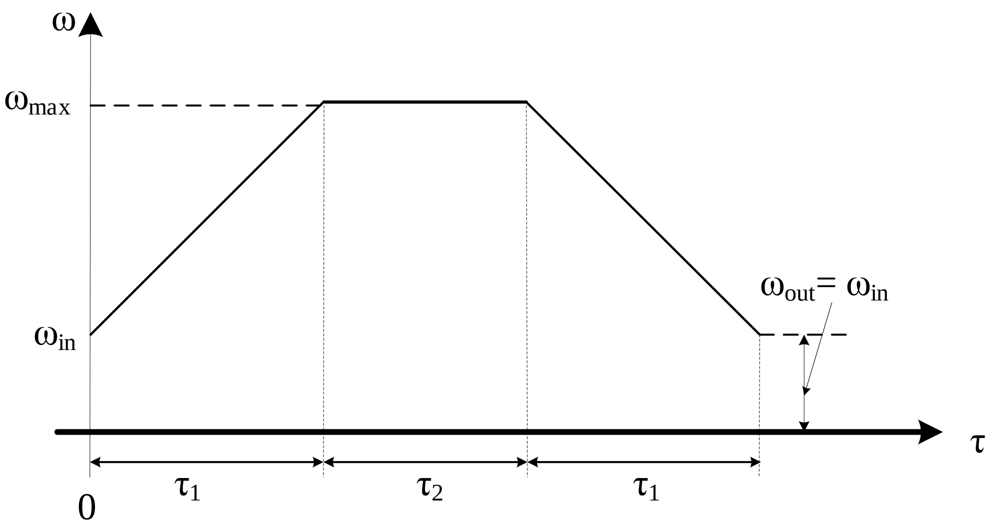


Figure R.10 – Change in angular rate for large angles of full turn ψ_{ft}

The boundary dividing the first and the second variants of the turn is the situation when the angular rate ω' (shown in Figure R.9) becomes equal to ω_{max}. Here

$$\tau_1 = \frac{\omega_{max} - \omega_{in}}{\dot{\omega}}. \tag{R.15}$$

Full turn angle ψ_{ft}, corresponding to the first variant, shall meet the condition:

$$\omega_{in}\tau_1 + \frac{\dot{\omega}\tau_1^2}{2} + \left(\omega_{in} + \dot{\omega}\tau_1\right)\tau_1 - \frac{\dot{\omega}\tau_1^2}{2} \ge \psi_{ft}. \tag{R.16}$$

Substituting (R.15) into (R.16), yields the condition under which the first variant of the turn maneuver shall be performed:

$$\frac{\omega_{max}^2 - \omega_{in}^2}{\dot{\omega}} \ge \psi_{ft}. \tag{R.17}$$

Otherwise the second variant shall be performed.

If intervals τ_1, τ_2, are specified, then turn angle ψ_t can be computed as per the formulae

$$\psi_t(\tau)=\begin{cases}\omega_{in}\tau+\dfrac{\dot{\omega}\tau^2}{2} \text{ for } 0<\tau\le\tau_1\\ \omega_{in}\tau_1+\dfrac{\dot{\omega}\tau_1^2}{2}+(\omega_{in}+\dot{\omega}\tau_1)(\tau-\tau_1)-\dfrac{\dot{\omega}(\tau-\tau_1)^2}{2} \text{ for } \tau_1<\tau\le 2\tau_1\end{cases} \quad \text{(R.18)}$$

for the first variant, where $\tau = T - T_{in}$, and as per the formulae

$$\psi_t(\tau)=\begin{cases}\omega_{in}\tau+\dfrac{\dot{\omega}\tau^2}{2} \text{ for } 0<\tau\le\tau_1\\ \omega_{in}\tau_1+\dfrac{\dot{\omega}\tau_1^2}{2}+\omega_{max}(\tau-\tau_1) \text{ for } \tau_1<\tau\le\tau_1+\tau_2\\ \omega_{in}\tau_1+\dfrac{\dot{\omega}\tau_1^2}{2}+\omega_{max}(\tau-\tau_1)-\dfrac{\dot{\omega}(\tau-\tau_1-\tau_2)^2}{2} \text{ for } \tau_1+\tau_2<\tau\le 2\tau_1+\tau_2\end{cases} \quad \text{(R.19)}$$

for the second variant.

Hence, if a user receives parameters T_{in}, τ_1, τ_2, ω_{in}, ω_{max}, $\dot{\omega}$, he can compute the value of turn angle ψ_t using (R.18) or (R.19). Opting for one of the formulae can be based on the analysis of parameter τ_2. If $\tau_2 = 0$, then formulae (R.18) shall be used. Otherwise formulae (R.19) shall be used.

R.5 Formal description of algorithm for computation of transformation matrix E for turn maneuver regime

A navigation message contains the following data used to compute transformation matrix **E** (see (R.2)) for turn maneuver regime:

T_{in} is the time of turn start (in signal time);

Ψ_{in} is yaw angle at T_{in};

sn is the sign of turn;

ω_{max} is the specified maximum angular rate of the turn;

ω_{in} is the angular rate of turn at time T_{in} ($\omega_{in} < \omega_{max}$);

$\dot{\omega}$ is the constant angular acceleration (deceleration) of the SV;

τ_1 is the interval between T_{in} and the instant of completion of angular rate increment with angular acceleration $\dot{\omega}$, or the interval of decrement of angular rate with angular acceleration $\dot{\omega}$ up to value $\omega_{out} = \omega_{in}$ at turn end;

τ_2 is the duration of the SV's turn with the specified maximum angular rate ω_{max}.

Transformation matrix **E** (see (R.2)) for turn maneuver regime shall be computed in accordance with the following algorithm:

1) Calculate T (in signal time) for the time the transformation matrix **E** shall be computed (see (R.2)):

$$T = T_{rm} - \rho/c, \tag{R.20}$$

where T_{rm} is the time in RT at the time to which the transformation matrix **E** based solution is referenced;

ρ is the pseudorange measured by the receiver at time T_{rm};

c is the speed of light.

2) Calculate interval τ between time T_{in} (in signal time) and the current time T:

$$\tau = T - T_{in} \tag{R.21}$$

3) If $\tau_2=0$

{

if $0 < \tau \le \tau_1$ $\psi_t(\tau) = \omega_{in}\tau + \frac{\dot{\omega}\tau^2}{2}$

otherwise $\psi_t(\tau) = \omega_{in}\tau_1 + \frac{\dot{\omega}\tau_1^2}{2} + (\omega_{in} + \dot{\omega}\tau_1)(\tau - \tau_1) - \frac{\dot{\omega}(\tau - \tau_1)^2}{2}$

}

otherwise

{

if $0 < \tau \le \tau_1$ $\psi_t(\tau) = \omega_{in}\tau + \frac{\dot{\omega}\tau^2}{2}$

otherwise

{

if $\tau_1 < \tau \le \tau_1 + \tau_2$ $\psi_t(\tau) = \omega_{in}\tau_1 + \frac{\dot{\omega}\tau_1^2}{2} + \omega_{max}(\tau - \tau_1)$

otherwise $\psi_t(\tau) = \omega_{in}\tau_1 + \frac{\dot{\omega}\tau_1^2}{2} + \omega_{max}(\tau - \tau_1) - \frac{\dot{\omega}(\tau - \tau_1 - \tau_2)^2}{2}$

}

}

4) Compute yaw angle Ψ at the instant T (in signal time) according to the formula (R.14).

5) Compute time T_{MT} (MT), which correspond to the instant T (in signal time) by introducing clock corrections, transmitted in the navigation message, into value T.

6) Compute Greenwich coordinates and components of velocity vector **v** using ephemeris data.

7) Compute unit vector components s $\mathbf{e}^r$ and $\mathbf{e}^v$ using formulae (R.3) and (R.6), accordingly.

8) Compute unite vector components ортов $\mathbf{e}^b$ and $\mathbf{e}^n$ using formulae (R.10) and (R.12), or (R.13), accordingly.

9) Generate transformation matrix **E** (R.2) from vector components $\mathbf{e}^r$, $\mathbf{e}^b$, $\mathbf{e}^n$.

Appendix S
Algorithm for computing direction cosines, distance to and coordinates of true and apparent Sun

Direction cosines ξ_s, η_s, $\Im_s$ and distance to the Sun r_s at the instant t_b of MT needed for integration of differential equations (J.1) shall be computed as per formulae

$$\begin{aligned}
\xi_s &= \cos\vartheta_s \cdot \cos\omega_s - \sin\vartheta_s \cdot \sin\omega_s , \\
\eta_s &= (\sin\vartheta_s \cdot \cos\omega_s + \cos\vartheta_s \cdot \sin\omega_s)\cos\varepsilon , \\
\Im_s &= (\sin\vartheta_s \cdot \cos\omega_s + \cos\vartheta_s \cdot \sin\omega_s)\sin\varepsilon , \\
r_s &= a_s \cdot (1 - e_s\cos E_s) .
\end{aligned} \tag{S.1}$$

where $\sin\vartheta_s = \dfrac{\sqrt{1-e_s^2}\sin E_s}{1-e_s\cos E_s}$;

$\cos\vartheta_s = \dfrac{\cos E_s - e_s}{1-e_s\cos E_s}$;

$E_s = q_s + e_s \sin E_s$ is the Kepler's equation for the eccentric anomaly which shall be solved by iteration, initial approximation $E_s = q_s$, until $|E_s - E_{s\,(prev.iter.)}|$ is less than 10^{-8};

$a_s = 1.49598 \cdot 10^8$ km is the major semi-axis of the Earth's orbit around the Sun;

$e_s = 0.016719$ is the eccentricity of the Earth's orbit around the Sun.

Solar nutation parameters at the instant t_b (GLONASS ephemeris setting) shall be computed using the following formulae:

- mean anomaly of the Sun, rad:

$$q_s = 6.2400601269 + 628.3019551714 \cdot T - 0.0000026820 \cdot T^2;$$

- mean tropic longitude of the Sun orbit perigee, rad:

$$\omega_s = -7.6281824375 + 0.0300101976 \cdot T + 0.0000079741 \cdot T^2;$$

- mean inclination of Earth equator to ecliptic, rad

$$\varepsilon = 0.4090926006 - 0.0002270711 \cdot T$$

- time from epoch 2000, 1 January, 12 hours (UTC(SU)) to the instant t_b in Julian centuries containing 36525 ephemeris days each:

$$T = (JD0 + (t_b - 10800) / 86400 - 2451545.0) / 36525, \tag{S.2}$$

where JD0 is the current Julian date for 00:00 in MT. The algorithm for calculation of JD0 based on navigation message data is provided in Appendix K;

t_b is the instant in MT to which GLONASS ephemerides are referenced, in seconds;

10800 is the difference between the MT and UTC(SU), in seconds;

2451545.0 is the Julian date for 12 hours, January 1, 2000 (UTC(SU)).

The coordinates of the true Sun at any arbitrary instant t_i in the orthogonal inertial geocentric coordinate system $OX_0Y_0Z_0$, whose origin O aligns to the origin of the PZ-90 coordinate system, axis OX_0 is directed to vernal equinox, axis OZ_0 is directed to the North Pole of the world and coincides with axis OZ of the PZ-90, and axis OY_0 completes a right-handed system, shall be computed using formulae

$$\begin{aligned} x_{s0} &= r_s \xi_s , \\ y_{s0} &= r_s \eta_s , \\ z_{s0} &= r_s \Im_s . \end{aligned} \tag{S.3}$$

where direction cosines ξ_s, η_s, $\Im_s$. are computed subject to substitution in (S.2) of t_b by t_i..

The coordinates of the apparent Sun at the instant t_i in the orthogonal inertial geocentric coordinate system $OX_0Y_0Z_0$ shall be computed using the following algorithm:

1) Compute the distance to the Sun r_s at the instant t_i using formulae (S.1);

2) Compute the light travel time from the Sun to the Earth using the formula

$$\Delta t = r_s / c ,$$

where c = 299792458 m/s is speed of light in vacuum.

3) Compute direction cosines and the distance to the Sun ξ_s, η_s, $\Im_s$, r_s using formulae (S.1) at the instant $(t_i - \Delta t)$;

4) Use the computed values ξ_s, η_s, $\Im_s$, r_s to calculate the coordinates of the apparent Sun using formulae (S.3).

Recalculating the coordinates of the apparent and the true Sun from the inertial coordinate system $OX_0Y_0Z_0$ into the PZ-90 coordinate system shall be performed using formulae

$$\begin{aligned}
x_s &= x_{s0} \cdot \cos S + y_{s0} \cdot \sin S, \\
y_s &= -x_{s0} \cdot \sin S + y_{s0} \cdot \cos S, \\
z_s &= z_{s0}, \\
S &= GMST + \omega_E \cdot (t_i - 10800),
\end{aligned} \qquad \text{(S.4)}$$

where GMST is the Greenwich mean sidereal time computed using the technique provided in Appendix K;

$\omega_E = 7.2921151467 \cdot 10^{-5}$ rad/s is the mean angular rate of the Earth relative to the vernal equinox.

Appendix T

Relativistic corrections in GLONASS

In GLONASS two relativistic corrections are used. The first one is caused by the difference of effective gravitational potentials on the surface of the Earth’s geoid and on the satellite orbit. This correction is accounted for with the frequency offset of the signal, generated by the 5.0 MHz frequency source within the SV, for the relative value $\Delta f / f_{st} = -4.36 \cdot 10^{-10}$, or $\Delta f = -2.18 \cdot 10^{-3}$ Hz. Thus the signal frequency of the source is 4.99999999782 MHz (for GLONASS circular orbit of 19,100 km nominal height) as it would appear to an observer located in SV, and that of 5 MHz – as it would appear to an observer on the ground.

The second relativistic correction is caused by the slight ellipticity of the GLONASS orbit. The current value of this correction for every orbit section of every SV is computed by the Control Segment and introduced into parameters $\tau^j(t_b)$, $\gamma^j(t_b)$, $\beta^j(t_b)$ transmitted within string Type 10 of navigation message.

Thus the relativistic corrections in GLONASS are accounted for automatically without user involvement.

Change Log

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For any further information regarding GLONASS Interface Control Document please contact Russian Rocket and Space Engineering and Information Systems Corporation, Joint Stock Company (Russian Space Systems, JSC).

e-mail: contact@spacecorp.ru

Website: http://russianspacesystems.ru
