Aerodynamic Optimization of an Ellipsoid

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To familiarize the author with the capabilities of geometric programming in an aircraft design context, an ellipsoid optimization problem was solved using GPkit. The ellipsoid represents an aircraft fuselage, and the goal was to optimize its fineness ratio (length/diameter) for minimum drag area. Two different problems were solved: one with a constraint on maximum cross-sectional area, and one with a constraint on internal volume. It was discovered that constraining the internal volume leads to a much higher optimal fineness ratio than constraining the maximum cross-sectional area.

1 Introduction

The problem of fuselage fineness ratio for minimum drag is of historical interest to aircraft designers. In his thesis, Raymer [1] lists fuselage fineness ratio as one of the eight most important aircraft design variables, alongside other better-known variables as thrust-to-weight ratio, wing loading, etc. Raymer goes on [1]:

> [The] constant diameter assumption [equivalent to the constant cross-sectional area assumption used here] gives an optimum [fineness ratio for minimum drag] of 3-4 . . . On the other hand, if volume is held constant then the optimum [fineness ratio] is somewhere between 6 and 8 - quite a different result! If an aircraft is volume-tight and is designed using the old suggested values of 3-4, the fuselage drag will be about 25-50% higher than possible with a fineness ratio of 6.0, according to this analysis.
>
> This ignores structural effects on the fuselage, which may push the multidisciplinary optimum solution towards a lower value. To find the true best fuselage fineness ratio, it must be included as a design variable in a multidisciplinary optimization.

2 Problem Formulation

The ellipsoid was parameterized in terms of two parameters: length l, and diameter d. Fineness ratio was constrained to be the quotient of the two: $f = l/d$.

The ellipsoid drag area was calculated using Equation 1:

$$C_D S = c_f * FF * SA \tag{1}$$

$C_D S$ is the drag area, c_f is the skin-friction coefficient, FF is the form factor (an adjustment for pressure drag), and SA is the ellipsoid surface area.

The skin-friction coefficient was computed as follows:

$$c_f = \frac{0.074}{Re^{0.2}} \tag{2}$$

$$Re = \frac{Vl}{\nu} \tag{3}$$

Equation 2 applies to a turbulent boundary layer [2]. Re is the Reynolds number and ν is the kinematic viscosity.

The form factor was computed using Equation 12.31 from Raymer [3], repeated here as Equation 4. This equation applies to a fuselage or a canopy. The same equation is given by Nicolai [4]; a comparison of different form-factor models is given by Gur et al [5].

$$FF = 1 + \frac{60}{f^3} + \frac{f}{400} \tag{4}$$

Equation 4 is a posynomial equality constraint, which is not compatible with GP. Posynomial equality relaxation was therefore used to obtain Equation 5, which is GP-compatible:

$$FF \geq 1 + \frac{60}{f^3} + \frac{f}{400} \tag{5}$$

Finally, a commonly used approximation for the surface area of an ellipsoid [6] is given as Equation 6:

$$SA = 4\pi \left[\frac{(ab)^p + (bc)^p + (ac)^p}{3}\right]^{1/p} \tag{6}$$

SA is the ellipsoid surface area, while a, b, and c are the three ellipsoid radii. p is approximately 1.6075. Equation 6 is exact to within about 1%.

By substituting the relations $a = \frac{l}{2}$, $b = \frac{d}{2}$, and $c = \frac{d}{2}$, then rearranging the result, we obtain Equation 7:

$$3\left(\frac{SA}{\pi}\right)^p = 2(ld)^p + d^{2p} \tag{7}$$

Like Equation 4, Equation 7 is a posynomial equality, which is not GP-compatible. Posynomial equality relaxation yields the GP-compatible posynomial inequality in Equation 8:

$$3\left(\frac{SA}{\pi}\right)^p \geq 2(ld)^p + d^{2p} \tag{8}$$

Two different problems were solved: one with a constraint on maximum cross-sectional area (Equation 9), and one with a constraint on internal volume (Equation 10). Both were implemented as monomial inequality constraints.

$$A_{max} \leq \frac{\pi d^2}{4} \tag{9}$$

$$V_{internal} \leq \frac{4\pi}{3}\left(\frac{l}{2}\right)\left(\frac{d}{2}\right)^2 \tag{10}$$

3 Results & Discussion

Constants that were common to both problems are listed in Table 1, while constants that differed between problems are listed in Table 2. Results are also in Table 2.

Table 1: Constants (common to both problems).

Parameter	Symbol	Value
Air density	ρ	1.225 kg/m^3
Kinematic viscosity	ν	1.4604e^{-5} m^2/s
Air velocity	V	25 m/s

Table 2: Constants (different between problems) and results.

Constraint type	Cross-sectional area	Internal volume
Constraint value	1.767 m^2	14.14 m^3
Optimal fineness ratio	5.53	9.25
Length	8.29 m	13.22 m
Diameter	1.50 m	1.429 m
Drag area	0.116 m^2	0.128 m^2
Drag	44.6 N	49.0 N

It can clearly be seen from Table 2 that the optimal fineness ratio with a cross-sectional area constraint (5.53) is much lower than the optimal fineness ratio with an internal-volume constraint (9.25). This is in accordance with the results from Raymer [1]. However, the fineness ratios discovered here are significantly greater than those from Raymer's study. In his work, the optimal fineness ratio with a cross-sectional area constraint was determined to be between 3 and 4, while the optimal fineness ratio with an internal-volume constraint was determined to be between 6 and 8. The reason for this discrepancy is unknown.

In an attempt to better understand the design space, Figure 1 was prepared. This plot uses the same GPkit models as described in Section 2, but with the fineness ratio constrained to a constant. The resulting plot is essentially of the same form as Figure 21 from Raymer [1], but the discrepancy in optimal fineness ratios still exists.

4 Conclusion

A GP-compatible model of an ellipsoid was created, and optimized for minimum drag area. Two different problems were solved: one with a constraint on maximum cross-sectional area, and one with a constraint on internal volume. It was discovered that constraining the internal volume leads to a much higher fineness ratio than constraining the maximum cross-sectional area. This result is in accordance with other research, although the resulting fineness ratios differ.

It is recommended that fuselage fineness ratio be incorporated as a variable into larger aircraft design optimization problems. In this manner, the effect of changing fuselage geometry (in terms of structures/weights, as well as aerodynamics) on the optimal design can be measured.

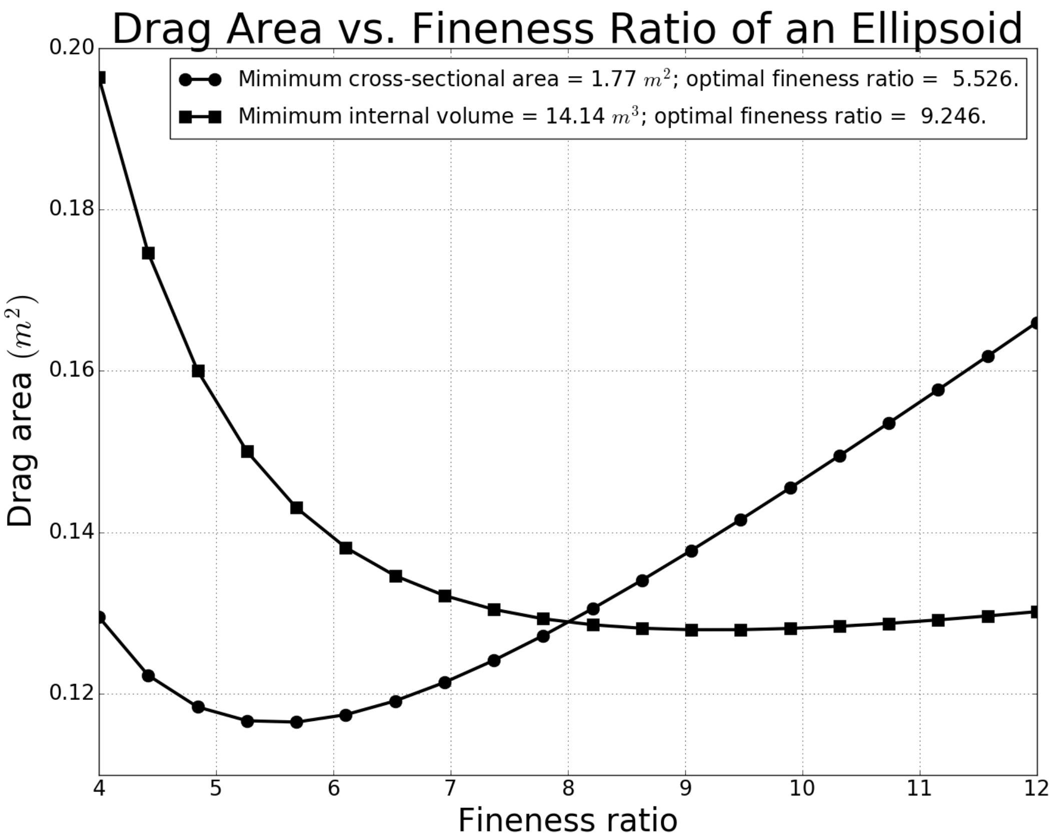

Figure 1: Ellipsoid drag vs. fineness ratio.

References

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