Compare Entropy Functions

See Coupled Functions file for the CoupledEntropy Function.

The Tsallis and Normalized Tsallis entropy functions are related to the non-root form of the Coupled Entropy

$$\text{TsallisNormalizedEntropy} = (1+\kappa)\ \text{CoupledEntropy}$$

$$\text{TsallisEntropy} = (1+\kappa)\ \text{Sum}\left(p_i^{1+\frac{\alpha\kappa}{1+\kappa}}\right)\text{CoupledEntropy}$$

In[64]:=
```
TsallisEntropy[dist_, κ_, α_ : 1, d_ : 1, limits_ : {-∞, ∞}, normalize_ : False] :=
  If[normalize,
   (1 + κ) CoupledEntropy[dist, κ, α, d, limits, False],
   (1 + κ) CoupledEntropy[dist, κ, α, d, limits, False] ∫_{limits[[1]]}^{limits[[2]]} PDF[dist, x]^(1+α κ/(1+κ)) ⅆx
  ];

TsallisRootEntropy[dist_, κ_, α_ : 1, d_ : 1, limits_ : {-∞, ∞}, normalize_ : False] :=
 If[normalize,
  (1 + κ)^(1/α) CoupledEntropy[dist, κ, α, d, limits, True],
  (1 + κ)^(1/α)
   CoupledEntropy[dist, κ, α, d, limits, True] ∫_{limits[[1]]}^{limits[[2]]} PDF[dist, x]^(1+α κ/(1+κ)) ⅆx
 ]
```

Plot Comparison

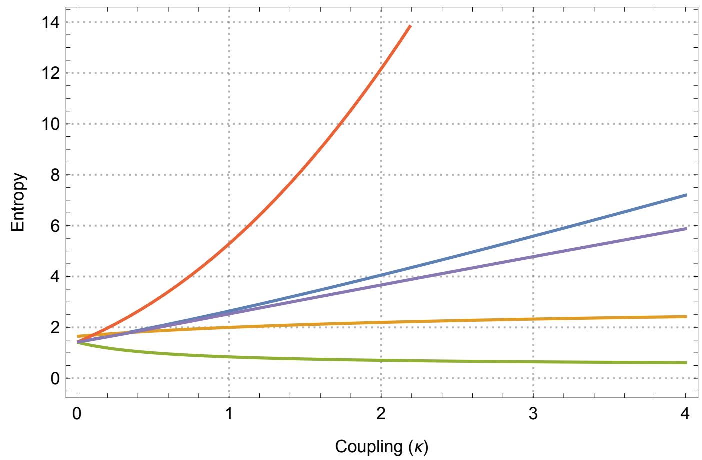
Compare entropies when the coupling matches the distribution

In[69]:=
```
PlotCoupledEntDist[2]
```

Out[69]=
```
$Aborted
```

Aborted because seemed to be taking unusually long; however, next result did complete.

Saved Plot to compare with Dec 17, 2020 update to CoupledEntropy

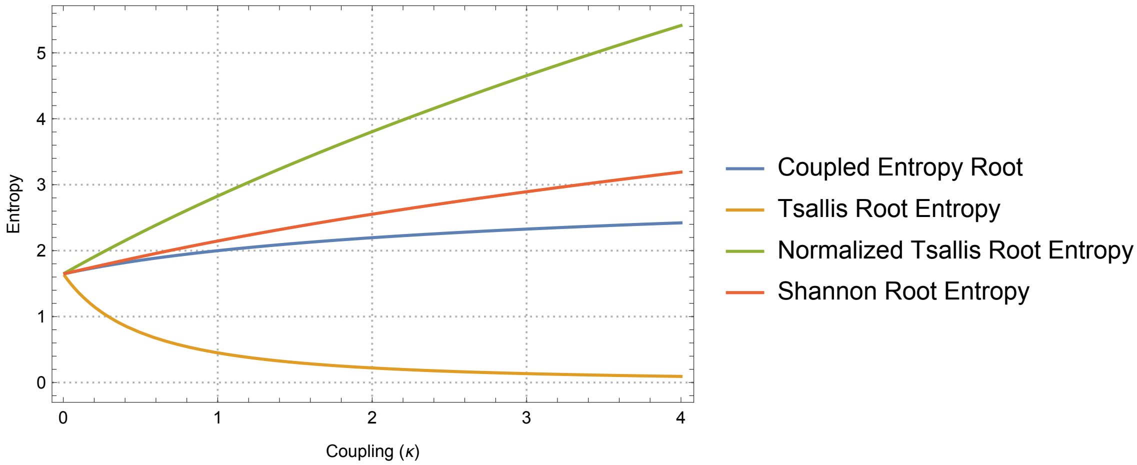


In[68]:=
```
PlotCoupledEntDist[α_] :=
 Module[{coupledDist},
  coupledDist = CoupledNormalDistribution[0., 1., κ];
  Plot[
   {
    CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
    CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
    TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, False]
   }, {κ, 0.01, 4},
   PlotLegends → {"Coupled Entropy", "Coupled Entropy Root",
"Tsallis Entropy", "Normalized Tsallis Entropy", "Shannon Entropy"},
   PlotTheme → "Detailed",
   FrameLabel → {"Coupling (κ)", "Entropy"}
  ]
 ]
```

Plot with all entropies taking root

In[67]:= `PlotCoupledEntDist[2]`

Out[67]=

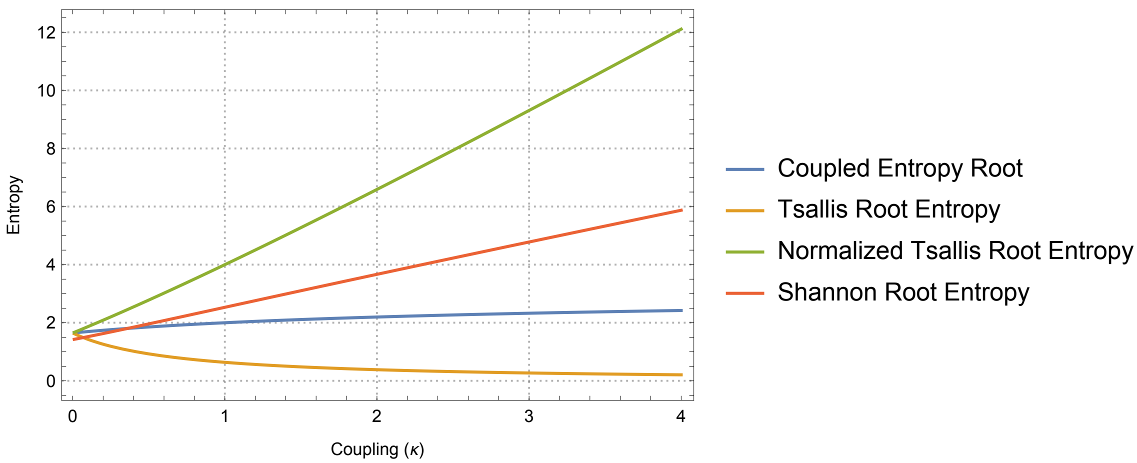


Saved Plot Dec 17, 2020

note that the Shannon Root Entropy mistakenly had $\alpha = 1$; should have been 2

and Tsallis entropies did not have the $(1+\kappa)$ term raised to a power which is required

Out[∘]=

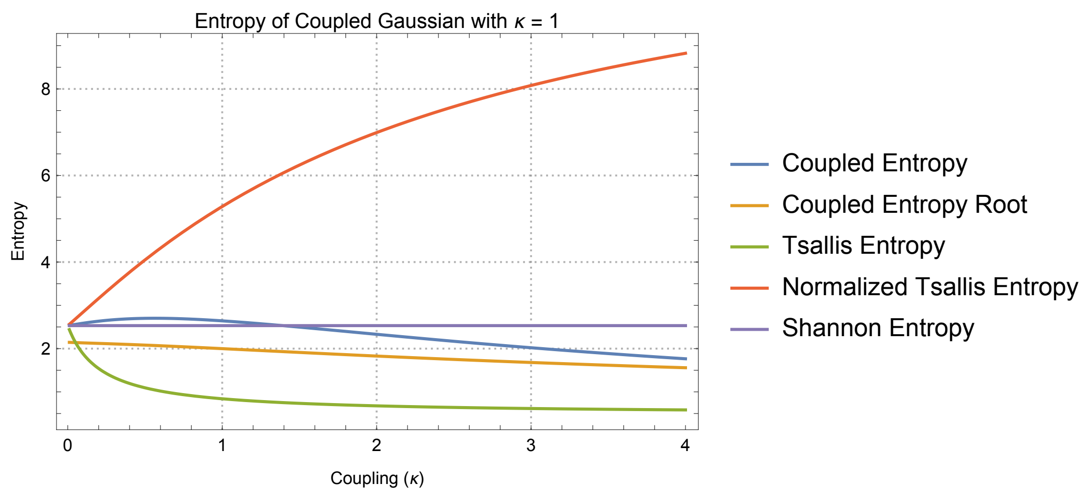


In[66]:=
```
PlotCoupledEntDist[α_] :=
 Module[{coupledDist},
  coupledDist = CoupledNormalDistribution[0., 1., κ];
  Plot[
   {
    CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
    TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    CoupledEntropy[coupledDist, 0, 2, 1, {-∞, ∞}, True]
   }, {κ, 0.01, 4},
   PlotLegends → {"Coupled Entropy Root", "Tsallis Root Entropy",
     "Normalized Tsallis Root Entropy", "Shannon Root Entropy"},
   PlotTheme → "Detailed",
   FrameLabel → {"Coupling (κ)", "Entropy"}
  ]
 ]
```

Compare Entropies for a Cauchy Distribution

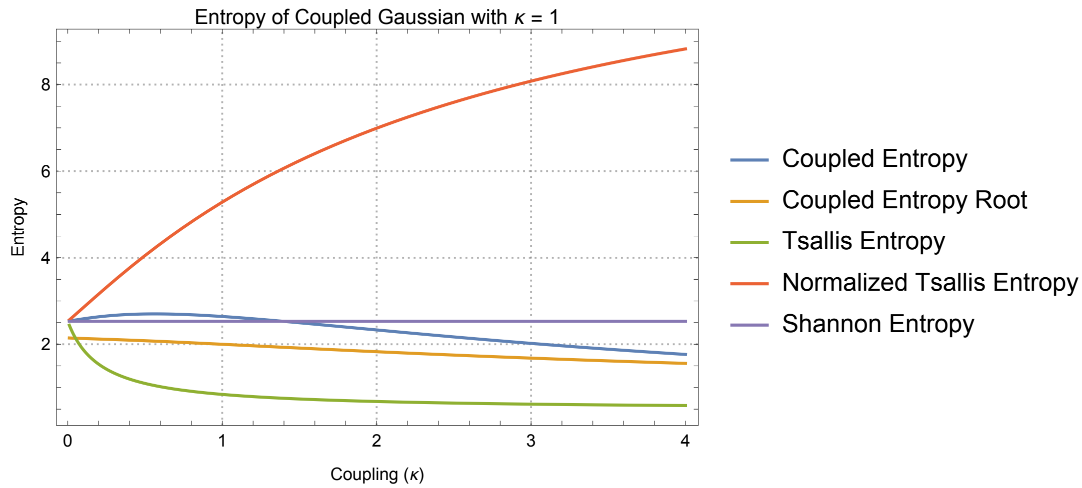
In[∘]:=
```
PlotCoupledEntDist[2, 1]
```

Out[∘]=



Saved Plot Dec 17, 2020

Out[•]=

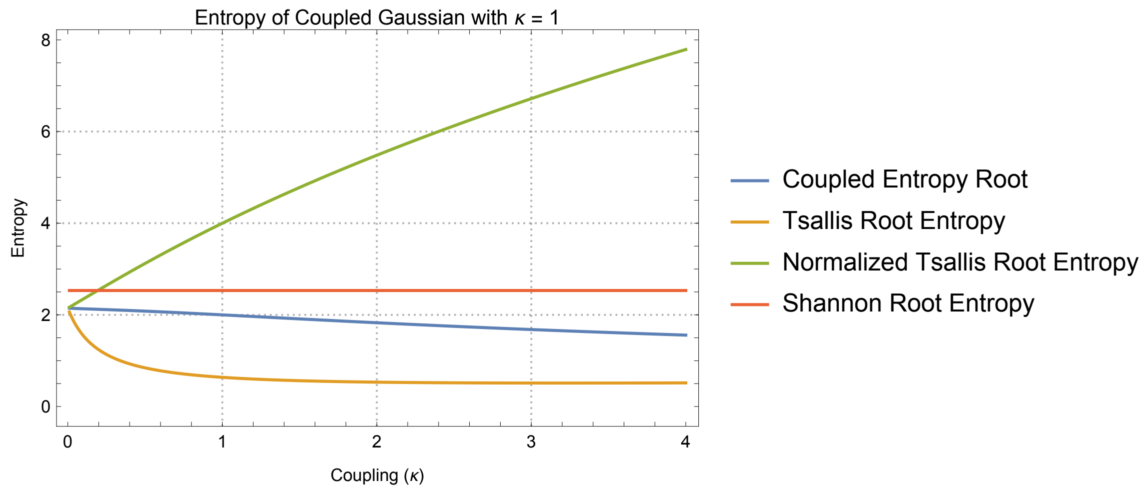


In[•]:=
```
PlotCoupledEntDist[α_, κDist_] :=
 Module[{coupledDist},
  coupledDist = CoupledNormalDistribution[0., 1., κDist];
  Plot[
   {
    CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
    CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
    TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, False]
   }, {κ, 0.01, 4},
   PlotLegends → {"Coupled Entropy", "Coupled Entropy Root",
"Tsallis Entropy", "Normalized Tsallis Entropy", "Shannon Entropy"},
   PlotTheme → "Detailed",
   FrameLabel → {"Coupling (κ)", "Entropy"},
   PlotLabel → "Entropy of Coupled Gaussian with κ = " <> ToString[κDist]
  ]
 ]
```

Comparison when other entropies also have a root term

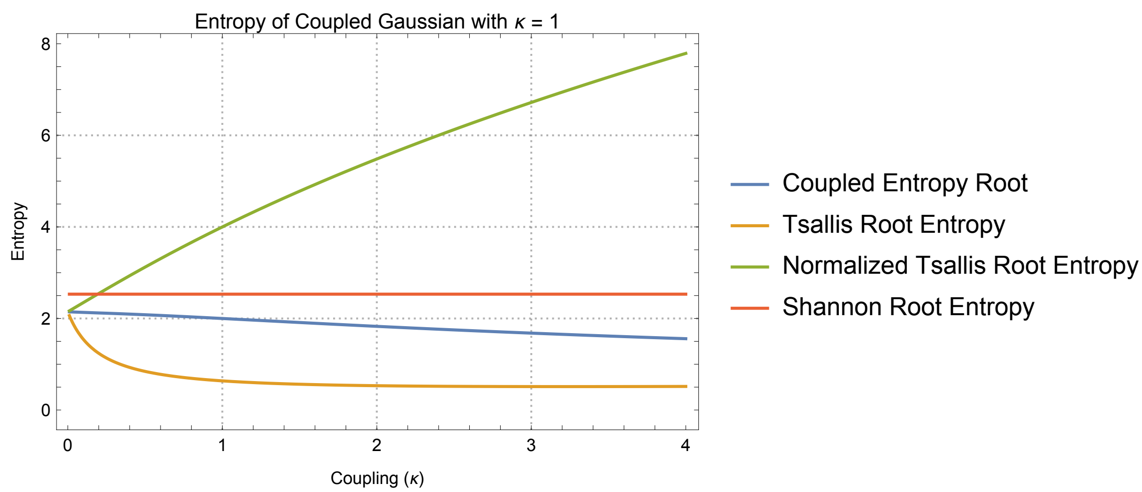
In[•]:= `PlotCoupledEntDist[2, 1]`

Out[•]=



Saved Plot Dec 17, 2020

Out[•]=



```
PlotCoupledEntDist[α_, κDist_] :=
 Module[{coupledDist},
  coupledDist = CoupledNormalDistribution[0., 1., κDist];
  Plot[
   {
    CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
    TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
    CoupledEntropy[coupledDist, 0, 2, 1, {-∞, ∞}, True]
   }, {κ, 0.01, 4},
   PlotLegends → { "Coupled Entropy Root", "Tsallis Root Entropy",
     "Normalized Tsallis Root Entropy", "Shannon Root Entropy"},
   PlotTheme → "Detailed",
   FrameLabel → {"Coupling (κ)", "Entropy"},
   PlotLabel → "Entropy of Coupled Gaussian with κ = " <> ToString[κDist]
  ]
 ]
```

Derivative as a function of coupling for the Coupled Entropy, Coupled Root Entropy, and Tsallis Entropy

```
In[•]:= PlotDerivativeCoupledEntDist[2, 1]
```

General: 0.01008151` is not a valid variable.

General: 0.09151008142857144` is not a valid variable.

General: 0.17293865285714288` is not a valid variable.

General: Further output of General::ivar will be suppressed during this calculation.

```
Out[•]= $Aborted
```

In[•]:=
```
PlotDerivativeCoupledEntDist[α_, κDist_] :=
 Module[{coupledDist},
  coupledDist = CoupledNormalDistribution[0., 1., κDist];
  Plot[
   DifferenceQuotient[
    {
     CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
     (*CoupledEntropy[coupledDist,κ,α,1,{-∞,∞},True],*)
     TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
     TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
     CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, False]
    },
    {κ, 0.01}],
   {κ, 0.01, 4},
   PlotLegends → {"Coupled Entropy", "Coupled Entropy Root",
"Tsallis Entropy", "Normalized Tsallis Entropy", "Shannon Entropy"},
   PlotTheme → "Detailed",
   FrameLabel → {"Coupling (κ)", "Entropy"},
   PlotLabel →
    "Derivative of Entropy of Coupled Gaussian with κ = " <> ToString[κDist]
  ]
 ]
```

Limit of Coupled Root Entropy

Need symbolic definition of Coupled Root Entropy

In[◦]:=
```
Limit[CoupledEntropy[CoupledNormalDistribution[0., σ, κ], κ, 2, 1, {-∞, ∞}, True],
 κ → ∞]
```

NIntegrate: The integrand

If[κ == 0, PDF[ProbabilityDistribution[1/(Piecewise[{≪2≫}, Times[≪5≫]] √Which[Greater[≪2≫], ≪6≫, Message[≪2≫]]), If[κ ≥ 0, {x$13681176, -DirectedInfinity[≪1≫], ∞}, {x$13681176, 0. + Times[≪2≫], 0. + √≪1≫}]], x], FullSimplify[PDF[ProbabilityDistribution[Times[≪2≫], If[≪3≫]], x]^(1-Times[≪3≫]) / ∫_{DirectedInfinity[≪1≫]}^{DirectedInfinity[≪1≫]} PDF[≪2≫]^{Plus[≪2≫]} dy]] √If[≪1≫]

has evaluated to non-numerical values for all sampling points in the region with boundaries {{-∞, 0.}}.

NIntegrate: The integrand

If[κ == 0, PDF[ProbabilityDistribution[1/(Piecewise[{≪2≫}, Times[≪5≫]] √Which[Greater[≪2≫], ≪6≫, Message[≪2≫]]), If[κ ≥ 0, {x$13681176, -DirectedInfinity[≪1≫], ∞}, {x$13681176, 0. + Times[≪2≫], 0. + √≪1≫}]], x], FullSimplify[PDF[ProbabilityDistribution[Times[≪2≫], If[≪3≫]], x]^(1-Times[≪3≫]) / ∫_{DirectedInfinity[≪1≫]}^{DirectedInfinity[≪1≫]} PDF[≪2≫]^{Plus[≪2≫]} dy]] √If[≪1≫]

has evaluated to non-numerical values for all sampling points in the region with boundaries {{-∞, 0.}}.

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[◦]= `$Aborted`

Test functionality of CoupledEntropy

In[◦]:=
```
$Assumptions = -1 < κ < ∞ && 0 < σ < ∞ && {x, κ, σ} ∈ Reals;
```

In[◦]:=
```
CoupledEntropy[CoupledNormalDistribution[0, 1, 1], κ, 2]
```

Out[◦]= `$Aborted`

In[◦]:=
```
CoupledEntropy[CoupledNormalDistribution[0, 1, 1], #, 2] & /@ {-0.5, 0., 0.5, 1., 1.5}
```

Integrate: Integral of $3.14159\,(1+y^2)^{1.}$ does not converge on $\{-\infty, \infty\}$.

Out[◦]=

$$\left\{-\int_{-\infty}^{\infty} -\left(\left(1.5708\left(\left(1+x^2\right)^2\right)^{0.5} \mathrm{If}\left[\left(1+x^2\right)^2 \geq 0, \mathrm{If}\left[-0.5 \neq 0, -\frac{\left(\pi^2\left(1+x^2\right)^2\right)^{-\frac{0.5}{1+1(-0.5)}}-1}{0.5}, \mathrm{Log}\left[\pi^2\left(1+x^2\right)^2\right]\right], \mathrm{Undefined}\right]\right) \Big/ \left(\int_{-\infty}^{\infty} 3.14159\left(\left(1+y^2\right)^2\right)^{0.5} dy\right)\right) dx,\ 2.53102,\ 2.69962,\ 2.64159,\ 2.4964\right\}$$

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