Generating Coupled Exponential Random Variables

Coupled Box-Müller Method

Multivariate Coupled Box-Müller Method

The method is based on polar version of the Box-Muller method. The advantage is there is a clear procedure for extending the method to multiple dimensions. The disadvantage is that method uses sample rejection to form a unit n-sphere. Since the n-sphere is based on $R^2 = x_1^2 + \ldots x_n^2$as n increases the number of rejected samples increases. This will make the algorithm very slow for large dimensions.

In[241]:=

```
Clear[CoupledVariate];
CoupledVariate::lsigma =
  "Length of scale `1` does not equal length of mean `2`";
CoupledVariate[μ_ : 0, Σ_ : 1, κ_ : 1, n_ : 1]  := Module[
   {UniformVariates, CoupledNormalVariates, radiusSquared,
    dimMean, dimScale, σAdj, j},
   dimMean = Length[μ];
   σAdj = If[(dimScale = Length[Σ]) ≠ dimMean,
     Message[CoupledVariate::lsigma, dimScale, dimMean];
     (*If[dimScale>dimMean,
      Σ[[;;dimMean]],
      PadRight[Σ,dimMean-dimScale,1],
      Σ
     ]*)
    ];
   UniformVariates = Table[0, n, Max[2, dimMean]];
   radiusSquared = Table[0, n];
   For[j = 1, j ≤ n, j++,
    While[Not[0 < radiusSquared[[j]] < 1],
     UniformVariates[[j]] = RandomReal[{-1, 1}, Max[2, dimMean]];
     (* Dividing by √Max[2,dimMean] seems to make solution closer to a
      multivarirate t distribution, but needs to be proven; also this factor
      shouldn't change radiusSquared from being a uniform distribution,
     but it will reduce the number of rejections; furthermore this should
      be the same a drawing from a domain reduced from from {-1,1} *)

     radiusSquared[[j]] = (* 1/√Max[2,dimMean] *) Sum[UniformVariates[[j, i]]^2, {i, 1, Max[2,dimMean]}];

    ]
   ];
```

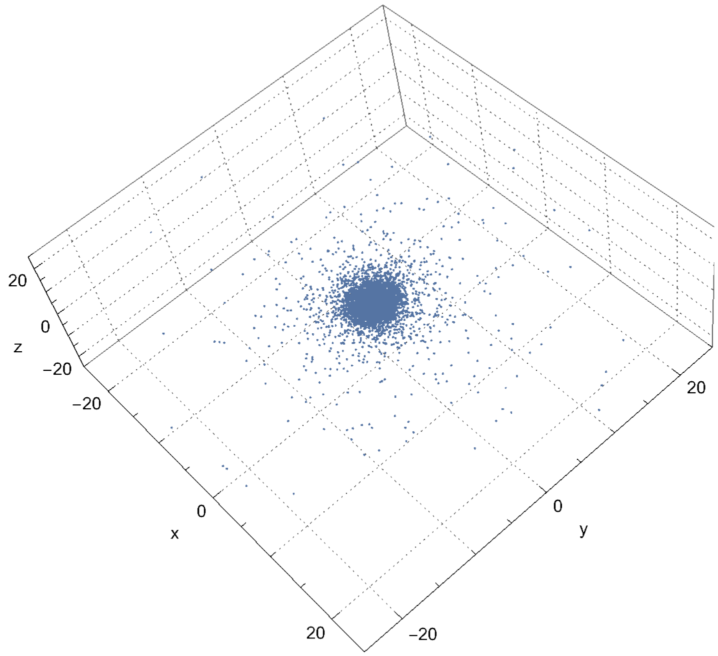
```
If[n == 1,
 CoupledNormalVariates =
  Table[Sqrt[CoupledLogarithm[radiusSquared[[1]]^-2, κ, 0] / radiusSquared[[1]]] UniformVariates[[1, i]],
   {i, dimMean}
  ];
 μ + CoupledNormalVariates.CholeskyDecomposition[Σ],
 Table[
  CoupledNormalVariates =
   Table[Sqrt[CoupledLogarithm[radiusSquared[[j]]^-2, κ, 0] / radiusSquared[[j]]] UniformVariates[[j, i]],
    {i, dimMean}
   ];
  μ + CoupledNormalVariates.CholeskyDecomposition[Σ],
  {j, n}
 ]

]
];
```

Examples compared with Mathematica MultivariateT generation

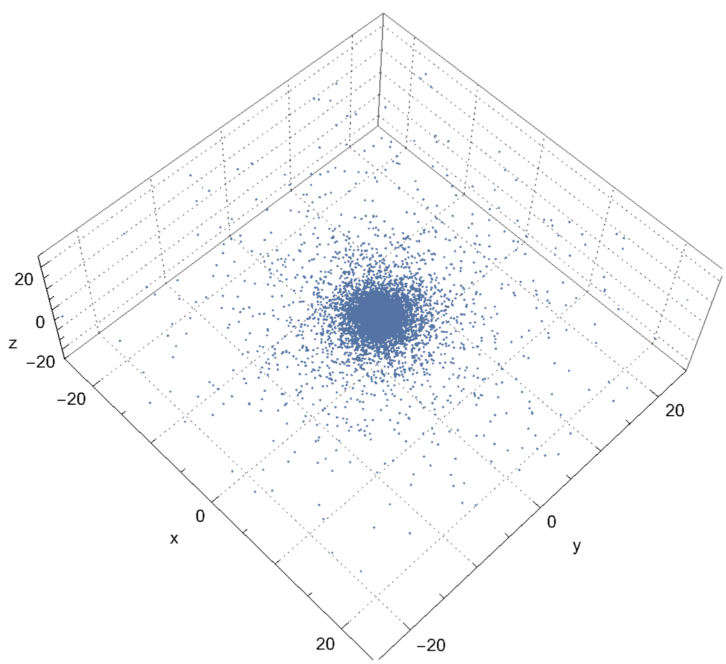
In[∘]:=
```
ListPointPlot3D[
 CoupledVariate[{0, 0, 0}, {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, 1, 10 000],
 PlotTheme → "Detailed",
 AxesLabel → {"x", "y", "z"},
 PlotRange → Table[{-25, 25}, 3]
]
```

Out[•]=



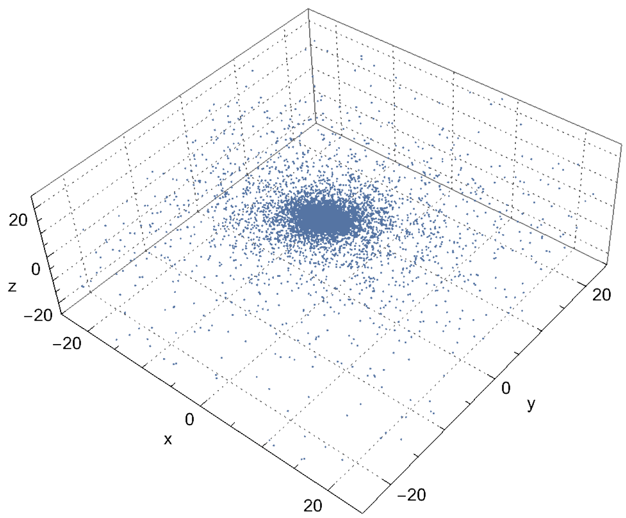
In[•]:=
```
ListPointPlot3D[RandomVariate[
   MultivariateTDistribution[{0, 0, 0}, {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, 1], 10 000],
  PlotTheme → "Detailed",
  AxesLabel → {"x", "y", "z"},
  PlotRange → Table[{-25, 25}, 3]
 ]
```

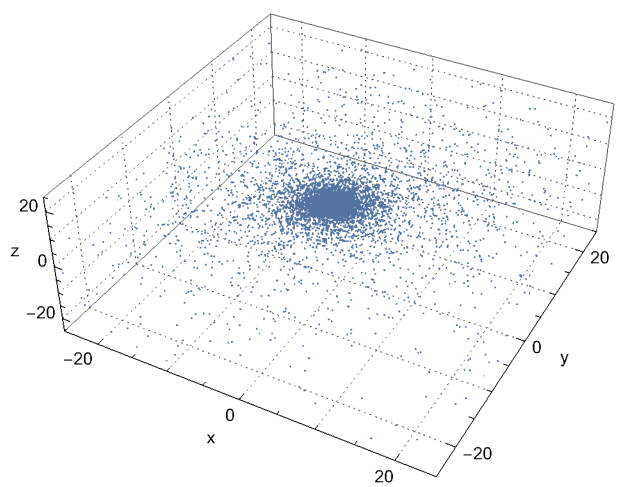
Out[•]=



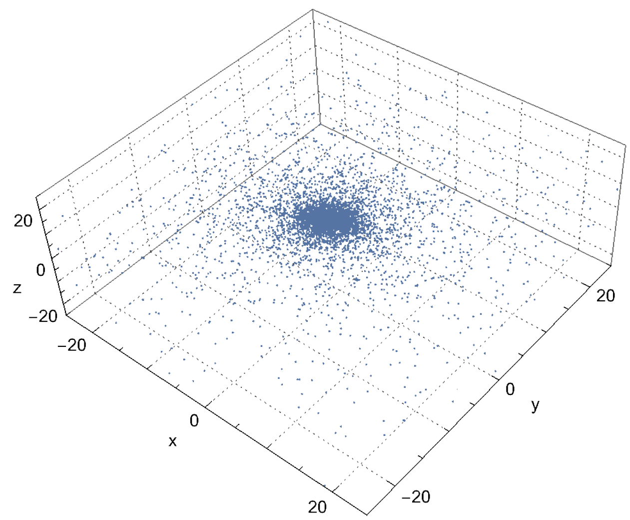
Examples side by side of CoupledVariate (left or top) and MultivariateT (right or bottom) variates show reasonable similarity

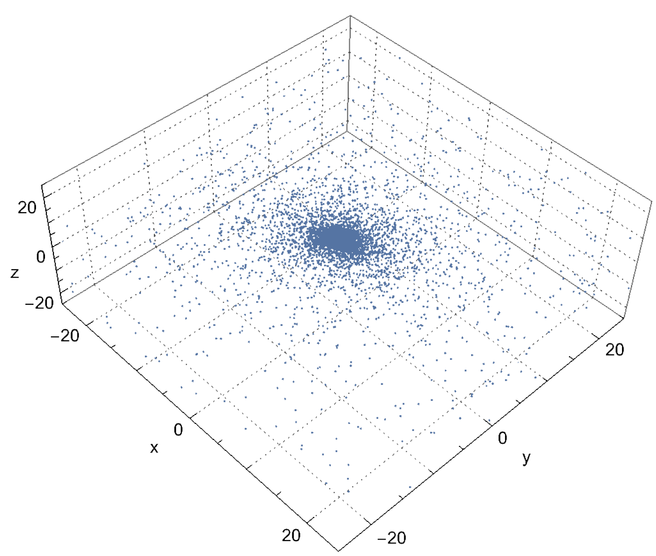
[{mean},{correlation matrix}, coupling, samples]
[{-2,1,1},{{4,1,0},{1,2,1},{0,1,1}},3,10000]



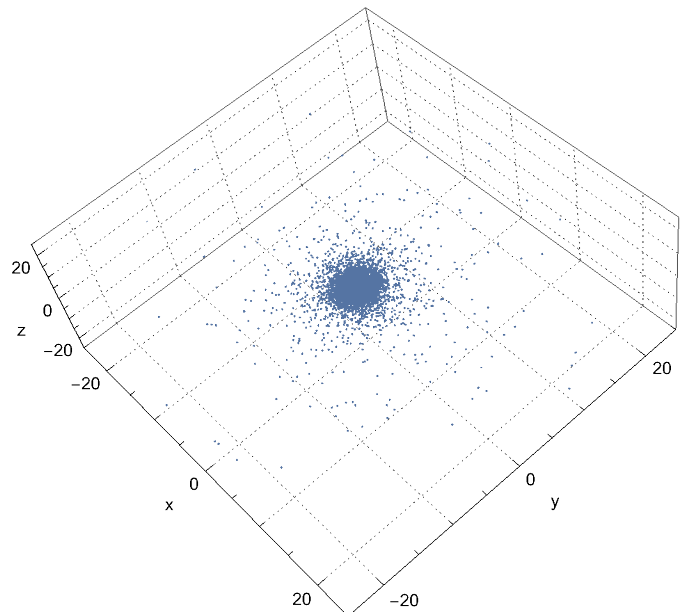


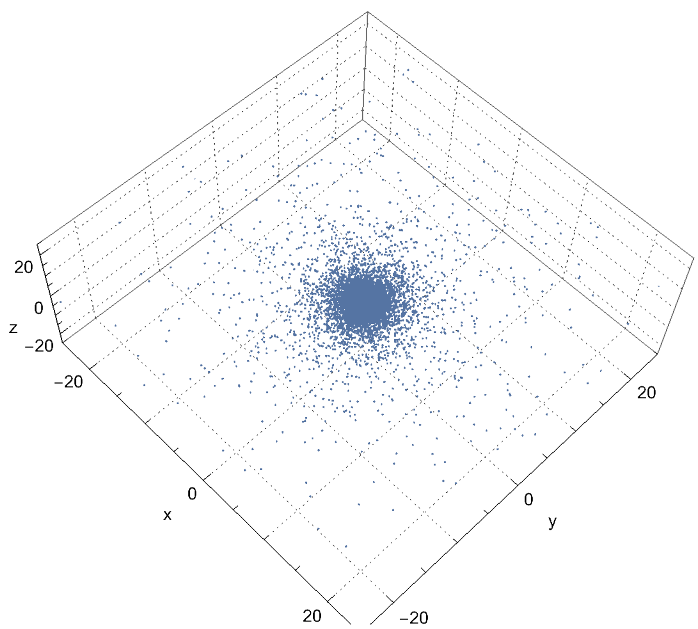
[{-2,1,1},{{4,1,0},{1,2,1},{0,1,1}},5,10000]





Cauchy Standard





Plots with $\sqrt{\kappa}$ multiplied by $1/\sigma$

Backup of Originals Generate and Plot Examples

Higher values of coupling

Visual Comparison Coupled Stretched Exp and Mittag-Leffler Distributions