Notes and Errata for

Calendrical Calculations: Third Edition

Nachum Dershowitz and Edward M. Reingold
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12:58pm, November 19, 2009

Do I contradict myself? Very well then I contradict myself.
(I am large, I contain multitudes.)

—Walt Whitman: *Song of Myself*

All those complaints that they mutter about... are on account of many places I have corrected. The Creator knows that in most cases I was misled by following... others whom I will spare the embarrassment of mention. But even were I at fault, I do not claim that I reached my ultimate perfection from the outset, nor that I never erred. Just the opposite, I always retract anything the contrary of which becomes clear to me, whether in my writings or my nature.

—Maimonides: *Letter to his student Joseph ben Yehuda* (circa 1190), *Iggerot HaRambam, I. Shilat, Maaliyot, Maaleh Adumim, 1987, volume 1, page 295* [*in Judeo-Arabic*]

If you find errors not given below or can suggest improvements to the book, please send us the details (email to reingold@iit.edu or hard copy to Edward M. Reingold, Department of Computer Science, Illinois Institute of Technology, 10 West 31st Street, Suite 236, Chicago, IL 60616-3729 U.S.A.). If you have occasion to refer to errors below in corresponding with the authors, please refer to the item by page and line numbers in the book, *not* by item number.

Unless otherwise indicated, line counts used in describing the errata are positive counting down from the first line of text on the page, excluding the header, and negative counting up from the last line of text on the page *including footnote lines.*

A list of functions mentioned in the errata is given at the end of this document, as is a list of errata in order by date added or last modified.

Our thanks to all those who pointed out errors or suggested improvements to the text. An index of specific contributions is given at the end of this document.

The latest version of this document can be obtained from the Web site `http://www.calendarists.com`

PRINTING HISTORY

Printing	Date	Notes
First	December, 2007	Paperback (2000) and hard-cover (250)

FRONTMATTER

1. Page xvi: Entries for inverse trigonometric functions should specify the principal values in each case, and the code should be modified to return (and use!) values in that range:

$\arcsin x$	Arc sine	Inverse sine of x, in degrees, in range $[-90°, 90°]$
$\arccos x$	Arc cosine	Inverse cosine of x, in degrees, in range $[0°, 180°]$
$\arctan x$	Arc tangent	Inverse tangent of x, in degrees, in range $(-90°, 90°)$

2. Page xx–xxii: Another calendar software failure from *The New York Times*, page B9 of the New York edition, January 1, 2009. Zune, Microsoft's portable media player, failed, according to the *Times* article because the software did not treat 2008 as a leap year; the truth is more complicated. Zune used the following code to compute the present year from the number of days elapsed since January 1, 1980

```
year = ORIGINYEAR; /* = 1980 */

while (days > 365)
{
    if (IsLeapYear(year))
    {
        if (days > 366)
        {
            days -= 366;
            year += 1;
        }
    }
    else
    {
        days -= 365;
        year += 1;
    }
}
```

This code fails on the last day of *any* leap year.

3. Page xx–xxii: Another example of poorly written calendar software is Notify Technology's code to synchronize mobile data devices (including many models of BlackBerry, PalmOS, Symbian, Windows Mobile, and Apple devices) with Peoplecube's *Meeting Maker*; it does not correctly handle monthly recurring events on the 29th, 30th, or 31st of the month because these dates do not occur in all months. (Courtesy of Ruth N. Reingold, July 30, 2009.)

CHAPTER 1: CALENDAR BASICS

4. Page 5, second line of footnote: Change "calandar" to "calendar". (Courtesy of Enrico Spinielli, March 12, 2008.)

5. Page 6, last paragraph: The reference for the 360-day Babylonian calendar is Wayne Horowitz, "The 360 and 364 Day Year in Ancient Mesopotamia," *Journal of the Ancient Near Eastern Society* **24** (1996), pp. 35–41.

6. Page 7, second complete paragraph: We should mention the Antikythera Mechanism which implements (among many other things) the Metonic cycle. See "The Antikythera Mechanism: A Computer Science Perspective," Diomidis Spinellis, *IEEE Computer* **41**, 5 (May, 2008), pp. 22–27. Also, see "Calendars with Olympiad display and eclipse prediction on the Antikythera Mechanism," Tony Freeth, Alexander Jones, John M. Steele, Yanis Bitsakis, *Nature* **454** (July 31, 2008), pp. 614–617 and "Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism," T. Freeth, Y. Bitsakis, X. Moussas, J. H. Seiradakis, A. Tselikas, H. Mangou, M. Zafeiropoulou, R. Hadland, D. Bate, A. Ramsey, M. Allen, A. Crawley, P. Hockley, T. Malzbender, D. Gelb, W. Ambrisco, M. G. Edmunds, *Nature* **444** (November 30, 2006), pp. 587–591. A popular treatment, with many interesting photographs, is given in Jo Marchant's *Decoding the Heavens*, William Heinemann, London, 2008.

7. Page 9, section 1.2: Interesting quotation from *Zichron Menachem* by Tzvi (Herman) Schvarcz, Kahn & Fried, Munkacs, Hungary, 5673 A.H. (= 1913–14), bottom of the second column of the back of the first leaf after the introduction: "An ordinary person cannot count each day, and say this is so many and so many days. Instead, the count uses a significant unit, that is, years." This is from a talk given in June, 1907 to honor the fortieth anniversary of Franz Josef I's rule of Hungary.

8. Page 11, line −14: Change "Atlcahualo 11" to "Panquetzaliztli 1". (Courtesy of Enrico Spinielli, June 30, 2008.)

9. Page 11, line −12: Change "Lunag" to "Luang". (Courtesy of Enrico Spinielli, July 28, 2008.)

10. Page 11, line −13: Change "9 Ozomatli" to "11 Atl". (Courtesy of Enrico Spinielli, June 30, 2008.)

11. Page 14, line 6: Add a footnote to the end of this line saying that historically scholars have mixed the notations, using negative years for the Julian calendar and the B.C.E./C.E. (B.C./A.D.) notations for Gregorian years, so one must be cautious in interpreting what a particular author means. The ambiguity has led to confusion and errors. (Courtesy of Svante Janson, July 22, 2008.)

12. Page 16, line 7: Change "1911" to "1912". Also, in the next edition we might want to give a table of such eras in which the Gregorian calendar is used, but not the Gregorian year such as Taiwan and North Korea (1912) Thailand (543 B.C.E.), and Japan (eras based on emperors). (Courtesy of Svante Janson, July 17, 2008.)

13. Page 17, line −2: The sentence "In general, $\lceil x \rceil = -\lfloor -x \rfloor$, so for example $\lfloor -\pi \rfloor = -4$." is misplaced; it belongs in line 1 of page 18, just before "For integers. . . ". (Courtesy of Svante Janson, July 12, 2008.)

14. Page 19, equation (1.17): It follows that $(-x) \bmod (-y) = -(x \bmod y)$.

15. Page 19, bottom: It would be convenient to extend the binary modulo operator mod to allow an interval as the modulus, and shift the remainder into that interval. We could define

$$x \bmod [a,b) \stackrel{\text{def}}{=} a + [(x-a) \bmod (b-a)] \tag{15-A}$$

$$x \bmod (a,b] \stackrel{\text{def}}{=} b - [(b-x) \bmod (b-a)] \tag{15-B}$$

and perhaps

$$m \bmod [a,b] \stackrel{\text{def}}{=} a + [(m-a) \bmod (b-a+1)].$$

The latter, with a closed interval for the range of shifted values, only makes sense for integers; it is the same as $m \bmod [a, b+1)$, so we don't need it. Notice that $x \bmod [a,b) = x \bmod (b,a]$ so we only need one of (15-A) and (15-B) . With this notation, $x \bmod y$ can be viewed as shorthand for $x \bmod [0, y)$, and our current "adjusted mod" notation $m \operatorname{amod} n$ is just shorthand for $m \bmod (0, n] = m \bmod [n, 0) = m \bmod [1, n]$. Some of the functions that would benefit from this notation are **estimate-prior-solar-longitude**, **lunar-altitude**, **fixed-from-hindu-lunar**, **fixed-from-astro-hindu-lunar**, and **hindu-tropical-longitude**.

16. Page 26: The left column of Persian month names is mis-aligned. (Courtesy of Enrico Spinielli, June 22, 2008.)

17. Page 27, definitions (1.45)–(1.50): For consistency with the way month names were defined (on the Gregorian and Hebrew calendars, for example), we should have defined the weekdays absolutely, not relative to Sunday:

$$\textbf{monday} \stackrel{\text{def}}{=} 1 \tag{1.45}$$

$$\textbf{tuesday} \stackrel{\text{def}}{=} 2 \tag{1.46}$$

$$\textbf{wednesday} \stackrel{\text{def}}{=} 3 \tag{1.47}$$

$$\textbf{thursday} \stackrel{\text{def}}{=} 4 \tag{1.48}$$

$$\textbf{friday} \stackrel{\text{def}}{=} 5 \tag{1.49}$$

$$\textbf{saturday} \stackrel{\text{def}}{=} 6 \tag{1.50}$$

Or, in Lisp (pages 361–362):

```
(defconstant monday
  ;; TYPE day-of-week
  ;; Residue class for Monday.
  1)
```

```
(defconstant tuesday
  ;; TYPE day-of-week
  ;; Residue class for Tuesday.
  2)
```

```
(defconstant wednesday
  ;; TYPE day-of-week
  ;; Residue class for Wednesday.
  3)
```

```
(defconstant thursday
  ;; TYPE day-of-week
  ;; Residue class for Thursday.
  4)
```

```
(defconstant friday
  ;; TYPE day-of-week
  ;; Residue class for Friday.
  5)
```

```
(defconstant saturday
  ;; TYPE day-of-week
  ;; Residue class for Saturday.
  6)
```

(Courtesy of Enrico Spinielli, June 22, 2008.)

18. Page 29, line 3: Change $x+1$ to $x-1$. (Courtesy of Svante Janson, July 12, 2008.)

19. Page 30, Table 1.3: Change the entry for "On or before d" from $d+\Delta-k$ to d. (Courtesy of Svante Janson, July 12, 2008.)

20. Page 39, equation (1.65). Another example of the use of this formula is in implementing the calendar of the Akan people of Ghana is based on a 42-day cycle of day names formed by two simultaneous cycles of six prefixes and seven stems (similar to the Mayan tzolkin calendar and the Chinese sexagesimal names). The prefixes are

(1) Nwona (care, wellness, surpass, innocence)
(2) Nkyi (passing, no restrictions)
(3) Kuru (sacred, complete)
(4) Kwa (ordinary, empty, freedom)
(5) Mono (fresh, new)
(6) Fo (generous, calm, love to another)

and the stems are

(1) Wukuo (cleansing, advocate, mean-spirited)
(2) Yaw (pain, suffering, bravery)
(3) Fie (depart from, come forth, travel)
(4) Memene (digest, satiety, creation, ancient)
(5) Kwasi (freedom, purify, smoke)
(6) Dwo (peaceful, cool, calm)
(7) Bene (well-cooked)

Together these prefixes and suffixes form a sequence of 42 day names, Nwonawukuo, Nkyiyaw, ..., Fobene. Representing Akan day names as pairs of positive integers,

$$\boxed{\mathit{prefix}}\boxed{\mathit{stem}},$$

where *prefix* and *stem* are integers in the ranges 1 to 6 and 1 to 7, respectively, the nth Akan day name is given by

$$\textbf{akan-day-name}\,(n) \stackrel{\text{def}}{=} \qquad (20\text{-A})$$

$$\boxed{n \ \text{amod}\ 6}\boxed{n \ \text{amod}\ 7}$$

Applying formula (1.65) with $c=6$, $d=7$, $\Gamma=\Delta=0$ but counting from 1 instead of 0, we find the Akan name $\boxed{a}\boxed{b}$ corresponds to name number

$$(a+36(b-a)) \ \text{amod}\ 42.$$

Determining the number of names between given Akan names is an instance of formula (1.66):

$$\textbf{akan-name-difference}\left(\boxed{\textit{prefix}_1 \mid \textit{stem}_1}, \boxed{\textit{prefix}_2 \mid \textit{stem}_2}\right) \stackrel{\text{def}}{=} \tag{20-B}$$

$$\left(\textit{prefix-difference} + 36 \times (\textit{stem-difference} - \textit{prefix-difference})\right) \text{ amod } 42$$

where

$$\textit{prefix-difference} = \textit{prefix}_2 - \textit{prefix}_1$$

$$\textit{stem-difference} = \textit{stem}_2 - \textit{stem}_1$$

Computing backwards from known dates in the present we find

$$\textbf{akan-day-name-epoch} \stackrel{\text{def}}{=} \textsc{r.d.}\ 37 \tag{20-C}$$

which allows us to write

$$\textbf{akan-name-of-day}\,(\textit{date}) \stackrel{\text{def}}{=} \tag{20-D}$$

$$\textbf{akan-day-name}\,(\textit{date} - \textbf{akan-day-name-epoch})$$

Now we can apply formula (1.54) to compute the R.D. date of the last date with a given Akan name before a given R.D. date:

$$\textbf{akan-day-name-on-or-before}\,(\textit{name}, \textit{date}) \stackrel{\text{def}}{=} \tag{20-E}$$

$$\textit{date} - \left(\left(\textit{date} + \textbf{akan-name-difference}\,(\textit{name}, \textbf{akan-day-name-epoch})\right) \bmod 42\right)$$

For more information, see Konadu, Kwasi, "The Calendrical Factor in Akan History," preprint; Bartle, Philip F. W., "The Forty Days: The Akan Calendar," *Africa* **48** (1978), 80–84; and Osei, Osafo K., *A Discourse on Akan Perpetual Calendar (for Religious Ceremonies and Festivals))*, Domak Press Ltd., Accra, Ghana, 1997. (Courtesy of Kwasi Konadu, October 23, 2009.)

21. Page 31, equation (1.66): The formula is correct, but is eqivalent to the simpler form $(a + 25(b - a))$ amod 60. (Courtesy of Svante Janson, July 12, 2008.)

22. Page 39, second full paragraph: The theoretical discussion here of how to approximate the year on an arithmetic calendar of a given fixed date, and then refine the approximation, is incorrect and does not correspond to what we actually do in the functions **alt-gregorian-year-from-fixed**, **hebrew-from-fixed**, and **arithmetic-french-from-fixed** (all of which are correct). Here is the correct method of approximation/refinement.

    Let n_0 be the calendar's fixed epoch (R.D. date of the start of year 1), Y its mean year length, **new-year**(i) the fixed starting day of year i, and $\delta = \max_i\{n_0 + (i-1) \times Y - \textbf{new-year}(i)\}$, the maximum number of days that a real year can begin before the mean year does. Assume that the actual year always begins before or simultaneously with the mean year; that is, assume that $\textbf{new-year}(i) \leq n_0 + (i-1) \times Y$. Assume further that $\delta \leq Y$. We first estimate the year of fixed date d to be the mean value $y = \lfloor (d - n_0)/Y \rfloor + 1$. Then, we check if $(d - n_0) \bmod Y \geq Y - \delta$ (d falls in the "twilight zone") and $d \geq \textbf{new-year}(y+1)$ (the estimate is actually wrong), in which case the estimate is off by one and the correct year is $y+1$. If the actual new year can begin *after* the mean new year, that is, if it can happen that $\textbf{new-year}(i) > n_0 + (i-1) \times Y$, the correct year could be $y-1$, as well as y or $y+1$. If $\delta > Y$, the correct year could be $y+2$.

    (Courtesy of Svante Janson, July 12, 2008.)

23. Section 1.13: Add a warning about floating point calculations being platform-dependent. For example, in the sample data on page 446, the solar longitude given for R.D. −214193, computed with the expression `(solar-longitude (+ -214193 0.5L0))`, is slightly different in Allegro Common Lisp on a Sun Blade than in CLisp on a (Dell) PC. Allegro Common Lisp gives 119.47497459934311 while CLisp gives 119.47497459928252184; LispWorks (ANSI CL) on a MacBook (Intel Core 2 Duo) gives the same result as Allegro Common Lisp. But such small differences, in this case about $6^\circ \times 10^{-11}$ of arc or 5×10^{-6} seconds of time, are most unlikely to affect the computations of dates. (Courtesy of Enrico Spinielli, August 2, 2009.)

24. Page 40: Add the caveat that we use degree-based trigonometric functions, so that for programming languages in which these functions are radian-based, conversions are necessary—see our Lisp code, for an example of such conversions. We also presume that the inverse trigonometric functions return the standard principal values as given in the modified table of Mathematical Notations (Erratum 1).

CHAPTER 2: THE GREGORIAN CALENDAR

25. Section 2.1: Isaac Newton had a different proposal for modifying the Julian calendar to make it more accurate. He suggested a 5000-year cycle in which years divisible by 4 would be leap years (February would have 29 days), except years divisible by 100 would not be leap years, except years divisible by 500 would be leap years; furthermore, years divisible by 5000 would be "double leap years" with 30 days in February. Newton also had a modified lunar cycle for determining the date of Easter. See Belenkiy, Ari and Eduardo Vila Echague, "History of one defeat: Reform of the Julian calendar as envisaged by Isaac Newton," *Notes & Records of the Royal Society*, **59** (2005), pp. 223–254.

26. Page 45: Pope Gregory XIII was not only responsible for the institution of the revised calendar, but he was also responsible for a bull *Vices eius nos* (September 1, 1577) organizing regular missionizing sermons by apostate Jews that the Jewish community of Rome was forced to attend and subsidize. His bull *Sancta mater ecclesia* (September 1, 1584) specified more precise conditions: beadles armed with rods made sure the Jews paid attention and checked that they had not put wax in their ears. These sermons took place throughout the Papal States and much of the Roman Catholic world, as well in the church nearest the Jewish quarter in Rome, San Gregorio della Divina Pietà (the front of this church

has an inscription in Hebrew and Latin, beside an image of the crucified Jesus, quoting from *Isaiah* 65:2–3, "I have spread out My hands all the day unto a rebellious people, that walk in a way that is not good, after their own thoughts; a people that provoke me to my face continually.").

27. Page 52, **day-number**: Replace (2.25) by

$$\textbf{day-number}\,(g\text{-}date) \stackrel{\text{def}}{=} \tag{2.25}$$

$$\textbf{gregorian-date-difference}\left(\begin{array}{|c|c|c|}\hline g\text{-}date_{\textbf{year}} - 1 & \textbf{december} & 31 \\ \hline \end{array}, g\text{-}date\right)$$

Or, in Lisp (page 370):

```
(defun day-number (g-date)
  ;; TYPE gregorian-date -> positive-integer
  ;; Day number in year of Gregorian date g-date.
  (gregorian-date-difference
   (gregorian-date (1- (standard-year g-date)) december 31)
   g-date))
```

(Courtesy of Andy Pepperdine, July 12, 2008.)

28. Page 52, **days-remaining**: Replace (2.26) by

$$\textbf{days-remaining}\,(g\text{-}date) \stackrel{\text{def}}{=} \tag{2.26}$$

$$\textbf{gregorian-date-difference}\left(g\text{-}date, \begin{array}{|c|c|c|}\hline g\text{-}date_{\textbf{year}} & \textbf{december} & 31 \\ \hline \end{array}\right)$$

Or, in Lisp (page 370):

```
(defun days-remaining (g-date)
  ;; TYPE gregorian-date -> nonnegative-integer
  ;; Days remaining in year after Gregorian date g-date.
  (gregorian-date-difference
   g-date
   (gregorian-date (standard-year g-date) december 31)))
```

(Courtesy of Andy Pepperdine, July 12, 2008.)

29. Page 56, **alt-gregorian-year-from-fixed**: We should recast this as per Erratum 22.

30. Page 58, **nth-kday**: The function must return **bogus** if $n = 0$:

$$\textbf{nth-kday}\,(n, k, g\text{-}date) \stackrel{\text{def}}{=} \tag{2.32}$$

$$\begin{cases} 7 \times n \\ \quad + \textbf{kday-before} \\ \qquad (k, \textbf{fixed-from-gregorian}\,(g\text{-}date)) & \textbf{if } n > 0 \\ 7 \times n \\ \quad + \textbf{kday-after}\,(k, \textbf{fixed-from-gregorian}\,(g\text{-}date)) & \textbf{if } n < 0 \\ \textbf{bogus} & \textbf{otherwise} \end{cases}$$

Or, in Lisp (page 371):

```
(defun nth-kday (n k g-date)
  ;; TYPE (integer day-of-week gregorian-date) -> fixed-date
  ;; If n>0, return the n-th k-day on or after
  ;; g-date.  If n<0, return the n-th k-day on or
  ;; before g-date.  If n=0 return bogus.  A k-day of
  ;; 0 means Sunday, 1 means Monday, and so on.
  (cond ((> n 0)
         (+ (* 7 n)
            (kday-before k (fixed-from-gregorian g-date))))
        ((< n 0)
         (+ (* 7 n)
            (kday-after k (fixed-from-gregorian g-date))))
        (t bogus)))
```

(Courtesy of Jonathan Leffler, January 2, 2009.)

CHAPTER 4: THE COPTIC AND ETHIOPIC CALENDARS

31. Page 76, line 3 of the section quote for section 4.3: Change "shits" to "shifts". (Courtesy of Jonathan Leffler, October 9, 2008.)

CHAPTER 6: THE ISLAMIC CALENDAR

32. Page 85, lines 3–4: Microsoft inexplicably calls this version the "Kuwaiti algorithm". (Courtesy of Robert H. van Gent, October 8, 2009.)

CHAPTER 7: THE HEBREW CALENDAR

33. Page 92, footnote 7: Mention that Savasorda and others say that equal-length hours, not variable-length hours, are intended (Hebrew calendar dates are unaffected).

34. Page 99, **hebrew-from-fixed**: We should recast the calculation of the year as per Erratum 22.

35. Page 105, **sh-ela** and **birkath-ha-hama**: These functions should be implemented as part of a Hebrew solar calendar, perhaps within the general framework of the general cyclical calendars of Chapter 12.

CHAPTER 8: THE ECCLESIASTICAL CALENDARS

36. Page 112, frontispiece. We might consider instead using Figure 36-A. A detailed explanation can be found in Denis Roegel's paper "An introduction to nomography: Garrigues' nomogram for the computation of Easter," *TUGboat*, **30**, 1 (2009), pp. 88–104.

37. Page 116, last line of **alt-orthodox-easter**: For robustness change "-272" to "$-273+$*gregorian-epoch*". (Courtesy of Irv Bromberg, September 11, 2008.)

CHAPTER 9: THE OLD HINDU CALENDARS

38. Page 128, first text line of section 9.3: Change "method which" to "method in which". (Courtesy of Enrico Spinielli, March 12, 2008.)

39. Page 129, line 18: Change "Pausha" to "Pauṣa". (Courtesy of Andy Pepperdine, July 24, 2008.)

CHAPTER 10: THE MAYAN CALENDARS

40. Page 146, line -14: Change "Heui" to "Huei". (Courtesy of Andy Pepperdine, August 1, 2008.)

41. Page 146: The table of xihuitl months is a confusion of that used in different parts of Mexico; we should follow Caso who gives:

(1) Izcalli (Sprout)
(2) Atlcahualo (Water left)
(3) Tlacaxipehualiztli (Man flaying)
(4) Tozoztontli (1-vigil)
(5) Huei Tozoztli (2-vigil)
(6) Toxcatl (Drought)
(7) Etzalcualiztli (Eating bean soup)
(8) Tecuilhuitontli (1-lords feast)
(9) Huei Tecuilhuitl (2-lords feast)
(10) Tlaxochimaco (Give flowers)
(11) Xocotlhuetzi (Fruit falls)
(12) Ochpaniztli (Road sweeping)
(13) Teotleco (God arrives)
(14) Tepeilhuitl (Mountain feast)
(15) Quecholli (Macaw)
(16) Panquetzaliztli (Flag raising)
(17) Atemoztli (Falling water)
(18) Tititl (Storm)
(19) Nemontemi (Full in vain)

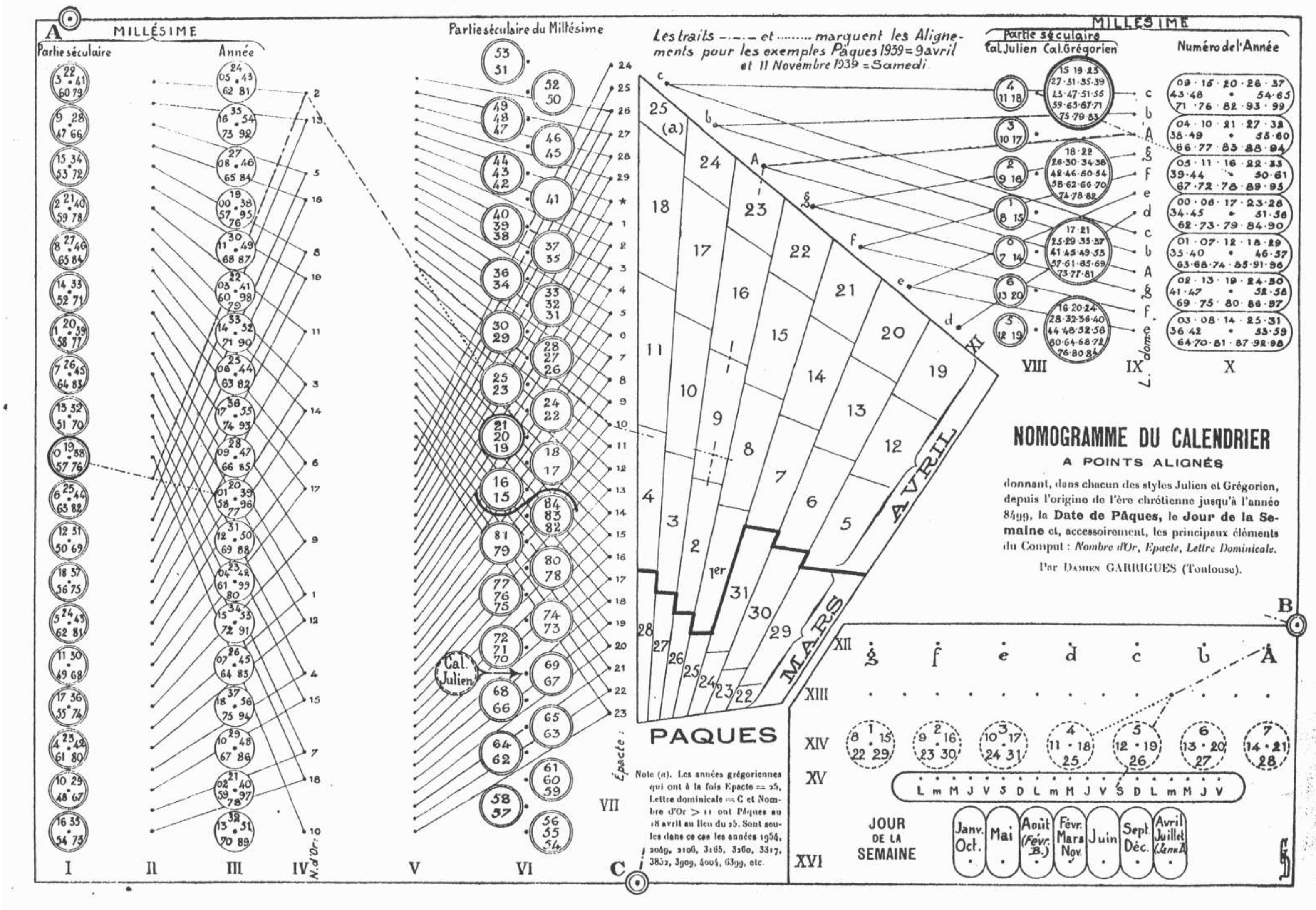

Figure 36-A: Nomogram for computing the dates of Julian and Gregorian Easter from Garrigues, Damien, "Généralization de la formule pascale de Gauss Nomogramme du calendrier perpétuel," *Annales Françaises de Chronométrie* **9** (1939), pp. 47–60.

Translations are taken from Edmundson (this is his calendar E on page 222). The placement of the nemontemi differs in different communities. Thanks to Susan Milbrath for helping straighten this out. (Courtesy of Andy Pepperdine, August 1, 2008.)

CHAPTER 12: GENERIC CYCLICAL CALENDARS

42. Page 166: A good example of a simple solar calendar to which the pattern of Section 12.1 can be applied is the 364-day Qumran calendar [Ben-Dov, Jonathan and Stéphane Saulnier, "Qumran Calendars: A Survey of Scholarship 1980–2007," *Currents in Biblical Research* **7** (2008) 124–168], and the similar calendars of the book of Jubillees and I Enoch. Each "season" consists of months of length 30, 30, 31, repeated four times to make a year. Thus every year has exactly 52 weeks, and every calendrical holiday always falls on the same day of the week.

    The appropriate parameters are $Y = 364$, $M = Y/12$, and $\delta_Y = \delta_M = 0$ in (12.1)–(12.6). No correlation, however, is known between this calendar and any other, so n_0 is unknown, and the calendar cannot be implemented for date conversion.

    At the same time, the Qumran calendar had a cycle of 24 weeks (based on I Chron. 2:4). The result is a 6-year repeating calendar. There was also a lunar cycle of alternating 29 and 30 day months, with one exceptional 30 day month at the end of the third year of the cycle, in which only the days of the arithmetical new and full moons were noted. This too can be represented with the formulas of this section.

43. Page 167, equation (12.42): Change amod to mod.

CHAPTER 13: TIME AND ASTRONOMY

44. A good quotation: "Time has no divisions to mark its passage, there is never a thunderstorm or blare of trumpets to announce the beginning of a new month or year. Even when a new century begins it is only we mortals who ring bells and fire off pistols." Thomas Mann, *The Magic Mountain*, ch. 5, "Whims of Mercurius," (1924), trans. by Helen T. Lowe-Porter (1928).

45. Page 177, **ephemeris-correction**: See

    `http://asa.usno.navy.mil/SecK/DeltaT.html`

    `http://eclipse.gsfc.nasa.gov/SEhelp/deltaT.html`

    `http://eclipse.gsfc.nasa.gov/SEhelp/deltatpoly2004.html`

    for more (and more recent) information.

46. Page 182, line -11: Change "Chapter 12" to "Chapter 20". (Courtesy of Enrico Spinielli, August 2, 2009.)

47. Page 186: There has recently been much worry in the fringe science community about the change in the tilt of the Earth's axis causing global calamity. *New Scientist* (9 August, 2008, page 56) refers to this as "fruitloopery".

48. Page 186, line -8: Change "latitude λ and longitude β" to "latitude β and longitude λ". (Courtesy of Kai Kuhlmann, August 13, 2008.)

49. Page 193, line 1: Change "such as noon midnight" to "such as noon or midnight". (Courtesy of Andy Pepperdine, August 28, 2008.)

50. Page 193, **mean-synodic-month**: The following value is what Meeus' *Astronomical Algorithms* has in the corrected 2nd ed. of June 15, 2005:

$$\textbf{mean-synodic-month} \stackrel{\text{def}}{=} 29.530588861 \tag{13.43}$$

(Courtesy of John Powers, September 27, 2008.)

51. Page 195–197, **nth-new-moon**: The following slightly modified version is what Meeus' *Astronomical Algorithms* has in the corrected 2nd ed. of June 15, 2005:

$$\textbf{nth-new-moon}\,(n) \stackrel{\text{def}}{=} \tag{13.44}$$

$$\textbf{universal-from-dynamical}\,(\mathit{approx} + \mathit{correction} + \mathit{extra} + \mathit{additional})$$

where

$$\begin{aligned}
n_0 &= 24724\\
k &= n - n_0\\
c &= \frac{k}{1236.85}\\
\mathit{approx} &= \textbf{j2000}\\
&\quad + \Big(5.09766\\
&\qquad + \textbf{mean-synodic-month} \times 1236.85 \times c\\
&\qquad + 0.0001437 \times c^2\\
&\qquad - 0.000000150 \times c^3\\
&\qquad + 0.00000000073 \times c^4\Big)\\
E &= 1 - 0.002516 \times c\\
&\quad - 0.0000074 \times c^2\\
\mathit{solar\text{-}anomaly} &= 2.5534^\circ\\
&\quad + 1236.85 \times 29.10535670^\circ \times c\\
&\quad - 0.0000014^\circ \times c^2\\
&\quad - 0.00000011^\circ \times c^3
\end{aligned}$$

$$
\begin{aligned}
\textit{lunar-anomaly} &= 201.5643^\circ \\
&\quad + 385.81693528 \times 1236.85^\circ \times c \\
&\quad + 0.0107582^\circ \times c^2 \\
&\quad + 0.00001238^\circ \times c^3 \\
&\quad - 0.000000058^\circ \times c^4
\end{aligned}
$$

$$
\begin{aligned}
\textit{moon-argument} &= 160.7108^\circ \\
&\quad + 390.67050284 \times 1236.85^\circ \times c \\
&\quad - 0.0016118^\circ \times c^2 \\
&\quad - 0.00000227^\circ \times c^3 \\
&\quad + 0.000000011^\circ \times c^4
\end{aligned}
$$

$$
\begin{aligned}
\Omega &= 124.7746 \\
&\quad + (-1.56375588 \times 1236.85) \times c \\
&\quad + 0.0020672 \times c^2 + 0.00000215 \times c^3
\end{aligned}
$$

$$
\begin{aligned}
\textit{correction} &= -0.00017^\circ \times \sin \Omega \\
&\quad + \sum \Big(\tilde{v} \times E^{\tilde{w}} \\
&\qquad \times \sin (\tilde{x} \times \textit{solar-anomaly} \\
&\qquad\qquad + \tilde{y} \times \textit{lunar-anomaly} \\
&\qquad\qquad + \tilde{z} \times \textit{moon-argument}) \Big)
\end{aligned}
$$

$$
\begin{aligned}
\textit{extra} &= 0.000325^\circ \\
&\quad \times \sin \Big(299.77^\circ \\
&\qquad + 132.8475848^\circ \times c \\
&\qquad - 0.009173^\circ \times c^2 \Big)
\end{aligned}
$$

$$\textit{additional} = \sum \left(\tilde{l} \times \sin (\tilde{\imath} + \tilde{\jmath} \times k) \right)$$

$\tilde{v}$ = (see Table 51-A)

$\tilde{w}$ = (see Table 51-A)

$\tilde{x}$ = (see Table 51-A)

$\tilde{y}$ = (see Table 51-A)

$\tilde{z}$ = (see Table 51-A)

$\tilde{\imath}$ = (see Table 51-B)

$\tilde{v}$	$\tilde{w}$	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$	$\tilde{v}$	$\tilde{w}$	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
−0.40720	0	0	1	0	0.00038	1	1	0	−2
0.17241	1	1	0	0	−0.00024	1	−1	2	0
0.01608	0	0	2	0	−0.00007	0	2	1	0
0.01039	0	0	0	2	0.00004	0	0	2	−2
0.00739	1	−1	1	0	0.00004	0	3	0	0
−0.00514	1	1	1	0	0.00003	0	1	1	−2
0.00208	2	2	0	0	0.00003	0	0	2	2
−0.00111	0	0	1	−2	−0.00003	0	1	1	2
−0.00057	0	0	1	2	0.00003	0	−1	1	2
0.00056	1	1	2	0	−0.00002	0	−1	1	−2
−0.00042	0	0	3	0	−0.00002	0	1	3	0
0.00042	1	1	0	2	0.00002	0	0	4	0

Table 51-A: Arguments for **nth-new-moon** (page 13).

$\tilde{\imath}$	$\tilde{\jmath}$	$\tilde{l}$	$\tilde{\imath}$	$\tilde{\jmath}$	$\tilde{l}$
251.88	0.016321	0.000165	34.52	27.261239	0.000047
251.83	26.651886	0.000164	207.19	0.121824	0.000042
349.42	36.412478	0.000126	291.34	1.844379	0.000040
84.66	18.206239	0.000110	161.72	24.198154	0.000037
141.74	53.303771	0.000062	239.56	25.513099	0.000035
207.14	2.453732	0.000060	331.55	3.592518	0.000023
154.84	7.306860	0.000056			

Table 51-B: Arguments for **nth-new-moon** (page 13).

$$\tilde{\jmath} \quad = \quad \text{(see Table 51-B)}$$

$$\tilde{l} \quad = \quad \text{(see Table 51-B)}$$

The differences in new moon times are only a fraction of a second for modern times, but can be 10 minutes different for 2000 years ago. (Courtesy of John Powers, September 27, 2008.)

52. Pages 195–197: The function **nth-new-moon** is based on Jean Meeus, *Astronomical Algorithms*, Willmann-Bell, Inc., Richmond, VA, 1991; the second edition (1998), which we cited but did not use, has more accurate coefficients and we should have used them. The comment at the bottom of page 197 regarding the time of first new moon after *j2000* only included the most significant term—the actual time is nearly 4 hours later. (Courtesy of Zhuo Meng, August 6, 2008.)

53. Page 199, last line: Change “distancefrom” to “distance from”. (Courtesy of Enrico Spinielli, March 12, 2008.)

54. Page 201, **moon-node**: The following function computes the angular distance of the node from the equinoctal point:

$$\mathbf{lunar\text{-}node}\,(\mathit{date}) \stackrel{\text{def}}{=} \tag{54-A}$$

$$\left(\left(\mathbf{moon\text{-}node}\,(\mathbf{julian\text{-}centuries}\,(\mathit{date})) + 90^\circ\right) \bmod 180^\circ\right) - 90^\circ$$

.

55. Page 201, middle: The following function computes sidereal lunar longitude:

$$\mathbf{sidereal\text{-}lunar\text{-}longitude}\,(t) \stackrel{\text{def}}{=} \tag{55-A}$$

$$\big(\mathbf{lunar\text{-}longitude}\,(t) - \mathbf{precession}\,(t) + \mathbf{sidereal\text{-}start}\big) \bmod 360$$

56. Page 208, **approx-moment-of-depression**: With the assumption that the inverse trigonometric functions return the principal value (see Erratum 1), the code can be simplified to

$$\mathbf{approx\text{-}moment\text{-}of\text{-}depression}\,(t, \mathit{location}, \alpha, \mathit{early?}) \stackrel{\text{def}}{=} \tag{13.65}$$

$$\begin{cases} \mathbf{local\text{-}from\text{-}apparent}\left(\mathit{date} + 12^{\text{h}} + \begin{Bmatrix} -1 & \textbf{if } \mathit{early?} \\ 1 & \textbf{otherwise} \end{Bmatrix} \times \left(6^{\text{h}} + \arcsin \mathit{value}\right), \mathit{location}\right) & \textbf{if } |\mathit{value}| \le 1 \\ \mathbf{bogus} & \textbf{otherwise} \end{cases}$$

where

$$try = \textbf{sine-offset}\,(t, \mathit{location}, \alpha)$$

$$date = \textbf{fixed-from-moment}\,(t)$$

$$alt = \begin{cases} date & \textbf{if } \alpha \geq 0 \text{ and } \mathit{early?} \\ date + 1 & \textbf{if } \alpha \geq 0 \\ date + 12^{\mathrm{h}} & \textbf{otherwise} \end{cases}$$

$$value = \begin{cases} \textbf{sine-offset}\,(alt, \mathit{location}, \alpha) & \textbf{if } \left| \; try \; \right| > 1 \\ try & \textbf{otherwise} \end{cases}$$

(Courtesy of Tony Finch, August 12, 2008.)

57. Page 209, **sine-offset**: The function **sine-offset** cannot be used at the poles, and the calculation loses significance close to the poles. At the North pole, $\phi = 90°$, and $\cos\phi = 0$. This function only has a useful value when $\delta = -\alpha$, but then, as written, the definition reduces to $\infty - \infty$ which is indeterminate. Rewriting it as $(\sin\phi\sin\delta + \sin\alpha)/(\cos\delta\cos\phi)$ only changes it to $0/0$, again indeterminate. The function **moment-of-depression** cannot be used to find the one sunrise or sunset in the year at a pole, because the function always returns bogus when $\phi = 90°$; very close to the pole the results will be inaccurate. (Courtesy of Andy Pepperdine, December 29, 2008.)

58. Page 213, **asr**: The code is designed only for northern termperate regions. It should be made robust so that it also works for the southern hemisphere and polar regions. (Courtesy of Andy Pepperdine, August 31, 2008.)

59. Page 213: The rising and setting times of the moon can be determined in a similar fashion to that of the sun. A standard value of refraction, taking elevation into account, is computed as follows:

$$\textbf{refraction}\,(t, \mathit{location}) \stackrel{\text{def}}{=} \tag{59-A}$$

$$50' + \mathit{dip} + 19'' \times \sqrt{h}$$

where

$$h = \max\{0\text{m}, \mathit{location}_{\textbf{elevation}}\}$$

$$R = 6.372 \times 10^{6}\text{m}$$

$$\mathit{dip} = \arccos\left(\frac{R}{R+h}\right)$$

(This can also be used within **sunrise** and **sunset**.) The time parameter t is not being used here, but could be used in a more refined calculation that takes average atmospheric conditions into account. Refraction is then used to adjust topocentric altitude:

$$\textbf{observed-lunar-altitude}\,(t, \mathit{location}) \stackrel{\text{def}}{=} \tag{59-B}$$

$$\textbf{topocentric-lunar-altitude}\,(t, \mathit{location}) + \textbf{refraction}\,(t, \mathit{location})$$

Moonrise and moonset are found by binary search, after estimating the time of the event, based on altitude at midnight and on whether the moon is waxing or waning:

$$\textbf{moonrise}\,(\mathit{date}, \mathit{location}) \stackrel{\text{def}}{=} \tag{59-C}$$

$$\begin{cases} \textbf{standard-from-universal}\,(\mathit{rise}, \mathit{location}) & \textbf{if } \mathit{rise} < t+1 \\ \textbf{bogus} & \textbf{otherwise} \end{cases}$$

where

$$t = \textbf{universal-from-standard}\,(\mathit{date}, \mathit{location})$$

$$\mathit{waning} = \textbf{lunar-phase}\,(t) > 180^\circ$$

$$\mathit{alt} = \textbf{observed-lunar-altitude}\,(t, \mathit{location})$$

$$\mathit{offset} = \frac{\mathit{alt}}{360^\circ}$$

$$\mathit{approx} = \begin{cases} t + 1 - \mathit{offset} & \textbf{if } \mathit{waning} \text{ and } \mathit{offset} > 0 \\ t - \mathit{offset} & \textbf{if } \mathit{waning} \\ t + \frac{1}{2} + \mathit{offset} & \textbf{otherwise} \end{cases}$$

$$\mathit{rise} = \underset{x \in \left[\mathit{approx} - 3^{\text{h}},\ \mathit{approx} + 3^{\text{h}}\right]}{\overset{p(l,u)}{\text{MIN}}} \left\{ \textbf{observed-lunar-altitude}\,(x, \mathit{location}) > 0^\circ \right\}$$

$$p\,(l, u) = u - l < \frac{1}{60}^{\text{h}}$$

$$\textbf{moonset}\,(\mathit{date}, \mathit{location}) \stackrel{\text{def}}{=} \tag{59-D}$$

$$\begin{cases} \textbf{standard-from-universal}\,(\mathit{set}, \mathit{location}) & \textbf{if } \mathit{set} < t+1 \\ \textbf{bogus} & \textbf{otherwise} \end{cases}$$

where

$$
\begin{aligned}
t &= \textbf{universal-from-standard}\,(date, location)\\
waxing &= \textbf{lunar-phase}\,(t) < 180^\circ\\
alt &= \textbf{observed-lunar-altitude}\,(t, location)\\
offset &= \frac{alt}{360^\circ}\\
approx &= \begin{cases} t + offset & \textbf{if } waxing \text{ and } offset > 0\\ t + 1 + offset & \textbf{if } waxing\\ t - offset + \frac{1}{2} & \textbf{otherwise}\end{cases}\\
set &= \underset{x \in \left[\substack{approx\\ -3^{\mathrm{h}}}, \substack{approx\\ +3^{\mathrm{h}}}\right]}{\overset{p(l,u)}{\text{MIN}}} \left\{ \begin{array}{l}\textbf{observed-lunar-altitude}\\ \qquad\left(x, location\right) < 0^\circ\end{array}\right\}\\
p\,(l, u) &= u - l < \frac{1}{60}^{\mathrm{h}}
\end{aligned}
$$

A **bogus** value is returned if on the day in question, the event does not occur, as happens about once a month. This function is not robust in the sense that it returns the time the moon gets closest to the horizon, when it does not appear to cross the horizon, as happens in polar latitudes.

CHAPTER 14: THE PERSIAN CALENDAR

60. Page 218, line 4: Change "13.5" to "15.3". (Courtesy of Andy Pepperdine, July 23, 2008.)

61. Page 219, line 2: Change "calendar 1925" to "calendar in 1925". (Courtesy of Enrico Spinielli, March 12, 2008.)

62. Page 219, beginning of section 14.2: Add the following quotation from Ovid's *Fasti*, Book I, lines 149–150 "*Die, age, frigoribus quare novus incipit annus, qui melius per ver incipiendus erat*?" [Come, say, why doth the new year begin in the cold season? Better had it begun in spring.] Translation by Sir James George Frazer, *Ovid's Fasti*, Harvard University Press, Cambridge, MA, 1931.

CHAPTER 15: THE BAHÁ'Í CALENDAR

63. Page 236, line −1: Change "Birth of 'Abdu'l-Bahā (also 'Aẓamat 7 = May 23)" to "Day of the Covenant (Qawl 4 = November 26)". (Courtesy of William P. Collins, June 20, 2008.)

CHAPTER 16: THE FRENCH REVOLUTIONARY CALENDAR

64. Page 243, **arithmetic-french-from-fixed**: We should recast the calculation of the year as per Erratum 22.

CHAPTER 17: THE CHINESE CALENDAR

65. Page 247, line −16: Detailed calculations of earlier forms of the Chinese calendar are given in Jean-Claudse Martzloff, *Le calendrier chinois: structure et calculs (104 av. J.-C.–1644)*, Éditions Champion, Paris, 2009.

66. Page 261–262: For compactness and consistency we should rename **chinese-name-of-day** to **chinese-day-name**, **chinese-name-of-month** to **chinese-month-name** and **chinese-name-of-year** to **chinese-year-name**. (Courtesy of Irv Bromberg, May 16, 2007.)

67. Page 268, lines −5 and −4: The ending year for the Meiji era should be 1912, not 1911; the ending year for the Keio era should be 1868, not 1866. (Courtesy of Peter Zilahy Ingerman, February 15, 2008.)

CHAPTER 18: THE MODERN HINDU CALENDARS

68. Page 275. The great mathematician Leonhard Euler was interested in Indian astronomy and he developed calendrical formulas for the Indian calendars. See Kim Plofker, "Euler and Indian Astronomy," *Leonhard Euler: Life, Work, and Legacy*, Robert E. Bradley and C. Edward Sandifer, eds., Elsevier B.V., 2007, pp. 147–166.

69. Page 275. A detailed discussion of Indian dates and eras is given in chapter VII (pp. 219–326) in *Indian Epigraphy* by D. C. Sircar, Motilal Banarsidass, Delhi, 1965.

70. Page 277, line 15: Change "Pausha" to "Pauṣa". (Courtesy of Andy Pepperdine, October 1, 2008.)

71. Page 278, line −8: Change "Mārgaśīrsha" to "Mārgaśīrṣa".

72. Page 278, line −15: Change "Mārgaśīra" to "Mārgaśīrṣa". (Courtesy of Andy Pepperdine, October 1, 2008.)

73. Page 289: The terms "Orissa rule", "Tamil rule", "Malayali rule", "Madras", and "Bengal rule" were not indexed.

74. Page 292: Change **hindu-locale** to **hindu-location** for consistency with the Chinese, Japanese, Korean, and Vietnamese calendars.

75. Page 293, **hindu-tropical-longitude**: The computation of *day* does not need the floor function because the parameter *date* in a *fixed-date* and hence an integer, and **hindu-epoch** is also an integer. It would make more sense to drop the floor and have this function take moment t instead of a fixed date (Courtesy of Andy Pepperdine, October 13, 2008.)

76. Page 306, line 20. Change "Mārgaśīra" to "Mārgaśīrṣa".

77. Page 305, line −9: Change "Chaitra" to "Caitra". (Courtesy of Andy Pepperdine, October 15, 2008.)

78. Page 306, line 13: Change "Chaitra" to "Caitra". (Courtesy of Andy Pepperdine, October 15, 2008.)

79. Page 309, line −6: Change **hindu-lunar-day-at-or-after**$(d, (n+1)/2)$ to **hindu-lunar-day-at-or-after**$((n+1)/2, d)$. (Courtesy of Andy Pepperdine, October 23, 2008.)

80. Page 312, line 15: Change *kaṛana* to *karaṇa*. (Courtesy of Andy Pepperdine, October 21, 2008.)

CHAPTER 19: THE TIBETAN CALENDAR

81. Page 315, quotation by Berzin: Add a footnote with our usual disclaimer that we do not necessarily subscribe to the opinions expressed in the quotation!

82. Page 315, first two lines of the first paragraph: Change "*Phugpa* or *Phukluk*" to "*Phug-lugs* (or *Phug-pa* or *Phukluk*)". (Courtesy of Dieter Schuh, August 18, 2009.)

83. Page 315, first paragraph: Add a sentence: "Historically, many other variants have been used, posing similar problems to the historian as do Hindu dates. Two important works on the Tibetan calendars are Petri and Tseng." (See Erratum 86.) (Courtesy of Dieter Schuh, August 18, 2009.)

84. Page 321, **tibetan-leap-month?**: The order of the parameters should be reversed so they are most significant to least significant.

85. Page 321: It is a simple matter to check whether a given Tibetan historical date might actually be a leap day with the following new function

$$\textbf{tibetan-leap-day?}\,(\textit{t-year}, \textit{t-month}, \textit{t-day}) \stackrel{\text{def}}{=} \qquad \text{(85-A)}$$

$$\begin{aligned}
&\textit{t-day} \\
&= \Bigg(\textbf{tibetan-from-fixed}\Bigg(\textbf{fixed-from-tibetan}\left(\begin{array}{|c|c|c|c|c|}\hline \textit{t-year} & \textit{t-month} & \text{false} & \textit{t-day} & \text{true} \\ \hline\end{array}\right)\Bigg)\Bigg)_{\textbf{day}} \text{ or} \\
&\textit{t-day} \\
&= \Bigg(\textbf{tibetan-from-fixed}\Bigg(\textbf{fixed-from-tibetan}\left(\begin{array}{|c|c|c|c|c|}\hline \textit{t-year} & \textit{t-month} & \begin{matrix}\textbf{tibetan-}\\ \textbf{leap-month?}\\ (\textit{t-month}, \textit{t-year})\end{matrix} & \textit{t-day} & \text{true} \\ \hline\end{array}\right)\Bigg)\Bigg)_{\textbf{day}}
\end{aligned}$$

Or, in Lisp (page 436):

```
(defun tibetan-leap-day? (t-year t-month t-day)
   ;; TYPE (tibetan-day tibetan-month tibetan-year) ->
   ;; tibetan-leap-day.  True if t-day is leap in Tibetan
   ;; month t-month and year t-year.
   (or
    (= t-day
       (tibetan-day
        (tibetan-from-fixed
         (fixed-from-tibetan
          (tibetan-date t-year t-month false t-day true)))))
    ; Check also in leap month if there is one.
    (= t-day
       (tibetan-day
        (tibetan-from-fixed
         (fixed-from-tibetan
          (tibetan-date t-year t-month
                        (tibetan-leap-month? t-month t-year)
                        t-day true)))))))
```

86. Page 322: Add the following references and remarks. Alexander Csoma de Kőrős (*A Grammar of the Tibetan Language*, Calcutta 1834, p. 148) wrongly fixed the first year of the first Rabbyuñ-cycle as 1026 instead of 1027. This was corrected in Berthold Laufer's "The Application of the Tibetan Sexagenary Cycle," *T'oung Pao*, vol. 14 (1913), pp. 569–596 and in Paul Pelliot's "Le Cycle Sexagénaire dans la Chronologie Tibétaine," *Journal Asiatique*, vol. 1 (1913), pp. 633-667. Other important references are Alan Wallace's translation of *The Kalachakra Tantra, Geshe Ngawang Dhargyey*, Library of Tibetan Works and Archives, Dharamsala, 1986, Winfried Petri's *Indo-tibetische Astronomic Flabilitationsschrift zur Erlangung der nia legendi fur das Fach Geschichte der Naturwissenschaften an der Hohen Naturwissenschaftlichen Fakultät der Ludwig Maximilians Universität zu München*, Munich, 1966 and Te-ming Tseng's *Sino-Tibetische Divinationskalkulationen (Nag-rtsis) dargestellt anhand des Werkes dPag-bsam Ijon-šiṅ von Blo-bzaṅ tshul-khrims rgya-mtsho*, Halle, 2005. Reference [2], Svante Janson's manuscript, can be found at `http://www.math.uu.se/~svante/papers/calendar/tibet.pdf`. (Courtesy of Dieter Schuh, August 18, 2009.)

CHAPTER 20: ASTRONOMICAL LUNAR CALENDARS

87. Page 326, line 7: Table 8.1 is on page 120, not page 119. (Courtesy of Steve Ward, July 27, 2008.)

88. Section 20.2: Another criterion for lunar crescent visibility has been suggested by Odeh, Mohammad Sh., "New criterion for lunar crescent visibility," *Experimental Astronomy* **18** (2004), pp. 39–64.

89. Page 326, **visible-crescent**: The code fails for high altitudes and polar regions. There are two problems. First, dusk may return **bogus**, but, the moon will eventually become visible at some point during the lunar month, so returning false is not appropriate. Perhaps we should use midnight in this case. Second, at the poles, if the first quarter of the moon coincides with the winter solstice, and the moon is the opposite side of the ecliptic, then it will not have been visible throughout the whole of the first quarter. One way to make the test robust, would be to increase the upper limit test from first-quarter to full (or something in between), but the moon will be gibbous when it does appear. When the sun remains above the horizon, then the observation may require a larger arc-of-light. (Courtesy of Andy Pepperdine, September 15, 2008.)

90. Page 327, **visible-crescent**: The function **visible-crescent** checks visibility on the eve of *date*, so we want:

$$\textbf{phasis-on-or-after}\,(\textit{date}, \textit{location}) \stackrel{\text{def}}{=} \qquad (20.4)$$

$$\underset{d \geq \tau}{\text{MIN}} \left\{ \textbf{visible-crescent}\,(d, \textit{location}) \right\}$$

where

$$\textit{mean} = \textit{date} - \left\lfloor \frac{\textbf{lunar-phase}\,(\textit{date}+1)}{360^\circ} \times \textbf{mean-synodic-month} \right\rfloor$$

$$\tau = \begin{cases} \textit{date} & \textbf{if } \textit{date} - \textit{mean} \leq 3 \text{ and not } \textbf{visible-crescent}\,(\textit{date}, \textit{location}) \\ \textit{mean} + 29 & \textbf{otherwise} \end{cases}$$

91. Page 328, line 4: Change "Finally, once" to "Finally, one". (Courtesy of Enrico Spinielli, March 12, 2008.)

92. Page 328: Change **islamic-locale** to **islamic-location** for consistency with the Chinese, Japanese, Korean, and Vietnamese calendars.

93. Page 330: It might be better to use Haifa as the location for determining visibility, because the crescent moon is more often visible from there, with the criterion we are using. So, define

$$\textbf{hebrew-location} \stackrel{\text{def}}{=} \textbf{haifa} \qquad (93\text{-A})$$

and use it in the Observational Hebrew code:

$$\textbf{observational-hebrew-new-year}\,(\textit{g-year}) \stackrel{\text{def}}{=} \qquad (93\text{-B})$$

$$\textbf{phasis-on-or-after} \left(\lfloor \textit{equinox} \rfloor - \begin{cases} 14 & \textbf{if } \textit{equinox} < \textit{set} \\ 13 & \textbf{otherwise} \end{cases}, \textbf{hebrew-location} \right)$$

where

$$\textit{jan}_1 = \textbf{gregorian-new-year}\,(\textit{g-year})$$

$$\textit{equinox} = \textbf{solar-longitude-after}\,(\textbf{spring}, \textit{jan}_1)$$

$$set = \textbf{universal-from-standard}\left(\textbf{sunset}\left(\lfloor equinox \rfloor, \textbf{hebrew-location}\right), \textbf{hebrew-location}\right)$$

$$\textbf{fixed-from-observational-hebrew}\left(\boxed{year \mid month \mid day}\right) \stackrel{\text{def}}{=} \tag{93-C}$$

$$\textbf{phasis-on-or-before}\,(midmonth, \textbf{hebrew-location}) + day - 1$$

where

$$year_1 = \begin{cases} year - 1 & \textbf{if } month \geq \textbf{tishri} \\ year & \textbf{otherwise} \end{cases}$$

$$start = \textbf{fixed-from-hebrew}\left(\boxed{year_1 \mid \textbf{nisan} \mid 1}\right)$$

$$g\text{-}year = \textbf{gregorian-year-from-fixed}\,(start + 60)$$

$$new\text{-}year = \textbf{observational-hebrew-new-year}\,(g\text{-}year)$$

$$midmonth = new\text{-}year + \text{round}\,(29.5 \times (month - 1)) + 15$$

$$\textbf{observational-hebrew-from-fixed}\,(date) \stackrel{\text{def}}{=} \tag{93-D}$$

$$\boxed{year \mid month \mid day}$$

where

$$crescent = \textbf{phasis-on-or-before}\,(date, \textbf{hebrew-location})$$

$$g\text{-}year = \textbf{gregorian-year-from-fixed}\,(date)$$

$$ny = \textbf{observational-hebrew-new-year}\,(g\text{-}year)$$

$$new\text{-}year = \begin{cases} \textbf{observational-hebrew-new-year}\,(g\text{-}year - 1) & \textbf{if } date < ny \\ ny & \textbf{otherwise} \end{cases}$$

$$month = \text{round}\left(\frac{1}{29.5} \times (\textit{crescent} - \textit{new-year})\right) + 1$$

$$year = (\textbf{hebrew-from-fixed}\,(\textit{new-year}))_{\textbf{year}} + \begin{cases} 1 & \textbf{if } \textit{month} \geq \textbf{tishri} \\ 0 & \textbf{otherwise} \end{cases}$$

$$day = \textit{date} - \textit{crescent} + 1$$

94. Pages 328–331, sections 20.3 and 20.4: Our code allows for a 31st day of an Islamic or Hebrew month, which is longer than allowed by the rules. Instead, that day would be the first of the following month—were the moon actually observed when the simple criterion we are using says it becomes visible. This shift can cascade for several months. We did not take this into account, since there is no way to determine when in fact the new moons are actually observed, and which months are affected.

The following code takes this rule into account.

$$\textbf{month-length}\,(\textit{date}, \textit{location}) \stackrel{\text{def}}{=} \textit{moon} - \textit{prev} \tag{94-A}$$

where

$$\textit{moon} = \textbf{phasis-on-or-after}\,(\textit{date} + 1, \textit{location})$$

$$\textit{prev} = \textbf{phasis-on-or-before}\,(\textit{date}, \textit{location})$$

$$\textbf{alt-fixed-from-observational-islamic}\left(\boxed{\textit{year} \mid \textit{month} \mid \textit{day}}\right) \stackrel{\text{def}}{=} \tag{94-B}$$

$$\begin{cases} \textit{date} - 1 & \textbf{if early-month?}\,(\textit{midmonth}, \textbf{islamic-location}) \\ \textit{date} & \textbf{otherwise} \end{cases}$$

where

$$\textit{midmonth} = \textbf{islamic-epoch} + \left\lfloor \left((\textit{year} - 1) \times 12 + \textit{month} - \tfrac{1}{2}\right) \times \textbf{mean-synodic-month} \right\rfloor$$

$$\textit{moon} = \textbf{phasis-on-or-before}\,(\textit{midmonth}, \textbf{islamic-location})$$

$$\textit{date} = \textit{moon} + \textit{day} - 1$$

$$\textbf{alt-observational-islamic-from-fixed}\,(\textit{date}) \stackrel{\text{def}}{=} \tag{94-C}$$

year	month	day

where

$$early = \textbf{early-month?}\,(date, \textbf{islamic-location})$$

$$long = early \text{ and } \textbf{month-length}\,(date, \textbf{islamic-location}) > 29$$

$$date' = \begin{cases} date + 1 & \textbf{if } long \\ date & \textbf{otherwise} \end{cases}$$

$$moon = \textbf{phasis-on-or-before}\,\big(date', \textbf{islamic-location}\big)$$

$$elapsed\text{-}months = \text{round}\left(\frac{moon - \textbf{islamic-epoch}}{\textbf{mean-synodic-month}}\right)$$

$$year = \left\lfloor \frac{1}{12} \times elapsed\text{-}months \right\rfloor + 1$$

$$month = (elapsed\text{-}months \mod 12) + 1$$

$$day = date' - moon - \left\{\begin{array}{ll} -2 & \textbf{if } early \text{ and not } long \\ -1 & \textbf{otherwise} \end{array}\right\}$$

alt-observational-hebrew-from-fixed (94-D)

$$(date) \stackrel{\text{def}}{=}$$

year	month	day

where

$$early = \textbf{early-month?}\,(date, \textbf{hebrew-location})$$

$$long = early \text{ and } \textbf{month-length}\,(date, \textbf{hebrew-location}) > 29$$

$$date' = \begin{cases} date + 1 & \textbf{if } long \\ date & \textbf{otherwise} \end{cases}$$

$$moon = \textbf{phasis-on-or-before}\left(date', \textbf{hebrew-location}\right)$$

$$g\text{-}year = \textbf{gregorian-year-from-fixed}\left(date'\right)$$

$$ny = \textbf{observational-hebrew-new-year}\,(g\text{-}year)$$

$$new\text{-}year = \begin{cases} \textbf{observational-hebrew-new-year}\,(g\text{-}year - 1) & \textbf{if } date' < ny \\ ny & \textbf{otherwise} \end{cases}$$

$$month = \text{round}\left(\frac{1}{29.5} \times (moon - new\text{-}year)\right) + 1$$

$$year = \left(\textbf{hebrew-from-fixed}\,(new\text{-}year)\right)_{\textbf{year}} + \begin{Bmatrix} 1 & \textbf{if } month \geq \textbf{tishri} \\ 0 & \textbf{otherwise} \end{Bmatrix}$$

$$day = date' - moon - \begin{Bmatrix} -2 & \textbf{if } early \text{ and not } long \\ -1 & \textbf{otherwise} \end{Bmatrix}$$

$$\textbf{alt-fixed-from-observational-hebrew}\left(\boxed{year \mid month \mid day}\right) \stackrel{\text{def}}{=} \tag{94-E}$$

$$\begin{cases} date - 1 & \textbf{if early-month?}\,(midmonth, \textbf{hebrew-location}) \\ date & \textbf{otherwise} \end{cases}$$

where

$$year_1 = \begin{cases} year - 1 & \textbf{if } month \geq \textbf{tishri} \\ year & \textbf{otherwise} \end{cases}$$

$$start = \textbf{fixed-from-hebrew}\left(\boxed{year_1 \mid \textbf{nisan} \mid 1}\right)$$

$$g\text{-}year = \textbf{gregorian-year-from-fixed}\,(start + 60)$$

$$new\text{-}year = \textbf{observational-hebrew-new-year}\,(g\text{-}year)$$

$$\begin{aligned} \textit{midmonth} &= \textit{new-year} + \text{round}\,(29.5 \times (\textit{month} - 1)) + 15 \\ \textit{moon} &= \textbf{phasis-on-or-before} \\ &\quad (\textit{midmonth}, \textbf{hebrew-location}) \\ \textit{date} &= \textit{moon} + \textit{day} - 1 \end{aligned}$$

95. Pages 331, after section 20.3: The classical Babylonian calendar was of the lunisolar type, from about 380 B.C.E., with a fixed 19-year Metonic cycle, and with the day of the true new moon determined by an approximate calculation, based on the lag time between sunset and moonset, which had to be at least 48 minutes. Prior to that date, leap years were irregular. See Parker, Richard A. and Waldo H. Dubberstein, *Babylonian Chronology 626 B.C.-A.D. 75*, Brown University Press, Providence, RI, 1956. The month names were

(1) Nisanu	(7) Tashritu
(2) Ayaru	(8) Arakhsamna
(3) Simanu	(9) Kislimu
(4) Du'uzu	(10) Tebetu
(5) Abu	(11) Shabatu
(6) Ululu	(12) Adaru

Lag time is simply:

$$\textbf{moonlag}\,(\textit{date}, \textit{location}) \stackrel{\text{def}}{=} \tag{95-A}$$

$$\begin{cases} \textbf{bogus} & \textbf{if } \textit{sun} = \textbf{bogus} \\ \textit{date} + 1 - \textit{sun} & \textbf{if } \textit{moon} = \textbf{bogus} \\ \textit{moon} - \textit{sun} & \textbf{otherwise} \end{cases}$$

where

$$\begin{aligned} \textit{sun} &= \textbf{sunset}\,(\textit{date}, \textit{location}) \\ \textit{moon} &= \textbf{moonset}\,(\textit{date}, \textit{location}) \end{aligned}$$

taking into account the possibility of the non-occurrence of sunset or moonset. We take Babylon

$$\textbf{babylon} \stackrel{\text{def}}{=} \tag{95-B}$$

32.4794°	44.4328°	26m	$3\frac{1}{2}^{\text{h}}$

as the determining location. The time of the new moon is found by linear search:

$$\textbf{babylonian-new-month-on-or-before}\,(\textit{date}) \stackrel{\text{def}}{=} \tag{95-C}$$

$$\underset{d \geq \tau}{\text{MIN}} \left\{ \textbf{moonlag}\,(d, \textbf{babylon}) > lag \right\}$$

where

$$approx = \left\lfloor date - \frac{\textbf{lunar-phase}\,(date)}{12^\circ} \right\rfloor$$

$$lag = \frac{48}{60}^{\text{h}}$$

$$\tau = \begin{cases} approx - 33 & \textbf{if } date - approx \leq 3 \text{ and } \textbf{moonlag}\,(date, \textbf{babylon}) \leq lag \\ approx - 3 & \textbf{otherwise} \end{cases}$$

We use the beginning of the Seleucid era, April 3, 311 B.C.E., as the calendar's epoch

$$\textbf{babylonian-epoch} \stackrel{\text{def}}{=} \qquad (95\text{-D})$$

$$\textbf{fixed-from-julian}\left(\boxed{311 \text{ B.C.E.}} \boxed{\textbf{april}} \boxed{3} \right)$$

The leap-year rule is similar to that of the Hebrew calendar (**hebrew-leap-year?**):

$$\textbf{babylonian-leap-year?}\,(b\text{-}year) \stackrel{\text{def}}{=} \qquad (95\text{-E})$$

$$((7 \times b\text{-}year + 13) \mod 19) < 7$$

The last month of the year, Adaru, was intercalated in years 1, 4, 7, 9, 12, and 15 of the cycle; the sixth month, Ululu, was intercalated instead during the 18th year. Taking this anomaly into account, the conversions are straightforward:

$$\textbf{fixed-from-babylonian}\left(\boxed{year} \boxed{month} \boxed{leap} \boxed{day} \right) \stackrel{\text{def}}{=} \qquad (95\text{-F})$$

$$\textbf{babylonian-new-month-on-or-before}\,(midmonth) + day - 1$$

where

$$month_1 = \begin{cases} month & \textbf{if } leap \text{ or } \{(year \mod 19) = 18 \text{ and } month > 6\} \\ month - 1 & \textbf{otherwise} \end{cases}$$

$$
\begin{aligned}
\textit{months} &= \left\lfloor \frac{1}{19} \times ((\textit{year} - 1) \times 235 + 13) \right\rfloor + \textit{month}_1 \\
\textit{midmonth} &= \textbf{babylonian-epoch} \\
&\quad + \text{round}\left(\textbf{mean-synodic-month} \times \textit{months}\right) + 15
\end{aligned}
$$

$$
\textbf{babylonian-from-fixed}\,(\textit{date}) \stackrel{\text{def}}{=} \tag{95-G}
$$

year	*month*	*leap*	*day*

where

$$
\begin{aligned}
\textit{crescent} &= \textbf{phasis-on-or-before}\,(\textit{date}, \textbf{babylon}) \\
\textit{months} &= \text{round}\left(\frac{\textit{crescent} - \textbf{babylonian-epoch}}{\textbf{mean-synodic-month}}\right) \\
\textit{year} &= \left\lfloor \frac{1}{235} \times (19 \times \textit{months} + 5) \right\rfloor + 1 \\
\textit{approx} &= \textbf{babylonian-epoch} \\
&\quad + \text{round} \\
&\qquad \left(\left\lfloor \tfrac{1}{19} \times ((\textit{year} - 1) \times 235 + 13) \right\rfloor \right. \\
&\qquad\quad \left. \times \textbf{mean-synodic-month} \right) \\
\textit{new-year} &= \textbf{babylonian-new-month-on-or-before} \\
&\qquad (\textit{approx} + 15) \\
\textit{month}_1 &= \text{round}\left(\frac{1}{29.5} \times (\textit{crescent} - \textit{new-year})\right) + 1 \\
\textit{special} &= (\textit{year} \mod 19) = 18 \\
\textit{leap} &= \begin{cases} \textit{month}_1 = 7 & \textbf{if } \textit{special} \\ \textit{month}_1 = 13 & \textbf{otherwise} \end{cases} \\
\textit{month} &= \begin{cases} \textit{month}_1 - 1 & \textbf{if } \textit{leap} \text{ or } \{\textit{special} \text{ and } \textit{month}_1 > 6\} \\ \textit{month}_1 & \textbf{otherwise} \end{cases} \\
\textit{day} &= \textit{date} - \textit{crescent} + 1
\end{aligned}
$$

Since it is not always certain how the evening of the new moon was determined, these dates should be considered approximate. See Parker and Dubberstein.

96. Pages 329, between sections 20.3 and 20.4: Saudi Arabia employs the *Umm al-Qura* calendar for some secular purposes, as an approximation of the Observational Islamic calendar. The rule—since March 2002—is that the month begins on the first evening after the conjunction on which the moon sets after the sun. See, for example, `http://www.phys.uu.nl/~vgent/islam/ummalqura.htm`. This criterion can be expressed as

$$\textbf{saudi-crescent}\,(date, location) \stackrel{\text{def}}{=} \tag{96-A}$$

$$\textbf{moonset}\,(eve, location) > \textbf{sunset}\,(eve, location) \text{ and } \textbf{lunar-phase}\left(\textbf{universal-from-standard}\,(\textbf{sunset}\,(eve, location), location)\right) < 180^\circ$$

where

$$eve = date - 1$$

The functions **fixed-from-saudi-islamic** and **saudi-islamic- from-fixed** are analogous to **fixed-from-saudi-islamic** and **saudi-islamic- from-fixed**, respectively, except that **saudi-crescent** at **saudi-location** (that is, Mecca) is used:

$$\textbf{saudi-location} \stackrel{\text{def}}{=} \tag{96-B}$$

$$\boxed{21^\circ 25.34' \mid 39^\circ 49.57' \mid 0\text{m} \mid 3^{\text{h}}}$$

$$\textbf{fixed-from-saudi-islamic}\left(\boxed{year \mid month \mid day}\right) \stackrel{\text{def}}{=} \tag{96-C}$$

$$\textbf{saudi-phasis-on-or-before}\,(midmonth, \textbf{saudi-location}) + day - 1$$

where

$$midmonth = \textbf{islamic-epoch} + \left\lfloor \left((year-1)\times 12 + month - \tfrac{1}{2}\right) \times \textbf{mean-synodic-month} \right\rfloor$$

$$\textbf{saudi-islamic-from-fixed}\,(date) \stackrel{\text{def}}{=} \tag{96-D}$$

$$\boxed{year \mid month \mid day}$$

where

$$\begin{aligned}
\textit{crescent} &= \textbf{saudi-phasis-on-or-before} \\
&\qquad (\textit{date}, \textbf{saudi-location}) \\
\textit{elapsed-months} &= \text{round}\left(\frac{\textit{crescent} - \textbf{islamic-epoch}}{\textbf{mean-synodic-month}}\right) \\
\textit{year} &= \left\lfloor \frac{1}{12} \times \textit{elapsed-months} \right\rfloor + 1 \\
\textit{month} &= (\textit{elapsed-months} \mod 12) + 1 \\
\textit{day} &= \textit{date} - \textit{crescent} + 1
\end{aligned}$$

BACKMATTER

97. Page 337, *armenian-day*: The range should be $1 \ldots 30$. (Courtesy of Uri Blass, July 31, 2008.)

98. Page 341, line −1: We never use the type *time* (we use *clock-time*, which needs to be changed to ⟨*hour*, *minute*, *second*⟩); it should be called *decimal-time*. The function **time-from-moment** has this result type. (Courtesy of Uri Blass, July 30, 2008.)

99. Page 358: The functions **mean-sidereal-year**, **mean-synodic-month**, and **mean-tropical-year** should have type *duration*. Also in the Lisp code (pages 399, 401, and 399, respectively.

100. Appendix B: We have not been consistent with the order of parameters passed to functions—our general rule should be that parameters are in decreasing order of siginificance of units (year before month, month before day, and so on). Among the functions in which this is not done are `last-day-of-hebrew-month`, `molad`, `hindu-expunged?`, `tibetan-leap-month?`, and `tibetan-leap-day?` (Erratum 85).

101. Page 381, line 3 of `last-day-of-hebrew-month`: Incorrect whitespace around ***h-month***. (Courtesy of Enrico Spinielli, June 22, 2008.)

102. Page 383, line 13 of `yom-ha-zikkaron`: Change “Friday or Saturday” to “Thursday or Friday”:

```
13          ;; If Iyyar 4 is Thursday or Friday, then Wednesday
```

(Courtesy of Enrico Spinielli, June 22, 2008.)

103. Page 395: We should rename `cosine-degrees` and `tangent-degrees` to `cos-degrees` and `tan-degrees` for consistency with other trigonometric functions.

104. Page 450, index entry “Adjusted remainder function”: Delete the extra spaces around “amod”.

105. Page 467, index entry “Modulus function”: Add “(mod)”. Also, add an index entry for “mod” that says “See Modulus function”.

CAMBRIDGE WEB SITE CODE

In addition to all of the code corrections above, the Lisp code available through a Cambridge University Press web site `http://www.cambridge.org/us/9780521702386` needs the following correction:

106. Line 131: The function name **yom-ha-zikkaron** is mis-spelled; change that line to

```
131            tishah-be-av birkath-ha-hama sh-ela yom-ha-zikkaron
```

(Courtesy of Matthew Sheby, May 19, 2009.)

ERRATA BY FUNCTION NAME

The following errata mention the indicated functions and constants. Erratum numbers are of three types: The erratum number with the definition is shown in **boldface**. The erratum number with the corresponding Lisp code is shown in *italics*. Erratum numbers of other occurrences are given in roman.

ERRATA BY DATE

The dates that errata were added to the list, or last significantly modified, are as follows:

2009/08/18: 7.
2009/08/27: 81, 83, 85, 86, 100.
2009/11/12: 2, 32, 54, 55, 65, 84, 90, 92, 93, 103.
2009/11/15: 20, 21, 35, 37, 75, 94.
2009/11/18: 14, 15, 95.
2009/11/19: 42, 59, 82, 96, 99.

ERRATA BY CONTRIBUTOR

Our thanks to the following people for contributing to the indicated errata:

Uri Blass: 97, 98.
Irv Bromberg: 37, 66, errata corrections.
William P. Collins: 63.
Tony Finch: 56.
Peter Zilahy Ingerman: 67.
Svante Janson: 11–13, 18, 19, 21, 22.
Kwasi Konadu: 20.
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Zhuo Meng: 52.
Susan Milbrath: 41.
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John Powers: 50, 51.
Ruth N. Reingold: 3.
Dieter Schuh: 82, 83, 86.
Matthew Sheby: 106.
Enrico Spinielli: 4, 8–10, 16, 17, 23, 38, 46, 53, 61, 91, 101, 102.
Robert H. van Gent: 32.
Steve Ward: 87.