HVAC systems

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Table of Contents

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Table of Content (follows)

An easy way to compute energy needs

There is a simple but approximate way to compute energy needs for heating and cooling buildings. It's based on the concept of day degrees (HDD and CDD): you just need to know the average heat transfer coefficient of the building (U_{bat}), the surface area of the building in contact with outdoor (A_{bat}), the heating/cooling period and a table of day degrees for heating and cooling at the building location.

Résultats **

Affichage en tableau Affichage en courbes

	Jan	Fév	Mar	Avr	Mai	Jun	Jui	Aoû	Sep	Oct	Nov	Déc	Total
2025	383	290	250	118	71	18	0	0	0	0	0	0	1 130
2024	393	265	210	170	65	14	0	0	54	68	253	429	1 921
2023	409	315	248	176	67	0	0	9	11	61	280	364	1 940
2022	456	298	241	168	27	1	0	0	60	39	255	374	1 918
2021	463	273	276	198	115	1	9	7	15	163	348	445	2 314
2020	387	278	251	82	45	26	0	6	41	187	266	394	1 963
2019	467	313	251	173	123	15	1	3	22	94	310	345	2 118
2018	351	434	301	92	61	4	0	2	10	131	270	376	2 032
2017	555	271	203	180	84	7	4	8	80	134	351	467	2 345
2016	371	325	321	186	109	16	7	2	19	189	292	446	2 282

Date de la dernière MAJ des données : 01/08/2025

Heating day degrees (HDD) and cooling day degrees (CDD) quantify how much the outdoor temperature deviates from a reference set-point temperature during a heating or a cooling contiguous periods: $\mathcal{P}_{\text{heating}} = \{d^h_{\text{ini}}, \ldots d^h_{\text{end}}\}$ and $\mathcal{P}_{\text{cooling}} = \{d^c_{\text{ini}}, \ldots d^c_{\text{end}}\}$, assuming that heating is needed when it's too cold (HDD) and cooling is needed when it's too hot (CDD).

Depending of available data, the daily average outdoor temperatures are computed as follows:

$$\bar{T}_{\text{out}}(d) = \frac{\hat{T}_{\text{out}}(d) + \check{T}_{\text{out}}(d)}{2}$$

or, for high-resolution data (e.g., hourly data):

$$\bar{T}_{\text{out}}(d) = \frac{1}{24} \sum_{h=0}^{23} T_{\text{out}}(h)$$

Additionally, we have a temperature set-points for heating (usually 18°C) and cooling (usually 24°C) periods. It's assumed that:

Assumption 1

The inertia of the envelop does not impact year heating energy needs

Assumption 2

The heat loss surfaces separating indoor from other zones (but not outdoor) are negligible compared to the heat loss surfaces separating indoor from outdoor.

Assumption 3: The times during the heating period where free heat gain is enough to reach or overtake the set point temperature are negligible.

Assumption 4: Neglecting $\Phi_{free}(t)$, is equivalent to consider an lower set-point $\bar{T}_{in}$

Assumption 5: The heat transfer from inside a building and outdoor can be globalized in a single heat transfer coefficient (meaning no join house, for example), the so-called U_{bat} (W/m².K).

There are tables, usually HDD18 (temperature setpoint = 18°C) and CDD24 (temperature setpoint = 24°C) available for different locations in the world, see for instance https://cegibat.grdf.fr/simulateur/calcul-dju.

Tables for any location in the world for any year can be generated by the BATEM Python code (see related workshop).

These set-points 18°C and 24°C are lower than the actual set-points because it's a way to compensate the numerous neglected heat gains.

Denoting $\mathrm{pos}(x)$ the function $\max(0, x)$, the energy needs Q_{heating} and Q_{cooling} for heating and cooling are:

$$Q_{\text{heating}} = \frac{U_{\text{bat}} \times A_{\text{bat}} \times 24 \times \mathrm{hdd}(\mathcal{P}_{\text{heating}})}{1000} [kWh]$$

with $\mathrm{hdd}(\mathcal{P}_{\text{heating}}) = \sum_{d \in \mathcal{P}_{\text{heating}}} pos(T_{\text{ref}}(d) - \bar{T}_{\text{out}})$

$$Q_{\text{cooling}} = \frac{U_{\text{bat}} \times A_{\text{bat}} \times 24 \times \mathrm{cdd}(\mathcal{P}_{\text{cooling}})}{1000} [kWh]$$

with $\mathrm{cdd}(\mathcal{P}_{\text{cooling}}) = \sum_{d \in \mathcal{P}_{\text{cooling}}} pos(\bar{T}_{\text{out}} - T_{\text{ref}}(d))$

Water radiators

A radiator is basically modeled by 2 equations:

- The heat loss from the fluid:

$$Q = \dot{m}_{\mathrm{w}} c_{\mathrm{pw}} (T_{\mathrm{inlet}} - T_{\mathrm{outlet}}) = m_{\mathrm{w}} c_{\mathrm{pw}} \Delta$$

- The heat transfer between the fluid and the environment:

$$Q = U_{\mathrm{radiator}} A_{\mathrm{radiator}} (T_{\mathrm{w}} - T_{\mathrm{indoor}})$$

where:

- U_{radiator} [W/m².K] is the heat transfer coefficient of the radiator
- A_{radiator} [m²] is the surface area of the radiator
- $T_{w,in}$ and $T_{w,out}$ are resp. the inlet and outlet temperatures of the fluid in [°C]
- $U_{\mathrm{radiator}} = \frac{1}{\frac{1}{h_{h,radiator} + h_{r,radiator}} + \frac{e_{\mathrm{radiator}}}{\lambda_{\mathrm{radiator}}}}$
- $\ldots$

- $h_{r,radiator}$ is the radiative heat transfer coefficient of the radiator
- $h_{h,radiator}$ is the convective heat transfer coefficient of the horizontal radiator
- e_{radiator} is the thickness of the radiator material
- $\lambda_{\text{radiator}}$ is the thermal conductivity of the radiator material
- Δ is the temperature difference between the water inlet and outlet.

The convection and radiation heat transfer coefficients inside the radiator are neglected because:

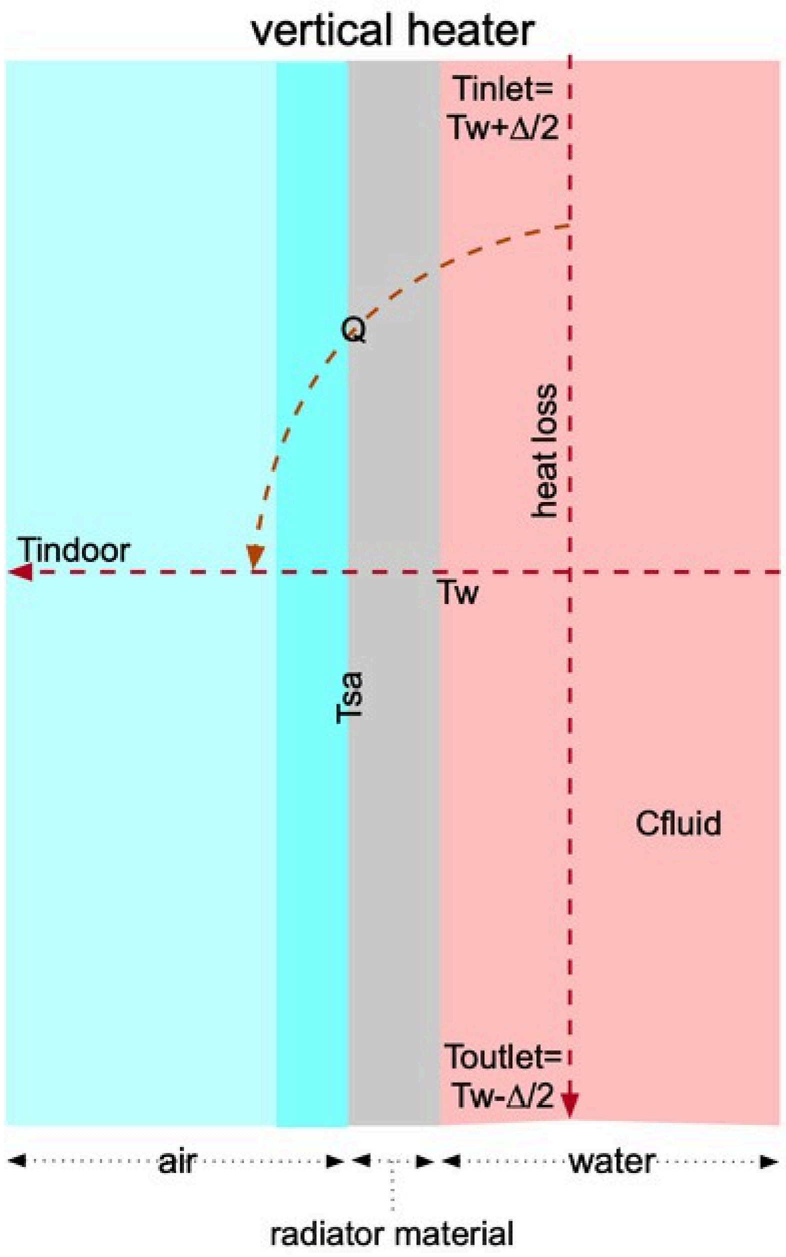
- the convection of water is negligible compared to the convection of air
- the temperature of the surface along water is almost similar inside the radiator

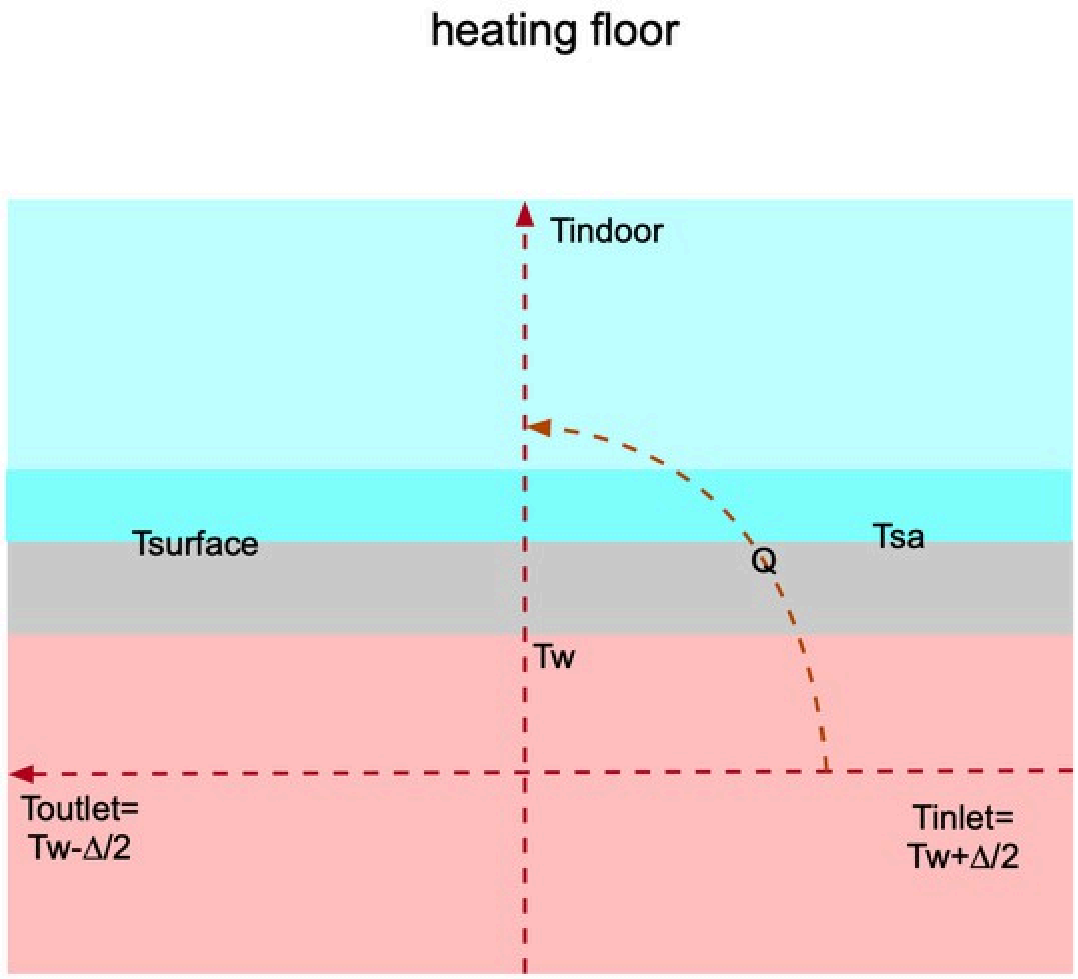
For a given difference $(T_\mathrm{w} - T_\mathrm{indoor})$, the heat exchange capacity is limited by the heat transfer rate from the fluid to the environment. It is proportional to the surface area and the material of the radiator and thermal conductivity and its thickness $U_\mathrm{radiator} A_\mathrm{radiator}$. The type of heater also matters (horizontal or vertical): it impacts the heat transfer coefficient U_radiator with h_i and h_r with

$$U_\mathrm{radiator} = \frac{1}{\frac{1}{h_{h,radiator} + h_{r,radiator}} + \frac{e_\mathrm{radiator}}{\lambda_\mathrm{radiator}}}$$

Let's determine a mass transfer of water $\dot{m}_w$ that satisfies the constraint: $T_w = T_{w,out} + \Delta/2$ ($\Delta \geq 0$), knowing that Q is the heat to be transferred and T_w, Δ and $\dot{m}_w$ are the unknown variables. We have:

$$\begin{cases} \dot{m}_w = \frac{Q}{c_{pw}\Delta} \\ \frac{2Q}{U_\mathrm{radiator} A_\mathrm{radiator}} = \Delta \end{cases}$$


vertical heater
Tinlet=
Tw+Δ/2
Q
heat loss
Tindoor
Tw
Tsa
Cfluid
Toutlet=
Tw-Δ/2
air
water
radiator material


heating floor
Tindoor
Tsurface
Tsa
Q
Tw
Toutlet=
Tw-Δ/2
Tinlet=
Tw+Δ/2

It yields:

$$\begin{cases} \dot{m}_w : 2\dot{m}_w c_{pw} - U_{\text{radiator}} A_{\text{radiator}} = 0 \\ \Delta : U_{\text{radiator}} A_{\text{radiator}} \Delta - 2Q = 0 \end{cases}$$

The solution is given by:

$$\boxed{\begin{cases} \dot{m}_w = \frac{U_{\text{radiator}} \cdot A_{\text{radiator}}}{2c_{pw}} \\ \Delta = \frac{2Q}{U_{\text{radiator}} \cdot A_{\text{radiator}}} \end{cases}}$$

Reminder

A dwelling can be simply modelled by:

- one global heat loss surface S in m^2, separating indoor from outdoor
- a unitary transmitivity coefficient U in W/m^2.K modeling the heat transfer between indoor and outdoor
- a controlled heating (or cooling) system with a set-point $T_{setpoint}$ in Celsius degrees
- indoor and outdoor temperatures $T_{indoor}(t)$ and $T_{outdoor}(t)$
- possibly an average internal heat gain $\Phi_{free}(t)$ in Watt
- the heat P_{in} that has to be supplied by the heating system in Watt
- the set of consecutive days D where the heating (or cooling) system is on

Heat pump: a promising HVAC system

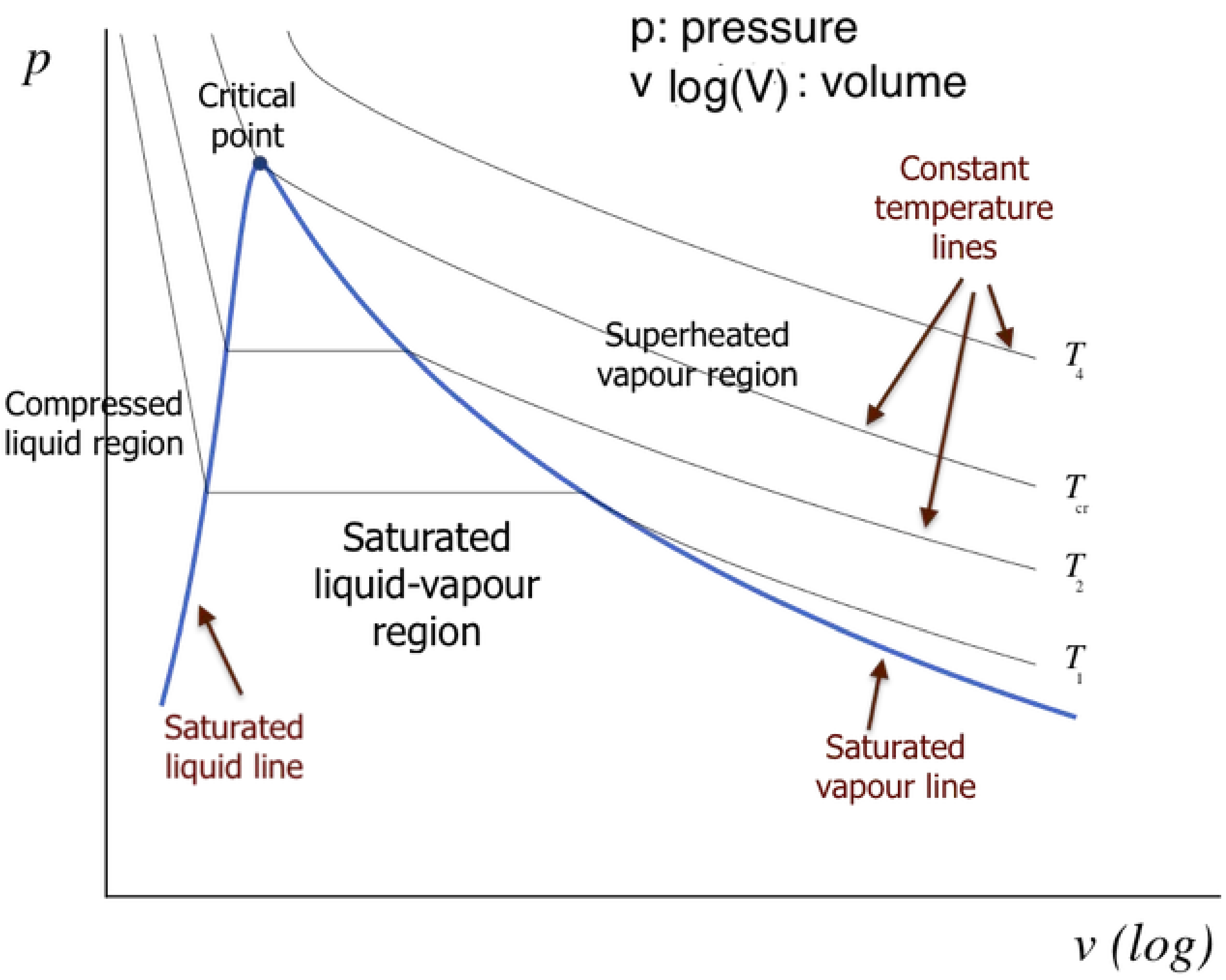
Heat pump, possibly invertible, is a very important technology to decarbonize the heating and cooling for buildings: it's becoming more and more popular because other heating or cooling technologies are consuming gas or oil. Photovoltaic energy requires large area properly directed: a heat pump is a mean to demultiply the precious PV power supplied: it's a low carbon intensity technology.

In France, the heat pump market is growing very quickly because the regulations are becoming more and more stringent and, except for the wood heating, which is rejecting thin particles (PM2.5) harmful for human health in dense urban areas, the heat pump is nowadays the only low carbon intensity technology that can be used to heat buildings.

From RT2012 regulation in France, heat pumps are even considered as a renewable energy plant despite they require electricity to operate. It's due to a coefficient of performance much greater than 1, practically from 2 to 7 kWh of heat produced with 1 kWh of electricity: a big part of the heat is collected in the environment. Nowadays, because of regulation, for most buildings, heat pumps are the only technology that can be used to comply with mandatory energy performance standards.

The principle behind heat pumps is that they do not generate heat, they just move the existing heat from outdoor to indoor or vice versa, which is not natural way for heat transfer. Like fridges, which include also heat pumps, they move heat from a low-temperature area (the heat source) to a high-temperature area (the heat sink). Most heat pumps are invertible, meaning that they can be used in heating mode or in cooling mode (see in the next).

- different phases of a fluid in a Mollier diagram



1. Compressed (or subcooled) liquid region

The fluid is entirely liquid, and its temperature is below the saturation temperature for its pressure. It is not ready to boil, even if pressure drops slightly. It is a stable and dense state

For example, consider liquid water at 20 °C and 1 atm.

2. Saturated state or equilibrium between liquid and vapor

It occurs at the phase-change boundary between liquid and vapor

a) Saturated liquid line: it is liquid at the boiling point for a given pressure, just about to evaporate.

b) Saturated mixture region: a coexistence of liquid and vapor in the dome. The fraction of vapor is called the quality $x \in [0, 1]$: $x = \frac{\text{mass of vapor}}{\text{total mass}}$

For example, consider boiling water at 100 °C with visible steam.

c) Saturated vapor line: vapor at the condensation point. Just about to condense if pressure increases.

3. Superheated vapor region: vapor at a temperature above the saturation temperature for a given pressure. It is not in contact with its liquid phase — behaves increasingly like an ideal gas.

For example, consider steam at 120 °C and 1 atm.

1. Critical point

The temperature and pressure at which saturated liquid and saturated vapor become indistinguishable.

For water: $T_{\text{critical}} \approx 374^{\circ}C, P_{\text{critical}} \approx 218.11\,\text{atm}$

4. 2-phase region: also called the dome. Any point inside this region corresponds to a saturated mixture.

Fundamentals of thermodynamics

An ideal gas satisfies the ideal gas law: $pV = nR_{mol}T$ where p is the pressure, V the volume, n the number of moles ($6.02214076 \times 10^{23}$ particles/mol: it's the Avogadro number), $R_{mol}(\approx 8.314$ J/mol.K$)$ the ideal gas constant and T the temperature.

Another form of the ideal gas law used by thermodynamical engineers is:

$$p = \rho R_{kg} T$$

where ρ is the density, and R_{kg} is the specific gas constant with $R_{kg} = \frac{8.314 \text{ J/kg.K}}{M}$ and M is the molar mass of the fluid (kg/mol).

A fluid is behaving as an ideal gas in the superheated vapor region.

It has the following properties:

1. molecules are point particles: their volume is negligible compared to the total volume
2. no intermolecular forces: they interact only via elastic collisions
3. there are elastic collisions only: no energy is lost during molecule-side or molecule-molecule collisions
4. random, constant motion: molecules move randomly in all directions with a distribution of speeds
5. internal energy depends only on temperature: $U = U(T)$ and $H = H(T)$

An ideal gas models properly a gas at low pressure, high temperature and far from phase change (no condensation or boiling). The isentropic (invertible transformation, without friction) relation for an ideal gas is: $pV^\gamma = \text{constant}$ where $\gamma = \frac{c_p}{c_v}$ is the adiabatic index:

- $c_v = \left(\frac{\partial U}{\partial T}\right)_v$ is the specific heat at constant volume [J/kg.K]
- $c_p = \left(\frac{\partial U}{\partial T}\right)_p$ is the specific heat at constant pressure [J/kg.K]

Variation of entropy is modeled by:

$$\Delta S = c_p \log\left(\frac{T_2}{T_1}\right) - R \log\left(\frac{p_2}{p_1}\right)$$

Let's assume we want to perform an isentropic compression of water vapor from 100kPa and 100°C to 1000kPa, considering the water vapor as an ideal gas.

Note that the isentropic condition is often chosen to represent an ideal compression because an isentropic process is:

- adiabatic: no heat exchange with surroundings
- reversible: no entropy generation i.e. no friction

We have: $\log\left(\frac{T_2}{T_1}\right) = \frac{R}{c_p}\log\left(\frac{p_2}{p_1}\right) \leftrightarrow T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{R}{c_p}}$

$c_p \approx 1860 \text{ J/kg.K}$ and $M_{\mathrm{H_2O}} = 0.01801528 \text{ kg/mol} \rightarrow R \approx 461.5 \text{ J/kg.K}$

Therefore,

$T_2 = 373.15\left(\frac{1000}{100}\right)^{\frac{461.5}{1860}} = 373.15 \times 1000^{0.248} = 373.15 \times 10^{0.248} = 448.15 \text{ K}$

In order to manage real gas, we need to use tables, for instance, for water vapor. Let's assume we want to perform an isentropic compression of water vapor from 100kPa and 150°C to 1000kPa.

```
import CoolProp.CoolProp as CP
enthalpy1 = CP.PropsSI('H','T',150+273.15,'P',100000,'Water')  # Enthalpy in J/kg
entropy1 = CP.PropsSI('S','T',150+273.15,'P',100000,'Water')  # Entropy in J/kg.K
temperature = CP.PropsSI('T','S',entropy1,'P',1000000,'Water')  # Temperature in K
enthalpy2 = CP.PropsSI('H','S',entropy1,'P',1000000,'Water')  # Enthalpy in J/kg
```

From the steam tables (see Python code above), we read the entropy 7614.7 J/kg and the enthalpy 2776602.5 J/kg. Looking for the state at pressures 100 bar and then 1000 bar with same entropy $s_2 = s_1$, we find the the enthalpy 3367500.6 J/kg and the temperature 721.3 K (448.15°C), which the same result as the previous calculation! The ideal gas law is valid for water vapor at low pressure and high temperature.

The internal energy U [J/kg] is the energy contained inside the molecules of a fluid. It depends on its current state (gas, liquid, vapor) and on temperature.

It is the sum of all microscopic forms of energy in a substance:

$$U = \text{kinetic energy (molecular)} + \text{Potential energy (intermolecular)}$$
$$\cdots + \text{other internal modes}$$

It includes:

- translational, rotational, and vibrational motion of molecules.
- intermolecular potential energy (especially important in liquids and solids).
- electronic energy (excitation states, if significant).
- chemical bond energy (if reactions occur).

In the first law of thermodynamics for a closed system, we have the change in internal energy:

$$\Delta U = Q - W$$

where:

- Q: the heat added to the system,
- W: the work done by the system

For an ideal gas, the internal energy depends only on temperature:
$U(T) = \int mc_v(T)\, dT$ where c_v is the specific heat at constant volume

For a calorically perfect gas (constant c_v): $U = mc_vT$

In contrast, real fluids (liquids, real gases) have an internal energy that depends on both temperature and specific volume (or pressure), due to intermolecular forces.

Let's compute now the internal energy of 1 liter of water vapor at 100 °C

- $100^{\circ}C \rightarrow 373.15K$
- assuming the air is saturated with water vapor at atmospheric pressure p=101325 Pa, from CoolProp, we get:

```
import CoolProp.CoolProp as CP
rho_g = CP.PropsSI('D', 'T', 373.15, 'Q', 1, 'Water')  # density in[kg/m³]
u_g = CP.PropsSI('U', 'T', 373.15, 'Q', 1, 'Water') * rho_g / 1000  # convert to kJ/l
```

- saturated vapor specific internal energy: $u_\mathrm{g} \approx 1499 \mathrm{J}/\mathrm{kg}$
- the mass of 1 liter (0.001 m³) of saturated vapor is: $v = \frac{V}{m} \Rightarrow m = \frac{V}{v} = \frac{0.001}{1.672} \approx 0.000598 \text{ kg}$
- the internal energy is: $U = m \cdot u = 0.000598 \times 2087.10^3 \approx 1248 \text{ kJ}$

- the enthalpy [J] is a thermodynamic property that represents the total heat contained by a system. It is the sum of the internal energy and the pressure-volume work:

$$H = U + pV$$

where:

- U [J] is the internal energy, it's the energy contained inside the molecules of a fluid. It depends on its current state (gas, liquid, vapor) and on temperature.
- p [Pa] is the pressure ($10^5 Pa = 1 bar$),
- V [m^3] is the volume.

Let's consider an evaporating refrigerant under constant pressure. This process involves no temperature change (isothermal), but a phase change from liquid to gas. The required energy is the enthalpy of vaporization ΔH_{vap}, which for water at 100°C is:

$$\Delta H_{\text{vap}} \approx 2257\,\text{kJ/kg}$$

This energy goes into breaking intermolecular forces, not into increasing temperature. The heat supplied is stored as enthalpy in the vapor. At constant pressure, all the heat added goes into increasing enthalpy: $Q_p = \Delta H$.

- the entropy is a thermodynamic property that represents the degree of disorder of a system. At equilibrium, entropy is maximized. It is defined as:

$$S = \frac{Q_{\text{reversible}}}{T} \; [J/K]$$

Entropy increases during vaporization because:

- the molecules in the vapor phase are more disordered than in the liquid.
- the system gains more microscopic configurations
- since the process is isothermal and reversible (in theory), we use: $\Delta S = \frac{\Delta H}{T}$

The heat transfer equation used when a fluid (typically a liquid or gas) undergoes a temperature change without phase change, under constant pressure conditions.

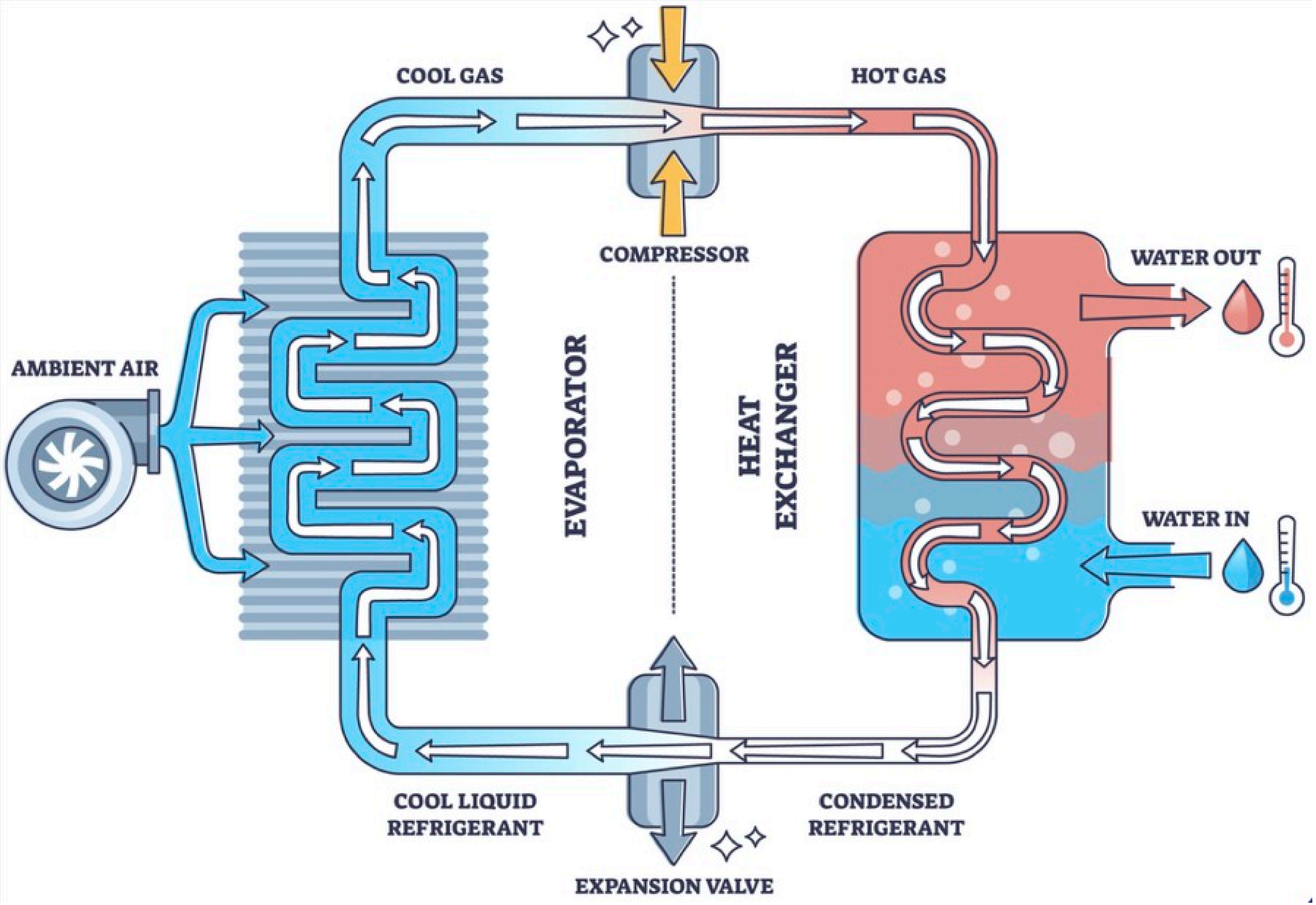
$$Q = m\, c_p\, (T_1 - T_2)$$

If c_p varies significantly with temperature, the integral form should be used:

$$Q = m \int_{T_1}^{T_2} c_p(T)\, dT$$

Basic operating principle of heat pumps

The basic principle is illustrated by this video: basic principle of heat pumps. The 4 main elements in a heat pump are illustrated in the following figure (heat source on the left and heat sink on the right). It transfers heat from a cold source to a hot sink thanks to a compression / expansion cycle of a fluid called refrigerant.


COOL GAS
HOT GAS
COMPRESSOR
WATER OUT
AMBIENT AIR
EVAPORATOR
HEAT
EXCHANGER
WATER IN
COOL LIQUID
REFRIGERANT
CONDENSED
REFRIGERANT
EXPANSION VALVE

A heat pump operates in a closed thermodynamic cycle that typically involves four key stages.

A. Evaporation (in the evaporator)

Process: The refrigerant absorbs heat from the heat source (e.g., outdoor air, ground, water) and evaporates, changing from liquid to vapor:

- thermodynamic transformation: Isothermal endothermic process at low pressure.
- result: Refrigerant becomes a low-pressure vapor.
- entropy increases due to heat absorption.

B. Compression (in the compressor)

Process: The vaporized refrigerant is compressed, increasing its pressure and temperature significantly.

- thermodynamic transformation: ideally isentropic (adiabatic and reversible).
- result: high-pressure, high-temperature superheated vapor.
- entropy remains constant (ideally), but may increase slightly due to irreversibilities.

C. Condensation (in the condenser)

Process: The hot, high-pressure vapor releases heat to the heat sink (e.g., indoor air or water) and condenses, returning to a liquid state.

- thermodynamic transformation: isothermal exothermic process at high pressure.
- result: refrigerant becomes high-pressure liquid.
- entropy decreases due to heat release.

D. Expansion (in the expansion valve)

Process: The liquid refrigerant expands rapidly, dropping in pressure and temperature, and may partially vaporize:

- thermodynamic transformation: Isenthalpic throttling process (no heat exchange, constant enthalpy).
- result: Low-pressure, low-temperature mixture (liquid + vapor).
- prepares refrigerant for evaporation again.

Different types of heat pumps

There are different types of heat pumps.

A. Air-Source Heat Pumps (ASHP)

These are the most common type. They work by extracting heat from the outside air. Because the mass specific heat of air is low ($c_{p,\mathrm{air}}$ = 1005 J/kg.K), a fan is needed to collect heat from a large mass of air.

- Standard ASHP transfers heat between the indoor air and outdoor air.
- Cold Climate ASHP are designed to operate efficiently in sub-freezing temperatures.
- Ducted (uses a system of air ducts) vs. Ductless (mini-splits: individual indoor units without duct work—ideal for retrofits or room-by-room control.)

B. Ground-Source Heat Pumps (GSHP) / Geothermal Heat Pumps

It collects heat from the ground or groundwater, where temperatures are more stable and mass specific heat much higher ($c_{p,\mathrm{water}}$= 4184 J/kg.K, $c_{p,\mathrm{ground}}$ $\approx$ 1000 kJ/kg.K).

There are 3 types of GSHP:

- horizontal Loop: Piping laid out horizontally in trenches, recharged by solar radiation
- vertical Loop: Piping installed in deep vertical boreholes—used when space is limited.
- pond/lake Loop: Uses a body of water as the heat exchange medium.

1. Water-Source Heat Pumps

Use a loop of water (e.g., from a cooling tower or central plant) as the heat exchange medium. Often used in large buildings with zoned systems.

2. Hybrid Heat Pumps (Dual-Fuel Systems)

Combine a heat pump with a gas furnace for backup. The system switches to the furnace during very cold conditions for efficiency and reliability.

3. Absorption Heat Pumps

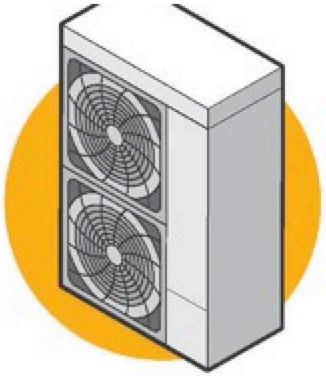
Instead of using electricity, these use a heat source (natural gas or solar power) and a refrigerant-absorbent pair (e.g., ammonia-water). Used in industrial or large-scale applications.

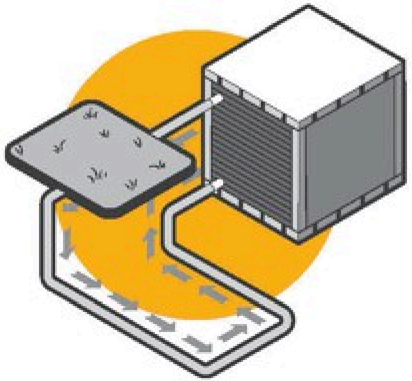
4. Exhaust Air Heat Pumps (EAHP)

Capture heat from ventilation exhaust air—common in multi-family residential or high-performance buildings.

Air-Source Heat Pumps

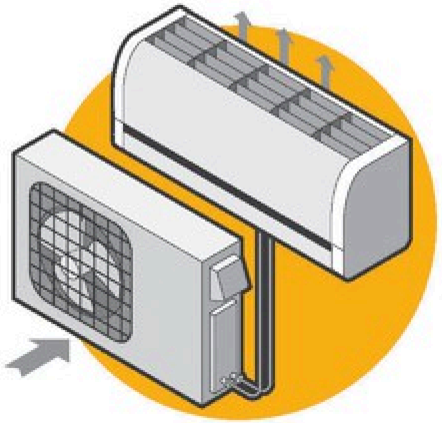
- Most commonly used heat pumps
- Moves heat rather than converting it from a fuel like combustion heating systems do
- Can reduce heating costs when compared to baseboard heaters or electric furnaces
- Newer, more efficient systems now represent a legitimate space heating alternative in colder regions like the Northeast and Midwest.





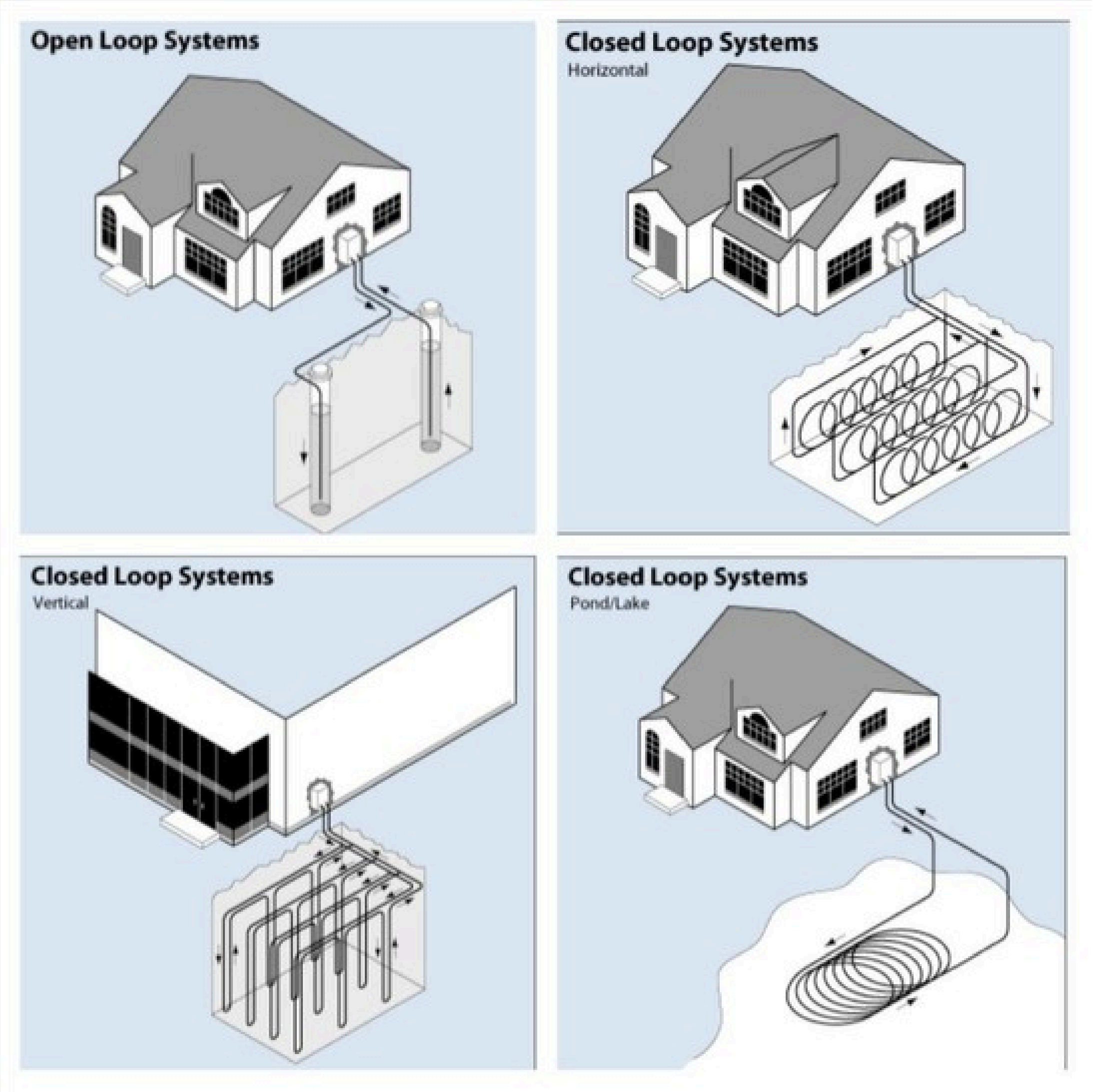
Geothermal Heat Pumps

- More expensive to install but provide more energy savings for heating and cooling
- Move heat through pipes buried underground
- When compared to a conventional heating system, can reduce energy use by 25 to 50 percent
- Effective in extreme climates



Ductless Mini-Split Heat Pumps

- Easier to install, quiet, small in size
- Flexible for heating and cooling individual rooms and smaller spaces
- No energy loss through ductwork, which accounts for more than 30 percent of a home's energy use for space heating/cooling.



A thermodynamical perspective of heat pumps

Thermodynamics is particularly useful to model the behavior of heat pump because it deals with fluids.

At constant pressure, if the variation of enthalpy is positive: $\Delta H > 0$, the related process is endothermic. It absorbs heat: it is named a heat source. Conversely, if $\Delta H < 0$, the process is exothermic. It releases heat: it is named a heat sink.

The process occurring in the evaporator is endothermic (heat source) and the one in the condenser is exothermic (heat sink). Switching from cooling to heating mode amounts to invert the positions of evaporator and condenser knowing that both are heat exchangers.

We have seen that there are different types of heat exchangers depending on the fluid, for instance, air or water (generally with glycol to avoid freezing, used for collecting or releasing energy in the outdoor environment. When comparing the internal energy of 1m^3 of air, considered as an ideal gas, and 1m^3 of water at a year average of 13°C*, it comes:

$$T = 273.15 + 13 \text{ K} = 286.25 \text{ K}$$

$$\begin{aligned}
\rho_{\text{air}} &= 1.293 \text{ kg/m}^3 \\
c_{v,\text{air}} &= 718 \text{ J/kg.K} \\
\rightarrow U_{\text{air}} &= \rho_{\text{air}} c_{v,\text{air}} T = 266 \text{ kJ} \\
c_{v,\text{water}} &= 4186 \text{ J/kg.K} \\
\rho_{\text{water}} &= 1000 \text{ kg/m}^3 \\
\rightarrow U_{\text{water}} &= \rho_{\text{water}} c_{v,\text{water}} T = 1197824 \text{ kJ} = 4503 U_{\text{air}}
\end{aligned}$$

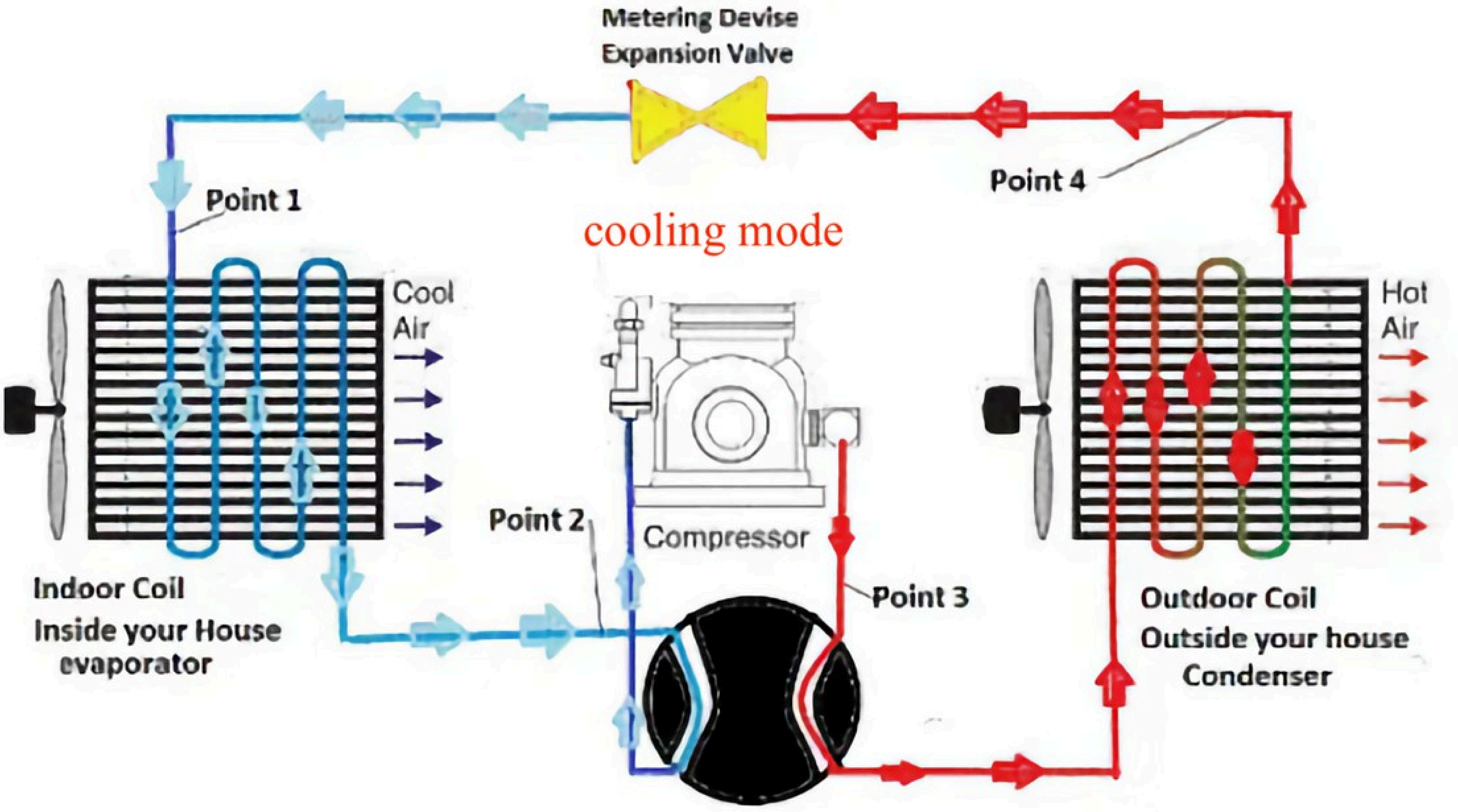
*reminder: T is the gas temperature in Kelvin. It can be deduced from celsius or Fahrenheit degrees with $T_K = 273.15 + T_{^\circ\text{C}} = 273.15 + 5/9(T_{^\circ\text{F}} - 32))$

The internal energy of the same volume of water is 4500 times greater than the internal energy of air: it explains why fans are required when air is used instead of water… and that energy performances are less.

Note that for ideal gas, $\Delta U = c_v \Delta T$ where c_v is the specific heat at constant volume (J/kg.K), or similarly, $\Delta U = c_p \Delta T$ at constant pressure with $c_p = c_v + R$ where $R = \frac{8.314}{M}$ and M, the molar mass at the considered pressure.

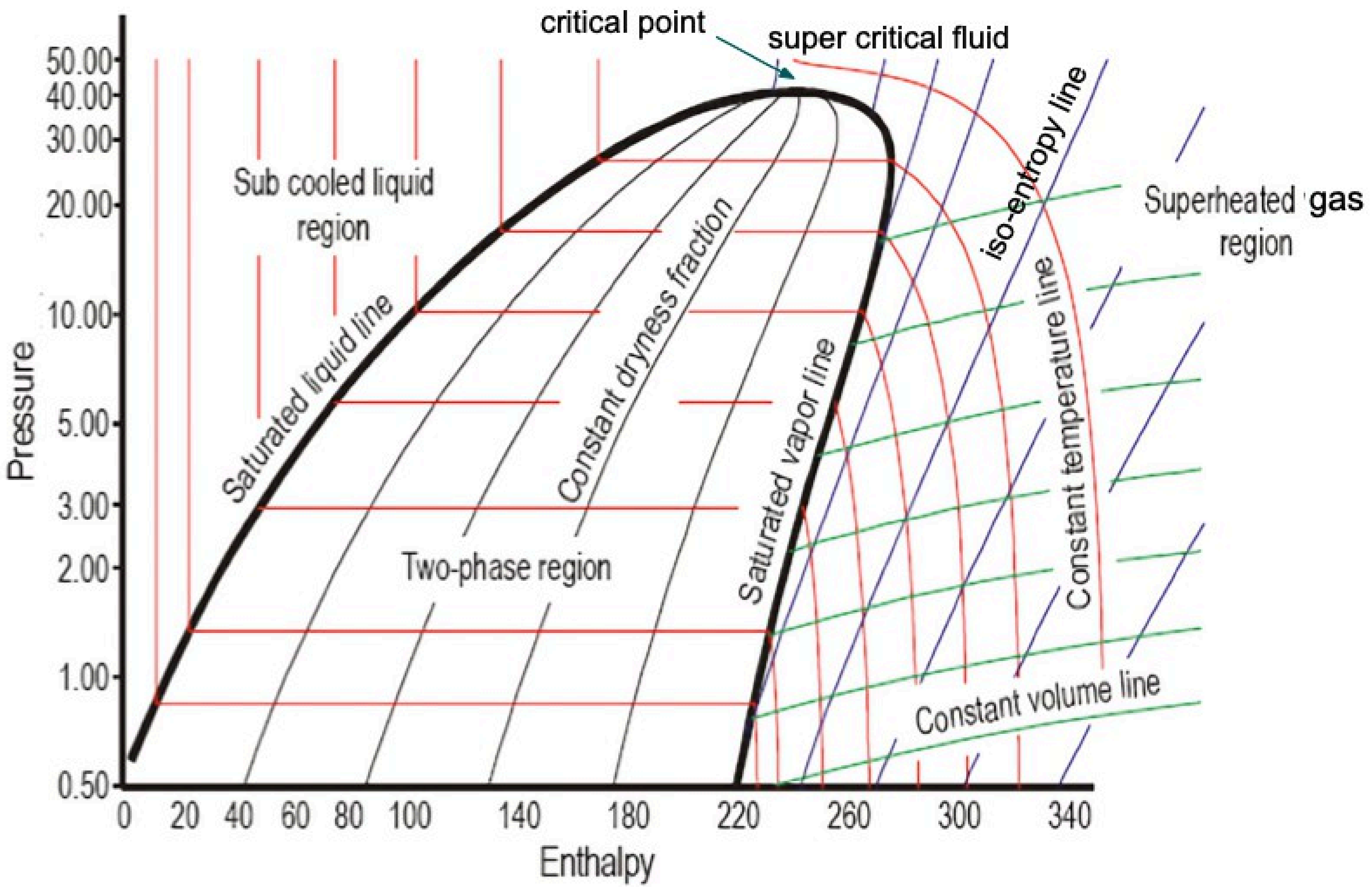
Reversible heat pump

During the heating mode, the pump extracts heat from the environment (air, water, or ground) therefore the heat source is outdoor and the heat sink is indoor. Conversely, in the cooling mode, it removes heat from indoor spaces therefore the heat source is indoor and the heat sink is outdoor. A heat pump is powered by a compressor, which is pump fed by electricity to drive a compression and expansion cycle.


Metering Devise
Expansion Valve
Point 1
Point 4
cooling mode
Cool
Air
Hot
Air
Point 2
Compressor
Point 3
Indoor Coil
Inside your House
evaporator
Outdoor Coil
Outside your house
Condenser

To find U, the internal energy of a real fluid, you can use steam tables, Mollier diagram or Python code like CoolProp for gas (see workshop06_systems)

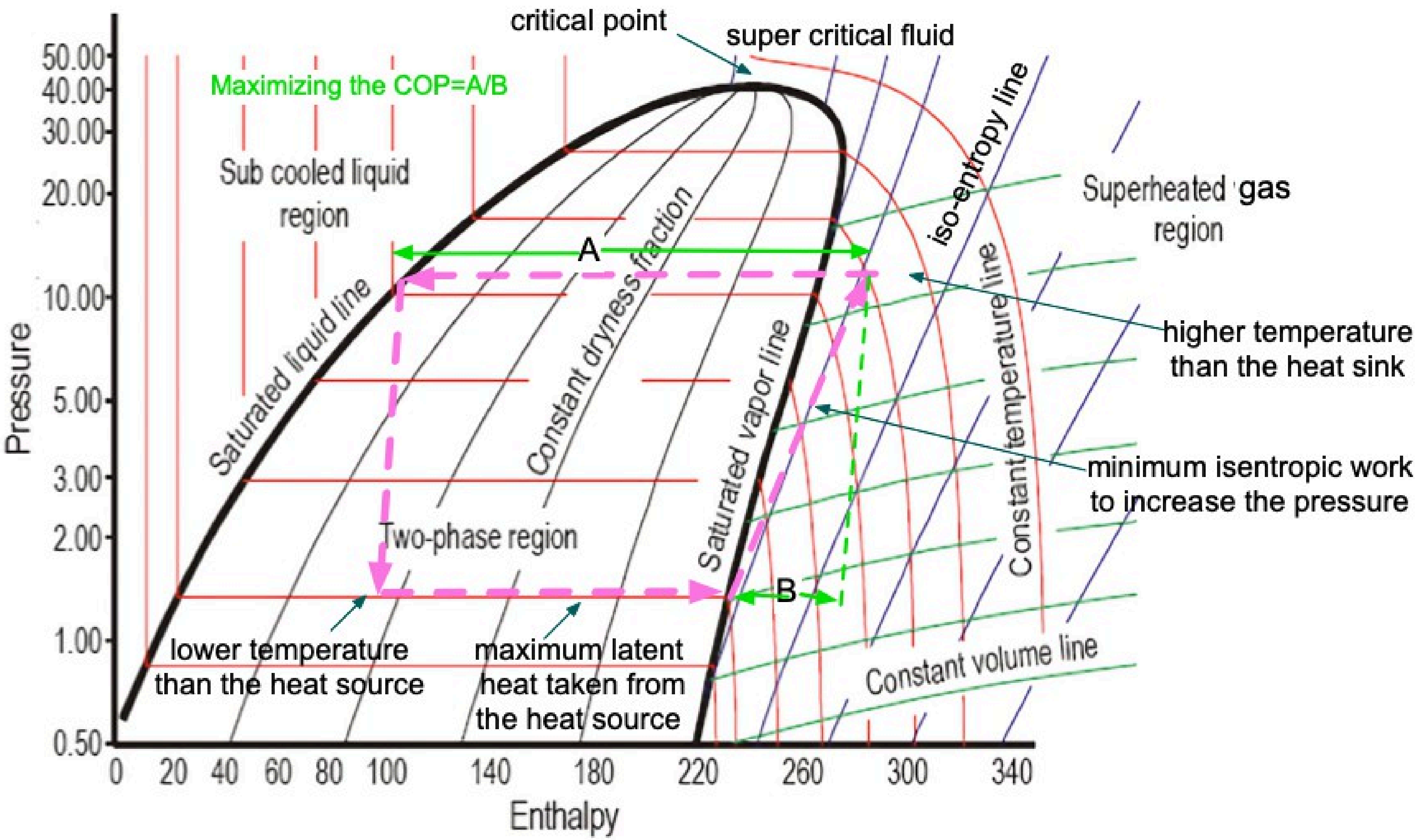
For a fluid, the Mollier diagram, which is a h-log(p) (i.e. usually x-axis → enthalpy and y-axis → log10(pressure)) diagram, looks like below.


critical point
super critical fluid
Sub cooled liquid region
Saturated liquid line
Constant dryness fraction
Two-phase region
Saturated vapor line
iso-entropy line
Superheated gas region
Constant temperature line
Constant volume line
Pressure
Enthalpy
50.00
40.00
30.00
20.00
10.00
5.00
3.00
2.00
1.00
0.50
0
20
40
60
80
100
140
180
220
260
300
340

A good refrigerant satisfies the following properties:

- it must evaporate at low temperatures e.g. a boiling temperature equal to -26.3°C for R134a, -29.5°C for R1234yf, -42°C for R290 (propane), -78.4°C for R744 (CO_2)
- it must have high values of latent heat of vaporization in order to absorb more heat during evaporation
- critical temperature and pressure should be above typical condenser temperature to avoid supercritical region
- it must easily condense with low-grade heat sinks
- it should not be harmful for the environment

Interesting properties of a refrigerant are reported on the following Mollier diagram:


critical point
super critical fluid
Maximizing the COP=A/B
Sub cooled liquid region
A
B
Saturated liquid line
Constant dryness fraction
Saturated vapor line
iso-entropy line
Superheated gas region
Constant temperature line
Constant volume line
Two-phase region
higher temperature than the heat sink
minimum isentropic work to increase the pressure
lower temperature than the heat source
maximum latent heat taken from the heat source
Pressure
Enthalpy
50.00
40.00
30.00
20.00
10.00
5.00
3.00
2.00
1.00
0.50
0
20
40
60
80
100
140
180
220
260
300
340

The pressure increase is operated by the compressor at minimum constant (the compressor is at steady state) entropy, because it should be as close as possible to a perfect the compressor i.e. without heat loss (adiabatic) and any irreversibility. Indeed, entropy, denoted S, quantifies how much heat is transferred (lost) per degree [K].

- when heat flows into a system, entropy increases.
- When heat leaves a system, entropy decreases.

For a reversible process, we have:

$$\Delta S_{1\to 2} = \frac{W}{T_2 - T_1}$$

where:

- ΔS [J/K], the change in entropy
- W_{rev} [J], the heat added reversibly
- T [K], the absolute temperature (in Kelvin)

The refrigerant is the working fluid in the heat pump. It is a fluid that can evaporate and condense at low temperature and pressure. The most common refrigerant is NH_3 (ammonia) but other refrigerants are used, for instance, R22, R134a, R152a, R290, R718. A refrigerant is selected based on the temperature level of the heat source and the heat sink, to match appropriate boiling and condensing temperatures, and because of its high latent heat of vaporization to ensure more heat transfer per unit mass.

Here is a list of common refrigerants.

Refrigerant	GWP (100 yrs)	ODP	Toxicity	Flammability	Operating Pressure (bar)	COP Potential	Environmental Regulation Status	Notes
Ammonia or NH_3 or R-717	0	0.0	Toxic	Non-flammable	6–20	High	Natural refrigerant	Efficient, toxic and corrosive, used in industrial systems
R-22: chlorodifluoromethane	1810	0.05	Low	Non-flammable	6–20	Medium	Phased out (HCFC)	Good performance, phased out due to ozone impact
R-32: hydrofluorocarbone	675	0.0	Low	Mildly flammable	10–30	High	Preferred alternative	Efficient with lower GWP, mildly flammable
R-134a: 1,1,1,2-tetrafluoroethane	1430	0.0	Low	Non-flammable	5–15	Medium	HFC with moderate GWP	Common in automotive AC, moderate GWP
R-152a: 1,1-difluoroethane	124	0.0	Low	Mildly flammable	5–15	Medium	Low GWP HFC	Lower GWP alternative to R-134a, flammable
R-290: propane	3	0.0	Low	Highly flammable	7–15	High	Natural refrigerant	Natural refrigerant with excellent efficiency, safety concerns
R-718: water	0	0.0	None	Non-flammable	0.03–0.1	Low	Natural refrigerant	Eco-friendly
R-744: CO_2	1	0.0	Low	Non-flammable	40–90	Medium	Natural refrigerant	Extremely low GWP, very high pressure system
R-410A: difluorométhane 50%, pentafluoroéthane 50%	2088	0.0	Low	Non-flammable	15–35	High	Phasing out	Widely used blend, high GWP
R1234yf: 2,3,3,3-Tetrafluoropropene	4	0.0	Very Low	Low flammability	6-33	High	Low GWP	replaces R-134a in the air conditioning systems of new automobile model

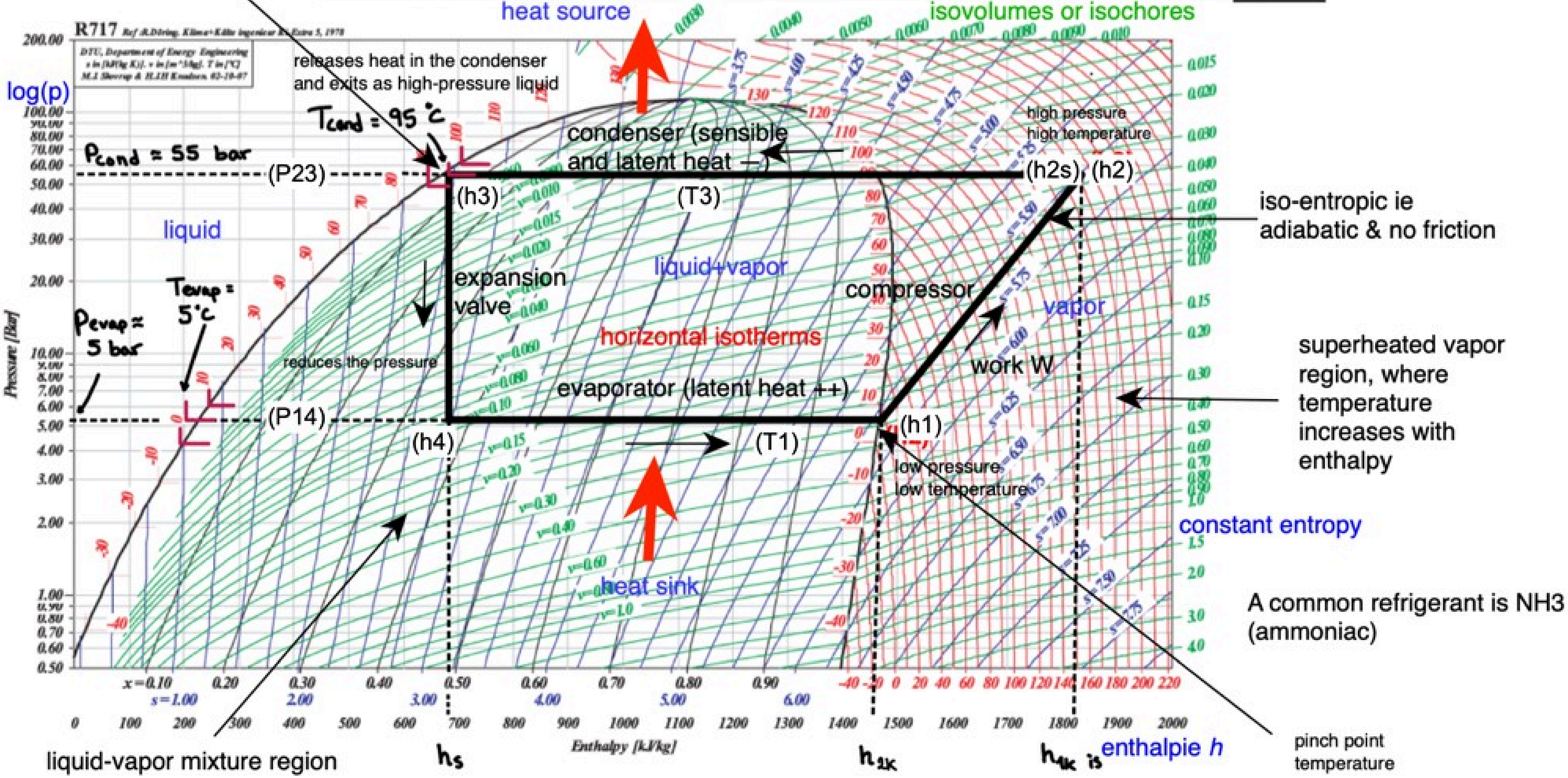
R32 is today the most common refrigerant in domestic heat pumps

The vapor-compression refrigeration (Carnot) cycle is the cycle followed by heat pumps. It is a closed cycle that uses a compressor to compress the refrigerant, a condenser to condense the refrigerant, an expansion valve to expand the refrigerant, and an evaporator to evaporate the refrigerant. The cycle is illustrated in the following $\log(p)/h$ Mollier chart, where p is the pressure in bar and h is the enthalpy 'see below).

The refrigerant, the heat source and the heat sink determined the performance of the heat pump. A good refrigerant should have a high latent heat of vaporization (heat source at low temperature) to ensure more heat transfer per unit mass. Regarding the low and high temperature heat sinks, they should be at a temperature close to the ambient temperature to avoid large temperature differences and therefore large heat losses.

1. **Refrigerant Properties:** Use refrigerant property tables or equations of state (e.g., REFPROP or CoolProp) to calculate h_1, h_2, h_3, h_4.

2. **Mass Flow Rate:** $\dot{m}_{ref}$ can depend on compressor speed and evaporator performance.



A heat pump follows a vapor-compression refrigeration cycle, typically involving these four key stages:

1. evaporator (low pressure, low temperature):

- refrigerant absorbs heat from the indoor/outdoor environment.
- entropy increases as the refrigerant vaporizes and gains heat.

2. compressor (raising pressure and temperature):

- compresses the refrigerant vapor to a high pressure.
- ideally isentropic (no change in entropy), but real compressors increase entropy slightly due to irreversibilities.

3. condenser (high pressure, high temperature):

- heat is released to the surroundings.
- entropy decreases as the refrigerant condenses.

4. expansion valve:

- drops in pressure and temperature rapidly.
- entropy remains roughly constant (isenthalpic process), but quality (vapor-liquid mix) changes.

In TESPy, a refrigerant is parameterized in different components by:

1	Declare in network	fluids=['R134a']
2	Set fluid at connection	fluid={'R134a': 1.0}
3	Define state	T=20, p=5 or h=220

A connection can have these attributes, set for instance with `component_reference.set_attr(attribute_name: value)`:

p	Pressure (in p_unit, e.g., bar or Pa)
T	Temperature (in T_unit, e.g., °C or K)
h	Specific enthalpy (in h_unit, e.g., kJ/kg)
s	Specific entropy (optional; rarely set directly)
x	Vapor mass fraction (optional; derived from T and p or h and p)
m / m0	Mass flow rate (can be m or m0, same function)
fluid	Fluid composition as a dictionary (e.g., {'R134a': 1.0}) where `'R134a': 1.0`,

Heat pump Coefficients Of Performance (COP)

The coefficients of performance (COP) of a reversible heat pump are defined by the energy needed (mainly by the compressor) divided by the heat delivered Q_{hot} and Q_{cold} (negative value):

$$COP_{\mathrm{heating}} = \frac{Q_{\mathrm{hot}}}{W_{\mathrm{input}}} = \frac{H_{\mathrm{condenser\ out}} - H_{\mathrm{condenser\ in}}}{H_{\mathrm{compressor\ out}} - H_{\mathrm{compressor\ in}}}$$

where $\dot{m}$ is the mass flow rate of refrigerant in [kg/s], and H, the specific enthalpy in [J/kg].

The Carnot COP is the maximum theoretical COP. It is independent of heat pump design:

$$COP_{\mathrm{heating\ Carnot}} = \frac{T_{\mathrm{sink}}}{T_{\mathrm{sink}} - T_{\mathrm{source}}}$$

For example, for an average outdoor temperature equal to 5°C and a heating floor surface of 30°C, it leads to a COP of 1.2. If instead of a heating floor, there is a radiator whose temperature should be higher because of the small heating surface, let's say at 50°C, the resulting theoretical COP is: 1.1 i.e. 1.1kW of heat for 1kW of electricity!

Regarding the cooling, we have:

$$COP_{\text{cooling}} = \frac{Q_{\text{cold}}}{W_{\text{input}}} = \frac{H_{\text{compressor in}} - H_{\text{valve in}}}{H_{\text{compressor out}} - H_{\text{compressor in}}}$$

The Carnot COP is:

$$COP_{\text{cooling Carnot}} = \frac{T_{\text{source}}}{T_{\text{sink}} - T_{\text{source}}}$$

For example, for an average outdoor temperature equal to 35°C and a heating floor surface of 15°C, it leads to a COP of -1.75. If instead of a heating floor, there is a radiator whose temperature should be higher because of the small heating surface, let's say at 5°C, the resulting theoretical COP is: -1.75 i.e. 1.17kW of extracted heat for 1kW of electricity!

Appendix A

A. Components of an HVAC system based on a heat pump

A.1 Compressor

The compressor is the driving force behind the refrigerant cycle, responsible for compressing the low-pressure refrigerant vapor exiting the evaporator into a high-pressure, high-temperature superheated gas. The compressor is a device that increases the pressure of a fluid. It is a mechanical device that uses a motor to drive a shaft that rotates an impeller or a screw. The impeller or screw increases the pressure of the fluid by increasing the kinetic energy of the fluid. The fluid then flows through the condenser, where it is cooled and condensed.

It can be modeled by:

$$W_{\mathrm{comp}} = \dot{m}(h_{\mathrm{comp},in} - h_{\mathrm{comp},out})$$

where W_{comp} is the mechanical power supplied by the pump, $\dot{m}$ is the mass flow rate of the refrigerant, $h_{\mathrm{comp},in}$ is the enthalpy of the refrigerant at the inlet of the compressor, and $h_{\mathrm{comp},out}$ is the enthalpy of the refrigerant at the outlet of the compressor.

Using the isentropic relations for an ideal gas with constant specific heats:

$$W_{\text{comp}} = \frac{\dot{m} c_p T_1}{\eta_{\text{comp}}} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

where:

- c_p, the specific heat at constant pressure
- $\gamma = c_p / c_v$, the heat capacity ratio
- η_{comp}, the: isentropic efficiency of the compressor (typically 85%)
- T_1, the inlet temperature
- p_2/p_1, the pressure ratio

The corresponding electrical power P_{comp} is:

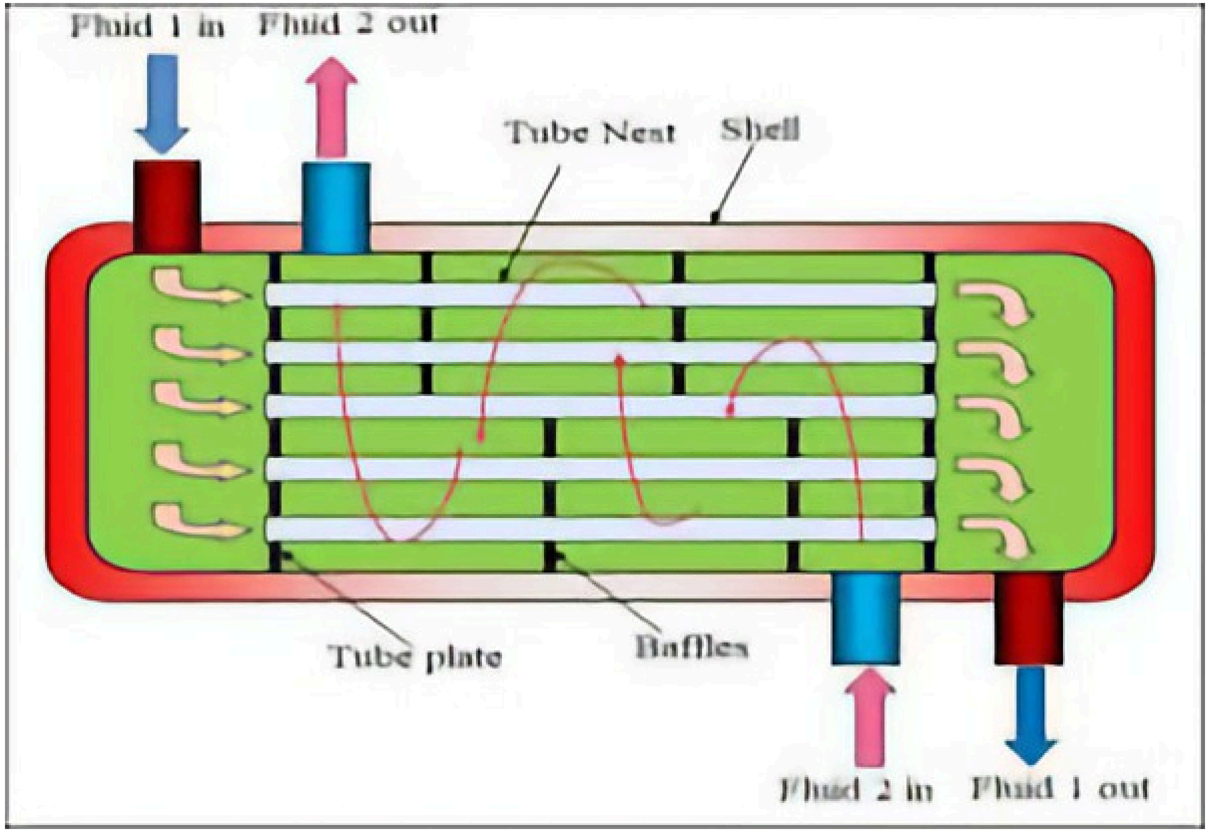
$$P_{\text{comp}} = W_{\text{comp}}\eta_{\text{comp}}$$

In `testpy`, a compressor is characterized by the following attributes, set for instance with `comporent_reference.set_attr(attribute_name: value)`:

pr	**Pressure ratio $\left(\frac{p_{out}}{p_{in}}\right)$**
eta_s	Isentropic efficiency $\eta_s \in (0, 1)$
P	Power input to the compressor (in W)

A.2 General model of a heat exchanger

A heat exchanger is a device that transfers heat between two fluids: the evaporator and the condenser are both heat exchangers. We are considering a counter-flow heat exchanger, which allows the smallest possible temperature difference (the pinch point) between the hot and cold streams.


Fluid 1 in
Fluid 2 out
Tube Nest
Shell
Tube plate
Baffles
Fluid 2 in
Fluid 1 out

1. Basic Energy Balance

Let's denote:

- $\dot{m}_h, \dot{m}_c$, the mass flow rates of hot and cold fluids
- $c_{p,h}, c_{p,c}$, the specific heats of hot and cold fluids
- $T_{h,in}, T_{h,out}$, the inlet and outlet temperatures of hot fluid
- $T_{c,in}, T_{c,out}$, the inlet and outlet temperatures of cold fluid

The heat exchanged is given by:

$$Q = \dot{m}_h c_{p,h}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{p,c}(T_{c,out} - T_{c,in})$$

For a counterflow heat exchanger having a surface A in m^2 and an overall heat transfer coefficient U in W/m^2K, we have:

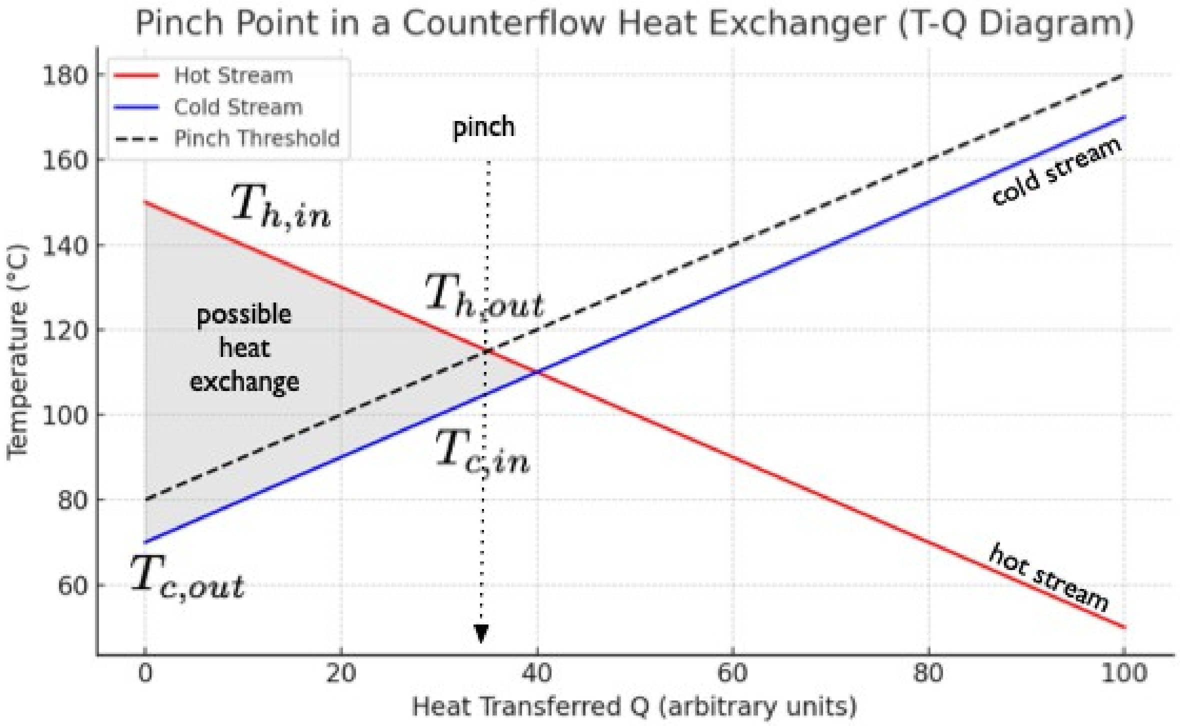
$$Q = UA \cdot \Delta T_{\text{lm}}$$

where

$$\Delta T_{\text{lm}} = \frac{(T_{h,\text{in}} - T_{c,\text{out}}) - (T_{h,\text{out}} - T_{c,\text{in}})}{\log\left(\frac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}\right)}$$

To size a heat exchanger, the pinch point method can be used: it ensures the design is both thermodynamically feasible and efficient. The pinch threshold $\Delta_{T_m in}$ is where the temperature difference between the fluids is minimal. It occurs at one extremity ($T_{h,\text{out}} - T_{c,\text{in}}$ for heat exchangers):

$$\Delta T_{\min} = \min(T_{h,\text{in}} - T_{c,\text{out}}, T_{h,\text{out}} - T_{c,\text{in}})$$



If $\Delta T_{\min}$ is small: $\Delta T_{\text{lm}} \rightarrow 0$ and therefore $Q \rightarrow 0$ therefore pinch temperature difference stands for a threshold. Thus, the pinch point limits the feasible heat transfer. It becomes the governing constraint in design.

For instance, let's set $\Delta T_{\min=10}$ and design the water/water heat exchanger with the following characteristics: $T_{h,in} = 120^\circ C, T_{h,out} = 60^\circ C, T_{c,in} = 50^\circ C, T_{c,out} = 80^\circ C$, $\lambda_{\text{copper}} = 0.38W/m.K$, $A = 0.3m^2$ and $d = 1mm$.

$$U_{\text{copper}} = \frac{\lambda_{\text{copper}}}{d} = 380W/m^2$$

It comes $\Delta T_{\text{lm}} = 27.3^\circ C$ and the heat exchanged is:

$$Q = 27.3^\circ C \times 380W/m^2.^\circ C \times 0.3m^2 = 3112.2W$$

The water flow for the hot and the cold streams are:

$$\dot{m}_h = \frac{Q}{c_{p,h}(T_{h,in} - T_{h,out})} = \frac{3112.2W}{4180J/kg.^{\circ}C \times 60^{\circ}C} = 44.7kg/h\dot{m}_c = \frac{Q}{c_{p,h}(T_{c,in} - T_{c,out})} = \frac{3112.2W}{4180J/kg \cdot {}^{\circ}C \times -30^{\circ}C} = -89.3kg/h$$

An approximated linear model can then be used:

$$Q = \varepsilon C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$$

with:

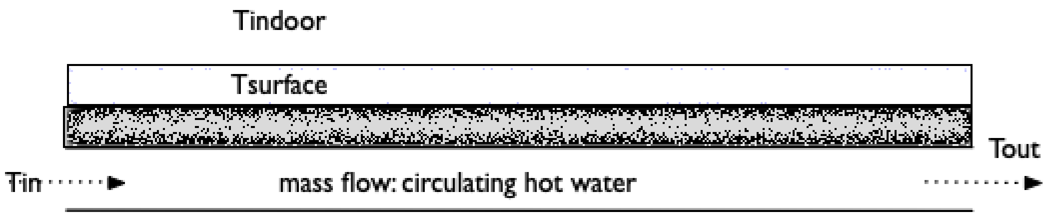
$$\varepsilon = \frac{1 - e^{-NTU(1-C_r)}}{1 - C_r e^{-NTU(1-C_r)}} C_h = \dot{m}_h c_{p,h} C_c = \dot{m}_c c_{p,c} C_{\min} = \min(C_h, C_c) C_{\max} = \max(C_h, C_c) C_r = \frac{C_{\min}}{C_{\max}} \text{NTU} = \frac{UA}{C_{\min}}$$

Regarding the previous example, we have: $C_h = 51.9W/^{\circ}C$ and $C_c = -103.74W/^{\circ}C$. We deduce that $C_{\min} = -103.74W/^{\circ}C$ and $C_{\max} = 51.9W/^{\circ}C$. Then $C_r = -2$, $NTU = -1.1$ and eventually, $\varepsilon = -0.47$ and $Q = 3432W$, which is relatively close to $3112.2W$ previously found.

In TESPy, heat exchangers are implemented using the following attributes, set for instance with `component_reference.set_attr(attribute_name: value)`:

kA	**Product of overall heat transfer coefficient and area $U \cdot A$ (W/K)**
ttd_u	Upper terminal temperature difference (K): $T_{h,\text{in}} - T_{c,\text{out}} = ttd_u$
ttd_l	Lower terminal temperature difference (K): $T_{h,\text{out}} - T_{c,\text{in}} = ttd_l$
pr1, pr2	Pressure loss ratios on sides 1 and 2
Q	Heat transfer rate (W)
efficiency	Heat exchanger effectiveness (optional)

A.3 Heating floor



The model of a heater describes how heat is transferred from circulating water through a purely radiative radiator to heat an living place with:

- T_{in}, the water inlet temperature [°C]
- T_{out}, the water outlet temperature [°C]
- $\dot{m}$, the mass flow rate of water [kg/s]
- A, the radiator surface area [m²]
- T_{outdoor}, the living place air temperature [°C]
- c_p, the the specific heat of water ≈ 4180 J/kg·K

1. Heat delivered by water (hydraulic model)

$$Q = \dot{m} c_p (T_{\text{in}} - T_{\text{out}})$$

It gives the total heat transferred from water to the radiator. For the power to be always the same direction, we have to satisfy $T_{\text{in}} - T_{\text{out}} > 0$ or with the pinch temperature $\Delta_{Tmin} > 0$:

$$T_{\text{in}} - T_{\text{out}} = 2\Delta_{Tmin} \dot{m} = \frac{Q}{2 c_p \Delta_{Tmin}}$$

Setting Δ_{Tmin} amounts to set the mass flow.

2. Heat transferred from floor to the indoor environment (thermal model)

Assuming steady state and uniform floor surface temperature, a heat transfer model can be used:

$$Q = UA(T_{\text{surface}} - T_{\text{indoor}})$$

where:

- U, the overall heat transfer coefficient between the radiator and room [W/m²·K]
- T_{surface}, the average floor surface temperature [°C] often approximated as:

$$T_{\text{surface}} \approx \frac{T_{\text{in}} + T_{\text{out}}}{2}$$

3. Final model

$$Q = UA\left(\frac{T_{\text{in}} + T_{\text{out}}}{2} - T_{\text{indoor}}\right)$$

$$Q = UA\left(T_{\text{in}} - \Delta_{\text{Tmin}} - T_{\text{indoor}}\right)$$

$$\frac{Q}{UA} + T_{\text{indoor}=T_{\text{in}}-\Delta_{\text{Tmin}}}$$

$$\frac{Q}{UA} + T_{\text{indoor}} - T_{\text{in}} = -\Delta_{Tmin}$$

It leads to:

$$T_{\text{in}} = \frac{Q}{UA} + T_{\text{indoor}} + \Delta_{Tmin}$$

$$T_{\text{out}} = T_{\text{in}} - 2\Delta_{Tmin}$$

$$\dot{m} = \frac{Q}{2c_p\Delta_{Tmin}}$$

A.4 Evaporator

The evaporator is a heat exchanger where the refrigerant absorbs heat from the heat source (lower temperature). It carries out the phase change of the refrigerant from a liquid to a low-pressure vapor by absorbing latent heat. It operates at low pressure to allow the refrigerant to boil.
It operates at constant temperature.

It is typically composed by copper piping and aluminum fins to
maximize surface area for the heat transfer. It requires a precise control of refrigerant flow to ensure efficient heat
absorption without liquid to carry-over into the compressor.

It can be modeled by:

$$Q_{\mathrm{evap}} = \dot{m}(h_{\mathrm{evap},in} - h_{\mathrm{evap},out})$$

where Q_{evap} is the heat absorbed by the evaporator, $\dot{m}$ is the mass flow rate of the refrigerant, $h_{\mathrm{evap},in}$ is the enthalpy of the refrigerant at the inlet of the evaporator, and $h_{\mathrm{evap},out}$ is the enthalpy of the refrigerant at the outlet of the evaporator.

In `TESPy`, an evaporator is typically modeled using the Condenser component (despite its name), since it represents a 2-stream heat exchanger where one fluid evaporates (gains heat) and the other releases heat (often air or water). TESPy treats evaporators and condensers symmetrically—you just reverse which side is evaporating.

The evaporator component has:

- side 1: refrigerant (typically the evaporating side) → in1, out1
- side 2: heat source (e.g., ambient air or water) → in2, out2

TESPy ensures that refrigerant enters as liquid and leaves as vapor during evaporation.

There are different options, set for instance with `component_reference.set_attr(attribute_name: value`), to describe an evaporator in `TESPy`:

A. Pinch Point Based (ttd_u)

B. Heat Transfer Rate (Q)

C. Heat Transfer Coefficient $U \cdot A$ (kA) with $Q = kA \cdot \Delta T_{lm}$

Optionally, the pressure drop can be specified for each side: ``evap.set_attr(pr1=0.98, pr2=0.95)```

A.5 Condenser

Similarly to the evaporator, the condenser is a heat exchanger where the refrigerant releases heat to the heat sink (higher temperature). In invertible systems, it releases latent heat either to the indoor air (in heating mode) or the outdoor environment (in cooling mode). It carries out the phase change of the refrigerant from a high-pressure vapor to a liquid by releasing latent heat. It is basically a coil of copper piping and aluminum fins, which operates at high pressure to allow the refrigerant to condense. The heat rejection occurs at constant pressure.

The refrigerant exits the condenser at high-pressure as a sub-cooled liquid ready for expansion.

It can be modeled by:

$$Q_{\text{cond}} = \dot{m}(h_{\text{cond},in} - h_{\text{cond},out})$$

where Q_{cond} is the heat released by the condenser, $\dot{m}$ is the mass flow rate of the refrigerant, $h_{\text{cond},in}$ is the enthalpy of the refrigerant at the inlet of the condenser, and $h_{\text{cond},out}$ is the enthalpy of the refrigerant at the outlet of the condenser.

In `TESPy`, the condenser component is defined by:

- Side 1: refrigerant (typically the evaporating side) → in1, out1
- Side 2: heat source (e.g., ambient air or water) → in2, out2

The attributes, set for instance with `component_reference.set_attr(attribute_name: value)`, are the same as the ones of the evaporator:

ttd_u	Pinch point temp diff (hot side outlet)
Q	Heat released by refrigerant
kA	Heat transfer area (A) × transmission coefficient (U)
pr1 / pr2	Pressure ratios for refrigerant and coolant

A.6 Expansion valve

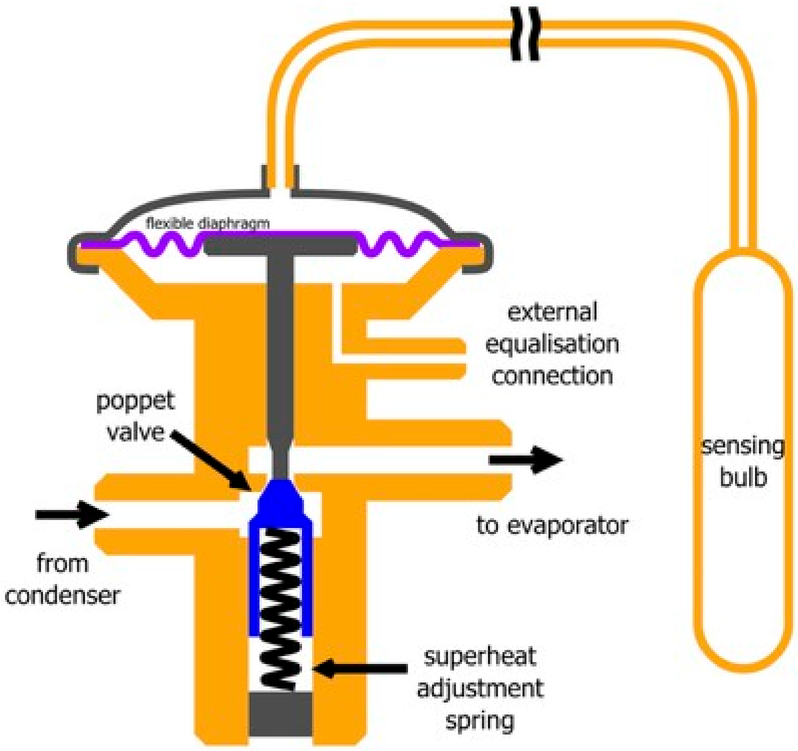
The expansion valve is a device that reduces the pressure of the refrigerant. It is a mechanical device causing a significant drop in temperature. This prepares the refrigerant to absorb heat in the evaporator.

The valve meters the refrigerant flow into the evaporator, ensuring proper evaporation and preventing flooding of the compressor. It converts high-pressure liquid refrigerant into a low-pressure, low-temperature mixture (part liquid, part vapor).

It operates under constant enthalpy, where the pressure drops without affecting the total heat content.

There are 3 types of expansion valves:

- the Thermostatic Expansion Valve (TXV), which adjusts the refrigerant flow based on the evaporator temperature.
- the Electronic Expansion Valve (EXV), which provides precise control via sensors and controllers.
- the Capillary Tubes, which are simplified, often used in smaller systems.


flexible diaphragm
external
equalisation
connection
poppet
valve
sensing
bulb
to evaporator
from
condenser
superheat
adjustment
spring

Modeling the expansion valve in a heat pump is typically done using thermodynamics, since it is a purely mechanical device with no moving parts or external work. The expansion valve is involved in an adiabatic process (no heat transfer with surroundings), with no work exchange, steady-state flow, and an isenthalpic process (enthalpy stays constant) but with a significant pressure drop between inlet and outlet.

Because enthalpy remains constant across the valve, tt comes:

$$h_1 = h_4$$

with:

- h_4, the specific enthalpy at inlet (high pressure)
- h_1, the specific enthalpy at outlet (low pressure)

The mass balance yields:

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

with:

- $\dot{m}_{\text{in}}$, the mass flow rate at inlet
- $\dot{m}_{\text{out}}$, the mass flow rate at outlet

The phase change leads to:

- a refrigerant typically partially vaporizes during throttling.
- an outlet quality $x_2 \in (0, 1)$, which is usually a liquid-vapor mixture.

The pressure drop from the high-pressure side (condenser outlet) to the low-pressure side (evaporator inlet) yields to:

$$p_2 \ll p_1$$

In `TESPy`, this is implemented via `pr1` and `pr2`.

The `TESPy` optional attributes, set for instance with `comporent_reference.set_attr(attribute_name: value`), are:

A. Using Pressure Ratio (pr)

B. Using Pressure Drop Coefficient (zeta). TESPy computes the pressure drop using: $\Delta p = \zeta \cdot \frac{\rho v^2}{2}$

A.7 Hydronic pump

A hydronic or circulation pump in thermodynamic modeling is a device that increases the pressure of a liquid by doing mechanical work on it. The mathematical model for a pump focuses on energy conservation, fluid properties, and optional loss modeling.

The mass flow in equals the mass flow out:

$$\dot{m}_{\mathrm{in}} = \dot{m}_{\mathrm{out}}$$

For steady-state, neglecting heat losses and kinetic/potential energy changes leads to:

$$\dot{m}(h_{\mathrm{out}} - h_{\mathrm{in}}) = \frac{P_{\mathrm{el}}}{\eta}$$

where:

- $\dot{m}$: mass flow rate [kg/s]
- $h_{\mathrm{in}}, h_{\mathrm{out}}$: specific enthalpies in/out [J/kg]
- P_{el}: electrical power input [W]
- η: pump efficiency (hydraulic or total, dimensionless)

The isentropic efficiency (η_s) is commonly used:

$$\eta_s = \frac{h_{\text{out,s}} - h_{\text{in}}}{h_{\text{out}} - h_{\text{in}}}$$

where $h_{\text{out,s}}$ is the ideal (isentropic) outlet enthalpy.

TESPy allows you to define either:

- eta_s (isentropic efficiency)
- eta (total efficiency)
- or even P (power input in W), depending on what you want to constrain.
- Outlet pressure p = p_{out}
- Pressure ratio: pr = $\frac{p_{\text{out}}}{p_{\text{in}}}$

A.8 Heat source

A Source component is used to introduce a fluid into the system. It has only one outlet connection and represents the entry point of mass, energy, and fluid composition into the network.

The attributes are set via .set_attr() on the component or its outlet Connection.

The fluid state is usually defined here:

A Source component is used to introduce a fluid into the system. It has only one outlet connection and represents the entry point of mass, energy, and fluid composition into the network.

The attributes are set via .set_attr() on the component or its outlet Connection.

The fluid state is usually defined here:

- p: pressure [Pa]
- T: temperature [K]

  or
- h: specific enthalpy [J/kg]
- m: mass flow rate [kg/s] *(optional, sometimes solved by the system)*
- fluid: dictionary of fluid mass fractions (must sum to 1)

A.9 Heat sink

A heat sink is the opposite of a Source: it is a terminal component where the fluid exits the system. It has only one inlet connection, and its main role is to absorb flow without returning it.

Like the Source, most important attributes are defined on the connection to the Sink, not on the Sink component itself.

You can specify:

- p – Pressure at the sink inlet [Pa]
- T – Temperature [K]
- h – Specific enthalpy [J/kg]
- m – Mass flow rate [kg/s] (optional)
- fluid – Dictionary of fluid composition (must sum to 1)

A.10 Connection

In `testpy`, components are connected using connections binding output port(s) (out1 and possibly out2) to input port(s) (in1 and possibly in2).

A connection binding a evaporator to a compressor is coded as:

```
connection = Connection(self.evaporator, "out2", self.compressor, "in1", label="1")
self.con_evaporator_compressor.set_attr(fluid={self._refrigerant.value: 1})
```

The attributes are:

- p: pressure [Pa]
- T: temperature [K]
- m: mass flow rate [kg/s] , for instance, `connection.set_attr({'water': 1.0, 'air': 0.0})`
- h: specific enthalpy [J/kg]
- fluid: dictionary of mass fractions for each fluid in the network
- x: vapor mass fraction (used with two-phase fluids like refrigerants)

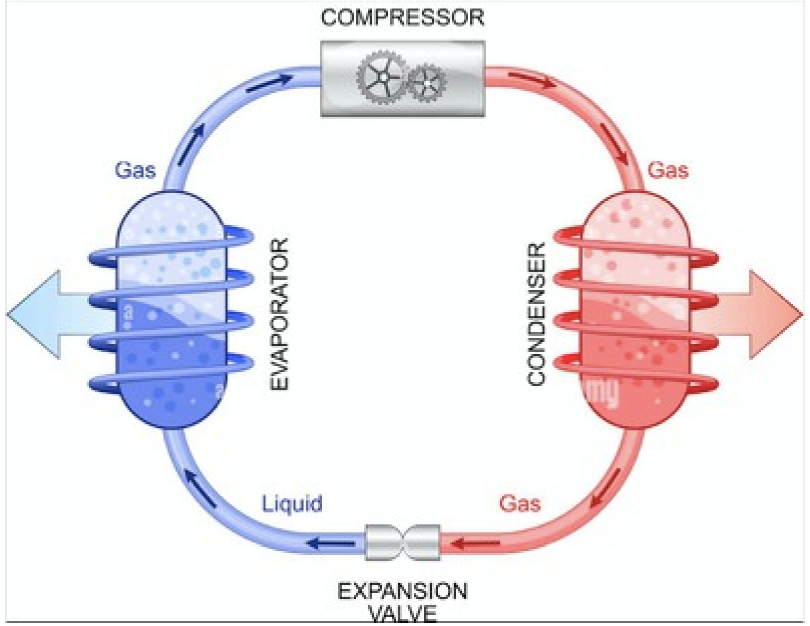
A.11 Developing a model with Python/TESPy

We are going to use the Python library TESPy that can model many different kinds of heat pumps. An alternative approach for the design of heat pomp systems is to use OpenModelica but it is only for Windows.

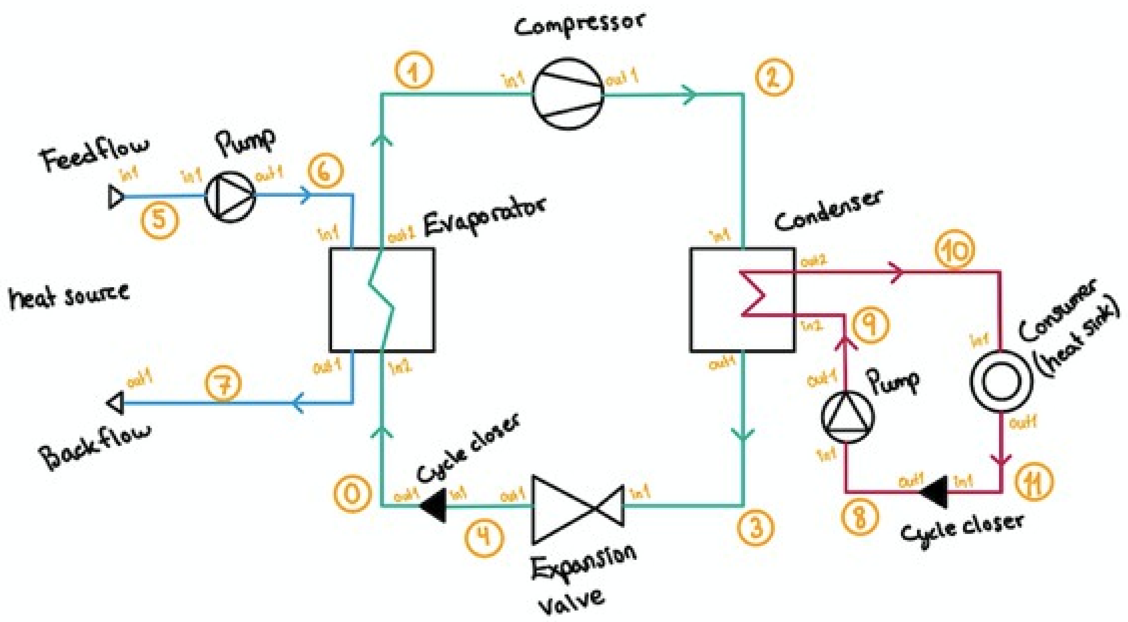
In TESPy, a model starts by defining a network, specifying the units of the different variables:

```
network = Network(T_unit="C", p_unit="bar", h_unit="kJ / kg", m_unit="kg / s")
```

As seen before, a heat pump is characterized by 4 main components: a compressor, a condenser, an evaporator and an expansion valve.



Designing a model with `TESPy` supposes to depict the structural diagram of the systems with components and how they are connected each other. If a circuit forms a closed loop, i.e. there is a continuity in flow and in temperature at any point, a "cycle closer" component must be added to close the circuit.



The elements of the system are:

```
# refrigerant loop
refrigerant_cycle_closer = CycleCloser("Refrigerant Cycle Closer")
compressor = Compressor("Compressor")
condenser = Condenser("Heat Sink Condenser")
valve = Valve("Expansion Valve")
evaporator = HeatExchanger("Heat Source Evaporator")
# heat source loop
heatsource_feedflow = Source("Heat Source Feed Flow")
heatsource_pump = Pump("Heat Source Recirculation Pump")
heatsource_backflow = Sink("Heat Source Back Flow")
# heat sink loop
heatsink_pump = Pump("Heat Sink Recirculation Pump")
heatsink = SimpleHeatExchanger("Heatsink")
heatsink_cycle_closer = CycleCloser("Heatsink Feed Flow")
```

Components have to be connected to form a network:

```
# #### refrigerant loop
connection_refrigerant_cycle_closer_evaporator = Connection(refrigerant_cycle_closer, "out1", evaporator, "in2", label="0")
connection_evaporator_compressor = Connection(evaporator, "out2", compressor, "in1", label="1")
connection_compressor_condenser = Connection(compressor, "out1", condenser, "in1", label="2")
connection_condenser_valve = Connection(condenser, "out1", valve, "in1", label="3")
connection_valve_refrigerant_cycle_closer = Connection(valve, "out1", refrigerant_cycle_closer, "in1", label="4")
# heat source loop
connection_feedflow_heatsource_pump = Connection(heatsource_feedflow, "out1", heatsource_pump, "in1", label="5")
connection_heatsource_pump_evaporator = Connection(heatsource_pump, "out1", evaporator, "in1", label="6")
connection_evaporator_heatsource_backflow = Connection(evaporator, "out1", heatsource_backflow, "in1", label="7")
# heat sink loop
connection_cycle_closer_heatsink_pump = Connection(heatsink_cycle_closer, "out1", heatsink_pump, "in1", label="8")
connection_heatsink_pump_condenser = Connection(heatsink_pump, "out1", condenser, "in2", label="9")
connection_condenser_heatsink = Connection(condenser, "out2", heatsink, "in1", label="10")
connection_heatsink_heatsink_cycle_closer = Connection(heatsink, "out1", heatsink_cycle_closer, "in1", label="11")
```

Then set the known temperatures and pressures in the heat source and the heat sink circuits:

```
T_heat_source_backflow = 10  # heat source out back flow
T_heat_source_feedflow = 15  # heat source in feed flow
T_heatsink_backflow = 50  # heat sink out
T_heatsink_feedflow = 90  # heat sink in
# heat sink circuit
connection_condenser_heatsink.set_attr(T=T_heatsink_feedflow, p=10, fluid={"water": 1})
connection_heatsink_cycle_closer.set_attr(T=T_heatsink_backflow)

# heat source circuit
connection_feedflow_heatsource_pump.set_attr(T=T_heat_source_feedflow, p=1, fluid={"water": 1})
connection_evaporator_heatsource_backflow.set_attr(T=T_heat_source_backflow, p=1)
```

`testpy` retrieves the data of the specified refrigerant:

```
# Fluid in the heat pump
refrigerant = "NH3"
#  evaporation to fully saturated gas
connection_evaporator_compressor.set_attr(x=1, fluid={refrigerant: 1})
```

Then set the pump efficiencies:

```
# parametrization components
# isentropic efficiency
heatsink_pump.set_attr(eta_s=0.8)
heatsource_pump.set_attr(eta_s=0.8)
compressor.set_attr(eta_s=0.85)
```

Then set the pressure ratios:

```
# pressure ratios
condenser.set_attr(pr1=0.98, pr2=0.98)
evaporator.set_attr(pr1=0.98, pr2=0.98)
heatsink.set_attr(pr=0.99)
```

Get the evaporation and condensation pressures thanks to the CoolProp database;

```
# evaporation pressure (see properties at http://www.coolprop.org/coolprop/HighLevelAPI.html#parameter-table)
# the first argument is the property to be calculated the pressure in Pa
# the second argument is the quality of the fluid in mol/mol
# the third argument is the temperature of the fluid in K
# the fourth argument is the refrigerant
evaporation_pressure_Pa = CoolProp.CoolProp.PropsSI("P", "Q", 1, "T", T_heat_source_backflow + 273.15, refrigerant)
connection_evaporator_compressor.set_attr(p=evaporation_pressure_Pa)

# condensation pressure
condensation_pressure_Pa = CoolProp.CoolProp.PropsSI("P", "Q", 0, "T", T_heatsink_feedflow + 273.15, refrigerant)
connection_condenser_valve.set_attr(p=condensation_pressure_Pa)
```

The heat power has to be set:

```
# consumer heat demand
supplied_heat_W = -3000
heatsink.set_attr(Q=supplied_heat_W)
```

Eventually, compute powers and COP:

```
# solve the network
network.solve("design")

# calculate the COP
cop = abs(heatsink.Q.val / (heatsink_pump.P.val + heatsource_pump.P.val + compressor.P.val))
print("the COP is: ", cop)
```

A.12 Choosing a heat pump

Let's consider a heat pump based on R32 refrigerant (difluoromethane: CH_2F_2), the most frequently used refrigerant in HVAC systems applications. It's a refrigerant with a low global warming potential (GWP=675 years) and a low ozone depletion potential (ODP=0).

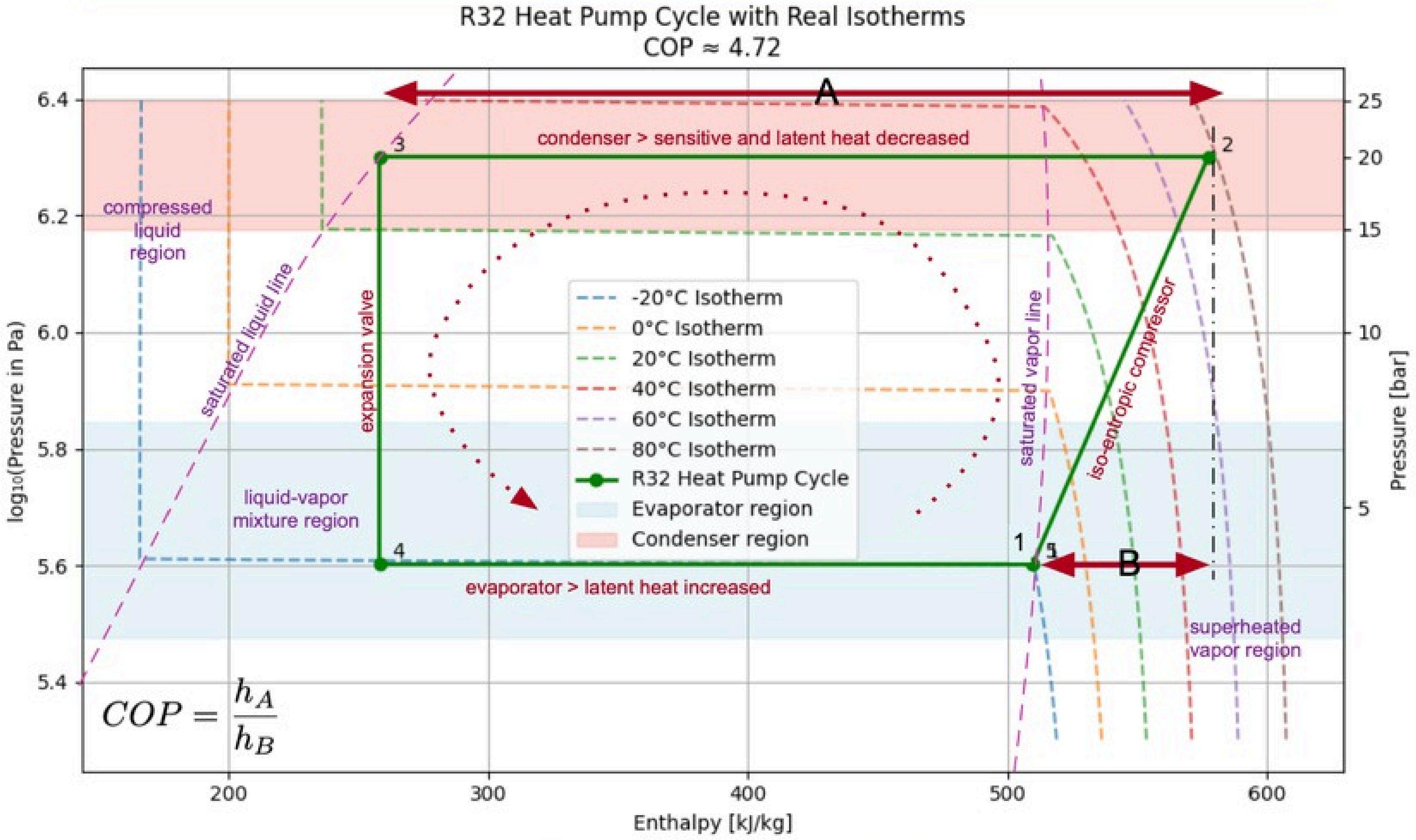
R32 enables high Seasonal Energy Efficiency Ratios (SEER) and Seasonal COPs (SCOP) in real conditions because:

- R32 has higher latent heat of vaporization than R410A (former leader) → more cooling/heating per unit mass flow.
- Higher volumetric cooling capacity → smaller compressors and heat exchangers.

The detailed properties of R32 are:

- molar mass: 52.02 g/mol
- Specific heat c_p: 1.42 kJ/kg·K
- Specific heat c_v: 1.09 kJ/kg·K
- Gas constant $R = c_p - c_v$: 0.33 kJ/kg·K
- critical temperature: 78.11 °C (351.26 K)
- critical pressure: 57.82 bar (5782 kPa)
- critical density: 424 kg/m³
- Triple point temperature: -136 °C (137.15 K)
- Triple point pressure: 0.00234 bar (0.234 kPa)
- Normal boiling point at 1 atm: -51.7 °C
- Saturation pressure @25°C: 13.9 bar (1390 kPa)
- Saturation pressure @0°C: 8.8 bar (880 kPa)
- Latent heat of vaporization @25 °C: 390 kJ/kg
- density (liquid) @25 °C: 958 kg/m³
- density (gas) @25 °C: 43 kg/m³
- Enthalpy of vapor (hg) @25 °C: 465 kJ/kg
- Enthalpy of liquid (hf) @25 °C: 75 kJ/kg
- Entropy of vapor (sg) @25 °C: 1.73 kJ/kg·K
- Thermal conductivity (gas) @25 °C: 0.0135 W/m·K
- Dynamic viscosity (gas) @25 °C: 12.1 µPa·s
- Prandtl number (gas) @25 °C: 0.76

heat (hot) sink (lower temperature than the refrigerant)



heat (cold) source (higher temperature than the refrigerant)

The COP depends of the temperature of the heat source and the heat sink.

Heat source temperature	Heat sink temperature	Heating COP
35°C	-20°C	4.7
25°C	-20°C	5.6
62°C	-20°C	2.1
35°C	0°C	11.2
35°C	-40°C	3.4

We can conclude that:

- the closer the heat source and the heat sink temperatures are, the higher the COP is
- the worst for the COP is when we increase the heat sink temperature (inlet temperature of the HVAC system)
- the refrigerant has to be chosen in function of local climate but also of the needed temperature range of the HVAC system
- heating floors are much more efficient than vertical radiators

The seasonal performance of heating and cooling systems is evaluated over an entire season. Indeed, the COP depends on many factors including the outdoor temperature. The Seasonal Coefficient of Performance (SCOP) is defined as:

$$\text{SCOP} = \frac{\text{Total useful heating energy delivered over the season}}{\text{Total electricity consumed over the season}}$$

In the EN 14825 EU standard / Ecodesign Directive, the heating seasonal performance is based on climate profiles:

- Average Climate: Strasbourg (default EU reference)
- Warmer Climate: Athens
- Colder Climate: Helsinki

Instead of simulated a whole season, the SCOP is calculated thanks to weighted average of the COP over several outdoor temperatures (e.g., -7°C, 2°C, 7°C, 12°C), each with predefined duration and load percentage.

Let's consider a building with a 50m^2 of heating floor defined by:

Weather:

- lowest yearly outdoor temperature: -15°C
- studied temperatures: -7°C, 2°C, 7°C, 12°C

Building:

- heat transfer surface: 300m^2
- Ubat: 0.25kW/m^2.K
- setpoint temperature: 20°C

Heating floor:

- heating floor area: 100.00m^2
- floor material: concrete
- thickness: 0.020m

Heat pump:

- refrigerant: R32
- assumed COP: 5
- max electric power: to be determined
- mass flow rate: to be determined

Let's size the heating floor inlet temperature: inlet_water: 22.20°C (Δ=2°C, $\dot{m}$=1.7kg/s)

The needed heat power is:

$$P_{-15^{\circ}C} = U_{bat} S_{bat} (T_{\text{indoor}} - \check{T}_{\text{outdoor}}) < 3000.0W$$

With a assumed COP of 5 (we will check later), the max electric power is: 600W

We can use a R32 standard air/water heat pump used for heating.

Let's compute the COP for outdoor temperatures of -7°C, 2°C, 7°C, 12°C. The water inlet temperature is 26°C.

Toutdoor	COP	Thermal power	Electrical power	Mass flow rate
-7°C	5.1	10325.23 W	2025.0 W	0.03223059 kg/s
2°C	8.5	17165.00 W	2025.0 W	0.05619888 kg/s
7°C	10.8	21871.79 W	2025.0 W	0.07348653 kg/s

Self-check questionnaire

12 questions — HVAC systems and heat pumps

Choose the best answer each time.

(In class: discuss in pairs; answers on the following slides.)

Q1 — Day degrees (HDD / CDD)

The degree-day method on the slides estimates annual needs using:

- A. $Q_{\text{heating}} \propto U_{\text{bat}}\, A_{\text{bat}}\, \mathrm{HDD}$ and $Q_{\text{cooling}} \propto U_{\text{bat}}\, A_{\text{bat}}\, \mathrm{CDD}$ (with HDD/CDD summing positive degree-days vs reference set-points)
- B. Only solar irradiance on windows, with no U_{bat}
- C. PMV comfort votes only
- D. Compressor isentropic efficiency only

Q2 — HDD vs CDD

Heating degree-days (HDD) and cooling degree-days (CDD) respectively quantify:

- A. How much outdoor daily mean temperature is below the heating reference (HDD) or above the cooling reference (CDD)
- B. Indoor humidity in g/kg only
- C. Wind speed against vertical walls
- D. Refrigerant quality x in the dome

Q3 — Heat pump principle

A heat pump for buildings is described on the slides as:

- A. Moving heat from a low-temperature source to a high-temperature sink using a refrigeration cycle (not “creating” heat from nothing)
- B. Burning fuel inside the evaporator
- C. A resistor with COP always equal to 1
- D. A device that only works in cooling mode

Q4 — Vapour-compression cycle (order)

The four main stages of a standard heat pump cycle are:

- A. Evaporation → Compression → Condensation → Expansion (throttling)
- B. Condensation → Expansion → Evaporation → Compression
- C. Only isothermal compression in the evaporator
- D. Expansion in the compressor and compression in the expansion valve

Q5 — Expansion valve

The expansion valve process is characterized on the slides as:

- A. Isenthalpic throttling (no heat exchange; pressure and temperature drop)
- B. Isothermal heat release to the room
- C. Isentropic compression of liquid
- D. Constant-volume heating of refrigerant

Q6 — COP (heating)

COP for heating is defined on the slides as:

- A. $COP_{\text{heating}} = Q_{\text{hot}}/W_{\text{input}}$ (useful heat delivered divided by compressor/electrical work)
- B. $W_{\text{input}}/Q_{\text{hot}}$ always less than 0.1
- C. Outdoor temperature in °C divided by indoor set-point
- D. Mass flow rate times c_p only, with no work term

Q7 — Carnot COP (units)

The Carnot heating COP uses absolute temperatures (kelvin):

- A. $COP_{\text{Carnot,heating}} = T_{\text{sink}}/(T_{\text{sink}} - T_{\text{source}})$
- B. $COP = T_{\text{sink}} - T_{\text{source}}$ with T in °C only
- C. $COP = 1$ regardless of temperatures
- D. $COP = P_{\text{electrical}}/Q_{\text{hot}}$ (inverse of the usual definition)

Q8 — Emitter temperature and COP

For a given outdoor source temperature, the slides note that raising the heat-sink / supply temperature (e.g. radiators vs low-temperature heating floor) tends to:

- A. Lower the achievable COP (larger temperature lift for the cycle)
- B. Increase COP without limit
- C. Have no effect on Carnot or real COP
- D. Eliminate the need for a compressor

Q9 — SCOP

SCOP (seasonal COP) on the slides is:

- A. Total useful heating energy over the season divided by total electricity consumed over the season
- B. Instantaneous COP at -15°C only
- C. The HDD table for Grenoble only
- D. Radiator surface area times U_{bat}

Q10 — Air-source vs ground-source

Air-source heat pumps often need a fan to collect heat because:

- A. Air has low volumetric heat capacity compared with water/ground; large volumes must be moved
- B. Air has no sensible heat
- C. Ground loops cannot store heat
- D. Fans replace the expansion valve

Q11 — Water radiator (two balances)

A water radiator is modeled with:

- A. $Q = \dot{m} c_p (T_{\text{inlet}} - T_{\text{outlet}})$ and $Q = U_{\text{radiator}} A (T_w - T_{\text{indoor}})$
- B. $Q = \sigma T^4$ only
- C. $Q = U_{\text{bat}} S_{\text{bat}}$ with no fluid term
- D. Q independent of water mass flow

Q12 — RT2012 / decarbonization

In the French context on the slides, heat pumps are emphasized because:

- A. They can deliver several kWh of heat per kWh of electricity (COP $\gg 1$), helping decarbonize heating vs direct fossil combustion
- B. They consume more gas than boilers by design
- C. They work without any heat source outdoors
- D. Regulations forbid any electrical input to HVAC

Answer key (1/2)

Q	Answer	Short rationale
1	A	$Q \propto U_{\text{bat}} A_{\text{bat}} \times$ HDD or CDD (×24 h, kWh).
2	A	HDD: cold days vs ref.; CDD: hot days vs ref.
3	A	Transfers heat source → sink via refrigerant cycle.
4	A	Evap → comp → cond → expand.
5	A	Throttling: isenthalpic, Δp, ΔT.
6	A	$COP_h = Q_{\text{hot}}/W$.

Answer key (2/2)

Q	Answer	Short rationale
7	A	Carnot: T in K; smaller lift → higher COP.
8	A	Higher supply/sink T → lower COP.
9	A	Seasonal useful heat / electricity.
10	A	Low c_p per m^3 of air → large airflow.
11	A	Fluid enthalpy balance + $UA\Delta T$ to room.
12	A	High COP; environmental heat + electricity.

It's time to start workshop06_system.ipynb