powerbox: A Python package for creating structured fields with isotropic power spectra

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Summary

The power spectrum is a cornerstone of both signal analysis and spatial statistics, encoding the variance of a signal or field on different scales. Its common usage is in no small part attributable to the fact that it is a *full* description of a purely Gaussian process – for such statistical processes, no information is contained in higher-order statistics. The prevalence of such processes (or close approximations to them) in physical systems serves to justify the popularity of the power spectrum as a key descriptive statistic in various physical sciences, eg. cosmology (Peacock 1998) and fluid mechanics (Monin and Yaglom 2007). It furthermore readily avails itself to efficient numerical evaluation, being the absolute square of the Fourier Transform.

Another feature of many approximate physical systems, especially those already mentioned, is that they are both homogeneous and isotropic (at least in some local sample). In this case, the n-dimensional power spectrum may be losslessly compressed into a single dimension, which is radial in Fourier-space. Such processes approximately describe for example the over-density field of the early Universe and locally isotropic turbulent flows. Thus it is of great use to have a numerical code which simplifies the dual operations of; (i) producing random homogeneous/isotropic fields (of arbitrary dimensionality) consistent with a given 1D radial power spectrum, and (ii) determination of the 1D radial power spectrum of random fields (or a sample of tracers of that field). `powerbox` exists to perform these duals tasks with both simplicity and efficiency.

Performing the first of these tasks is especially non-trivial. While the power spectrum can be evaluated on any field (though it may not fully describe the given field), the precise machinery for *creating* a field from a given power spectrum depends on the probability density function (PDF) of the process itself. The machinery for creating a Gaussian field is well-known. However, other PDF's – especially those that are positively bounded – are extremely useful for describing such physical entities as density fields. In these cases, the *log-normal* PDF has become a standard approximation (Coles and Jones 1991), and `powerbox` makes a point of supporting the machinery for generating log-normal fields (Beutler et al. 2011) for this purpose. Indeed, `powerbox` is *primarily* geared towards supporting cosmological applications, such as measuring and and producing samples of galaxy positions in a log-normal density field (while account for standard effects such as shot-noise and standard normalisation conventions). It is nevertheless flexible enough to support research in any field (with its own particular conventions) that is based on the homogeneous and isotropic power spectrum.

`Powerbox` is a pure-Python package devoted to the simple and efficient solution of the previous considerations. As the most popular language for astronomy, Python is the natural language of choice for `powerbox`, with its focus on cosmological applications, and it also provides for great ease-of-use and extensibility. As an example of the former, all

functions/classes within `powerbox` are able to work in arbitrary numbers of dimensions (memory permitting), simply by setting a single parameter n. As an example of the latter, the class-based structure of the field-generator may be used to extend the generation to fields with PDF's other than either Gaussian or log-normal (indeed, the log-normal class is itself sub-classed from the Gaussian one). `powerbox` does not sacrifice efficiency for its high-level interface. By default, the underlying FFT's are performed by `numpy`, which uses underlying fast C code. In addition, if the `pyFFTW` package is installed, `powerbox` will seamlessly switch to using its optimized C code for up to double the efficiency. It is also written with an eye for conserving memory, which is important for the often very large fields that may be required.

`Powerbox` was written due to research-demand, and as such it is highly likely to be suited to the requirements of research of a similar nature. Furthermore, as previously stated, every effort has been made to sufficiently generalize its scope to be of use in related fields of research. It has already been instrumental in several publications (Murray, Trott, and Jordan 2017; Wolz et al. 2018), and we hope it will be a useful tool for approximate theoretical simulations by many others.

Acknowledgements

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