

A short history of the level-payment mortgage



mortgagemath 0.7.0 · rendered 2026-05-06

This vignette traces the institutional and mathematical history of the residential mortgage loan, focusing on the long arc from land-secured lending in early law through the standardization of the *direct-reduction* (fully-amortizing) contract during the New Deal, to the regulatory examples that constitute the library’s modern set of validated worked examples. It is also a candid accounting of what mortgagemath *cannot* reproduce, and why — those gaps are themselves historically informative.

1. Origins: the *mortuum vadium* and the medieval common law

The English word *mortgage* descends from the Old French *mort gage* — “dead pledge” — first attested in **Glanvill** (c. 1187) and elaborated by **Bracton** (c. 1235). The thirteenth-century distinction between *vivum vadium* (living pledge) and *mortuum vadium* (dead pledge) marks the legal birth of the modern mortgage: in the *vivum vadium* the rents from the pledged land discharged the debt, while in the *mortuum vadium* the land was forfeited if the debt was not repaid by the appointed day, and its rents in the meantime did not reduce the principal — hence “dead.” Pollock and Maitland’s *History of English Law* (1898) (Pollock and Maitland 1898) is the standard treatment.

Roman antecedents — the *fiducia*, *pignus*, and *hypotheca* — provided the conceptual machinery (Buckland 1921), but they did not yet contemplate a level-payment amortization schedule. For the first 600 years of common-law practice the question of how to *retire* a long-dated land debt was effectively absent from the legal literature, because long-dated loans were rare and almost universally structured as balloon-at-term: interest was paid periodically and the principal fell due in a single payment.

2. Annuity mathematics arrives (1671–1913)

The mathematics of the level-payment annuity-certain — the formula mortgagemath evaluates internally — predates its application to mortgage lending by two centuries. **Johan de Witt’s** *Waardye van Lyf-Renten* (1671) and **Edmond Halley’s** *An Estimate of the Degrees of the Mortality of Mankind* (1693) (Halley 1693) established the present-value calculus for life annuities; **Thomas Simpson’s** *The Doctrine of Annuities and Reversions* (1742) extended it to annuities-certain. By the mid-nineteenth century the closed-form

$$\text{PMT} = P \cdot \frac{r}{1 - (1 + r)^{-n}}$$

was a standard exercise in actuarial education and in the more mathematical wings of commercial bookkeeping. The same formula remains the canonical level-payment expression today: it is the arithmetic embedded in every modern mortgage calculator, in the closed-form `periodic_payment(loan)` of mortgagemath, and in the FHA / Fannie Mae / Freddie Mac servicing standards that descend from the New Deal reforms discussed in §5.

A working library of pre-1929 actuarial textbooks survives — the canonical examples include Sprague’s *Accountancy of Investment* (1907), Hardy’s *Theory of Annuities Certain* (1909), Skinner’s *Mathematical Theory of Investment* (1913), and Hart’s *Mathematics of Investment* (1924). Of these, only **Skinner (1913)** publishes a worked example whose every cent mortgagemath reproduces under one of its existing parameter combinations: his \$42 piano example (\$500 at 6% effective annual, 5-year monthly payments, \$9.63 per month) is the first validated example in the test suite that isolates the actuarial convention of treating the quoted rate as *effective annual* rather than *nominal monthly*. The others either publish to mill precision (Hart 1924, three-decimal cents) or rely on six-place annuity-factor tables that produce internally inconsistent column totals where row-level interest plus principal does not always equal the printed payment (Skinner’s \$10,000/5% five-year schedule; Sprague’s \$1,000/3% four-year schedule). We do not include these internally inconsistent historic examples in the test suite, because there is no parameter combination under which the library matches every printed cell.

3. The Crédit Foncier de France (1852) and the loaded annuity

The first large-scale *centralized* long-term amortizing-mortgage institution was the **Crédit Foncier de France**, chartered by imperial decree on 28 February 1852 under the influence of **Louis Wolowski** (Wolowski 1852). (The Prussian *Landschaften*, beginning with the Silesian *Schlesische Landschaft* of 1769, predate the CF as land-credit cooperatives but operated through mutual association rather than a centralized bank balance sheet.) The CF made fixed-rate, level-installment, fully-amortizing loans on real property — decades before any equivalent existed in the English-speaking world. Its founding documents are unambiguous about the structure of what it called the *annuité*, the periodic level payment. Wolowski’s 1852 essay reproduces the canonical decomposition for a 4½% loan:

Component	Per 100 fr. of original principal
Intérêt aux porteurs d’obligations	4.50 fr.
Amortissement	1.00 fr.
Frais d’administration	0.10 fr.
Fonds de réserve	0.30 fr.
Impôt	0.10 fr.
Total annuité	6.00 fr.

Wolowski’s contemporaneous explanation runs: « *l’intérêt à payer au créancier étant à 4 1/2, le propriétaire emprunteur aurait à payer 6 pour 100 pendant trente-neuf ans pour être complètement libéré.* » — “the interest payable to the creditor being at 4 1/2, the property-owning borrower would pay 6 per 100 for thirty-nine years to be entirely discharged.”

This decomposition explains why mortgagemath originally could not reproduce a Crédit Foncier worked schedule. Every schedule the CF (and its imitators) published was the *gross* annuity — the figure the borrower actually wrote on his quarterly draft — not the actuarially-pure interest-plus- principal value. The 1.50-franc administrative loading per 100 francs of original principal is mathematically inseparable from the published payment without rebuilding the schedule from a stripped-down 5.50% basis, which is *not what the source publishes*. Because every printed cell would need to match for a worked example to be admitted to the test suite, no canonical CF table — neither in Wolowski’s 1852 essay, nor in Bellet’s 1854 *Guide de l’emprunteur*, nor in Josseau’s 1872 *Traité du crédit foncier*, nor in the U.S. Senate Document 214 of 1913 (*Agricultural Cooperation in Europe*, which reproduces CF testimony but only bond-side schedules) — currently passes validation. The structural pattern recurs: when an institution’s contract embeds servicing fees or taxes into the *annuité* itself, a pure interest+principal model cannot reproduce it. The library now exposes that structure through `fee_per_period`; see §10 below.

4. American building & loan associations (1831–1934)

The American mortgage market evolved on a wholly different substrate. The **Oxford Provident Building Association**, founded in Frankford, Pennsylvania in **January 1831**, is conventionally recognized as the first U.S. building and loan (B&L) association (Bodfish 1931; Rose and Snowden 2013). By 1893 the U.S. Commissioner of Labor’s *Ninth Annual Report* (Wright 1894) documented several thousand such associations nationwide, with significant concentrations in Pennsylvania, Ohio, and the upper Midwest. Dayton, Ohio in particular was a leading center: the **Mutual Home and Savings Association** of Dayton, chartered 1873, was for several decades among the largest B&Ls in the country (Crew 1909).

The crucial fact for mortgagemath is that the dominant 19th- century B&L lending scheme was *not* direct-reduction amortization. It was **share-accumulation** (Rose and Snowden 2013):

Under the share-account sinking-fund plan, the \$3,000 loaned to the borrower technically remains outstanding in full during the entire period of the loan, to be paid off in full at maturity. ... The borrower agrees to purchase \$3,000 worth of stock whose par value is \$100, by additional payments of \$15 each month. Thus, each monthly payment, covering both interest and the payment on shares, totals \$30. (Federal Home Loan Bank Board 1935, p. 188)

Each monthly remittance was bisected: half (the “interest installment”) went to service the unchanging principal, the other half (the “dues” or “payment on shares”) accumulated in a share account that earned dividends until it grew to equal the loan, at which point the loan and the share account were canceled against each other. The borrower’s *principal balance* never declined

month-to-month; what declined was the *gap* between the constant principal and the growing share account. This is a fundamentally different bookkeeping structure from the modern direct-reduction loan, and *mortgagemath* does not model it.

There were also several intermediate schemes — the *premium plan*, the *cancel-and-endorse* or *drop-share* plan, the *permanent plan*, the *serial plan* — each a different compromise between the share-accumulation tradition and the direct-reduction logic emerging in the 1880s. Rose and Snowden (Rose and Snowden 2013) document that direct-reduction lending began appearing within the B&L industry **in the 1880s** and reached “moderate use by the 1920s,” but remained a minority practice until the New Deal.

A point of historiographic confusion deserves comment. **S. Rufus Jones’s 1896 paper “The Dayton Plan”** (Jones 1896), delivered at the Fourth Annual Convention of the U.S. League of Local Building and Loan Associations, gave its name to a withdrawable-share *permanent* B&L variant — itself a share-accumulation product, not direct-reduction. Some later twentieth-century literature retrofits “Dayton plan” onto the direct-reduction loan, which Jones’s 1896 usage does not support. Care is required when the term is used in modern secondary sources without specifying the era.

5. The New Deal turning point (1933–1935)

The Depression-era foreclosure crisis exposed the fragility of the share-accumulation contract: borrowers who had paid in for years discovered that, under the *Pennsylvania rule* (followed in 40 of 49 jurisdictions), their share payments were not credits on the loan but capital subscriptions sharing pro rata in the association’s losses. A nominally-near-paid-off loan could be declared in default and the borrower’s accumulated share balance extinguished alongside the institution. The *Federal Home Loan Bank Review* of March 1935 (Federal Home Loan Bank Board 1935, pp. 197–198) is explicit:

The use of the direct-reduction loan eliminates a grave risk to which the borrower under the share-account sinking-fund plan in most States is subject. This risk is the loss of all his payments on his home in the event that the building and loan association from which he has borrowed becomes insolvent.

The federal response unfolded across three pieces of legislation: the **Home Owners’ Loan Corporation** (HOLC), authorized by the Home Owners’ Loan Act of 1933, which refinanced more than a million distressed mortgages on level-payment direct-reduction terms; the **Federal Housing Administration** (FHA), created by the National Housing Act of 1934, which insured long-amortizing mortgages issued by approved lenders on standardized terms; and the **Federal Savings and Loan Insurance Corporation** and the *federal* savings-and-loan charter, which required participating associations to use the direct-reduction plan (Federal Home Loan Bank Board 1935, p. 187):

... the direct-reduction plan has been made compulsory for Federal savings and loan associations.

The same FHLBB *Review* article publishes parallel schedules for a typical \$3,000, 6%-per-annum loan under all four contemporaneous plans (Serial Sinking-Fund, Drop-Share, Direct-Reduction Plan A with monthly interest credit, and Direct-Reduction Plan B with semiannual interest credit). These are the earliest published direct-reduction schedules from a U.S. federal authority. Yet they are *not* in the *mortgagemath* test suite — for a historically illuminating reason. The 1935 *Review* designed all four schedules around a fixed payment of *exactly \$30 per month* (precisely 1% of the original principal), with the loan term allowed to run to a non-integer number of months and a small final-payment trueup absorbing the residual. Plan A retires in “138 monthly payments of \$30 each with an additional 139th payment of \$29.27” (Federal Home Loan Bank Board 1935, p. 196). The library, by contrast, follows the modern closed-form convention: the borrower chooses an integer term and the level payment is *derived* — not chosen — from P , r , and n . For a 139-month, 6%, \$3,000 loan the closed-form payment is \$29.9964..., which any cents rounding mode lifts to \$30.00, and the resulting trueup at month 139 is \$29.35 — not \$29.27.

The eight-cent gap is a residue of a historical convention that has since vanished from American practice: the 1935 lender chose the payment first and let the term and trueup follow; the modern lender chooses the term first and derives the level payment. *mortgagemath* reproduces the modern convention, so the 1935 *Review* schedule, despite being available in full at cents-level precision, sits just outside the policy.

6. The post-war standardization (1944–1968)

The **Servicemen’s Readjustment Act of 1944** (“GI Bill”) extended FHA-style direct-reduction lending to veterans through the VA loan-guarantee program, completing the transition: by 1950 the level-payment, fully-amortizing, 25-to-30-year residential mortgage was the *de facto* American standard (Green and Wachter 2005). The chartering of **Fannie Mae** as a federal agency

in 1938, and its conversion to a private shareholder-owned secondary-market entity in 1968 by the Charter Act amendments, institutionalized the convention nationally; Freddie Mac (1970) extended it to thrift-originated loans. The closed-form payment formula — what `mortgagemath` evaluates in `periodic_payment(loan)` — became the canonical arithmetic of American residential housing finance.

The library’s worked examples drawn from this regulatory regime are: post-1968 standardized regime:

- **CFPB H-25(B)** Closing Disclosure form sample, promulgated under the 2014 TRID rule integrating TILA and RESPA disclosures. (\$162,000 / 3.875% / 30 yr.)
- **Reg Z Sample H-14** at 12 CFR Part 1026 Appendix H, specifying the disclosure of a Variable-Rate Mortgage Sample. (\$10,000 / 17.41% initial / 30 yr 1/1 ARM, with 1-year CMT index, 3 pp margin, 2 pp annual cap, 5 pp lifetime cap; the schedule traces 1982–1996 historical 1-year CMT.)
- **Fannie Mae Multifamily Guide §1103** Tier 2 SARM amortization. (\$25 M / 5.5% / 10 yr term on 30 yr amortization basis, Actual/360.)

7. Adjustable-rate mortgages (1981–1996)

The **Garn-St. Germain Depository Institutions Act of 1982** — acting through implementing OCC and FHLBB regulations — for the first time permitted federally-chartered banks and thrifts to originate adjustable-rate mortgages, which had previously been restricted by Regulation Q-era rate ceilings. By 1985 ARMs were roughly 60% of new originations (Green and Wachter 2005). The canonical published worked example of an ARM is **Reg Z Sample H-14**, which traces a \$10,000-per-unit 1/1 ARM through the actual 1-year CMT history from 1982 through 1996 — the period in which the U.S. ARM market matured. The Reg Z H-14 trajectory is in the library as the `regz_apph_h14_arm_10k_1741_360mo` worked example, and the library reproduces every published value (initial rate, post-cap rate at each annual adjustment, recast monthly payment, year-end remaining balance) to the cent under the library’s `RateChange` API.

A related innovation was the **payment-capped ARM with optional negative amortization**, in which the periodic payment is bounded above by a fixed multiplicative factor on the prior period’s payment (typically 1.075, i.e. a 7.5% annual cap) regardless of where the recast payment would otherwise land. When the cap binds and accrued interest exceeds the capped payment, the unpaid interest is capitalized into the principal balance and the loan *grows*. The library models this through the optional `payment_cap_factor` parameter on `RateChange`; the canonical worked example is the ProEducate ARM payment-cap example (\$65,000 at 10% rising to 12%, 7.5% annual cap, with \$420.90 of explicit cumulative negative amortization in year 2) (ProEducate 2014).

8. International conventions

The American closed-form, monthly-compounded, monthly-payment mortgage is one regional convention among several. The library supports the major contemporary alternatives:

Canada — semi-annual compounding (*Interest Act* §6). Section 6 of the federal *Interest Act* (R.S.C., 1985, c. I-15, derived from §3 of the original 1880 statute) requires that interest on residential mortgages be quoted as compounded *no more frequently than semi-annually* (Parliament of Canada 1880). A “5% / 30 yr” Canadian mortgage and a “5% / 30 yr” American mortgage therefore have meaningfully different periodic rates and payments. The library’s `Compounding.SEMI_ANNUAL` mode handles this; the worked worked examples are drawn from Olivier’s *Business Math* (Olivier 2021, the Chans first term and renewal) and eCampus Ontario’s *Mathematics of Finance* §4.4.1 (the quarterly-payment \$297,500 first term and renewal).

France — direct CF descendants. The *Crédit Foncier de France* itself was wound down in 2013, but the institutional model — a long-term, fixed-rate, fee-loaded *annuité* — persists in French residential mortgage practice. Modern French residential *tableaux d’amortissement* in actual *offres de prêt* (the borrower-facing PDF that lenders are statutorily required to provide) are row-level cents-precision documents that the library reproduces directly; v0.7.0’s `fee_per_period` field covers the *assurance emprunteur* loading that is the dominant fee component of a modern French *échéance*. The MoneyVox fixture validates that modern shape against a full row-level published table. Public bank educational webpages (*Crédit Agricole*, *Meilleurtaux*, *ANIL*, *service-public.fr*) typically show only year-aggregate totals without a per-month table, and the rounding convention behind those aggregates is not documented; several different row-level conventions can produce identical year sums, so those pages are not fixture-grade sources. Historical *Crédit Foncier* validation still awaits retrieval of a row-level CF table.

Australia and the Australasian colonial period. Carl Pinschof’s 1892 paper “The Credit Foncier System,” delivered in Melbourne and reported in *The Argus* of 18 November 1892, advocated the establishment of a state-owned mortgage bank on the

CF model. Victoria adopted a qualified version with the *Credit Foncier Act 1896* (Vic.), and the term *credit foncier* persisted in Australian mortgage-banking parlance for decades thereafter (Pinschof 1892).

Germany — Pfandbrief-funded fixed-rate mortgages. The *Hypothekenbanken* of the German tradition issued covered mortgage bonds (Pfandbriefe) against pools of long-dated fixed-rate amortizing mortgages, a structure that influenced both the Crédit Foncier’s *obligations foncières* and, indirectly, the American GSE-issued mortgage-backed securities of the 1970s and later (Kohn 1999).

United Kingdom — building societies and endowment mortgages. The British **Benefit Building Societies Act 1836** (6 & 7 Will. IV c. 32) provided the statutory frame for what would become the U.K.’s dominant retail mortgage lender through most of the twentieth century, with successor legislation in 1874, 1894, and 1986. Where American B&Ls converged toward direct-reduction in the 1930s, U.K. building societies continued to offer interest-only loans paired with separate share-accumulation savings vehicles — the *endowment mortgage* — as a major product class through the 1990s. The library models the level-payment direct-reduction case, which by the 1990s had also become the building-society default.

9. What mortgagemath does and does not model

The library’s algorithmic core is the closed-form annuity-certain of De Witt, Halley, and Simpson, evaluated under the rounding, day-count, and balance-tracking conventions adopted in modern American (and selectively non-American) regulatory and lender practice:

- The closed-form payment derived from (`principal`, `annual_rate`, `term_months`), with optional non-monthly compounding (Canadian *Interest Act* §6, actuarial effective-annual) and non-monthly payment frequency (semi-monthly through annual);
- Round-each-balance schedule construction (the U.S. lender convention) or carry-precision (the textbook / Excel default);
- The three principal cents-rounding modes (`ROUND_UP`, `ROUND_HALF_UP`, `ROUND_HALF_EVEN`);
- 30/360 and Actual/360 day counts;
- Multi-rate ARM schedules with optional payment caps and capitalized negative amortization.

Three classes of historically-important contracts are deliberately *not* modeled, and the reasons are themselves part of the history:

1. **Share-accumulation B&L loans** (the dominant U.S. structure pre-1934). The library’s principal balance declines each period; the share-accumulation principal does not. Modeling the share account as a parallel ledger would require an entirely separate data structure that has no application to any post-1934 loan.

An earlier-flagged exclusion has since been closed:

- **Given-payment, find-term contracts** like the FHLBB *Review 1935 Plan A* schedule. The 1935 convention chose payment first and let the term and final-payment trueup follow. v0.6.0’s `LoanParams.payment_override` field reproduces this cell-for-cell against the FHLBB Plan A fixture (\$3,000 / 6% / \$30 monthly; 138 full payments + 139th of \$29.27).
- **Loaded-annuity contracts** where a fixed fee rides on top of principal and interest. The `LoanParams.fee_per_period` field reproduces the modern French MoneyVox schedule including *assurance emprunteur*. Historical Crédit Foncier tables still need a retrievable row-level source before they can become fixtures.

The shipped scope now covers the post-1934 American direct-reduction convention plus the Skinner 1913 piano single-anchor pre-New-Deal exception, the FHLBB 1935 given-payment convention (via `payment_override`), the French flat-fee convention (via `fee_per_period`), the Canadian *Interest Act* §6 semi-annual convention, and the SOA Exam FM annual schedules. Within that window the library reproduces 41 published worked-example schedules cell-for-cell; on the small number of historical sources that themselves carry an internal arithmetic typo (notably two rows of the Geltner CRE Ch 20 schedule), the divergent rows are documented in the fixture notes field rather than forced into the corpus.

10. Flat per-period fees

The Crédit Foncier source gap from §3 remains consequential for 19th-century European mortgage practice. The structural problem is straightforward: CF rows publish $\text{annuité} = \text{interest} + \text{amortissement} + \text{frais} + \text{réserve} + \text{impôt}$, with the last three components rolled into a single flat-amount loading on top of the actuarially-pure interest-plus-principal value.

The library now models the fixed-loading case with optional `fee_per_period` on `LoanParams` and a corresponding fee field on the `Installment` record:

```

@dataclass(frozen=True)
class LoanParams:
    # ... all existing fields ...
    fee_per_period: Decimal = Decimal("0") # flat amount added to each
                                           # period's published payment

@dataclass(frozen=True)
class Installment:
    number: int
    payment: Decimal      # = interest + principal + fee (gross annuity)
    interest: Decimal
    principal: Decimal
    fee: Decimal          # default 0.00; non-zero only for fee-loaded loans
    balance: Decimal

```

Semantics:

- `periodic_payment(loan)` continues to return the *actuarially-pure* P+I closed-form value. The `fee_per_period` is added by `amortization_schedule` to each `Installment.payment` after the schedule is built, so the principal/interest split is unchanged.
- `Installment.fee` exposes the loading cleanly, so consumers can recover either the gross annuity (the published figure) or the stripped P+I (what the underlying interest rate accrues against) as needed.
- The fee is treated as a *per-period flat amount in the loan's currency*, not a percentage. Sources that publish the loading as a percentage of original principal (CF: 0.50 fr. per 100 fr. per year) require a one-line preprocessing step: `fee_per_period = original_principal * fee_pct / payments_per_year`.

The trigger source for shipping the feature is the MoneyVox French schedule: €10,000 at 5% over 12 months with a published €2.92 monthly *assurance emprunteur* column and gross mensualité of €858.99 for rows 1-11 and €859.04 on the final trueup row. For Wolowski's 1852 $4\frac{1}{2}\%$ / 39-year canonical example the math works out cleanly: the closed-form payment at 4.50% annual over 39 years on 100 fr. of principal is 5.485 fr. \approx 5.50 fr., and a fixed 0.50-fr. annual fee lifts the published *annuité* to 6.00 fr., matching the source. Whether the full Wolowski schedule (or any other CF schedule) reproduces row-by-row at that rate is an empirical question we can answer only by reading the source itself — and the row-level CF tables in Bellet's 1854 *Guide* and Josseau's 1872 *Traité* are currently behind JavaScript walls on Gallica and Trove that have resisted automated retrieval.

The same field also accepts several non-French-mortgage examples (e.g. early-twentieth-century Australian *credit foncier*-style state-bank schedules) where a fixed periodic loading rides on top of the closed-form payment, once a verifiable row-level source surfaces. Variable-fee structures (commission-de-gestion proportional to outstanding balance, or stepped-fee schedules) are left for a later extension if any specific verifiable source motivates them.

Bibliography

Note

The full machine-checkable bibliography of validated worked-example sources is generated automatically from the [source] blocks of `tests/schedules/*.toml`; see the *Validation* vignette for the data-driven version. The references below cover the historical literature this vignette draws on.

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