

An Introduction to String Theory

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ABSTRACT: This set of notes is based on the course “Introduction to String Theory” which was taught by Prof. Kostas Skenderis in the spring of 2009 at the University of Amsterdam. We have also drawn on some ideas from the books *String Theory and M-Theory* (Becker, Becker and Schwarz), *Introduction to String Theory* (Polchinski), *String Theory in a Nutshell* (McMahon) and *Superstring Theory* (Green, Schwarz and Witten), along with the lecture notes of David Tong, sometimes word-for-word.

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