

MicroGridsPy – Mathematical Formulation

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1 Introduction

MicroGridsPy Planning is a bottom-up, open-source optimization tool developed for the planning of energy systems in remote and underserved areas. The model is implemented in **Linopy**, a Python-based optimization framework, and provides an open and transparent approach to the sizing and dispatch of mini-grids. MicroGridsPy explicitly addresses the challenges of energy system scaling, technology selection, and operational planning in contexts characterized by limited data availability, high uncertainty, and strong economic constraints.

1.1 An Integrated Modelling Platform

MicroGridsPy is part of a broader **integrated modelling platform** designed to address the multiple, interrelated challenges of rural electrification within the theoretical framework of the **Comprehensive Energy System Planning (CESP)** methodology. The platform is structured as a collection of interoperable, open-source Python tools, each targeting a specific stage of the electrification planning process.

Ideally, the workflow begins with the design of the **distribution grid topology**, where different network configurations are explored using detailed GIS data. Through customizable heuristics and user-defined settings, the platform supports the identification of users to be connected to the grid and those better served by stand-alone systems, ensuring both **electrical feasibility** (e.g. acceptable losses and voltage drops) and **financial viability** (e.g. distribution infrastructure costs).

Once the set of connected users is defined, a dedicated **load demand assessment tool** is available, featuring the **RAMP** framework for high-resolution demand simulation. This component explicitly captures **stochasticity**, **seasonality**, and long-term demand evolution. For residential users, built-in rural archetypes are provided, allowing demand estimation based on user tiers, climatic zones, and latitude in cases where detailed appliance-level data are unavailable.

A further module is dedicated to **resource assessment**, leveraging the **PVGIS API** to automatically retrieve solar irradiation and wind speed time series when required. After completing demand and resource characterization, the **MicroGridsPy optimization model** is used to perform planning analyses, yielding optimal system sizing and the **least-cost energy mix** for the selected case study. The model supports multiple mini-grid configurations and advanced modelling features, including the explicit representation of uncertainty.

Finally, the platform includes a **dispatch simulation tool** to evaluate alternative operational strategies (e.g. load-following, cycle-charging). This allows refinement of planning results by accounting for **component degradation**, inverter-level control strategies, and realistic operational constraints. In addition, a **power flow analysis tool** enables the assessment of congestion and energy flows in the distribution network once generation and demand nodes are defined, further improving the realism and robustness of the analysis.

1.2 Reference Energy System

The reference energy system considered within MicroGridsPy primarily integrates **renewable energy technologies**, an **energy storage system** and **backup generation systems**.

The renewable energy component is designed allowing a generic renewable technology to be modeled using general parameters. This approach enables the system to represent a variety of renewable sources, such as photovoltaic (PV) panels, wind turbines (including power curve modeling), hydro power, or any renewable source based on production time series data and basic cost parameters.

The energy storage system is represented as a single battery bank, modeled with specific parameters related to the state of charge (SOC), depth of discharge (DoD), and efficiency. This battery bank can store excess energy produced by renewable sources and discharge it to meet the electric load when renewable generation is insufficient.

Finally, the system includes a backup generation component, which is designed to produce energy in a flexible manner. This backup system can represent various generation technologies, such as diesel generators, biomass generators, or any other dispatchable source capable of meeting the load when renewable energy and storage are inadequate.

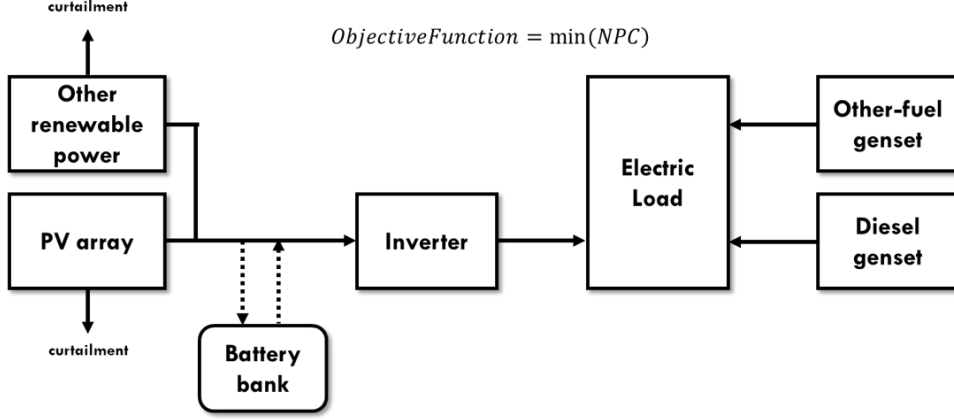


Figure 1: Reference energy system scheme

This modular representation allows the model to capture the technical and economic interactions between generation, storage, and dispatch decisions in off-grid and weak-grid contexts.

2 Planning Modes

The **MicroGridsPy planning tool** supports two complementary modelling modes, designed to address different levels of temporal complexity and data availability:

- **Multi-year planning mode** (dynamic formulation),
- **Typical-year planning mode** (steady-state formulation).

Both modes are **investment-oriented** and rely on an **annuity-based cost formulation**, making them suitable for long-term, multi-scenario techno-economic planning of mini-grid systems. In both cases, the model minimizes the **expected total system cost**, which includes annuitized investment costs (with capital recovery), operating costs, and optional externalities.

The optimization is performed at **hourly time resolution** and can simultaneously account for multiple **scenarios** representing alternative realizations of time-dependent parameters such as electricity demand, renewable resource availability, or grid conditions. Each scenario is assigned a probability reflecting its likelihood of occurrence, resulting in a **single-stage stochastic optimization problem**. System sizing decisions are shared across all scenarios, ensuring a robust design capable of performing effectively under diverse future conditions, while operational decisions are optimized separately for each scenario to minimize expected costs.

Both planning modes can represent either a **fully off-grid (isolated)** system or a **weakly grid-connected** system. Grid interaction is governed by a **grid availability matrix**, which defines whether electricity can be imported from or exported to the national grid at each time step and year of the project. When enabled, the system can exchange electricity with the grid subject to technical limits, pricing assumptions, and reliability constraints.

2.1 Multi-Year Planning Mode

The **multi-year planning mode** represents the most comprehensive formulation implemented in **MicroGridsPy**. It is defined over a multi-year planning horizon $y = 1, \dots, H$, where both system decisions

and exogenous parameters may evolve over time. Time-dependent inputs such as electricity demand, renewable resource availability, prices, and grid conditions are explicitly indexed by year and scenario.

The model is formulated as a **single-stage stochastic capacity expansion problem** and it captures three key effects:

- **Monotone capacity expansion:** investments are phased across predefined planning steps, allowing the system to grow over time while enforcing non-decreasing installed capacity and explicitly modeling technology roll-out and replacement cycles.
- **Intertemporal economic valuation:** all system costs are evaluated in present value terms using a **dual-rate logic**. Capital recovery is computed using technology-specific financial discount rates ($WACC_j$), reflecting opportunity costs of capital, while system-level intertemporal discounting is performed using a **social discount rate** r_s .
- **Residual value of long-lived assets:** an economically consistent **salvage value** is included for assets whose technical lifetime exceeds the model horizon. This avoids short-horizon bias and ensures neutrality between early and late investments in durable, capital-intensive technologies.

2.1.1 Objective Function

The objective of the multi-year planning mode is to minimize the **expected Net Present Welfare Cost (NPWC)** of the system over the planning horizon. The NPWC accounts for long-term investment decisions, system operation, and external effects, all evaluated in present-value terms.

Formally, the objective function can be expressed as:

$$\min \sum_{y=1}^H \frac{\sum_j \text{Annuity}_{y,j} + \sum_{\omega \in \Omega} p_{\omega} \sum_j (\text{OPEX}_{y,j,\omega} + A_{y,j,\omega}^{\text{ext}})}{(1 + r_s)^y} - \sum_j \frac{\text{SV}_j}{(1 + r_s)^H} \quad (1)$$

where $\omega \in \Omega$ denotes the set of scenarios, each associated with a probability p_{ω} .

2.1.2 Economic Interpretation

The multi-year planning formulation is grounded in standard principles of energy system economics and welfare-based planning. All monetary quantities entering the objective function are expressed in **real (inflation-free) terms**, measured in today's money. This implies that all prices and costs are net of general inflation and that intertemporal discounting is performed using real discount rates. As a result, economic values occurring at different points in time are directly comparable and unaffected by changes in the nominal price level.

Investment costs are represented using an **annuity-based accounting framework** combined with an explicit representation of **investment steps**. In the multi-year formulation, capacity expansion is not restricted to the initial year but can occur at discrete investment steps τ , each corresponding to a specific commissioning year within the planning horizon. Each investment step therefore defines a distinct **investment cohort**, characterized by its installation time, technical lifetime, and financial parameters.

Rather than charging the full capital expenditure at the time of installation, each investment cohort is converted into an equivalent stream of constant annual payments over its technical lifetime. These annuities are activated starting from the year in which the investment occurs and contribute to system costs only for the years in which the asset is operational. This structure allows the model to represent phased investments, delayed capacity deployment, and replacement cycles in a consistent economic framework.

For a generic investment cohort of technology j , with present investment cost $I_{j,\tau}$, technical lifetime LT_j , and weighted average cost of capital $WACC_j$, the corresponding annualized investment cost (annuity) is defined as:

$$\text{Annuity}_{j,\tau} = I_{j,\tau} \cdot \text{CRF}_j \quad (2)$$

where the **capital recovery factor (CRF)** is given by:

$$\text{CRF}_j = \frac{WACC_j (1 + WACC_j)^{LT_j}}{(1 + WACC_j)^{LT_j} - 1} \quad (3)$$

The total annuity paid in a given model year is obtained by summing the contributions of all investment cohorts that are still within their technical lifetime. This ensures that capital costs are accounted for only when the corresponding assets are available to the system.

This formulation reflects a standard financing logic in which each capital outlay is recovered over time through fixed annual payments that cover both depreciation and the opportunity cost of capital. The **Weighted Average Cost of Capital (WACC)** represents this opportunity cost and is defined as:

$$\text{WACC}_j = \frac{E_j}{E_j + D_j} K_j^E + \frac{D_j}{E_j + D_j} K_j^D (1 - T) \quad (4)$$

where E_j and D_j are the equity and debt shares, K_j^E and K_j^D are the costs of equity and debt, and T is the corporate tax rate.

Note

The annuity-based formulation implicitly assumes that capital repayment is evenly spread over the **entire technical lifetime** of each technology, implying that the economic lifetime used for capital recovery coincides with the technical lifetime. This corresponds to a fully amortized investment with constant annual payments over the asset's operational life. In the current implementation, no distinction is made between technical and economic lifetime, and financing is assumed to extend until the end of technical operation. This modelling choice ensures internal consistency and limits the number of free parameters in the planning problem. However, the formulation can be naturally extended in future developments to allow users to specify an explicit **economic lifetime** or repayment period for each technology, independent of the technical lifetime.

Note

In the current formulation, the weighted average cost of capital is assumed constant over time for each technology. However, the cohort-based investment structure allows, in principle, the specification of time-dependent WACC values. This extension can be used to represent changes in financing conditions, risk profiles, or policy support over the project lifetime.

Although this annuity-based structure mirrors project finance and private investment appraisal, it is not restricted to a private-investor perspective. In MicroGridsPy, the WACC is treated as an explicit and configurable parameter, allowing the model to represent different financing environments, including concessional finance, public-sector investments, or policy-driven de-risking mechanisms. Moreover, by explicitly incorporating externalities into the objective function, the formulation naturally extends to **social cost-benefit analysis** and welfare-oriented planning.

Intertemporal evaluation of system costs follows a **dual-rate discounting logic**. While capital recovery for individual technologies is governed by their respective WACC values, all system-level cash flows entering the objective function—including investment annuities, operational costs, and externalities—are discounted to present value using a **social discount rate**. This separation allows the model to distinguish between financial opportunity costs at the asset level and societal time preferences at the system level, which is particularly relevant in long-term energy planning contexts involving intergenerational trade-offs.

In welfare-based analysis, the social discount rate r_s is commonly defined following the Ramsey formulation:

$$r_s = \rho + \eta g \quad (5)$$

where ρ denotes the pure rate of time preference, η the elasticity of marginal utility of consumption, and g the expected long-term growth rate of per-capita consumption. This expression highlights the normative nature of social discounting, which reflects societal preferences over time rather than market-based financing conditions.

Finally, the formulation includes an **economically consistent salvage value** for assets whose technical lifetime exceeds the planning horizon. By crediting the residual value of these assets at the end of the horizon, the model avoids end-of-horizon bias that would otherwise penalize long-lived, capital-intensive technologies. Economic consistency implies that the annualized cost of an asset is invariant with respect to the chosen project horizon: evaluating an investment over a truncated horizon with salvage

yields the same annual cost as evaluating it over its full lifetime. This property ensures neutrality between early and late investments and supports efficient long-term system design.

Formally, for an investment cohort of technology j commissioned in year τ , the salvage value is defined as:

$$SV_{j,\tau} = \begin{cases} \text{InvPresent}_{j,\tau} \cdot \frac{(1 + \text{WACC}_j)^{LT_j} - (1 + \text{WACC}_j)^{H-\tau}}{(1 + \text{WACC}_j)^{LT_j} - 1}, & H - \tau < LT_j, \\ 0, & H - \tau \geq LT_j. \end{cases} \quad (6)$$

This expression credits the fraction of unrecovered capital associated with the remaining technical lifetime of the asset beyond the model horizon, discounted consistently using the technology-specific WACC_j .

2.2 Typical-Year Planning Mode

The **typical-year planning mode** represents a tractable steady-state approximation of long-term system economics. In this formulation, the system is described by a single representative operating year that is assumed to repeat identically over time. As a result, the objective function reduces to the minimization of the **expected annual (welfare) cost** of the system.

This formulation is particularly suitable for analyses where the system is assumed to have already reached a long-term equilibrium and where key time-varying parameters, most notably electricity demand, are not expected to evolve significantly over time.

The typical-year formulation can be interpreted as a collapsed version of the multi-year model under the assumption of **single-year operation**: load demand, resource availability, technology performance, and operating conditions are assumed to be identical in every year. Inter-annual variability, demand growth, technology learning, and degradation effects are neglected.

Under this assumption, the annuity-based investment cost becomes mathematically equivalent to an **infinite discounted sequence of identical replacements**. Rather than modeling a finite planning horizon with explicit reinvestments and salvage values, the system is implicitly assumed to operate indefinitely in a steady-state regime, where each asset is replaced by an identical one at the end of its technical lifetime.

2.2.1 Objective Function

The objective of the typical-year planning mode is to minimize the **expected equivalent annual cost (EAC)** of the system. Investment decisions are shared across all scenarios, while operational decisions and costs are scenario-specific.

Formally, the objective function is expressed as:

$$\min \sum_j [\text{CAPEX}_j \cdot \text{CRF}_j(\text{WACC}_j, LT_j)] + \sum_{\omega \in \Omega} p_\omega \sum_j (\text{OPEX}_{j,\omega} + \text{FuelCost}_{j,\omega} + A_{j,\omega}^{\text{ext}}) \quad (7)$$

where the first term represents the annualized investment cost and the second term represents the expected annual operating cost and externalities across scenarios.

Note

In contrast to the multi-year formulation, **intertemporal discounting does not affect system sizing** in the typical-year model. This is because all costs are evaluated on an annual basis and the system is assumed to operate indefinitely under stationary conditions. The use of annuity-based capital recovery already embeds the effect of discounting at the asset level through the WACC. As a result, the choice of discount rate does not influence relative investment decisions beyond its role in determining the annualized capital cost of each technology. System sizing is therefore driven exclusively by the trade-off between annualized investment costs and expected annual operating costs.

The typical-year planning mode provides a computationally efficient alternative to the dynamic formulation and is particularly well suited for:

- steady-state or mature systems,

- early-stage feasibility studies,
- comparative technology assessments,
- stochastic analyses with multiple scenarios, where computational tractability is a priority.

While this formulation neglects long-term dynamics, it retains the core economic structure of the full model and can be interpreted as its steady-state limit under time-invariant conditions.

3 Cost Accounting

This section describes how investment and operation costs are computed for each technology in MicroGridsPy, consistently with the objective functions introduced in the previous sections. The formulation follows a bottom-up cost accounting approach, where all cost components are explicitly parameterized and mapped to decision variables.

The same conceptual structure is adopted in both planning modes; however, the dimensionality of costs differs between the **typical-year (steady-state)** and **multi-year (dynamic)** formulations.

For each technology j , total system cost is decomposed into three main components:

- **Annualized investment costs**, including fixed operation and maintenance;
- **Variable operational costs and revenues**, dependent on dispatch decisions;
- **Externalities**, such as emissions and unserved energy penalties.

Investment-related costs depend exclusively on capacity sizing decisions and are therefore **scenario-independent**, while operational costs and externalities depend on scenario-specific dispatch variables.

3.1 Investment Cost

Investment costs are annualized using a CRF-based annuity formulation. For each technology j , installed capacity is defined as unit-based technologies (kW or kWh):

$$C_j = N_j \cdot P_j, \quad (8)$$

where N_j is the number of installed units and P_j the nominal capacity per unit. The annualized investment cost is then computed as:

$$\text{INV}_j^{\text{ann}} = C_j \cdot \left[\text{CRF}_j \cdot \text{CAPEX}_j^{\text{eff}} + \text{CAPEX}_j \cdot \text{FOM}_j \right] \quad (9)$$

where:

- $\text{CAPEX}_j^{\text{eff}} = (1 - g_j) \text{CAPEX}_j$ accounts for possible investment grants g_j ,
- FOM_j is the fixed O&M cost expressed as a fraction of CAPEX,
- CRF_j is the capital recovery factor defined from WACC_j and the technical lifetime.

In the **typical-year formulation**, investment costs are computed once and interpreted as steady-state annual costs. In the **multi-year formulation**, investment costs are defined for each investment step and translated into year-dependent annuities, allowing phased expansion and replacements.

3.2 Operational Costs

Operational costs depend on dispatch variables and are computed at hourly resolution, then aggregated at annual level. For each scenario ω , the following components are considered:

- **Fuel costs:**

$$\text{FuelCost}_\omega = \sum_{t,g} f_{t,\omega,g} \cdot c_g^{\text{fuel}} \quad (10)$$

- **Grid interaction costs and revenues** (if enabled):

$$\text{GridCost}_\omega = \sum_t \left(e_{t,\omega}^{\text{imp}} \cdot c_{t,\omega}^{\text{imp}} - e_{t,\omega}^{\text{exp}} \cdot c_{t,\omega}^{\text{exp}} \right) \quad (11)$$

- **Renewable production subsidies:**

$$\text{Subsidy}_\omega = \sum_{t,r} p_{t,\omega,r}^{\text{ren}} \cdot s_r \quad (12)$$

- **Lost-load penalties:**

$$\text{LLCost}_\omega = \sum_t \ell_{t,\omega} \cdot c^{\text{LL}} \quad (13)$$

The expected annual operational cost is obtained as the probability-weighted sum across scenarios.

3.3 Externalities

Externalities are accounted for explicitly and include both operational and embodied emissions. For each scenario ω :

$$\text{EXT}_\omega = c^{\text{CO}_2} \left(\sum_{t,g} f_{t,\omega,g} \cdot \epsilon_g^{\text{fuel}} + \sum_j \frac{C_j \cdot \epsilon_j^{\text{emb}}}{LT_j} \right) \quad (14)$$

where:

- the first term represents direct operational emissions,
- the second term represents annualized embodied emissions from installed capacity.

Note

The two planning modes differ in how time is represented, but not in the underlying economic logic. In the **typical-year formulation**, investment costs are annualized over the technical lifetime, yielding steady-state annual equivalents that are independent of the chosen time horizon. In the **multi-year formulation**, time is modeled explicitly. Investment costs, operational costs, and externalities are all resolved on a year-by-year basis and discounted to present value. Despite these differences, both formulations rely on the same bottom-up cost structure and parameterization, ensuring conceptual consistency and allowing the typical-year model to be interpreted as the steady-state limit of the dynamic formulation.

4 Renewable Technologies Characterization

Renewable energy technologies in MicroGridsPy are modeled using a generic and technology-agnostic formulation based on installed capacity, resource availability, and conversion efficiency. This approach allows the representation of different renewable sources, such as photovoltaic systems, wind turbines, or hydropower plants, using a common mathematical structure driven by time-series inputs and a limited set of techno-economic parameters.

4.1 Renewable Production Constraint

Renewable electricity production is limited by the installed capacity and by the availability of the primary energy resource. For each time period t , scenario ω , and renewable technology r , production is constrained as:

$$E_{t,\omega,r} \leq A_{t,\omega,r} \cdot \eta_r \cdot P_r \cdot N_r \quad (15)$$

where:

- $E_{t,\omega,r}$ is the renewable electricity production,
- $A_{t,\omega,r}$ is the normalized resource availability time series (e.g. solar irradiation, wind capacity factor),
- η_r is the inverter or conversion efficiency,
- $P_r \cdot N_r$ represents the installed nominal capacity.

This constraint enforces that renewable generation cannot exceed the maximum power output allowed by both installed capacity and resource availability at each time step.

4.2 Maximum Installable Capacity

An optional upper bound can be imposed on the maximum installable capacity of each renewable technology to reflect physical, spatial, or regulatory constraints. When enabled, the following constraint applies:

$$N_r \cdot P_r \leq \overline{C}_r \quad (16)$$

where $\overline{C}_r$ denotes the maximum allowable installed capacity for technology r .

5 Battery Storage Characterization

MicroGridsPy models the battery energy storage system as a single aggregated battery bank, parameterized through an installed energy capacity and operational constraints on charging, discharging, and state of charge (SOC). The battery is represented through three groups of constraints: **flow constraints**, **SOC dynamics**, and **capacity bounds**. The mathematical structure is identical in both planning modes; however, in the multi-year formulation parameters (e.g. costs, lifetime assumptions, or availability) can be indexed by year if required.

Let $t \in \mathcal{T}$ denote the set of time periods (typically hourly), $\omega \in \Omega$ the scenario set, and let the installed battery energy capacity be:

$$C^{\text{bat}} = N^{\text{bat}} \cdot \overline{E}^{\text{unit}} \quad (17)$$

where N^{bat} is the number of battery units and $\overline{E}^{\text{unit}}$ is the nominal energy capacity per unit (kWh).

Battery operation is described by charging power $P_{t,\omega}^{\text{ch}}$, discharging power $P_{t,\omega}^{\text{dis}}$, and stored energy (SOC) $\text{SOC}_{t,\omega}$, all expressed in consistent energy units over the time step (for hourly resolution, kW and kWh can be used interchangeably through $\Delta t = 1$).

5.1 Battery Flow Constraints

Charging and discharging power are bounded using **time-to-full** parameters, which define the maximum admissible charge/discharge rate as a fraction of installed energy capacity:

$$P_{t,\omega}^{\text{ch}} \leq \frac{C^{\text{bat}}}{t_{\text{ch}}} \quad \forall t, \omega \quad (18)$$

$$P_{t,\omega}^{\text{dis}} \leq \frac{C^{\text{bat}}}{t_{\text{dis}}} \quad \forall t, \omega \quad (19)$$

where t_{ch} and t_{dis} are the charging and discharging times (hours) required to fully charge or discharge the battery at maximum rate.

5.2 Battery State of Charge

The SOC dynamics are modeled with a standard energy balance including charging and discharging efficiencies. For an hourly time step ($\Delta t = 1$ h), SOC evolves as:

$$\text{SOC}_{0,\omega} = \text{SOC}_0 \cdot C^{\text{bat}} + \eta_c P_{0,\omega}^{\text{ch}} - \frac{P_{0,\omega}^{\text{dis}}}{\eta_d} \quad \forall \omega \quad (20)$$

$$\text{SOC}_{t,\omega} = \text{SOC}_{t-1,\omega} + \eta_c P_{t,\omega}^{\text{ch}} - \frac{P_{t,\omega}^{\text{dis}}}{\eta_d} \quad \forall t = 1, \dots, |\mathcal{T}| - 1, \forall \omega \quad (21)$$

where:

- η_c and η_d are charging and discharging efficiencies,
- $\text{SOC}_0 \in [0, 1]$ is the initial SOC fraction.

A cyclic boundary condition is enforced to avoid end-of-horizon artifacts and ensure consistent operation across repeated years:

$$\text{SOC}_{|\mathcal{T}|-1,\omega} = \text{SOC}_0 \cdot C^{\text{bat}} \quad \forall \omega \quad (22)$$

5.3 Battery Capacity Constraints

The SOC is bounded between a minimum and maximum admissible stored energy, based on the usable capacity and depth-of-discharge limit:

$$\text{SOC}_{t,\omega} \leq C^{\text{bat}} \quad \forall t, \omega \quad (23)$$

$$\text{SOC}_{t,\omega} \geq (1 - \text{DoD}) C^{\text{bat}} \quad \forall t, \omega \quad (24)$$

where $\text{DoD} \in [0, 1]$ denotes the maximum depth of discharge.

Note

In the current MicroGridsPy planning formulation, battery degradation is treated implicitly through a **calendar-lifetime assumption**: the battery technical lifetime (in years) drives the annualized investment cost (via CRF) and replacement logic in the multi-year formulation. In other words, degradation does not directly constrain operation, and the battery maximum usable capacity is assumed constant during its lifetime.

A natural extension, still compatible with linear optimization, is to model **exogenous capacity fade** by introducing a time-dependent usable capacity multiplier $\alpha_y \in (0, 1]$, so that the effective capacity becomes $C_y^{\text{bat}} = \alpha_y C^{\text{bat}}$. In the multi-year formulation this enables linear constraints such as:

$$\text{SOC}_{t,\omega,y} \leq \alpha_y C^{\text{bat}}, \quad \text{SOC}_{t,\omega,y} \geq (1 - \text{DoD}) \alpha_y C^{\text{bat}},$$

and similarly for charging/discharging limits in (18)–(19). The trajectory α_y can be provided as an input series derived from empirical calendar-aging models, manufacturer data, or scenario assumptions.

Furthermore, temperature-driven derating can be incorporated in the same manner by defining α_y (or a higher-resolution $\alpha_{t,y}$) as a deterministic function of ambient temperature (or battery temperature) profiles. This preserves linearity as long as the fade/derating factors are treated as exogenous parameters, while allowing users to capture first-order impacts of operating environment on usable storage capacity.

6 Backup Generator and Fuel Characterization

Dispatchable backup generation is modeled through one or more generator technologies $g \in \mathcal{G}$. Each generator is represented by a unit-based sizing variable and an hourly production variable. Fuel consumption is modeled explicitly and linked to electrical production through either a nominal-efficiency relationship or a partial-load efficiency curve.

6.1 Installed Capacity and Production Limit

Generators are sized in discrete units N_g , each with nominal electrical capacity P_g (kW). The total installed generator capacity is:

$$C_g = N_g \cdot P_g \quad (25)$$

Hourly generator production is bounded by installed capacity:

$$E_{t,\omega,g}^{\text{gen}} \leq N_g \cdot P_g \quad \forall t, \omega, g \quad (26)$$

6.2 Fuel–Power Relationship (Nominal Efficiency)

Fuel consumption is expressed in volumetric units (e.g. liters), and is linked to electrical production through the fuel lower heating value (LHV) and an efficiency factor.

For generators without partial-load modeling, the relationship is enforced as an equality using a nominal conversion efficiency η_g^{nom} :

$$E_{t,\omega,g}^{\text{gen}} = F_{t,\omega,g} \cdot \text{LHV}_g \cdot \eta_g^{\text{nom}} \quad \forall t, \omega, g \in \mathcal{G}^{\text{noPL}} \quad (27)$$

where $F_{t,\omega,g}$ is the fuel consumption and LHV_g is the energy content per unit of fuel.

6.3 Partial-Load Efficiency and Piecewise Linear Approximation

When partial-load modeling is enabled, generator efficiency becomes output-dependent. MicroGridsPy implements a **convex piecewise linear lower bound** on fuel consumption as a function of electrical production. This ensures a linear formulation while capturing the increase in specific fuel consumption at low load.

Let $r \in [0, 1]$ denote the generator loading fraction (relative output), and let a set of breakpoints $\{r_p\}_{p=0}^P$ be provided together with the corresponding efficiencies $\eta_{g,p}$. The electrical power at each breakpoint is:

$$p_{g,p} = P_g \cdot r_p \quad (28)$$

and the corresponding fuel consumption per unit is:

$$f_{g,p} = \frac{p_{g,p}}{\eta_{g,p} \text{LHV}_g} \quad (29)$$

For each segment k connecting consecutive breakpoints $(p_{g,k}, f_{g,k})$ and $(p_{g,k+1}, f_{g,k+1})$, the segment slope is:

$$m_{g,k} = \frac{f_{g,k+1} - f_{g,k}}{p_{g,k+1} - p_{g,k}} \quad (30)$$

Fuel consumption is then constrained to lie above all segment lines (epigraph form), scaled by the number of installed units:

$$F_{t,\omega,g} \geq m_{g,k} (E_{t,\omega,g}^{\text{gen}} - N_g p_{g,k}) + N_g f_{g,k} \quad \forall t, \omega, g \in \mathcal{G}^{\text{PL}}, \forall k \quad (31)$$

This construction yields a convex piecewise linear approximation of the true nonlinear fuel curve and guarantees that fuel consumption is not underestimated. In practice, it enables realistic modeling of part-load performance while preserving linearity and tractability of the optimization problem.

Note

The partial-load formulation adopted in MicroGridsPy is valid under the assumption that the generator fuel consumption curve is **convex** with respect to electrical output. In practical terms, this implies that marginal fuel consumption increases as the generator operates at lower load levels, a behavior that is consistent with most internal combustion engines and small-scale diesel generators commonly used in mini-grid applications.

The current formulation is deliberately minimal to maintain tractability. However, several consistent extensions are possible in future developments:

- **Mixed-integer unit commitment:** introducing binary on/off variables would allow explicit modeling of minimum load, start-up costs, and non-convex efficiency regions, at the cost of increased computational complexity.
- **Piecewise efficiency with minimum load:** a lower bound on generator output (e.g. $E^{\text{gen}} \geq \alpha P_g N_g$) could be added to reflect technical operating limits without altering the convex structure.
- **Technology-specific degradation:** efficiency parameters $\eta_{g,p}$ could be made time-dependent to reflect aging or maintenance conditions, remaining compatible with the piecewise linear structure.

These extensions preserve the conceptual separation between **capacity planning** and **operational realism**, allowing MicroGridsPy to progressively increase fidelity while retaining a transparent and modular economic structure.

7 Grid Connection and Availability

MicroGridsPy can represent a **weakly grid-connected** mini-grid in addition to the fully isolated off-grid configuration. When grid connection is enabled, the system can import electricity from the national grid and, optionally, export surplus electricity back to it. Grid interaction is constrained by (i) the physical

exchange capacity of the interconnection line and (ii) the **time-dependent grid availability matrix**, which captures outages and reliability of the upstream grid.

Let $E_{t,\omega,y}^{\text{imp}}$ and $E_{t,\omega,y}^{\text{exp}}$ denote imported and exported electricity at time t , scenario ω , and year y . Grid exchange is subject to the following parameters:

- $A_{t,y}^{\text{grid}} \in \{0, 1\}$: grid availability (1 if grid is available, 0 if outage),
- $\bar{P}^{\text{grid}}$: maximum active power exchange capacity of the interconnection line (kW),
- $\eta^{\text{imp}}, \eta^{\text{exp}}$: import/export efficiencies accounting for transformer/conversion losses.

In typical-year mode, the year index y is omitted and the availability matrix reduces to a single representative year A_t^{grid} .

7.1 Grid Import and Export Capacity

When on-grid operation is enabled, grid imports are bounded by the product of availability, line capacity, and efficiency:

$$E_{t,\omega,y}^{\text{imp}} \leq A_{t,y}^{\text{grid}} \bar{P}^{\text{grid}} \eta^{\text{imp}} \quad \forall t, \omega, y \quad (32)$$

If export is enabled, a symmetric constraint is imposed for exports:

$$E_{t,\omega,y}^{\text{exp}} \leq A_{t,y}^{\text{grid}} \bar{P}^{\text{grid}} \eta^{\text{exp}} \quad \forall t, \omega, y \quad (33)$$

These constraints enforce that grid exchange is possible only when the upstream grid is available and within the rated capacity of the interconnection.

7.2 Grid Availability Simulation

For realistic modelling of weak-grid contexts, MicroGridsPy supports the generation of an exogenous **grid availability matrix** $A_{t,y}^{\text{grid}}$ over the entire project horizon. The availability matrix is stored as a CSV file with:

- one column per project year $y = 1, \dots, H$,
- one row per time period $t \in \mathcal{T}$ (e.g. 8760 rows for hourly resolution),
- binary entries $A_{t,y}^{\text{grid}} \in \{0, 1\}$ indicating whether the grid is available.

The simulation logic proceeds as follows:

1. **Pre-connection years.** If the grid is assumed to become available only after a connection year, all years prior to the first connection year are set to zero availability:

$$A_{t,y}^{\text{grid}} = 0 \quad \forall t, y < y_{\text{conn}}.$$

2. **Ideal grid special case.** If the average number of outages per year or the average outage duration is set to zero, the grid is treated as fully reliable after connection:

$$A_{t,y}^{\text{grid}} = 1 \quad \forall t, y \geq y_{\text{conn}}.$$

3. **Outage event generation.** Otherwise, an outage process is simulated by generating alternating sequences of:

- **time between outages** (grid available, $A^{\text{grid}} = 1$),
- **outage durations** (grid unavailable, $A^{\text{grid}} = 0$).

Both quantities are sampled from Weibull distributions to produce a realistic variability of outage patterns over time. The resulting sequence is then scaled such that, over the connected years, the expected total number of outages and total outage hours match the user-specified reliability targets (average outages per year and average outage duration).

4. **Matrix assembly.** The final binary sequence is reshaped into a $|\mathcal{T}| \times (H - y_{\text{conn}} + 1)$ availability matrix for the connected years and concatenated with the zero-availability pre-connection block.

This approach yields a flexible representation of grid reliability: the model can reproduce fully unavailable grids (off-grid), perfectly reliable grids, or weak grids characterized by stochastic outages while preserving full transparency of the underlying availability assumptions.

8 System Constraints

This section presents the system-level constraints that couple all technologies and ensure feasibility of supply, reliability, and policy or resource limits. Constraints are enforced at hourly resolution for each scenario, while additional aggregate constraints may apply at annual level.

8.1 Energy Balance Constraint

At each time period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$, the energy balance enforces that total net supply equals demand. Let:

- $\sum_r E_{t,\omega,r}^{\text{res}}$ be total renewable production,
- $\sum_g E_{t,\omega,g}^{\text{gen}}$ be total generator production,
- $E_{t,\omega}^{\text{imp}}$ and $E_{t,\omega}^{\text{exp}}$ be grid import/export (if enabled),
- $P_{t,\omega}^{\text{dis}}$ and $P_{t,\omega}^{\text{ch}}$ be battery discharge/charge,
- $\ell_{t,\omega}$ be lost load (unserved energy),
- $D_{t,\omega}$ be the electrical demand.

MicroGridsPy adopts the convention that storage charging and grid export act as demand-side sinks, while storage discharging and grid import act as supply-side sources. The balance is implemented as:

$$\sum_r E_{t,\omega,r}^{\text{res}} + \sum_g E_{t,\omega,g}^{\text{gen}} + E_{t,\omega}^{\text{imp}} - E_{t,\omega}^{\text{exp}} + P_{t,\omega}^{\text{dis}} - P_{t,\omega}^{\text{ch}} + \ell_{t,\omega} = D_{t,\omega} \quad \forall t, \omega \quad (34)$$

If the system is off-grid, then $E_{t,\omega}^{\text{imp}} = E_{t,\omega}^{\text{exp}} = 0$. If grid connection is enabled but export is disabled, then $E_{t,\omega}^{\text{exp}} = 0$ and only imports are allowed. In all cases, Eq. (34) retains the same structure.

8.2 Minimum Renewable Penetration

A minimum renewable penetration constraint can be enforced at system level to represent policy targets or sustainability requirements. Define:

- E^{tot} as the total annual electricity supplied to meet demand (renewables, generators, and grid imports),
- E^{ren} as the annual renewable contribution (renewable production plus any renewable share of dispatchable generation, if modeled).

The constraint is expressed as:

$$E^{\text{ren}} \geq \alpha_{\min}^{\text{ren}} E^{\text{tot}} \quad (35)$$

where $\alpha_{\min}^{\text{ren}} \in [0, 1]$ is the minimum renewable penetration fraction. In the implemented accounting, grid imports contribute to E^{tot} and are treated as non-renewable unless explicitly modeled otherwise.

8.3 Maximum Lost-Load Share

System reliability can be enforced through an upper bound on the share of demand that can remain unserved. Let total annual lost load and demand be:

$$LL^{\text{tot}} = \sum_{t,\omega} \ell_{t,\omega}, \quad E^{\text{dem}} = \sum_{t,\omega} D_{t,\omega} \quad (36)$$

The constraint implemented in MicroGridsPy is:

$$LL^{\text{tot}} \leq \alpha_{\max}^{\text{LL}} E^{\text{dem}} \quad (37)$$

where $\alpha_{\max}^{\text{LL}} \in [0, 1]$ is the maximum admissible fraction of unmet demand.

8.4 Land Availability Constraint (Renewables)

When spatial limitations are relevant, MicroGridsPy can enforce an upper bound on the total land area required by renewable technologies. Let a_r denote the specific land requirement per installed kW for renewable technology r (m^2/kW), and let total installed renewable capacity be $C_r = N_r P_r$. The land-use constraint is:

$$\sum_{r \in \mathcal{R}} C_r a_r \leq A^{\max} \quad (38)$$

where $A^{\max}$ is the maximum land area available (m^2). This constraint is applied only when a positive land limit is specified.