Mathematical formulation

1 The integrated energy system

The integrated energy system with all the generator and prosumer agents, and their physical interactions is shown in Fig. 1. For the sake of simplicity, we only show the physical system of a single prosumer in detail, while the system of other prosumer and generator agents is represented by their electric power consumption or generation.

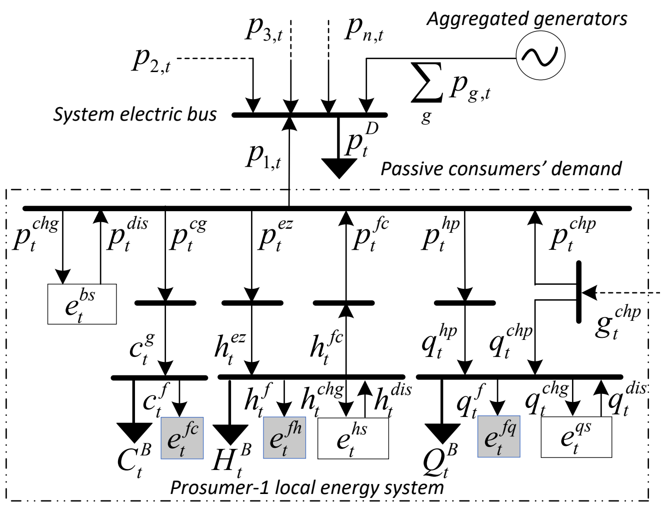

Figure 1: Integrated energy system and physical interactions between agents

2 Nomenclature

Sets

T	set of timesteps
G	set of generator agents
N	set of prosumer agents

Parameters

α_g, β_g	Generator cost coefficients [€/MWh^2], [€/MWh]
λ_t^g	Natural gas prices [€/MWh]
$\overline{A_{g,t}}$	Generator available capacity [MW]
p_t^D	Electricity demand of passive consumers [MW]
Q_t^B, H_t^B, C_t^B	Base heat, hydrogen, and cooling demands [MW]
$\underline{E_t^{fq}}, \overline{E_t^{fq}}$	Min, Max energy consumption of flexible heat demand [MWh]
$\underline{E_t^{fh}}, \overline{E_t^{fh}}$	Min, Max energy consumption of flexible hydrogen demand [MWh]
$\underline{E_t^{fc}}, \overline{E_t^{fc}}$	Min, Max energy consumption of flexible cooling demand [MWh]
$\overline{I^{hp}}, \overline{I^{chp}}$	Heat pump and CHP installed capacity [MW]
$\overline{I^{ez}}, \overline{I^{fc}}$	Electrolyzer and fuel cell installed capacity [MW]
$\overline{I^{cg}}$	Cooling generation device installed capacity [MW]
$\overline{Q^{chg}}, \underline{Q^{dis}}$	Heat storage charge and discharge capacity [MW]
$\underline{E_t^{qs}}, \overline{E_t^{qs}}$	Min, Max heat storage energy capacity [MWh]
$\overline{H^{chg}}, \underline{H^{dis}}$	Hydrogen storage charge and discharge capacity [MW]
$\underline{E_t^{hs}}, \overline{E_t^{hs}}$	Min, Max hydrogen storage energy capacity [MWh]
n_e^{chp}, n_q^{chp}	CHP electric and thermal efficiency [%]
COP^{hp}	HP coefficient of performance [-]
n^{ez}, n^{fc}	Electrolyzer and fuel cell efficiency [%]
n^{cg}	Cooling device electric efficiency [%]
n_{chg}^q, n_{dis}^q	Heat storage charge and discharge efficiency [%]
n_{chg}^h, n_{dis}^h	Hydrogen storage charge and discharge efficiency [%]

Variables

$p_{g,t}$	Electric power generation from generators [MW]
g_t^{chp}	CHP natural gas consumption [MW]
$p_{n,t}$	Electric power of prosumers [MW]
p_t^{chp}	CHP electric power generation [MW]
p_t^{fc}	Fuel cell electric power generation [MW]
p_t^{dis}	Battery discharge power [MW]
p_t^{chg}	Battery charge power [MW]
p_t^{cg}	Cooling device electric power consumption [MW]
p_t^{ez}	Electrolyzer electric power consumption [MW]
p_t^{ez}	Heat pump electric power consumption [MW]
q_t^{hp}	Heat pump heat generation [MW]
q_t^{chp}	CHP heat generation [MW]
q_t^{dis}	Heat storage discharged heat [MW]
q_t^{chg}	Heat storage charged heat [MW]

q_t^f	Flexible heat demand [MW]
e_t^{fq}	Flexible heat demand energy energy level [MWh]
e_t^{qs}	Heat storage energy energy level [MWh]
h_t^{ez}	Electrolyzer hydrogen generation [MW]
h_t^{dis}	Hydrogen storage discharged hydrogen [MW]
h_t^{chg}	Hydrogen storage charged hydrogen [MW]
h_t^{fc}	Fuel cell hydrogen consumption [MW]
h_t^f	Flexible hydrogen demand [MW]
e_t^{fh}	Flexible hydrogen demand energy energy level [MWh]
e_t^{hs}	Hydrogen storage energy energy level [MWh]
c_t^g	Cooling supply from cooling generation device [MW]
c_t^f	Flexible cooling demand [MW]
e_t^{fh}	Flexible cooling demand energy energy level [MWh]

3 Mathematical model

3.1 Objective function, generator capacity limit, and system power balance constraints

In centralized co-optimization, a central entity (the central operator) minimizes the total costs of satisfying the demand for all energy end-uses subject to system constraints.

$$\min \sum_{g,t} \Big(\alpha_g \cdot p_{g,t}^2 + \beta_g \cdot p_{g,t} \Big) + \sum_t \Big(\lambda_t^g \cdot g_t^{chp} \Big) \tag{1}$$

$$s.t. \qquad 0 \le p_{g,t} \le \overline{A_{g,t}}, \quad \forall \quad g,t \tag{2}$$

$$\sum_g p_{g,t} + \sum_n p_{n,t} \ge p_t^D \quad \forall \quad t \tag{3}$$

3.2 Prosumer agent-1: (ID=01 in the ‘generic-config-test‘)

In addition to the generator agents’ capacity limit constraints and the system electric power balance constraints, the central operator also takes into account the constraints of each prosumer’s local energy system. Each prosumer interacts with the rest of the integrated energy system through its electric power, which is negative when the prosumer has a net withdrawal from the system, and positive when it has a net injection as described in *Equation 4 and 5.*

$$s.t. \qquad p_{1,t} = \Big(p_t^{chp} + p_t^{fc} + p_t^{dis} \Big) - \Big(p_t^{chg} + p_t^{cg} + p_t^{ez} + p_t^{hp} \Big) \quad \forall \quad t \tag{4}$$

$$p_{1,t} : \textit{free} \quad \forall \quad t \tag{5}$$

In the ‘generic-config-test‘, prosumer agent-1 has demand for the following end-uses: space-heating, hydrogen generation, and cooling (could be any other end-use demand such as lighting, transport, etc.). For each of these end-uses, the following constraints are enforced:

space-heating end-use:

$$q_t^{hp} + q_t^{chp} + q_t^{dis} - q_t^{chg} \geq Q_t^B + q_t^f \quad \forall \quad t \tag{6}$$

$$e_t^{fq} = e_{t-1}^{fq} + q_t^f, \quad \forall \quad t \tag{7}$$

$$\underline{E_t^{fq}} \leq e_t^{fq} \leq \overline{E_t^{fq}}, \quad \forall \quad t \tag{8}$$

$$e_t^{qs} = e_{t-1}^{qs} + n_{chg}^q \cdot q_t^{chg} - 1/n_{dis}^q \cdot q_t^{dis}, \quad \forall \quad t \tag{9}$$

$$p_t^{chp} = n_e^{chp} \cdot g_t^{chp} \quad \forall \quad t \tag{10}$$

$$q_t^{chp} = n_q^{chp} \cdot g_t^{chp} \quad \forall \quad t \tag{11}$$

$$q_t^{hp} = COP^{hp} \cdot p_t^{hp} \quad \forall \quad t \tag{12}$$

$$0 \leq p_t^{hp} \leq \overline{I^{hp}} \quad \forall \quad t \tag{13}$$

$$0 \leq g_t^{chp} \leq \overline{I^{chp}} \quad \forall \quad t \tag{14}$$

$$0 \leq q_t^{chg} \leq \overline{Q^{chg}} \quad \forall \quad t \tag{15}$$

$$0 \leq q_t^{dis} \leq \overline{Q^{dis}} \quad \forall \quad t \tag{16}$$

$$\underline{E^{qs}} \leq e_t^{qs} \leq \overline{E^{qs}} \quad \forall \quad t \tag{17}$$

hydrogen generation end-use:

$$h_t^{ez} + h_t^{dis} - h_t^{chg} - h_t^{fc} \geq H_t^B + h_t^f \quad \forall \quad t \tag{18}$$

$$e_t^{fh} = e_{t-1}^{fh} + h_t^f, \quad \forall \quad t \tag{19}$$

$$\underline{E_t^{fh}} \leq e_t^{fh} \leq \overline{E_t^{fh}}, \quad \forall \quad t \tag{20}$$

$$e_t^{hs} = e_{t-1}^{hs} + n_{chg}^h \cdot h_t^{chg} - 1/n_{dis}^h \cdot h_t^{dis}, \quad \forall \quad t \tag{21}$$

$$p_t^{fc} = n^{fc} \cdot h_t^{fc} \quad \forall \quad t \tag{22}$$

$$h_t^{ez} = n^{ez} \cdot p_t^{ez} \quad \forall \quad t \tag{23}$$

$$0 \leq p_t^{ez} \leq \overline{I^{ez}} \quad \forall \quad t \tag{24}$$

$$0 \leq h_t^{fc} \leq \overline{I^{fc}} \quad \forall \quad t \tag{25}$$

$$0 \leq h_t^{chg} \leq \overline{H^{chg}} \quad \forall \quad t \tag{26}$$

$$0 \leq h_t^{dis} \leq \overline{H^{dis}} \quad \forall \quad t \tag{27}$$

$$\underline{E^{hs}} \leq e_t^{hs} \leq \overline{E^{hs}} \quad \forall \quad t \tag{28}$$

cooling (or lighting or any other) end-use:

$$c_t^g \geq C_t^B + c_t^f \quad \forall \quad t \tag{29}$$

$$e_t^{fc} = e_{t-1}^{fc} + c_t^f, \quad \forall \quad t \tag{30}$$

$$\underline{E_t^{fc}} \leq e_t^{fc} \leq \overline{E_t^{fc}}, \quad \forall \quad t \tag{31}$$

$$c_t^g = n^{cg} \cdot p_t^{cg} \quad \forall \quad t \tag{32}$$

$$0 \leq p_t^{cg} \leq \overline{I^{cg}} \quad \forall \quad t \tag{33}$$