Orthogonal Transformations

- spatial relationships in 3D

Def – An $n \times n$ matrix A is an orthogonal transformation I.F.F (if and only if)

- It has n mutually perpendicular rows or columns with unit length
  - $\perp$ rows must be independent (can't be multiples of each other)

    ex $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ → linearly dependent

    $\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ → independent but not $\perp$
  - to be perpendicular, the dot product must be 0

dot product: $x \cdot y = \sum_{i=1}^{n} x_i y_i$

$x \cdot y = 0 \longleftrightarrow x \perp y$ (perp.)

  - rows/colums must have unit length

    ↳ $\|x\| = \sqrt{\sum_i x_i^2} = \sqrt{x \cdot x}$

- The rows or columns of A form an orthonormal basis of $\mathbb{R}^n$
  - basic for space – set of vectors that can combine to create any vector in a space
  - basically first point with more words

- $AA^T = A^T A = I$ ⇝ transpose $[\ ]^T$ switches the rows and columns
- $A^{-1} = A^T$

  ex: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

* more about transpose on next page