Adjusted Geopotential Height from Orogeny or Glaciation

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July 6, 2017

We can compute the geopotential height relative to the Earth's radius at sea level:

$$\Phi = \int g(z)\,\mathrm{d}z \tag{1}$$

$$z = R_\oplus + h \tag{2}$$

$$\Phi = GM \int_0^h \frac{\mathrm{d}r}{(R_\oplus + r)^2} \tag{3}$$

$$= -GM \left(\frac{1}{R_\oplus + r} \right) \Bigg|_0^h$$

$$= GM \left(\frac{1}{R_\oplus} - \frac{1}{R_\oplus + h} \right)$$

$$\Phi_0 = \frac{GM}{R_\oplus} \left(\frac{h}{R_\oplus + h} \right) \tag{4}$$

We thus consider a perturbed geopotential, such that instead of h we have $h + \delta z$. For simplicity, we'll express this using a Taylor series expansion.

$$\Phi' = \frac{GM}{R_\oplus} \left(\frac{h + \delta z}{R_\oplus + h + \delta z} \right) \tag{5}$$

$$\approx \Phi_0 + g_0\,\delta z - \frac{g}{R_\oplus + h}\,\delta z^2 \tag{6}$$

We can get h by rearranging Equation 4 to get

$$h = \frac{R^2\Phi_0}{GM - R\Phi_0}. \tag{7}$$