Study Design Reviewer Feedback

The manuscript presents a rigorous theoretical framework utilizing principal component analysis to determine optimal intervention strategies in network games. While the mathematical decomposition provides elegant insights into the relationship between strategic complements/substitutes and network structure, I recommend addressing specific concerns regarding the sensitivity of the conclusions to the underlying objective functions and the robustness of the findings when applied to empirical network structures.

Comments

1. **Sensitivity of the "Simple Intervention" Conclusion to the Welfare Function:** The manuscript relies heavily on "Property A," where the planner's objective (aggregate equilibrium utility) is proportional to the sum of squared actions. While the authors note this facilitates analysis, they claim it is "not essential." However, in many standard policy contexts (e.g., crime reduction, education, vaccination), a utilitarian planner is primarily concerned with the *sum* of actions (or the mean), rather than the variance or sum of squares. The Online Appendix (Corollary OA1) suggests that when maximizing the sum of actions with linear costs, the optimal strategy shifts toward targeting specific individuals rather than a distribution proportional to an eigenvector. The authors should explicitly discuss in the main text whether the "Simple Intervention" logic (proportionality to the first/last principal component) holds if the objective function is linear in actions. If the qualitative nature of the optimal policy changes from a distributed intervention (eigenvector-based) to a focused intervention (key-player based) under different standard welfare definitions, the claim that Property A is "not essential" may be logically overstating the universality of the principal component characterization.

2. **Robustness to Network Measurement Error:** The model assumes the planner possesses perfect information regarding the adjacency matrix G. In practical applications, social and economic networks are often measured with significant noise (missing links or spurious links). The logic of targeting specific eigenvectors—particularly the last principal component in the case of strategic substitutes—relies on precise knowledge of the local network topology to minimize crowding out. I recommend the authors discuss or demonstrate (perhaps via a robustness check or simulation) how the efficacy of the optimal intervention degrades if the planner targets the eigenvectors of an *observed* network that differs slightly from the *true* network. Does the "Simple Intervention" remain robust, or does it become counter-productive if the spectral structure is sensitive to measurement error?

3. **Empirical Relevance of Spectral Gaps:** Proposition 2 establishes that the sufficiency of "simple interventions" depends on the spectral gap (for complements) or the bottom gap (for substitutes). While the theoretical bound is sound, the paper would benefit significantly from evidence regarding whether real-world economic networks actually exhibit the necessary gaps to make simple interventions viable at moderate budgets. I suggest applying the spectral gap condition to a set of standard empirical networks (e.g., co-authorship networks, trade networks, or village social networks) to determine if the "simple intervention" approximation is likely to hold in realistic settings, or if the gaps are typically small enough to require complex, multi-component interventions.

4. **Feasibility of Negative Interventions in Strategic Substitutes:** The analysis for strategic substitutes suggests that the optimal intervention (aligned with the last principal component) often involves "taxing" or reducing the incentives of certain agents to minimize crowding out. In many of the cited applications (e.g., R&D investment, public goods provision), the planner may be constrained to non-negative interventions (subsidies only). The authors should clarify the logical implications of a non-negativity constraint on the intervention vector y^*. Does the logic of targeting the last principal component break down if the planner cannot reduce the standalone marginal benefits of the "crowding" nodes?

5. **Cost Function Convexity and Solution Structure:** The derivation of the optimal intervention vector y^* as proportional to network eigenvectors is driven by the quadratic cost function. If the costs of intervention were linear (as discussed briefly in the appendix), the solution would likely be a "bang-bang" corner solution rather than an interior solution proportional to a vector. The main text should more clearly delineate that the "shape" of the optimal intervention is a direct consequence of the assumed convexity in costs. This is a critical boundary condition for the theory: if a policy maker faces linear costs (e.g., a fixed price per unit of subsidy), the principal component logic may not apply in the way presented.

Reproducibility Reviewer Feedback

The manuscript presents a rigorous theoretical framework using principal component analysis to determine optimal intervention strategies in network games. While the mathematical derivation is elegant, there are significant concerns regarding the robustness of the spectral decomposition to measurement error and the reproducibility of the numerical illustrations that must be addressed to meet the standards of a high-impact publication.

Comments

1. **Sensitivity of Policy to Network Measurement Error:** The central finding relies on the precise identification of the principal components (eigenvectors) of the adjacency matrix $\boldsymbol{G}$. In practical applications, network data is rarely observed without error; links are often estimated with statistical noise. It is a known result in spectral graph theory (e.g., Davis-Kahan $\sin\theta$ theorem) that eigenvectors can be highly unstable under perturbation, particularly when the spectral gap (the difference between eigenvalues) is small.

    - The authors discuss the spectral gap in the context of convergence to "simple" interventions (Proposition 2). However, they must also address how measurement error in $\boldsymbol{G}$ affects the *validity* of the optimal intervention $\boldsymbol{y}^*$.
    - Specifically, if the spectral gap is small, the principal components may rotate significantly with minor noise, potentially rendering a targeted intervention ineffective or counter-productive. The authors should provide a sensitivity analysis or a theoretical bound describing how robust the optimal intervention is to perturbations in $\boldsymbol{G}$.

2. **Reproducibility of Numerical Illustrations:** Figures 1, 2, and 3 present numerical results based on specific network topologies (circle, hubs, etc.) and specific parameter sets. To ensure the reproducibility of these findings:

    - The authors should provide the code (e.g., MATLAB, Python, R) used to generate these figures and the specific adjacency matrices used in the examples.
    - The manuscript should explicitly state the exact numerical values for all parameters used in these figures (e.g., the specific entries of the status quo vector $\hat{\boldsymbol{b}}$ used in Figure 2), as visual representation alone is insufficient for replication.

3. **Handling of Non-Generic Spectra (Repeated Eigenvalues):** Assumption 2 posits that all eigenvalues of $\boldsymbol{G}$ are distinct. While the authors state this holds "generically," many networks of theoretical and economic interest (such as regular lattices, complete bipartite graphs, or block models often used to stylize community structures) possess high symmetry and thus repeated eigenvalues.

    - The authors should clarify how the principal component decomposition and the resulting policy recommendations change when the eigenspace has dimension > 1. Does the choice of basis vector within the repeated eigenspace matter for the cost or the welfare outcome? This is critical for the theoretical robustness of the method when applied to idealized network structures.

4. **Tightness of Bounds in Proposition 2:** Proposition 2 provides a sufficient condition on the budget C for the optimal intervention to be well-approximated by a simple intervention. The bound depends on the status quo incentives $||\hat{\boldsymbol{b}}||^2$ and the spectral gap terms.

    - The authors should address whether this bound is tight. If the bound is very loose, its practical utility for a planner determining whether a simple intervention is sufficient is limited.
    - I recommend including a numerical assessment (potentially added to Figure 3) comparing the theoretical bound derived in Proposition 2 against the actual convergence rate observed in the simulations to demonstrate the tightness (or lack thereof) of the derived inequality.

5. **Clarification of "Variance Control" in Section 5:** In Section 5 (Incomplete Information), the planner is modeled as choosing a random variable $\mathcal{B}$ with a cost associated with the covariance matrix.

    - The interpretation of this mechanism requires more rigor. In standard policy contexts, a planner sets rules or taxes (first moment shifts) or perhaps insurance schemes (reducing variance). It is unclear how a planner practically "chooses" the covariance structure of shocks $\Sigma_{\mathcal{B}}$ as a direct policy instrument, particularly creating specific correlation structures aligned with network eigenvectors.
    - The authors should clarify if this is a reduced-form approximation for a specific economic mechanism or a purely mathematical extension. If the latter, the language should be tempered to reflect that this is a theoretical upper bound on welfare under variance control, rather than a readily implementable policy prescription.

Limitations & Context Reviewer Feedback

This manuscript offers a rigorous and novel theoretical framework for targeting interventions in networks using principal component analysis, providing clear insights into the dichotomy between strategic complements and substitutes.

Comments

1. **Contextualization of "Simple" Interventions:** The manuscript defines "simple" interventions (Proposition 2) as those proportional to a single principal component (e.g., eigenvector centrality for complements). While mathematically elegant, this definition of "simplicity" assumes the planner has the computational capacity and data to calculate the spectral decomposition of the full network G. In the context of practical policy implementation, "simple" often implies heuristic-based targeting (e.g., targeting based on degree centrality or threshold rules). It would be valuable for the authors to contextualize their findings against these common heuristics. Does the "optimal simple" intervention significantly outperform a degree-based heuristic in networks with specific spectral properties? Discussing this would better situate the theoretical results within the literature on practical network seeding and targeting.

2. **Economic Interpretation of the Last Principal Component:** The result regarding strategic substitutes targeting the last principal component (associated with the smallest eigenvalue) is a strong theoretical contribution. However, the economic interpretation of this intervention requires further elaboration in the discussion. The text notes that this component captures local structure and bipartiteness, often requiring interventions with opposite signs on adjacent nodes. The authors should discuss the practical limitations or feasibility of implementing such "anti-correlated" interventions in real-world socioeconomic contexts, where political or ethical constraints might prevent a planner from simultaneously subsidizing one agent and taxing their neighbor to optimize local crowding-out effects.

3. **Limitations regarding Network Uncertainty:** Section 5 provides a thoughtful extension regarding incomplete information about agents' standalone marginal returns (b). However, a pervasive limitation in network intervention literature is uncertainty regarding the network structure (G) itself (e.g., missing links or measurement error). Given that the optimal intervention relies heavily on the spectral properties of G (specifically the spectral gap and specific eigenvectors), the authors should discuss how robust these interventions might be to noise or mis-specification of the adjacency matrix. If the planner observes a perturbed G, does the "simple" intervention strategy remain robust? A discussion of this limitation is warranted to define the scope of applicability.

4. **Justification of Cost Function Convexity:** The main results, particularly the interior solutions that allow for the "mixing" of principal components, rely heavily on the quadratic cost function (L_2 norm). While the Online Appendix (OA3.3) addresses linear costs, the main text's narrative regarding the "profile" of interventions is driven by this convexity. The authors should explicitly acknowledge in the main text (likely in the Model or Discussion sections) that linear or fixed costs would likely lead to corner solutions (targeting a single node or a specific subset), thereby altering the "principal component mixing" intuition. This is essential for readers to understand the boundary conditions of the theoretical mechanism.

5. **Discussion of Endogeneity:** The current framework treats the network G as fixed while the planner intervenes on incentives b. The conclusion briefly mentions interventions that alter the matrix of interaction as a future direction. However, the authors should also acknowledge the limitation regarding the *endogeneity* of the network in response to incentive interventions. In many economic contexts (e.g., R&D collaborations or social public goods), changing the returns to action b_i might induce agents to form or sever links, altering G. A brief discussion on the time-horizon validity of the static network assumption would strengthen the limitations section.