Semi-analytic modeling of transient multi-layer flow with TTim

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Abstract TTim is a free code for the semi-analytic simulation of transient flow in multi-layer systems consisting of an arbitrary number of layers. No grid or time-stepping is required, nor does a closed model boundary need to be specified in any of the layers. Currently, TTim includes multi-layer wells and line-sinks, which may be used to simulate transient flow to a variety of hydrogeologic features, including wells with a skin and wellbore storage, incompletely sealed abandoned wells, streams with leaky beds, vertical faults, and horizontal wells; transient forcing needs to be represented by a step function. Other features that may be simulated include vertical anisotropy and the delayed response of the water table. Behind the scenes of TTim, the Laplace-transform analytic element method is applied. TTim is written in Python, with Python scripts used as input files. TTim has many practical applications, including the design of riverbank filtration systems, analysis of aquifer tests near surface-water bodies, design and evaluation of recirculation wells, and modeling of the transient pressure response of proposed carbon geologic sequestration projects. In addition, the short and simple input files and the one-to-one link between analytic elements and hydrogeologic features make TTim well suited for education.

Keywords Groundwater modeling · Analytic element method · Transient flow · Multi-layer

Introduction

A semi-analytic approach has been developed for the modeling of transient flow in multi-layer systems (Hemker and Maas 1987; Bakker and Kuhlman 2011; Bakker 2013). As the approach is analytic, no grid, time-stepping, or closed model boundary are needed. The approach is based on the Laplace-transform analytic element method (Furman and Neuman 2003; Kuhlman and Neuman 2009) and is implemented in the free and open-source computer code TTim (pronounced: tee-tim) (Bakker 2012). Analytic element models have been used for the simulation of steady flow at a variety of scales, from screening models to large regional models, and for a variety of problems from wellhead protection to groundwater/surface-water interaction (Strack 2003; Hunt 2006). The objective of this note is to discuss the current capabilities of the analytic element code TTim for modeling transient multi-layer flow (version 0.2). This note consists of three parts. First, the main approximations and the main features of the TTim code are presented. Second, TTim is benchmarked against both a semi-analytic solution and a numerical solution; and third, a practical application is presented to demonstrate how TTim may be used to analyze a pumping test near a river with a leaky bed.

A few alternative codes are available for the semi-analytic simulation of transient head variations in multi-layer systems; the large number of numerical codes that apply, for example, the finite difference or finite element method are not reviewed here. PhreFlow is a free program for the simulation of transient three-dimensional (3D) flow in an unconfined aquifer including partially penetrating wells and ellipsoidal inhomogeneities; elastic storage is neglected in PhreFlow (Janković and Barnes 2001). MLU is a commercial code for the simulation of transient multi-layer flow to wells. It applies the same theory as used in TTim with a few different choices in implementation. MLU has a nice Windows interface and includes automatic parameter estimation and a free two-layer version is available (Hemker and Post 2011; Carlson and Randall 2012). Cihan et al. (2011) developed a free code with a subset of the capabilities of MLU (no skin effect, no well bore storage, no user interface, and no parameter estimation) but with abandoned wells; the FORTRAN code may be obtained from the authors. AnAqSim is a commercial code for transient multi-layer modeling that includes wells, line-sinks, and inhomogeneities. AnAqSim is less analytic than TTim and MLU as the spatial distribution of the leakage between aquifers and the release from storage is approximated by radial basis functions and transient flow is approximated with time stepping. However, AnAqSim may be used to simulate variable transmissivity in unconfined aquifers, anisotropic horizontal hydraulic conductivity and, in principle, non-linear effects such as ephemeral streams (Fitts 2010; McLane 2012).

Received: 25 July 2012 / Accepted: 6 March 2013
Published online: 25 April 2013

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 DOI 10.1007/s10040-013-0975-2

Main features and approximations

TTim is developed for the simulation of transient flow in linear multi-layer systems. A solution for the head change as a function of time at any point is obtained through superposition of elementary solutions in both space and time. Each elementary solution is called an analytic element and represents a hydrogeologic feature in the aquifer system (e.g., Strack 2003; Bakker 2013). Currently (version 0.2), flow may be simulated using superposition of two types of elements: wells and line-sinks. Wells may be used to simulate flow to a variety of multi-layer wells. Line-sinks may be used to simulate flow to or through linear features such as streams, horizontal wells, and vertical faults.

TTim is applicable to systems that are linear (e.g., confined systems) or that may be approximated as linear (e.g. unconfined aquifers where the variation of the saturated thickness is limited) so that the effects of different transient events may be superimposed in time. A TTim solution represents the head change due to all transient forcing that starts at time $t=0$. Transient forcing that started before $t=0$ is not simulated, which is the same as treating flow in the aquifer at $t=0$ as steady state. The aquifer domain is infinite with the head change at infinity equal to zero in all aquifer layers. Neither the steady-state conditions at time $t=0$ nor the response of any transient activities before $t=0$ are simulated, but these can be superimposed.

Multi-layer systems

TTim is developed to simulate transient flow in an aquifer system consisting of an arbitrary number of aquifer layers and leaky layers. Both aquifer layers and leaky layers are numbered from the top of the aquifer system to the bottom; the first leaky layer is on top of the first aquifer layer. The resistance to vertical flow is neglected within aquifer layers (the Dupuit approximation) but flow is still three-dimensional (e.g., Strack 1984, 1989). Each aquifer layer is defined by a transmissivity and a storage coefficient, which are both homogeneous and constant through time. Flow in leaky layers is approximated as vertical. Each leaky layer is defined by a resistance to vertical flow and a storage coefficient, which are both homogeneous and constant through time. The resistance c_{n} [T] of leaky layer n with thickness B_{n} [L] and vertical hydraulic conductivity κ_{n} [L T^{-1}] is computed as

$$c_{\mathrm{n}} = B_{\mathrm{n}}/\kappa_{\mathrm{n}} \tag{1}$$

so that the vertical flux q_{n} [L T^{-1}] through leaky layer n may be computed as

$$q_{\mathrm{n}} = \frac{h_{\mathrm{n}} - h_{\mathrm{n-1}}}{c_{\mathrm{n}}} \tag{2}$$

where h_{n} [L] is the head in layer n. Note that some codes (e.g., MODFLOW, Harbaugh 2005) require specification of the leakance L [T^{-1}], which is the reciprocal of the resistance: $L=1/c$.

When two aquifer layers are not separated physically by a leaky layer, the leaky layer in TTim has a zero thickness and a resistance to vertical flow that is computed from the vertical hydraulic conductivities of the aquifer layers using a standard finite difference scheme (e.g., Harbaugh 2005)

$$c_{\mathrm{n}} = \frac{H_{\mathrm{n-1}}}{2k_{\mathrm{v,n-1}}} + \frac{H_{\mathrm{n}}}{2k_{\mathrm{v,n}}} \tag{3}$$

where H_{n} [L] and $k_{\mathrm{v,n}}$ [L T^{-1}] are the thickness and vertical hydraulic conductivity of aquifer layer n, respectively. The resistance between two aquifer layers may also be computed as the combined effect of the resistance to vertical flow of the aquifer layers and the leaky layer, in which case c_{n} is the sum of Eqs. (1) and (3). It is noted, however, that the vertical resistance of an aquifer is commonly much smaller than the vertical resistance of a leaky layer so that Eq. (3) is small compared to Eq. (1).

Three different boundary conditions have been implemented for the top of the aquifer system. First, no transient leakage is induced through the top of aquifer layer 1 by the elements in the model. Second, aquifer layer 1 may be bounded on top by a leaky layer, which represents a low-permeability layer on top of the first aquifer layer. Water exchange between this layer and aquifer layer 1 may occur through the release from or increase of storage in the leaky layer. Third, aquifer layer 1 is bounded on top by a leaky layer with a fixed and constant water level above it. This is commonly referred to as a semi-confined condition (e.g., Strack 1989, section 14). The fixed water level may represent a surficial aquifer with an artificially fixed water level through, for example, ditches and drains.

All aquifer layers are assumed to remain confined, as the vertical flux between aquifer layers is a function of the difference in head between the two aquifer layers (Eq. 2). When the top aquifer layer represents unconfined conditions, an average transmissivity needs to be specified. Models with such an average transmissivity often yield accurate results as compared to models where the transmissivity varies with the head (e.g., Haitjema 1995, p. 158). The delayed water-table response (delayed yield) may be simulated through the addition of a thin layer with phreatic storage above the top aquifer layer, as demonstrated in the first benchmark problem.

Wells

Well elements have been implemented to simulate four types of wells:

1. Wells with a specified discharge that is the same in each layer. In practice, this element is applied in one layer only and multi-layer wells are simulated with one of the other well types.

 DOI 10.1007/s10040-013-0975-2

2. Multi-layer wells representing wells that are screened in multiple layers for which only the total discharge is known. TTim distributes the discharge across the screens such that the head inside the well is the same in all layers, while wellbore storage may be taken into account. Observation wells that are screened in multiple layers may be simulated through specification of a zero total discharge, so that water may flow into the observation well in certain layers and out of the well in other layers.
3. Incompletely sealed abandoned wells that are screened in multiple layers with a non-zero resistance to vertical flow inside the well. These wells are similar to multi-layer wells with a zero total discharge, but the vertical flow inside the well is equal to the head difference between two screened layers inside the well multiplied by the vertical conductance C_w [L^2 T^{-1}] inside the well defined as

$$C_{\mathrm{w}} = \pi r_{\mathrm{w}}^2 / c_{\mathrm{v}} \tag{4}$$

   where c_v [T] is the resistance to vertical flow inside the well and r_w [L] is the radius of the well.
4. Wells with a specified head. The same head is used for each layer that the element is screened in. TTim computes the discharge from each aquifer layer that is needed to maintain the specified head. This well type may be used to compute the required discharge of a well for a desired drawdown or a prescribed injection pressure.

The specified discharge or head may vary stepwise through time for all well types. All well types may optionally have an entry resistance. The well discharge Q_n [L^3 T^{-1}] (positive for water flowing from the aquifer into the well) in layer n is computed as

$$Q_{\mathrm{n}} = 2\pi r_{\mathrm{w}} H_{\mathrm{n}} \frac{h_{\mathrm{n}} - h_{\mathrm{w}}}{c_{\mathrm{e}}} \tag{5}$$

where c_e [T] is the entry resistance, and h_w and h_n are the head inside the well and just outside the well in aquifer layer n, respectively. An alternative definition of the skin effect is given by, e.g., Kruseman and De Ridder (1990) who define the additional drawdown s_n [L] in layer n caused by the dimensionless skin effect parameter c_s as

$$s_{\mathrm{n}} = \frac{Q_{\mathrm{n}}}{2\pi T_{\mathrm{n}}} c_{\mathrm{s}} \tag{6}$$

where T_n [L^2 T^{-1}] is the transmissivity of layer n. For the case of a single pumping well, the skin effect parameter may be expressed in terms of the entry resistance as

$$c_{\mathrm{s}} = \frac{T_{\mathrm{n}}}{H_{\mathrm{n}} r_{\mathrm{w}}} c_{\mathrm{e}} \tag{7}$$

The simulation of wellbore storage requires the specification of the radius r_c of the caisson. The head inside the well is computed from a water balance inside the well

$$\pi r_{\mathrm{c}}^2 \frac{\mathrm{d}h_{\mathrm{w}}}{\mathrm{d}t} = \sum_{\mathrm{n}} Q_{\mathrm{n}} - Q_{\mathrm{w}} \tag{8}$$

where Q_w is the discharge of the well (positive for pumping water out of the aquifer) and Q_n is the discharge entering the well through the screen in layer n as computed with Eq. (5).

Line-sinks

Line-sinks may be used to simulate flow to or from linear features such as narrow streams, horizontal wells, or vertical faults. The term line-sink is also used when the discharge of the line-sink is negative and the line-sink acts as a source. In TTim version 0.2, the inflow is spatially uniform along a line-sink, so that each feature with a potentially spatial variation of the inflow needs to be discretized into a string of line-sinks. Line-sink strings may be used to model the following features with TTim:

1. Streams with a known stage. The inflow/outflow from each line-sink is computed such that the head is equal to the specified head (stage) at the center of the line-sink. When the stage is constant through time, the head along the line-sink is specified as zero since TTim simulates the head change due to the elements in the model. For cases of a well pumping near a stream, the time-varying discharge of the line-sink string represents the stream depletion.
2. Horizontal wells or ditches with a known total discharge. A horizontal well may be simulated through discretization of an aquifer in multiple aquifer layers and by placing a line-sink string in the layer at the depth of the horizontal well; ideally the thickness of this layer is equal to the diameter of the well. The inflow of each line-sink in the string is computed in such a fashion that the heads at the centers of the line-sinks are equal, and the sum of the discharges of all line-sinks in the string is equal to the discharge of the horizontal well. A similar approach may be used to simulate flow to a ditch that cuts through one or more aquifer layers.
3. Anisotropic linear faults with a high vertical hydraulic conductivity but low horizontal hydraulic conductivity (e.g., Anderson and Bakker 2008). The vertical fault cuts through multiple aquifer layers. Flow inside the fault is approximated as vertical. The fault may be modeled with a string of line-sinks. The inflow/outflow of each segment is computed such that the vertical flow between two layers inside the fault is equal to the head difference between the two layers inside the fault (at the center of each line-sink) multiplied by the

 DOI 10.1007/s10040-013-0975-2

vertical conductance C_{ls} [L^2 T^{-1}] inside the fault segment defined as

$$C_{ls} = Lw/c_v \tag{9}$$

where c_v [T] is the resistance to vertical flow inside the fault, and L and w are the length and width of the fault segment, respectively. The total inflow summed over the exposed layers is zero (similar to the condition for an abandoned well). When the resistance c_v is set to zero, the heads are the same in all layers inside the fault.

A resistance c_e to inflow into a line-sink may be specified for each type of line-sink string. The discharge Q_n of a line-sink screened in layer n is a function of the difference between the head h_{ls} inside the line-sink and the head h_n just outside the line-sink

$$Q_n = Lw\frac{h_n - h_{ls}}{c_e} \tag{10}$$

where L is the length of the line-sink, and w is the distance over which water enters the line-sink. This equation is applied at the center of the line-sink. A zero entry resistance means that the head in the stream is equal to the head in the aquifer. The value of w depends on the feature that is modeled. For example, when modeling a narrow stream that only penetrates the top part of the aquifer, w is the width of the stream and c_e represents the combined effect of stream bed resistance and partial penetration. When modeling the boundary of a lake or wide stream where water enters from one side only, w is the thickness of the aquifer. The specified distance w and the entry resistance c_e are perfectly negatively correlated so that during calibration the ratio w/c_e needs to be varied; this ratio is sometimes called the conductance per unit length.

TTim code

TTim is written in Python, a free and open-source computer language with simple yet powerful syntax. Together with the NumPy and SciPy packages, Python provides a ‘computational ecosystem’ for scientific computing that is quickly gaining in popularity (Oliphant 2007; Pérez et al. 2011). Graphical output is generated with the matplotlib package (Hunter 2007). Some of the computationally demanding functions in TTim are written in FORTRAN and compiled into Python extensions using f2py. Python and the appropriate packages need to be installed to run TTim; TTim installers are available for several Python versions. Free Python installers that include the required packages are available at, e.g., Pythonxy (2013) and Enthought (2013). The design of the TTim code is based on the object-oriented design developed by Bakker and Kelson (2009).

The input for a TTim model consists of a (commonly short) Python script with the definition of the aquifer system and the specification of the wells and line-sinks that represent the hydrogeologic features in the aquifer. The TTim script for the second benchmark problem is shown in the Appendix as an example. After the model is solved, the head may be evaluated at any point and at any time between the minimum and maximum times that are specified in the script. For every point in space, TTim computes an analytic element solution in the Laplace domain and uses this solution to obtain a solution at an arbitrary number of times in the time domain using the algorithm of De Hoog et al. (1982), which allows for the accurate and efficient superposition of solutions in time. This is all done behind the scenes. The only thing the user needs to do is to specify the minimum and maximum times of the simulation. (The number of terms used in the algorithm of De Hoog et al. may also be specified but the default value is sufficient for most cases.) Details of the numerical implementation are presented in Bakker and Kuhlman (2011) and Bakker (2013).

Benchmark problems

The well element in TTim has been benchmarked against a number of analytic, semi-analytic, and finite difference solutions. Louwyck et al. (2012) compare TTim to (semi)-analytic solutions and finite difference solutions (using the program MAxSym) for fully penetrating and partially penetrating wells with either a constant discharge or a discharge that is a delta function (as in a slug test). Bakker (2013) compares TTim output to an analytic solution by Neuman (1972) for flow to a constant-discharge well in an unconfined aquifer with a delayed response of the water table, and to a well with a periodic discharge in a multi-aquifer system. The implementation of line-sinks is benchmarked against a finite row of wells (a consistency check) and against results from MODFLOW (Harbaugh 2005) in Bakker (2012). In this article, two new benchmarks are presented for the implementation of line-sinks. The first concerns a comparison against a semi-analytic solution for a horizontal well and the second a comparison against MODFLOW for a problem with a well, a canal, and a fault.

Benchmark against semi-analytical solution for a horizontal well

Consider a horizontal well in an unconfined aquifer. The thickness of the aquifer is, initially, 20 m, the horizontal and vertical hydraulic conductivity are 10 and 2 m/d, respectively, the specific storage coefficient is 2×10^{-5} m^{-1}, and the specific yield of the water table is 0.2. A 40-m-long horizontal well is located along the x-axis with its center at $(x,y)=(0,0)$ and at an elevation $z=10$ m from the bottom of the aquifer. The well starts pumping at $t=0$ with a discharge of 1,000 m^3/d.

 DOI 10.1007/s10040-013-0975-2

TTim is benchmarked against the semi-analytical solution for a horizontal well in an unconfined aquifer of Zhan and Zlotnik (2002). Inflow along the horizontal well is uniform and the well has an infinitesimal radius. Following Neuman (1972), they assume that the drawdown of the water table is small compared to the initial saturated thickness of the aquifer, which means that it is appropriate to fix the transmissivity and apply the water-table condition along the top of the aquifer. They obtain a solution in the Laplace domain through numerical integration of the 3D solution for a point-sink, and a solution in the time domain through numerical inversion using the Stehfest algorithm (Stehfest 1970). The solution of Zhan and Zlotnik (2002) is computed with the program WHI (Zhan 2013).

A TTim model of the 3D head distribution is obtained by discretizing the aquifer in 11 uniform model layers with elastic storage plus a thin model layer (1 cm) with phreatic storage at the top of the model. The horizontal well is simulated with one line-sink element screened in the middle layer of the model to facilitate comparison with the solution of Zhan and Zlotnik (2002). TTim is used to compute the head variation at $(x,y)=(20,20)$ at two elevations (Fig. 1). The blue line represents the head in the middle of the aquifer, while the red line is the head at the top of the aquifer, which represents the water table. On a log-log plot, the delayed response of the water table is clearly visible in the head-response in the middle of the aquifer (blue line in Fig. 1). The TTim solution gives a very close match with the solution of Zhan and Zlotnik (dots in Fig. 1). An almost perfect match may be obtained by using more layers of uniform thickness or by using layers that are thinner near the horizontal well. A layer with a thickness equal to the diameter of the well is needed to simulate an accurate head inside the well (as was done for steady flow by Bakker et al. 2005).

TTim can also be used to approximate flow to a horizontal well with a uniform head, which is physically more realistic than a uniform inflow. For this purpose, the horizontal well is discretized in 10 line-sinks, using longer line-sinks in the middle part of the horizontal well and shorter line-sinks towards the end using the distribution proposed by Janković and Barnes (1999). The difference between uniform inflow and uniform head is negligible at $(x, y)=(20,20)$, but a difference is observed near the well. TTim head contours are shown at $t=1$ day at an elevation of 10 m in Fig. 2. Contours corresponding to a uniform head are shown in the left half of the figure, while contours for a uniform inflow are shown in the right half of the figure. The difference between uniform head and uniform inflow is relatively small for this case, but is expected to be larger for longer wells (e.g., constructed with micro tunneling) and when other features cause flow in the aquifer such as nearby wells or streams.

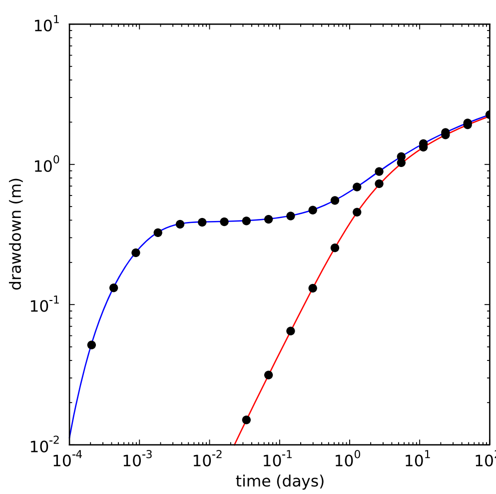

Fig. 1 Comparison of TTim results (*lines*) vs. semi-analytical solution for a horizontal well in an unconfined aquifer (*dots*) at $(x, y)=(20,20)$ at the top of the aquifer (*red*) and in the middle of the aquifer (*blue*)

Benchmark against finite-difference solution with a well, canal and fault

TTim is benchmarked against MODFLOW (Harbaugh 2005), arguably the most popular finite difference groundwater code in the world. The benchmark problem consists of a system with two aquifers, bounded on top by a leaky layer and a fixed water level (semi-confined conditions). The aquifer and leaky layer properties are given in Table 1; a cross-section is shown in Fig. 3. A pumping well is located at $(x,y)=(0,0)$ and starts pumping with a discharge $Q=500$ m^3/d at time $t=0$. On the right side of the well is a straight canal which runs from $(x,y)=(200,-410)$ to $(x,y)=(200,410)$ and is open to layer 1 (dashed black line in Fig. 4a); the head in the canal is fixed and hence the head change caused by the pumping well is zero. (The coordinates are chosen such that they fit perfectly to a uniform MODFLOW grid and no additional error is introduced due to a mismatch between geometry and grid.) On the left side of the well is a straight fault which runs from $(x,y)=(-200,-410)$ to $(x,y)=(-200,410)$ and connects layers 1 and 2 (solid black line in Fig. 4a). The resistance to vertical flow inside the fault is zero. The TTim input file is shown in the Appendix as an example. The MODFLOW model consists of 201 × 201 cells of 20 × 20 m so that the surrounding no-flow boundaries are far enough away to have an insignificant influence on the solution. The fault is

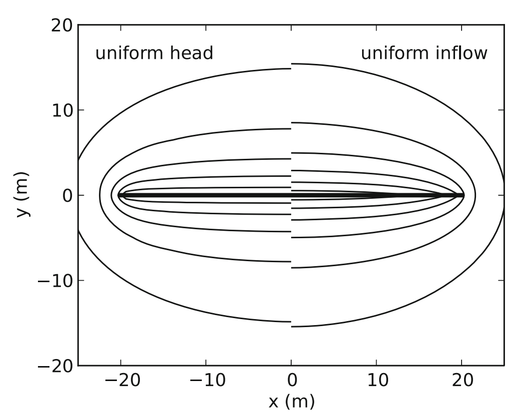

Fig. 2 TTim head contours for a horizontal well at the elevation of the well and at $t=1$ day. Uniform head (*left half*) vs. uniform inflow (*right half*) boundary condition. Contour interval is 0.5 m

 DOI 10.1007/s10040-013-0975-2

Table 1 Aquifer data used in TTim-MODFLOW benchmark

Layer	T (m^2/d)	S_s (m^{-1})	c (d)
Leaky layer 1	–	0	1,000
Aquifer 1	100	1e-4	–
Leaky layer 2	–	0	1,000
Aquifer 2	200	1e-4	–

simulated by specifying a high vertical conductance between the two layers for the 41 cells that represent the fault. A solution is obtained using a time step of 0.02 days.

Results of the TTim model and the MODFLOW model are shown in Fig. 4. Head contours at time t=0.2 day are shown for the area around the well in Fig. 4a; TTim contours are shown in the top half (red for layer 1 and blue for layer 2), while MODFLOW contours are shown in the bottom half (green for layer 1 and magenta for layer 2). Note that the heads in the top and bottom aquifers are equal along the fault. Heads along the x-axis are plotted for t=0.2 and t=2 in Fig. 4b for layer 1 and in Fig. 4c for layer 2 using the same colors as in Fig. 4a. The TTim and MODFLOW lines almost coincide, except near the well in layer 2. The drawdown near the well in the TTim model is significantly larger than in the MODFLOW model as can be expected, since the well has a radius of 0.1 m, while the cell in MODFLOW containing the well is 20 × 20 m. The MODFLOW results may be improved by reducing the cell size near the well.

The TTim solution is used to compute the variation of the discharge from the canal into the aquifer (the canal depletion) as a function of time (dashed line in Fig. 5), and the discharge from the top aquifer to the bottom aquifer through the fault (solid line in Fig. 5). The discharge through the fault reacts much quicker to the pumping well, as the fault cuts through the leaky layer separating the two aquifers, while the canal is open to aquifer 1 only so that it is separated from the bottom aquifer by a leaky layer. Steady state is approached after 10 days of pumping. At that time, the downward discharge through the fault is 26% of the discharge of the well and the discharge from the canal into the aquifer is 25.2% of the discharge of the well. The same quantities are computed from the MODFLOW head solution and are shown with dots in Fig. 5 (one dot every 5 MODFLOW time steps); the TTim and MODFLOW solutions match well.

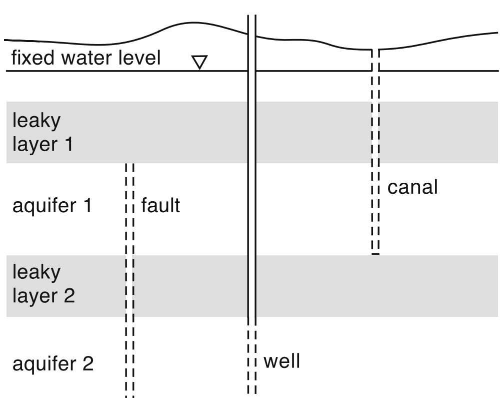

Fig. 3 Cross-section of the TTim-MODFLOW benchmark problem

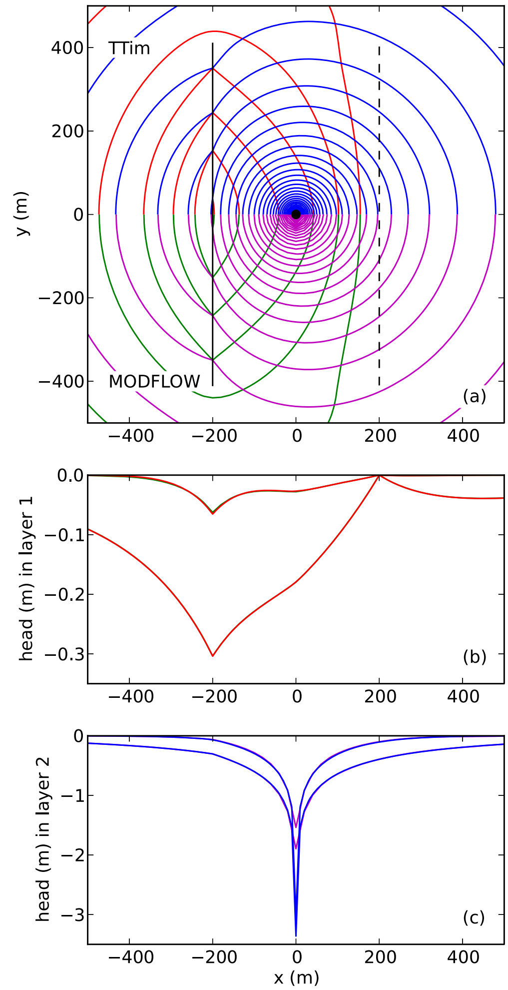

Fig. 4 TTim results (*red* for layer 1 and *blue* for layer 2) and MODFLOW results (*green* for layer 1 and *magenta* for layer 2) for the TTim–MODFLOW benchmark. **a** Head contours at t=0.2 days with a contour interval of 5 cm. Head along y=0 at t=0.2 and t=2 days in **b** layer 1 and **c** layer 2

Practical application: an aquifer test near a river

A TTim simulation of an aquifer test conducted near a river is presented as a practical example. The aquifer test was performed next to the Black River, 2 km NorthEast of Poplar Bluff, Missouri (USA), to assess the potential yield of a riverbank filtration system for water supply. A test well and five observation wells were installed at the site, as shown in Fig. 6. The boring logs indicate that the aquifer consists of sand and gravel confined on top by a 12-m-thick clay layer and below by bedrock consisting of low-permeability material. The aquifer has a fairly uniform thickness of 15 m across the well field. The test well and the observation

 DOI 10.1007/s10040-013-0975-2

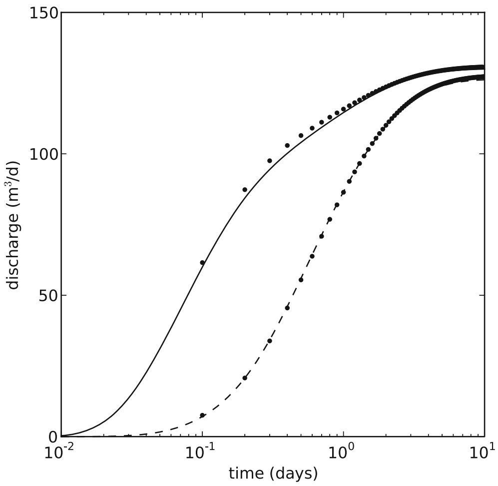

Fig. 5 TTim results for total discharge through the fault from aquifer layer 1 to aquifer layer 2 (*solid*) and total discharge from canal into the aquifer (*dashed*); MODFLOW results are shown with *dots* every 0.1 days

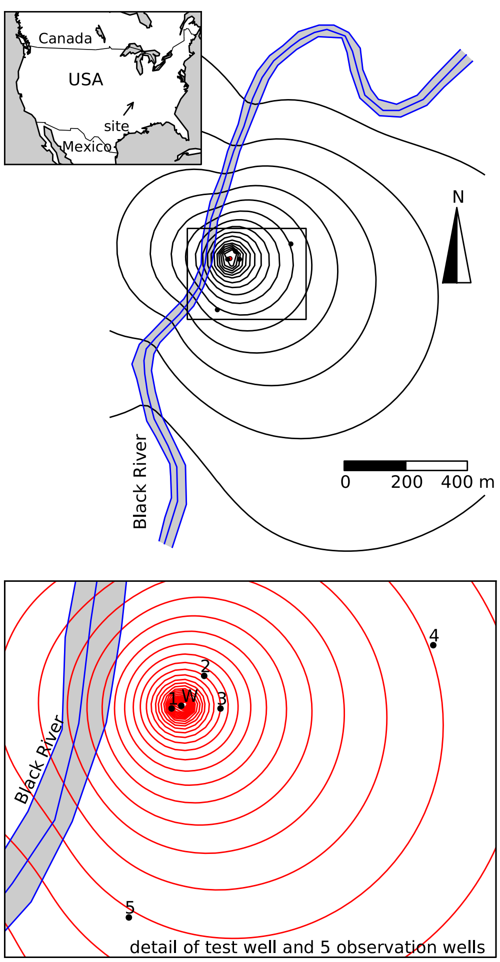

Fig. 6 Map of the test well (indicated by *w*), five observation wells (numbered *1–5*) and the river (*grey area*) for the practical application. The river is simulated by three strings of line-sinks (*blue*). Contours of the calibrated model are shown using a contour interval of 0.25 m after 3 days of pumping

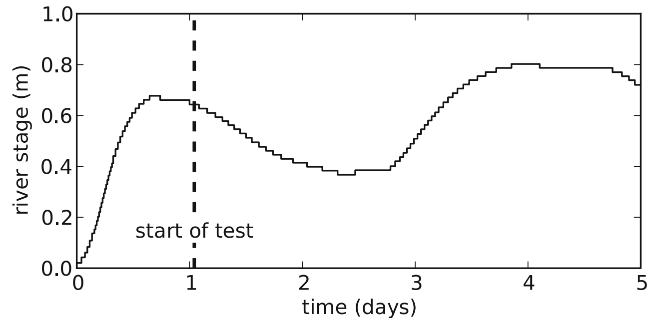

Fig. 7 River stage variation. The TTim model starts 1,500 min before the start of the test

wells are screened over the bottom 6 m of the aquifer, except for well 1, which is screened over 9 m. The test well has a diameter of 0.3 m. The river lies on top of the aquifer and cuts partly through the confining clay layer.

A 72-h constant rate aquifer test was conducted with an average pumping rate of 3,143 m^3/d. Water levels were recorded with automatic data loggers every minute in wells 1 through 4. Manual measurements of water levels were taken 11 times during the aquifer test in well 5. Prior and during the pumping test, the river stage fluctuated over 0.8 m (Fig. 7).

The TTim model of the site consists of two model layers of 9 and 6 m, respectively, and with the same aquifer properties. The two layers are used to simulate the partial penetration of the pumping well and observation wells. The pumping well and all observation wells are screened in the bottom aquifer layer. The river is approximately 36 m wide in the model area and is simulated with three strings of line-sinks with a specified head, one along each edge of the river and one along the centerline. Each string is given the same bottom resistance c_e and width w=12 m (see Eq. 10). River stage variations prior to the start of the test were taken into account by starting the TTim simulation 1,500 min before

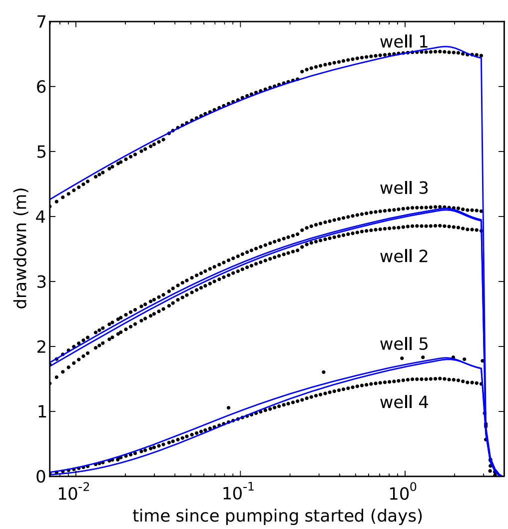

Fig. 8 Observations (*dots*) and calibrated TTim results (*lines*) for the pumping test of the practical application

DOI 10.1007/s10040-013-0975-2

the start of pumping. The specified head varies stepwise every time the river stage increased or decreased by 1.5 cm, resulting in 85 steps (see Fig. 7). During calibration, 100 observations (approximately equally spaced in log time starting 10 min after the beginning of the test) are used for wells 1–4, and all 11 observations of well 5 (the dots in Fig. 8). Simulated drawdowns at observation wells were normalized against the simulated drawdown caused by the river stage variation at the start of the test.

The TTim model is calibrated using the least squares approach. Four parameters are optimized during the calibration: the specific storage coefficient (S_s), the horizontal (k_h), and vertical (k_v) hydraulic conductivity of the aquifer, and the resistance of the streambed (c_e). The optimized parameters are k_h=24 m/d, k_v=12 m/d, S_s= 1.0 × 10^{-5} m^{-1}, and c_e=34 days. The fit between the model (solid line) and observations used for calibration (dots) is shown in Fig. 8; the match is considered reasonably good for an aquifer test. The root mean square error of the calibrated model is 14 cm.

Two issues need further attention: the areal extent of the model and the simulation of the river. TTim models extend to infinity, but only give physically realistic results to the extent where hydrogeologic features are included in the model. The extent of the drawdown caused by the pumping well at the end of pumping (3 days) is shown in Fig. 6. The drawdown does not extend significantly beyond the river segment simulated in the model; the extent of the TTim model is adequate for analyzing the aquifer test.

The river is simulated with three strings of line-sinks, which represent a discretization of the spatial variation across the river of the leakage through the riverbed. Such a coarse discretization is acceptable when the leakage does not vary significantly over the width of the river. The distance over which leakage through the streambed varies significantly may be expressed in terms of the leakage factor λ [L] defined as (e.g., Verruijt 1970; Bakker 2007)

$$\lambda = \sqrt{Tc_e} \tag{11}$$

where T is the transmissivity of the aquifer. The vertical leakage q through the stream bed reduces exponentially away from the edge of the river as

$$q = q_0 e^{-d/\lambda} \tag{12}$$

where q_0 is the leakage at the edge of the river and d is the distance from the edge of the river. For the case considered here, and using the calibrated parameters k_h and c_e, $\lambda \approx 110$ m, while the width of the river is 36 m. Hence, the leakage does not vary significantly across the width of the river; along the centerline of the river, the leakage has reduced to 85% of the leakage along the edge and simulation with three strings of line-sinks is adequate. In cases where the leakage factor is much smaller than the width of the river, it may be necessary to discretize the riverbed with more line-sink strings. Ideally, an element is used that simulates distributed leakage across an area such as the lake element for steady multi-layer flow (Bakker 2007); this approach may be modified to work for transient flow with TTim.

Conclusions

The capabilities of the free code TTim were discussed for the semi-analytic modeling of transient multi-layer flow. The main distinguishing features of TTim are that no grid, time-stepping or closed model boundary are needed. Head variations may be computed semi-analytically at any point in the aquifer system. Currently (version 0.2), two types of elements are available: wells and line-sinks, and transient forcing is represented by a step function. These elements may be used to simulate a variety of hydrogeologic features, including multi-aquifer wells, abandoned wells, streams, horizontal wells, and vertical faults. The input for a simple TTim model consists of only a few lines of Python (see, for instance, the Appendix). For more complicated models, the input may be read from, for example, geographic information system (GIS) shape files. GIS-based TTim user interfaces are under development. Two new benchmark problems were presented: a benchmark against a semi-analytic solution for flow to a horizontal well and a benchmark against a MODFLOW solution. Finally, TTim was applied to a real-world case of an aquifer test near a stream with a leaky bed. The capabilities of TTim may be expanded in the future through the addition of line elements for the simulation of impermeable or leaky walls (included in version 0.21), areal elements for the simulation of transient recharge and distributed leakage, and inhomogeneities to simulate piecewise changes in aquifer properties. TTim is available from www.ttim.googlecode.com (Bakker 2012).

Acknowledgements TTim development was funded by Layne Hydro in Bloomington, Indiana (USA), and by the US EPA Ecosystems Research Division in Athens, Georgia (USA), under contract QT-RT-10-000812 to SS Papadopulos and Associates in Bethesda, Maryland (USA). Brad Shroeder of Layne Hydro provided the field data of the practical application. Brad Shroeder and Erik Anderson of Layne Hydro developed the TTim model of the practical application.

Appendix

TTim script for the finite difference benchmark

An example TTim input file is presented to illustrate that TTim input files are short and relatively easy to read. This input file is for the second benchmark problem consisting of a well, a canal and a fault in a two-aquifer system (see

 DOI 10.1007/s10040-013-0975-2

Fig. 3 and Table 1). Most commands are self-explanatory. Detailed input instructions are given in the TTim manual (Bakker 2012).

```
# Import required packages
from TTim import *
from numpy import *
# TTim model
ml = ModelMaq( kaq = [10, 20],
               z = [30, 25, 15, 10, 0],
               c = [1000, 1000],
               Saq = [1e-4, 1e-4],
               topboundary = 'semi',
               tmin = 1e-2,
               tmax = 10 )
w = Well( ml, xw = 0, yw = 0, rw = 0.1,
          tsandQ = [(0,500)], layers = 2 )
xfault = -200 * ones(21)
yfault = linspace( -410, 410, 21 )
ls1 = ZeroMscreenLineSinkString( ml,
      xy = zip(xfault,yfault), layers=[1,2] )
xcanal = 200 * ones(21)
ycanal = linspace( -410, 410, 21 )
ls2 = ZeroHeadLineSinkString( ml,
      xy = zip(xcanal,ycanal), layers=[1] )
ml.solve()
# Compute head: ml.head(x,y,t)
ml.head(100,100,[1,2,3,4,5])
```

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