Design of a 3.5 meters Rotor Two Bladed Horizontal Axis Wind Turbine

Gómez G., Carlos; Macías G, Bulmaro; Azucena C, Honorato; Palomino M., Amparo
Benemérita Universidad Autónoma de Puebla

Abstract

In this work we reviewed some aerodinamical concepts of Horizontal Axis Wind Turbines. We focused in power extraction and how it is related with the blade geometry. We designed a 3.5 meters two bladed rotor which maximizes the power coefficient when it is operating at variable speed. Once we got the blade geometric parameters we analyzed the performance of the rotor to know its behavior at different tip speed ratios. The airfoil used in the rotor blades is the NACA5317.

Key Words: aerodynamics, airfoil, power coefficient, Tip Speed Ratio.

1. Introduction

Mechanical wind power has been used at least since the early middle age and maybe already three millennia ago[1][2]. A wind turbine is a machine which converts the power of the wind into mechanical power [3]. Wind turbine power production depends on the interaction between the rotor and the wind.

Wind turbine can be classified by its shaft orientation in vertical axis wind turbine (VAWT) and horizontal axis wind turbine (HAWT). In HAWT's, the conversion process uses the aerodynamic force of lift to produce a net positive torque on a rotating shaft to produce mechanical power and then transform it into electricity in a generator. The aerodynamics of wind turbine has been developed since 1865, when W. J. M. Rankine described by the first time the momentum theory for an actuator disk [1], until our days and it continues under investigation in big experiments like the project MexNet [4].

The principal subsystems of a typical horizontal axis wind turbine are the rotor (includes the blades and the supporting hub), the drive train (it usually consist of shafts, gearbox, coupling, a mechanical brake and the generator), the nacelle and main frame (including wind turbine housing, bedplate, and the yaw system), the tower and the foundation, the machine controls and the balance of the electrical system [3]. The main element in a HAWT is the rotor; its aerodynamics determines how much power can be extracted from the wind. We are focus in this subsystem.

The HAWT power output changes with the wind speed. Each wind turbine has its own performance curve which relates the tip speed ratio (TSR) λ, with the power coefficient, C_p. Figure 1 shows the power coefficients of some wind turbines.

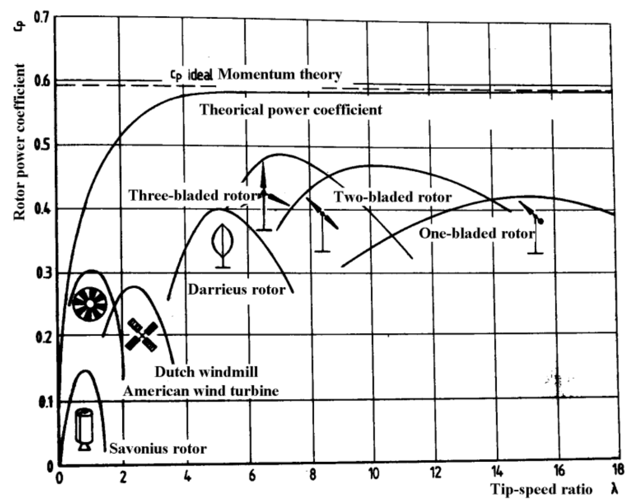

Figure 1. Power coefficients of different designs wind rotors [5].

2. Aerodynamics of HAWT

In practical HAWT rotor design airfoils are used to transform the kinetic energy from the wind into useful kinetic energy. Momentum theory and blade element theory are combined into a strip theory that enables calculation of the performance characteristics of an annular section of the rotor. The characteristics for the entire rotor are gotten by integrating the values obtained from each annular section.

The wind turbine is represented by a uniform actuator (Figure 2) disc which creates a discontinuity in the air pressure, where U is the wind velocity, p is the air pressure, ρ is the air density; the subscripts ∞, d, and w denotes the positions at upstream, in the disc, and in the wake, respectively.

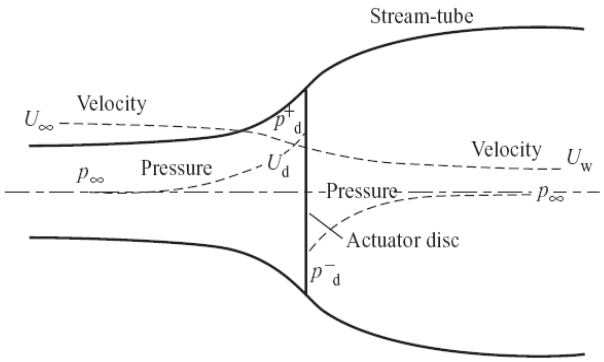

Figure 2. Energy extracting actuator disc and stream tube [2].

Momentum Theory

The change of rate of momentum is denoted by $(U_\infty - U_d)\rho A_d U_d$, the force causing this change of momentum comes entirely from the pressure difference across the actuator disc which can be determined by applying the Bernoulli equation [6]

$$F = (p_d^{\,+} - p_d^{\,-})A_d = 2\rho A_d U_\infty^{\,2} a(1-a) \tag{1}$$

where a is the axial flow inductor factor. The power obtained by this force in the actuator disc is FU_d. The power coefficient C_p is defined as the ratio between the power extracted and the power available in the wind [7].

$$C_p = 4a(1-a)^2 \tag{2}$$

The maximum value for C_p is known as the *Betz limit* [8] and it occurs when $a = 1/3$, and then we have $C_p = 0.593$. The force in the actuator disc can be adimensionalized to obtain a thrust coefficient C_T [7]

$$C_T = 4a(1-a) \tag{3}$$

The theory of momentum is not valid for $a \geq 0.5$ anymore.

Betz limit is reduced because of wake rotation, a finite number of blades, tip losses and drag coefficient C_d of airfoil.

Blade element momentum theory, BEM.

The flow field, characterized by axial and tangential induction factors that are in function with the rotor power extraction and thrust, will be used to define the airflow at the rotor airfoils. Momentum theory refers to a control volume analysis of the forces at the blade based on the conversion of linear and tangential (angular) momentum. Blade element theory refers to an analysis of forces at a section of the blade, as a function of the blade geometry. These analysis can be merged to create the blade element momentum theory.

We consider annular rings of radio r and wide $\mathrm{d}r$, with annular area $A_a = 2\pi r \mathrm{d}r$, ω is the angular velocity imparted to the wake and Ω is the angular velocity of the rotor. The thrust in the annular element is [2]

$$\mathrm{d}T = 4a'(1+a')\frac{1}{2}\rho\Omega^2 r^2 2\pi r \mathrm{d}r \tag{4}$$

where a' is the tangential flow induction factor, and $a' = a(1-a)/(\lambda^2\mu^2)$ [2], λ is defined by $\lambda = \Omega R/U_\infty{}^2$, and $\mu = r/R$ is the normalized radio.

The torque in each annular ring is the rate of change of angular momentum of the air crossing the ring [2]

$$\mathrm{d}Q = 4a'(1-a)\frac{1}{2}\rho U_\infty \Omega r^2 2\pi r \mathrm{d}r \tag{5}$$

In blade element theory we assume that the forces in the blade element can be expressed as function of lift and drag coefficients of the airfoil (C_l and C_d, respectively) and the attack angle α. Figure 3 shows the velocities and forces present in an airfoil where c is the chord, W is the relative wind, β is the pitch angle and ϕ is the flow angle.

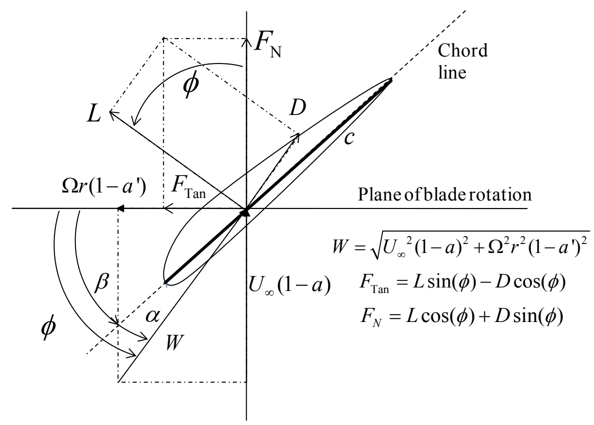

Figure 3. Geometry of the blade for the blade element analysis [3]

From Figure 3 we can get the following relations

$$\tan(\phi) = \frac{1-a}{(1+a')\lambda_r} \tag{6}$$

$$\phi = \beta + \alpha \tag{7}$$

$$W = \frac{U_\infty(1-a)}{\sin(\phi)} \tag{8}$$

$$\mathrm{d}F_N = \mathrm{d}F_L\cos(\phi) + \mathrm{d}F_D\sin(\phi) \tag{9}$$

$$\mathrm{d}F_{Tan} = \mathrm{d}F_L\sin(\phi) - \mathrm{d}F_D\cos(\phi) \tag{10}$$

where $\mathrm{d}F_L = C_l\frac{1}{2}\rho U_\infty{}^2 c\mathrm{d}r$ and $\mathrm{d}F_D = C_d\frac{1}{2}\rho U_\infty{}^2 c\mathrm{d}r$.

If the rotor has N blades, the total normal force on the section at a distance r, from the center is

$$\mathrm{d}F_N = N\frac{1}{2}\rho \mathrm{W}^2\left(C_l\cos(\phi) + C_d\sin(\phi)\right)c\mathrm{d}r \tag{11}$$

The differential torque due to the tangential force operating at a radio r, is obtained by

$$\mathrm{dQ} = N\frac{1}{2}\rho \mathrm{W}^2 \left(C_l \sin(\phi) + C_d \cos(\phi)\right) cr\mathrm{d}r \tag{12}$$

We can express the power contribution an annular ring from equation (5) as

$$\mathrm{d}P = \Omega \mathrm{d}Q = \frac{1}{2}\rho A_a U_\infty^3 \left[\frac{8}{\lambda^2} a'(1-a)\lambda_r^3 \mathrm{d}\lambda_r\right] \tag{13}$$

thus, we can write the power coefficient as

$$C_p = \frac{8}{\lambda^2}\int_0^\lambda a'(1-a)\lambda_r^3 \mathrm{d}\lambda_r \tag{14}$$

From equation (11) a new expression for the thrust coefficient can be obtained as

$$C_T = \frac{\sigma_r (1-a)^2 \left(C_l \cos\phi + C_d \sin^2\phi\right)}{\sin^2\phi} \tag{15}$$

3. Design of the Geometry of the Rotor Blades for variable speed operation

We need to choose some parameters as number of blades, and size of the rotor, but the most important is the design tip speed ratio. A turbine operating at variable speed operation can maintain constant the TSR for the maximum power coefficient to be developed regardless of wind speed. We can consult the existing wind turbines performance and use the following expression to choose the design TSR [9]:

$$\lambda = \sqrt{\frac{80}{N}} \tag{16}$$

We define the chord solidity σ_r as the ratio of the total chord length and the circumference at a specific radio,

$$\sigma_r = \frac{N}{2\pi}\frac{c}{r} = \frac{N}{2\pi\mu}\frac{c}{R} \tag{17}$$

From equations (4) and (11) we can get the geometric parameters for an optimum operation, with $a = 1/3$ the normalized chord is given by [2]

$$\frac{c}{R} = \frac{8}{9\frac{N}{2\pi}\lambda C_l \sqrt{\frac{4}{9} + \left(\lambda\mu\left(1 + \frac{2}{9\lambda^2\mu^2}\right)\right)^2}} \tag{18}$$

we also can determine the variation of the flux angle for an optimum operation as [2]

$$\tan(\phi) = 2 \Big/ \left(3\lambda\mu\left(1 + \frac{2}{9\lambda^2\mu^2}\right)\right) \tag{19}$$

In general, an optimum rotor blade has not an easy geometry to be manufactured, so we can make an approximation, we propose a conic planar shape as follows

$$\frac{c_u}{R} = m_{c_u}\mu + \mathrm{offset}_{c_u} \tag{20}$$

with $m_{c_u} = \frac{\frac{c}{R}(\mu_2) - \frac{c}{R}(\mu_1)}{\mu_2 - \mu_1}$ and $\mathrm{offset}_{c_u} = \frac{c}{R}(\mu_1) - m_{c_u}\mu_1$.

Equation (20) is the general equation of a line where m_{c_u} is the slope of the line.

According to compensate this change in chord variation we need to modify the lift coefficient

$$C_l = \frac{8/9}{\frac{Nc_u\lambda}{2\pi}\sqrt{\frac{4}{9} + \lambda^2\mu^2\left(1 + \frac{2}{9\lambda^2\mu^2}\right)^2}} \tag{21}$$

by modifying the lift coefficient we can adjust the attack angle which used to be constant [6], α_{opt} is the optimum attack angle in degrees.

$$\alpha \approx \frac{C_l}{0.1} - \alpha_{\mathrm{opt}} \tag{22}$$

4. Analysis of performance

We can determine the performance of HAWT rotors by analyzing its geometry. We need to know the axial and tangential induction factors, once we get them we can obtain the power coefficient. To calculate a and a' in each section blade (the blade is divided in S sections for its analysis) we need to use iterative methods and we have to consider additional factors as Prandtl tip losses and Glauert correction [10] for turbulent wake states.

Prandtl tip losses factor is calculate by the following equation [3]

$$F = \frac{2}{\pi}\cos^{-1}\left[\exp\left(-\left\{\frac{(N/2)(1-\mu)}{\mu\sin(\phi)}\right\}\right)\right] \tag{23}$$

A turbulent wake state is considered when the thrust coefficient is above 0.96, or similarly when $a \geq 0.4$. Glauert correction is done with measured tests to complete the momentum theory. First, C_T is computed with equation (15), if $C_T \geq 0.96$ we need to write a according to [3]

$$a = \frac{1}{F}\left[0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_T)}\right] \tag{24}$$

From BEM theory we can deduce two equations to calculate a and a'. These equations are

$$a = 1 \Big/ \left(1 + \frac{4F\sin^2(\phi)}{\sigma_r C_l \cos(\phi)}\right) \tag{25}$$

$$a' = 1 / \left(\frac{4F\cos(\phi)}{\sigma_r C} - 1 \right) \tag{26}$$

The values can be computed by the flux diagram in Figure 4.

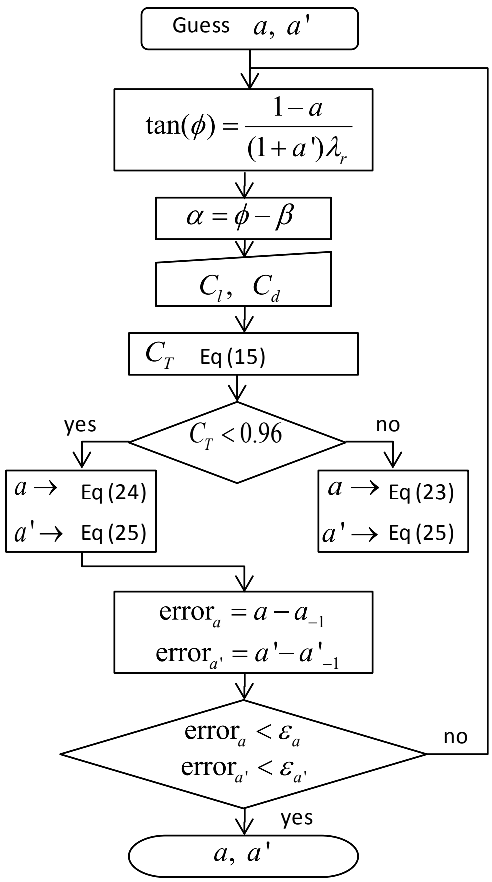

Figure 4. Flux diagram to determine the induction factors.

In words, we guess the initial values of the induction factors and then we determine the angle of attack, read the values of C_l and C_d of the airfoil at that angle of attack, calculate C_T with equation (15) and use the Glauert correction when necessary. We need to compare the new values for the induction factors with the old ones, if the error is less than a tolerance we have the value of a and a' for the blade section under analysis. Once we have the induction factors for all blade sections we can compute the power coefficient for a specific tip speed ratio λ, by using the equation (14) .

5. Results

We select the TSR according with equation (16) and existing C_p wind turbine performance, choosing a TSR of 7. The design TSR will determine the geometry of the blades.

The most important for the blade design is to determine the airfoil to be used; according with Izli, 2007 [11] we will use the NACA5317 airfoil which has the maximum C_l/C_d at 5° of attack angle, at that point $C_l = 1.2$ and $C_d = 0.0225$. We will design the blades considering 10 sections S to form a rotor of 3.5 meters diameter. In resume, we have the following design specifications:

Airfoil: NACA5317
N=2
R=1.75 m
Design tip speed ratio= 7
Design C_l= 1.2
Design C_d=0.0225
Sections=10

Applying equation (18) we obtain the optimal variation of the chord for our blades; with this result we compute a new chord form from equation (20). In Figure 5 is shown the variation chord along the blade, the adjust to the lift coefficient obtained from equation (21) and the modified angles (attack, pitch and flux) determined by equations (22) and (7). Table 1 resumes these results; these are the geometric parameters for blade manufacture.

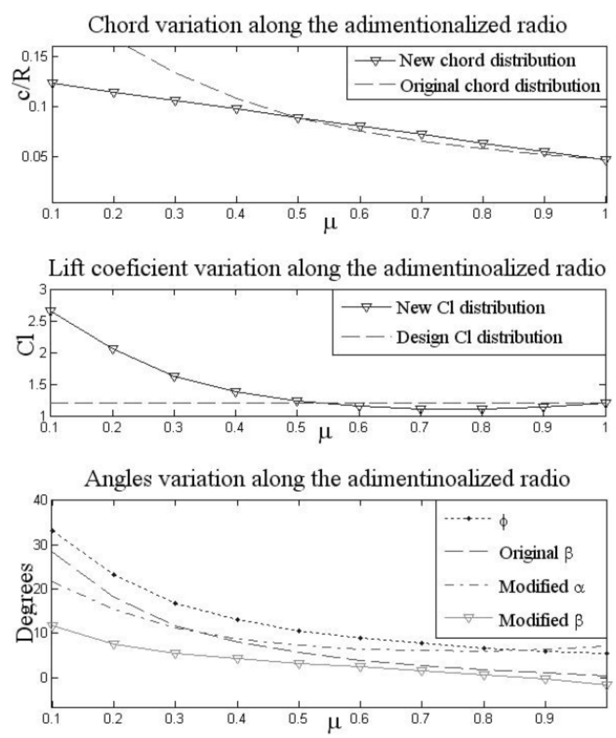

Figure 5. Obtained geometric parameters of the blades

μ	c/R	β
0.1	0.123	11.64
0.2	0.115	7.66
0.3	0.106	5.53
0.4	0.098	4.23
0.5	0.089	3.27
0.6	0.081	2.43
0.7	0.072	1.61
0.8	0.064	0.72
0.9	0.055	-0.32
1	0.046	-1.66

Table 1. Geometric parameters in function of normalized radio

With these parameters we can determine the performance by following the flux diagram of Figure 4 for tip speed ratios from 4 to 9.

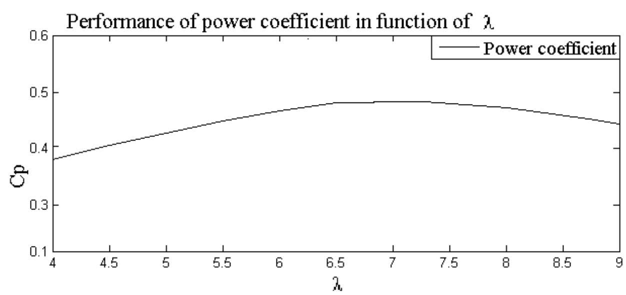

Figure 6. Variation of power coefficient for different values of tip speed ratio

5. Conclusions

We note that our rotor performance is in an acceptable range according to existing wind turbines, the maximum power coefficient is 0.5. The geometric parameters obtained from design are in a good range for manufacturing. The pitch angle has a net variation of 13.3º, and it is almost linear. So, we are now in possibilities to manufacture our HAWT rotor, the next step is to characterize the rotor and design the pitch and yaw control.

6. References

[1] Snel, Herman. "A Shrot History of Wind Turbine Aerodynamics, or: From Betz to Better". *The second conference on the Science of Making Torque from Wind*. ECN, Denmark, 2007. ECN -L-07-068

[2] Burton, Tony. *Wind Energy Handbook*. John Wiley & Sons, England, 2001. ISBN 0-471-48997-2

[3] Manwell, McGowan, and Rogers, *Wind Energy Explained, Theoroy, Design and Application*. John Wiley and Sons, USA 2002. ISBN 0470-84612-7

[4] Development, and Deployment of Wind Energy Systems of the IEA Executive Committee for the Implementing Agreement for Co-operation in the Research, "IEA WIND ENERGY, Annual Report 2008". USA, 2009. ISBN 0-9786383-3-6

[5] Hau, Erick. *Wind Turbines. Fundamentals, Technologies, Application, Economics*. Springer, United Kingdom, 2006. ISBN 3-540-24240-6

[6] Anderson, J. *Fundamentals of aerodynamics*. McGraw-Hill, USA, 2007. ISBN 0-07-001679-8

[7] Lissaman, R. E.; Wilson, *Applied Aerodynamics of Wind Power Machines*. Oregon State University. USA, 1974.

[8] Rosato, Mario. *Diseño de máquinas eólicas de pequeña potencia*. Progensa. España, 1991. ISBN 84-86505-35-6

[9] Vardar and Kurtulmus. "Aerodynamic Analysis of Different Wind Turbine Profiles". *Journal of applied sciences*. Uludag University. Turkey, 2007. ISSN 1812-5654

[10] Glauert, H. "Airplane propellers. Aerodynamic theory". *Julius Springer*. Germany, 1935.

[11] Nazmi Izli, Ali Vardar, and Ferhat K., "A study on aerodynamic properties of some NACA profiles used on wind turbine blades". *Journal of applied sciences*. Uludag University. Turkey, 2007. ISSN 1812-5654