Sun, Earth and heat transfer

author: stephane.ploix@grenoble-inp.fr

reviewer: mircea_stefan.simoiu@upb.ro

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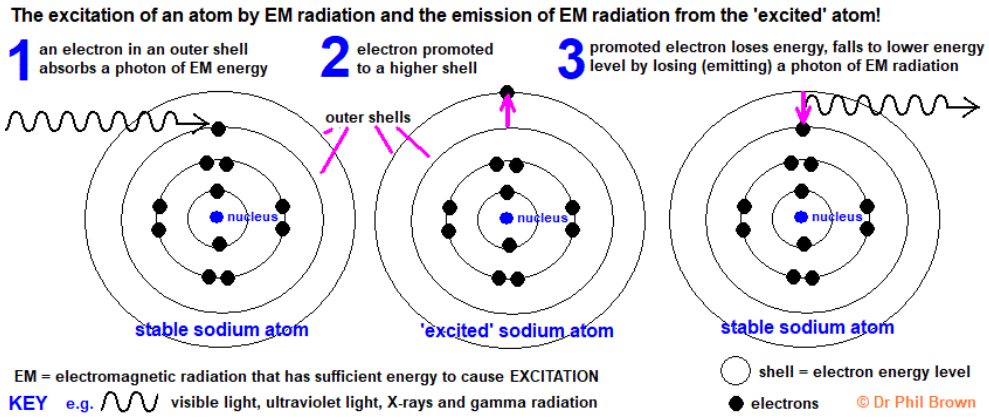
1. Fundamentals of electro-magnetic radiations

1.1 Basic concepts

Each material with a temperature above absolute zero emits electromagnetic radiation.

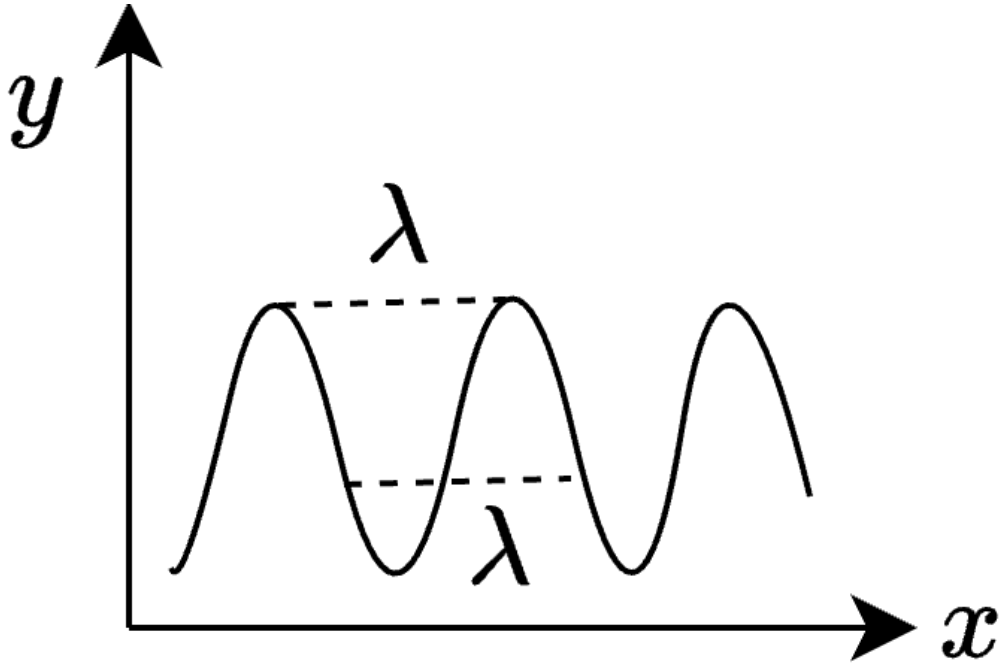
Thermal electro-magnetic radiation are emitted by a material due to incident radiations coming from other atoms exciting the incident one which in turn emits electromagnetic vibrations that transfer radiative energy.

- It depends on the temperature and on the surface of the material (except for black bodies, which depends only on the surface temperature).

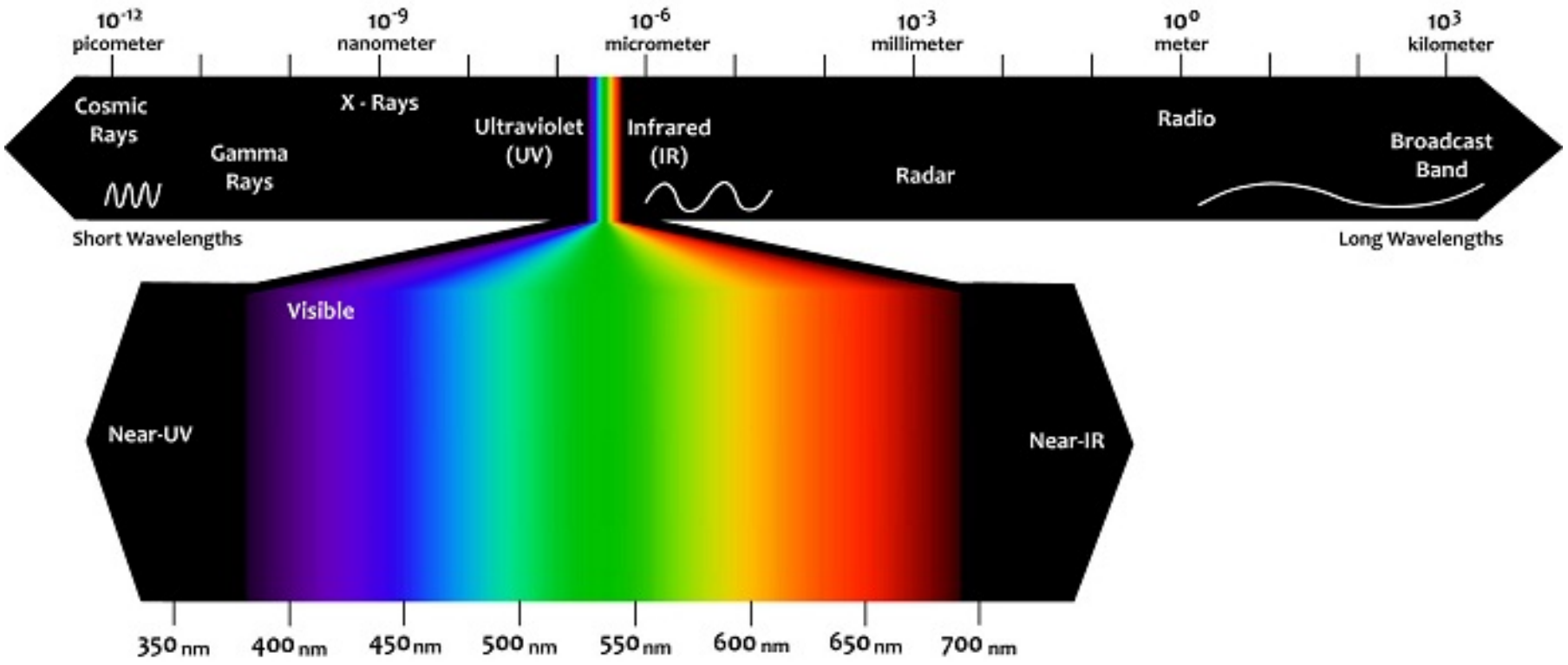


1.1. Basic concepts

Wavelength - the distance $[m]$ between two consecutive points in the same phase of a wave.



1.2. Example: the electromagnetic spectrum



- Mobile wave lengths: 1 to 43cm
- Micro-wave length; 12.2cm
- WiFi wave length; 0.5 to 13cm

Frequency spectrum of the Sun

The Sun emits electromagnetic radiation over a very broad spectrum, from gamma rays to radio waves, but most of its emitted power is concentrated in the visible and near-infrared ranges.

To first approximation, the solar spectrum is close to that of a blackbody at about $T_\odot \approx 5778\ \mathrm{K}$

The spectral radiance of a blackbody is described by Planck's law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

where:

- $B_\nu(T)$ is the spectral radiance per unit frequency
- ν is a frequency
- h is the Planck constant: $6.62607015 \times 10^{-34} J \cdot s$
- c is the speed of light: 299,792,458 m/s
- k_B is the Boltzmann constant: $1.380649 \times 10^{-23} \mathrm{J} \cdot \mathrm{K}^{-1}$
- T is the absolute temperature of the black body

Using Wien's displacement law expressed in frequency form, the maximum of B_ν occurs at approximately:

$$\nu_{\max} \approx 2.82 \frac{k_B T}{h}$$

For the Sun:

$$\nu_{\max} \approx 3.4 \times 10^{14} \text{ Hz}$$

This corresponds to a wavelength of roughly:

$$\lambda \approx \frac{c}{\nu} \approx 880 \text{ nm}$$

This is in the near infrared.

However, if the spectrum is expressed per unit wavelength, B_λ, the maximum occurs at:

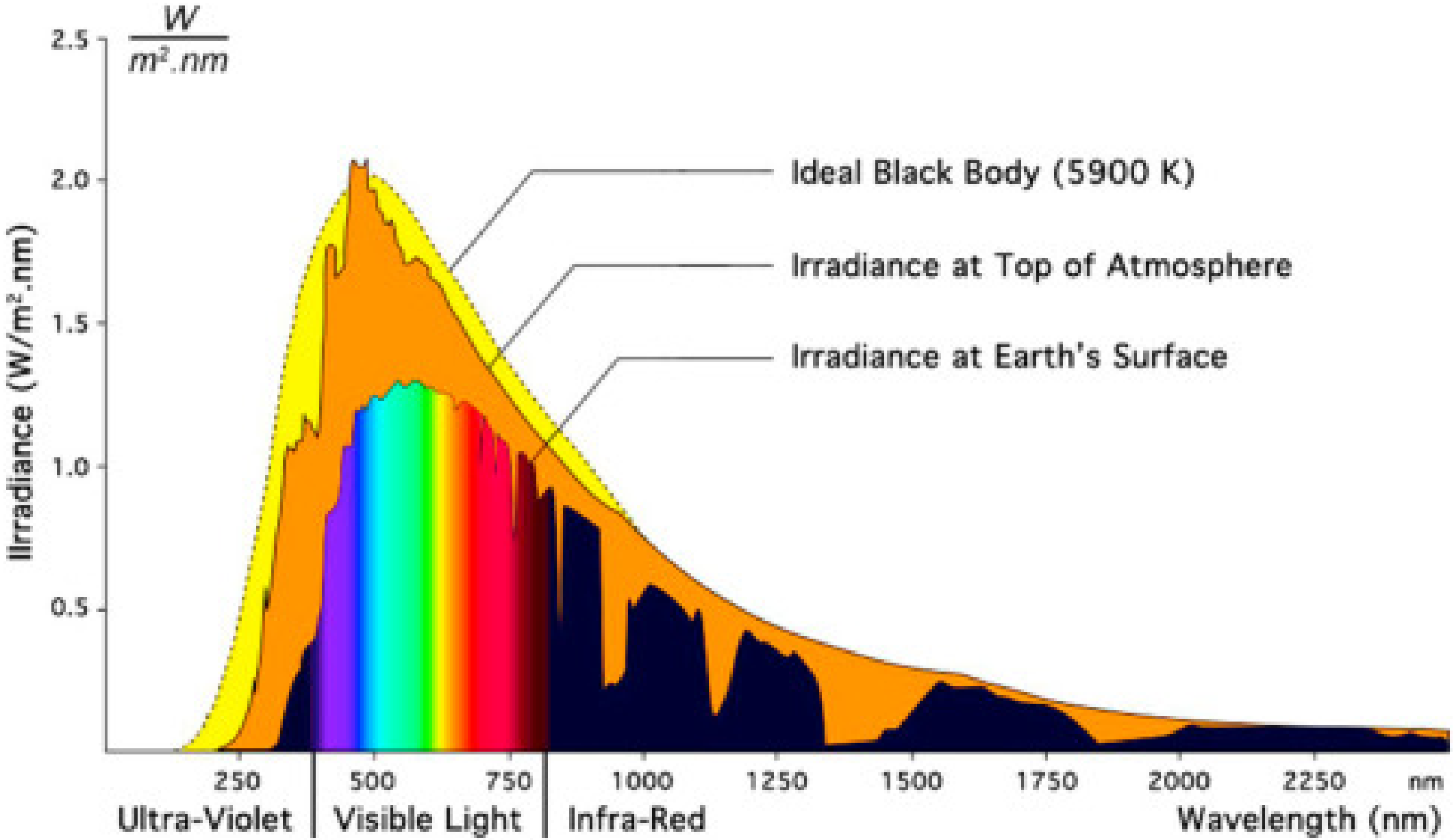
$$\lambda_{\text{max}} \approx \frac{2.898 \times 10^{-3}}{T}$$

For the Sun:

$$\lambda_{\text{max}} \approx 500 \text{ nm}$$

This is in the green-visible range.

The difference comes from the fact that the maximum depends on whether the spectrum is plotted per unit frequency or per unit wavelength.


W / m².nm
2.5
2.0
1.5
1.0
0.5
Irradiance (W/m².nm)
Ideal Black Body (5900 K)
Irradiance at Top of Atmosphere
Irradiance at Earth's Surface
250
500
750
1000
1250
1500
1750
2000
2250
nm
Ultra-Violet
Visible Light
Infra-Red
Wavelength (nm)

Main solar spectral ranges

Spectral range	Approximate wavelength	Approximate frequency	Importance in solar radiation
Ultraviolet C	$100–280\ \mathrm{nm}$	$1.1–3.0 \times 10^{15}\ \mathrm{Hz}$	Mostly absorbed by atmospheric ozone and oxygen
Ultraviolet B	$280–315\ \mathrm{nm}$	$9.5 \times 10^{14}–1.1 \times 10^{15}\ \mathrm{Hz}$	Partly reaches the surface; biologically active
Ultraviolet A	$315–400\ \mathrm{nm}$	$7.5–9.5 \times 10^{14}\ \mathrm{Hz}$	Mostly reaches the surface
Visible	$400–700\ \mathrm{nm}$	$4.3–7.5 \times 10^{14}\ \mathrm{Hz}$	Large fraction of solar energy; human vision
Near infrared	$700–2500\ \mathrm{nm}$	$1.2–4.3 \times 10^{14}\ \mathrm{Hz}$	Large fraction of solar heat input
Mid/far infrared	$> 2500\ \mathrm{nm}$	$< 1.2 \times 10^{14}\ \mathrm{Hz}$	Smaller part of direct solar spectrum
Microwave/radio	much longer wavelengths	much lower frequencies	Very weak part of total solar power

Approximate distribution of solar energy

At the top of the atmosphere, the solar spectrum is often summarized approximately as:

Range	Share of solar power
Ultraviolet	about 8%
Visible	about 43%
Infrared	about 49%

So, although we often associate sunlight with the visible spectrum, nearly half of the incoming solar energy is in the infrared.

Important distinction: solar radiation vs terrestrial radiation

The Sun emits mainly shortwave radiation:

$$\lambda_{\odot} \sim 300 - 3000 \text{ nm}$$

The Earth, being much colder, emits mainly longwave infrared radiation:

$$\lambda_{\oplus} \sim 4000 - 100000 \text{ nm}$$

Source	Approximate temperature	Main emission range
Sun	5778 K	shortwave: visible and near infrared
Earth	$255–288 \text{ K}$	longwave infrared

The atmosphere is relatively transparent to much of the incoming shortwave solar radiation, but greenhouse gases absorb and re-emit parts of the outgoing longwave infrared radiation emitted by the Earth.

Approximate distribution at the surface

For clear-sky surface solar radiation, a useful order of magnitude is:

Band	Approximate share of ground-level solar energy
Ultraviolet	3–5%
Visible	40–45%
Near infrared	50–55%

Terestral radiations

At usual indoor temperatures (**15–30 °C**), walls, furniture and other objects emit mainly **long-wave infrared radiation**, with a peak wavelength at about **9600–10100 nm**.

Wien's displacement law gives: $\lambda_{\max} = \frac{2898}{T} \quad [\mu\mathrm{m}]$ where T is the absolute temperature in Kelvin.

Surface temperature	Peak wavelength
15 °C	10100 nm
20 °C	9900 nm
25 °C	9700 nm
30 °C	9600 nm

At 20°C, the emitted radiations falls between **5000 and 25000 nm**.

Glass filtering

For ordinary clear window glass, most of the visible solar spectrum passes through, part of the near infrared passes through, but most ultraviolet below about 300–320 nm and most longwave thermal infrared are blocked.

Band	Approximate wavelength	Through ordinary clear glass?	Notes
UV-C	$100–280\ \mathrm{nm}$	No	Already almost fully removed by the atmosphere; glass also blocks it
UV-B	$280–315\ \mathrm{nm}$	Mostly no	Strongly attenuated by ordinary soda-lime glass
UV-A	$315–400\ \mathrm{nm}$	Partly	Some UV-A passes, especially above $\sim 330–350\ \mathrm{nm}$
Visible	$380–780\ \mathrm{nm}$	Yes, strongly	Clear glass typically transmits most visible light
Near infrared, solar IR	$780–2500/3000\ \mathrm{nm}$	Partly	A significant part passes, but with increasing absorption at longer wavelengths
Mid/far infrared, thermal IR	$> 3\ \mu\mathrm{m}$	Mostly no	Ordinary glass is almost opaque to terrestrial longwave infrared

In climate/building terms

- $\text{solar shortwave radiation (including near infrared)} \approx 300\text{–}3000\ \text{nm}$ can partly pass through glass, while:
- $\text{terrestrial far longwave infrared} \approx 4000\text{–}100000\ \text{nm}$ is largely blocked by glass.

This is why a room, greenhouse, or car behind glass can heat up: incoming solar radiation enters mainly as visible and near-infrared radiation, is absorbed by interior surfaces, then re-emitted as longwave infrared, which is much less able to escape through ordinary glass.

Ordinary clear glass

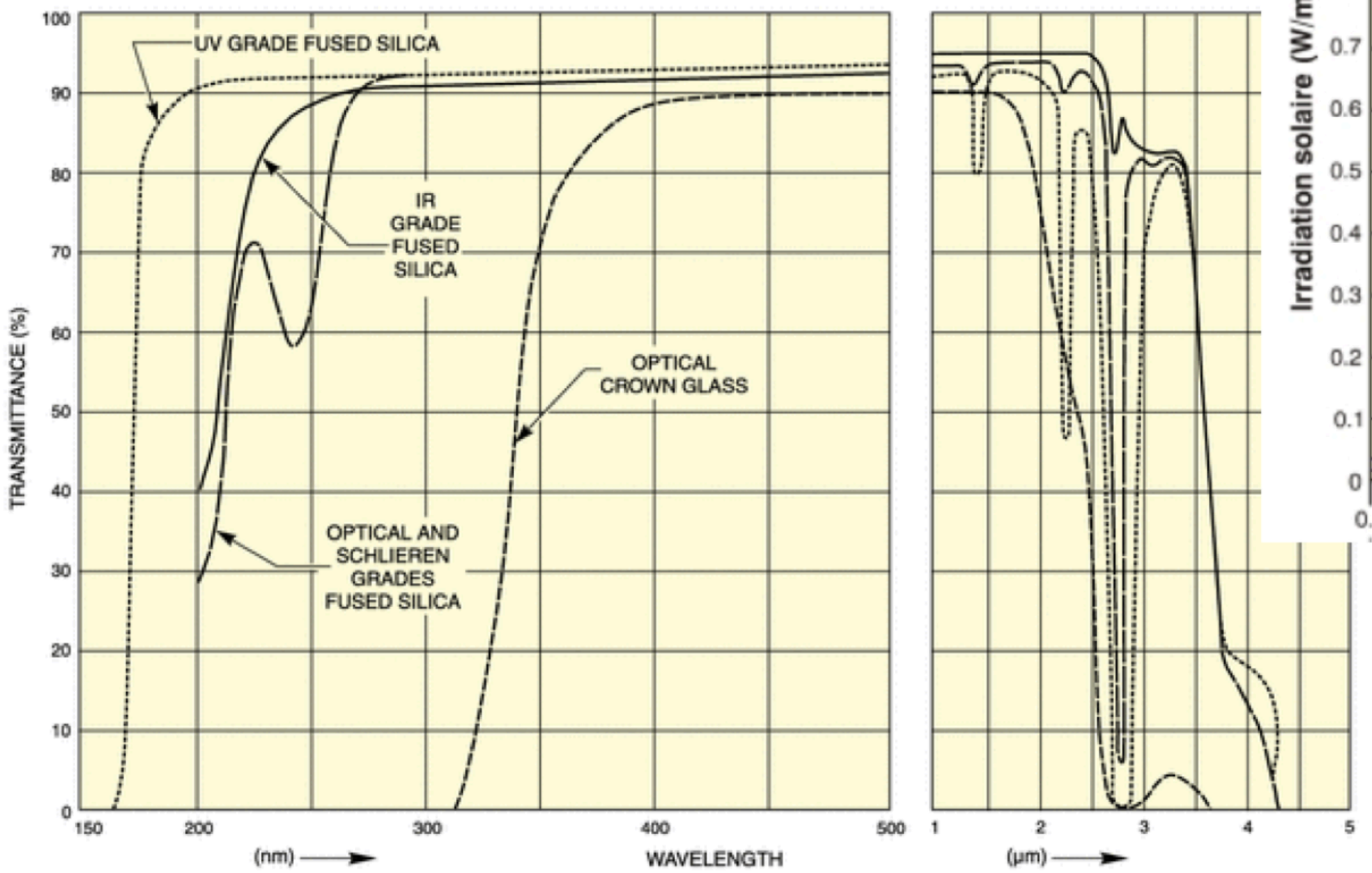
For a simple clear soda-lime window pane, the transmission is approximately:

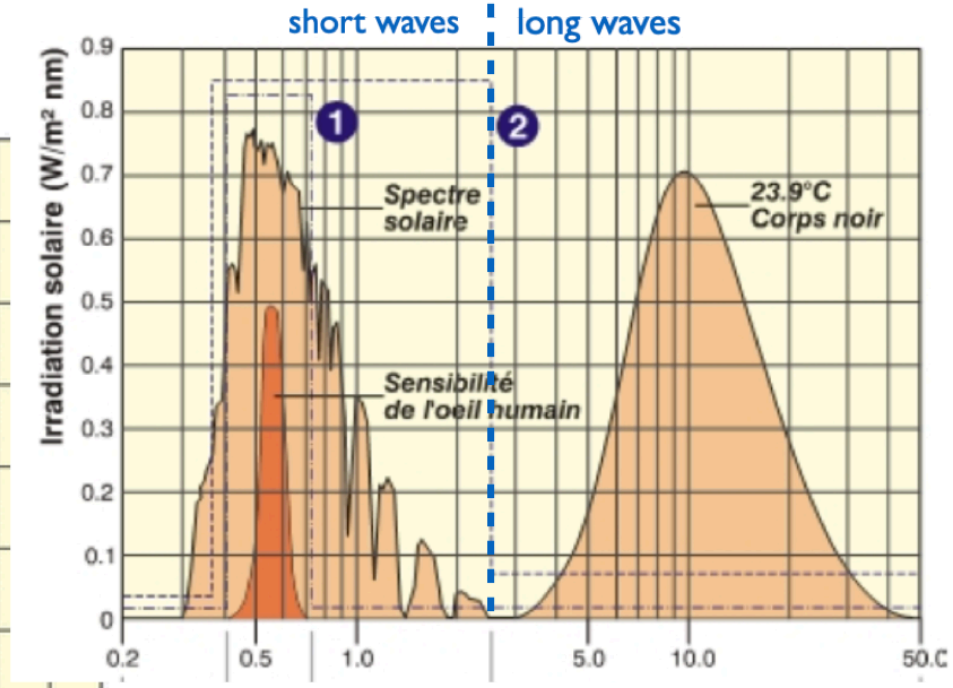
Quantity	Typical order of magnitude
Visible light transmittance	~ 80–90 for a single clear pane
Solar energy transmittance	lower than visible transmittance, because some near-IR is absorbed/reflected
UV transmittance	low below $\sim 320\ \mathrm{nm}$, partial in UV-A
Longwave IR transmittance	very low

The wave lenghts passing through normal glass is composed of:

- Visible light is approximately 380–$780\ \mathrm{nm}$,
- Near-infrared solar radiation from about 780 to $3000\ \mathrm{nm}$ at the Earth's surface.

transmitivity coefficient of glasses





Low-emissivity and solar-control glazing

Not all glass behaves like ordinary clear glass.

Glass type	Spectral behaviour
Clear single glass	High visible transmission; partial near-IR transmission; blocks most UV-B and longwave IR
Double glazing	Similar spectral shape, but lower total transmission because there are more glass surfaces
Low-emissivity glazing	Transmits visible light but reflects much thermal infrared; can be designed for high or low solar gain
Solar-control glazing	Designed to reduce near-infrared solar transmission, often to limit overheating
Tinted glazing	Absorbs selected visible and near-IR bands; changes colour and solar gain
Laminated glass	Often blocks more UV because of the interlayer

Low-solar-gain low-emissivity glass reduces near-infrared transmission, while high-solar-gain low-emissivity glass can allow more near-infrared solar radiation through while still reflecting thermal infrared.

Greenhouse-effect

Glass is relatively transparent to incoming shortwave solar radiation, especially visible light, but relatively opaque to outgoing longwave thermal infrared radiation emitted by warmed interior surfaces.

More precisely:

$$\lambda_{\text{Sun at ground}} \approx 0.3\text{–}3\ \mu\text{m}$$

partly passes through glass, while:

$$\lambda_{\text{Earth/room thermal emission}} \approx 4\text{–}100\ \mu\text{m}$$

is mostly absorbed or reflected by glass.

1.3. Human perception of electro-magnetic radiations

- through eyes: $\lambda \in [300nm, 700nm]$, so called *short wave lengths*
- through skin: $\lambda \in [700nm, 100\mu m]$, so called *long wave lengths*

The latter includes:

- near infrared: $0.7–1.4\ \mu\mathrm{m}$;
- short/mid infrared: $1.4–3\ \mu\mathrm{m}$;
- thermal infrared: $3–100\ \mu\mathrm{m}$.

Note:

- *ultraviolet radiations are mostly absorbed by ozone (O_3) in the troposphere and the stratosphere but the little quantity that reaches the ground is important.*
- *exposure to ultraviolet radiation is essential for good health, as it stimulates the production of vitamin D in the body but too much exposure might lead to skin cancer.*

The skin is especially sensitive to infrared radiation because it is absorbed and converted into heat.

Radiation in these ranges can activate thermoreceptors indirectly because it raises the temperature of skin tissue.

The skin does not have “infrared vision”. It can not identify a specific wavelength nor its origin or something like a specific sensory colour.

Instead, it detects the resulting temperature change:

$$\text{absorbed radiation} \rightarrow \text{skin heating} \rightarrow \text{warmth sensation}$$

1.4. Energy of a photon

The energy of a photon is given by:

$$E = h\nu = \frac{hc}{\lambda} \quad [J]$$

where

- h is the Planck's constant $(6.62607015 \times 10^{-34} m^2 kg/s)$,
- ν is the frequency,
- c is the speed of light $(299792458 m/s)$,
- λ is the wavelength.

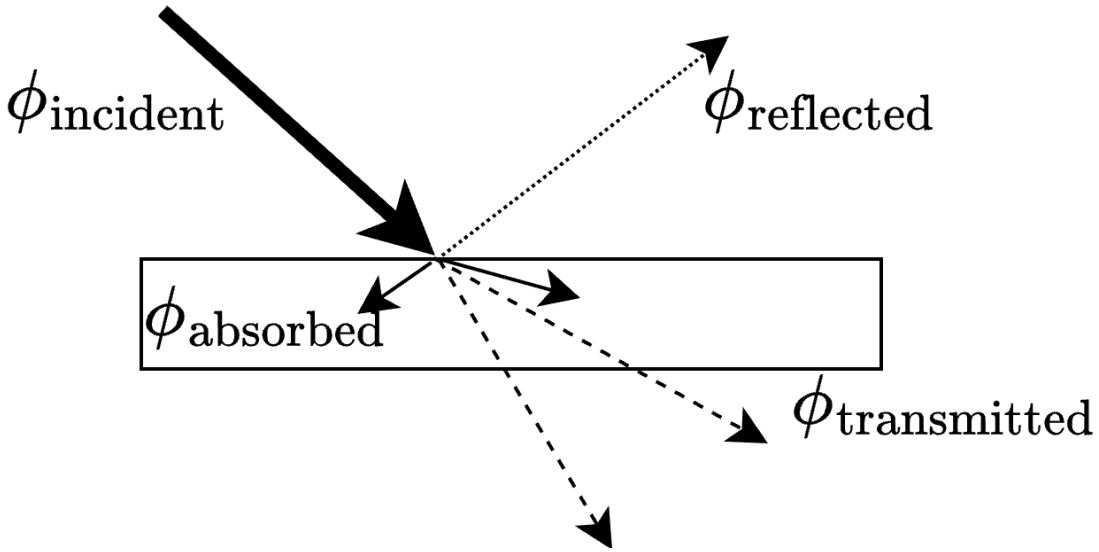
Note: very short wave radiations implies high power -> they are harmful for human body (X-rays, gamma radiations)

2. Radiative heat transfer basics

2.1 Interaction of electro-magnetic radiations with matter: basic concepts

When electromagnetic radiation ϕ_{incident} interacts with matter, it can be:

- Reflected $\phi_{\text{reflected}}$
- Transmitted $\phi_{\text{transmitted}}$
- Absorbed ϕ_{absorbed}



As energy depends on the wave length, the conservation of energy is given by:

$$\phi_{\text{incident}}(\lambda) = \phi_{\text{reflected}}(\lambda) + \phi_{\text{transmitted}}(\lambda) + \phi_{\text{absorbed}}(\lambda)$$

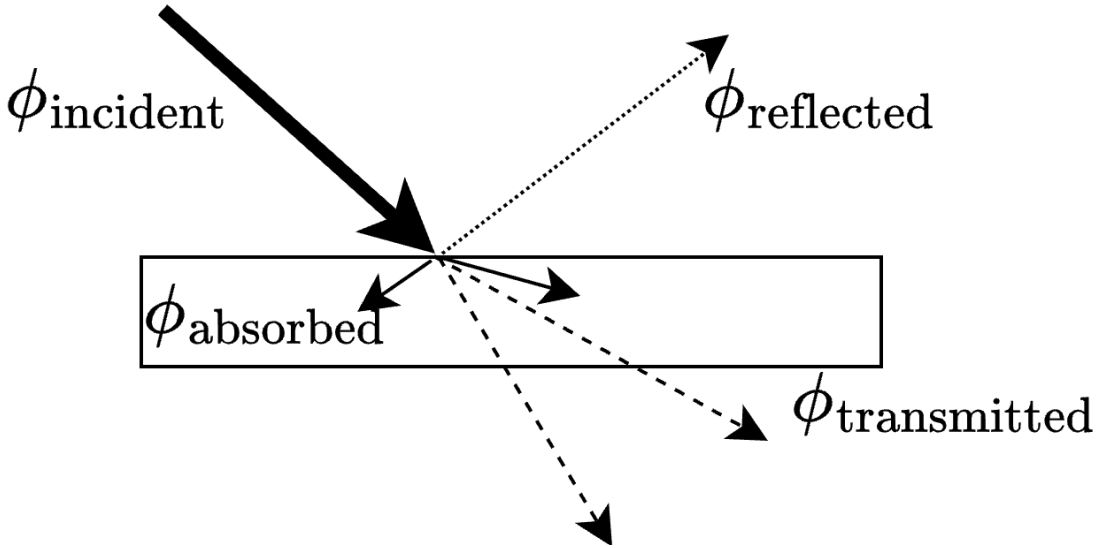
2.1 Interaction of electro-magnetic radiations with matter: basic concepts

The conservation of energy can also be written as:

$$1 = \rho(\lambda) + \tau(\lambda) + \alpha(\lambda)$$

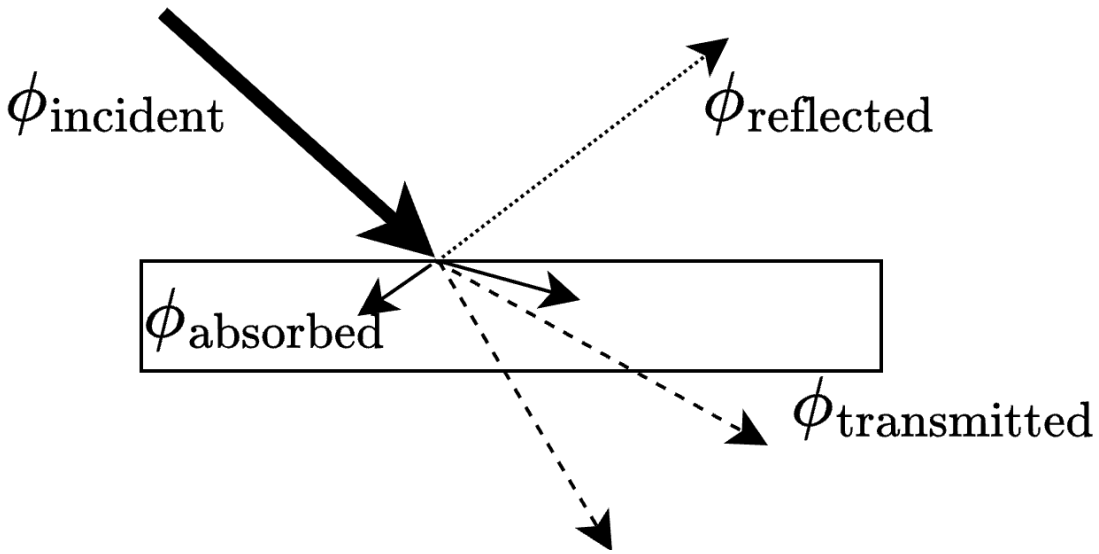
where

- $\rho(\lambda) = \frac{\phi_{\text{reflected}}(\lambda)}{\phi_{\text{incident}}(\lambda)} \rightarrow$ reflectance
- $\tau(\lambda) = \frac{\phi_{\text{transmitted}}(\lambda)}{\phi_{\text{incident}}(\lambda)} \rightarrow$ transmittance
- $\alpha(\lambda) = \frac{\phi_{\text{absorbed}}(\lambda)}{\phi_{\text{incident}}(\lambda)} \rightarrow$ absorptance



2.2 Practical implications

When a radiation beam is hitting a surface, the absorbed energy is given to the surface as a power source.



In practice, we often work with wavelength ranges rather than single wavelengths, such as visible short wavelength radiation from the Sun (380nm to 4μm) or long wavelength radiation from the Earth (4μm to 0.314mm).

2.3 Emittance and illuminance

Emittance (or power exitance) is the areal density of luminous flux *emitted* by a surface per unit area. It is the power emitted in all directions by per unit of emitting surface:

$$E = \frac{d\varphi}{dS} [W/m^2]$$

Illuminance is the radiated energy *received* by a surface.

There is no physical difference between these quantities, only the observer's point of view.

3. Materials properties and examples

3.1 Reflectance of materials

For the visible range, the reflectance ρ of the materials is given in the table (it comes from energy+).

Because most of these materials have no transmittance, the absorptance can be deduced.

painting colors	reflectance
white	0.7 to 0.8
yellow	0.5 to 0.7
green	0.3 to 0.6
grey	0.35 to 0.6
brown	0.25 to 0.5
blue	0.2 to 0.5
red	0.3 to 0.35
black	0.04

3.1 Reflectance of materials

construction materials	reflectance
white plaster	0.7 to 0.8
clean white marble	0.8 to 0.85
clean white brick	0.62
red brick	0.1 to 0.2
old red brick	0.05 to 0.15
polished aluminium	0.6 to 0.75
mate aluminium	0.55 to 0.6
white enamel	0.65 to 0.75
glazing	0.08 to 0.4

woods	reflectance
light birch, maple	0.55 to 0.65
light varnished oak	0.4 to 0.5
dark varnished oak	0.15 to 0.4
mahogany, walnut	0.15 to 0.4

wallpapers	reflectance
very light (white, cream)	0.65 to 0.75
light (grey, yellow, blue)	0.45 to 0.6
dark (black, dark blue, dark grey, green, red)	0.05 to 0.36

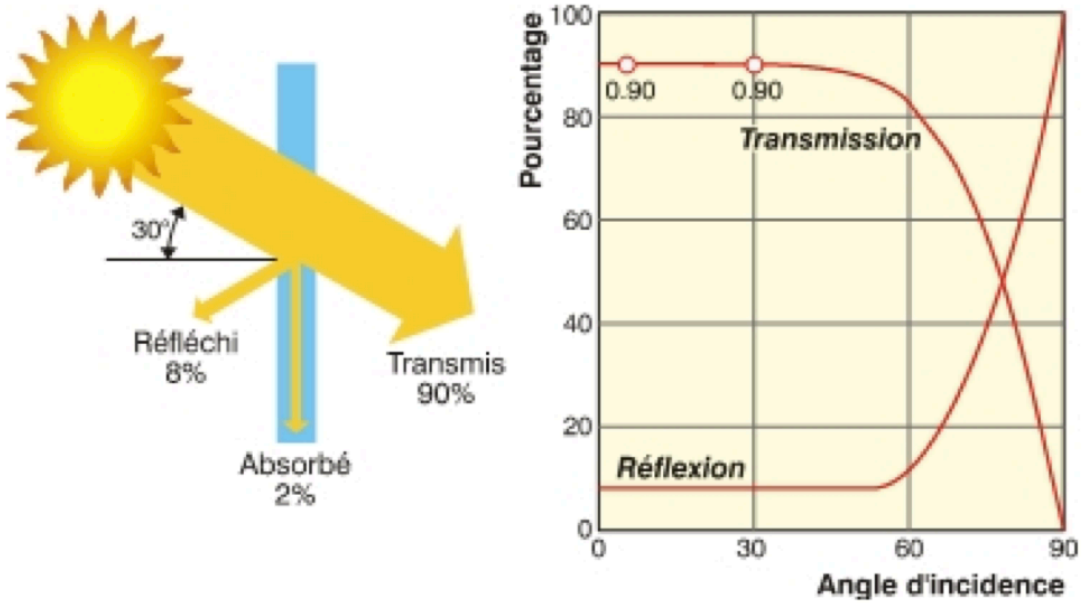
3.1 Reflectance of materials

construction materials	reflectance
new white render (crépi)	0.7 to 0.8
old white render (crépi)	0.3 to 0.6
new concrete	0.4 to 0.5
old concrete	0.05 to 0.15

3.2 Reflectance of glazing for solar radiation

Glazing (window glass) has specific optical properties:

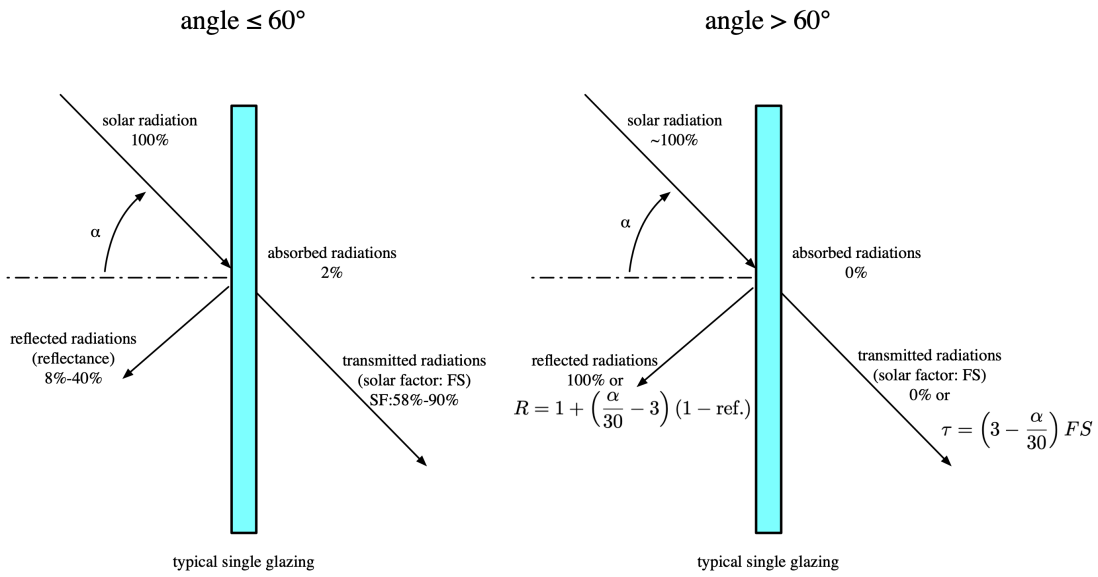
- It is mostly transparent to visible light (depending on the glass type)
- It is generally opaque to infrared radiation



However, representing it as a single coefficient is an approximation, as the actual transmittance varies with the angle of incidence of solar radiation.

Repartition of solar power (short waves reaching Earth):

- near-IR: 49%
- visible: 43%
- UV: 7%
- X-ray, gamma: 1% absorbed by atmosphere



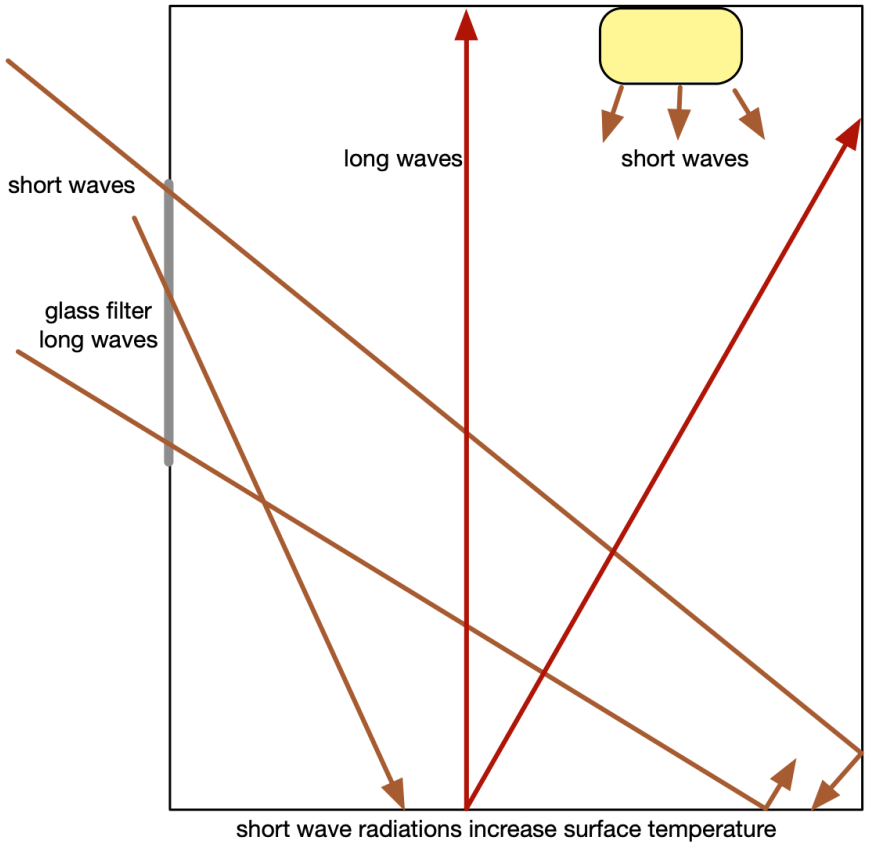
Solar irradiance reaching Earth and passing through standard glazing: 99%~100%

3.3 Solar Factor (SF) through glazing

The Solar Factor (SF) measures how much solar energy passes through or is absorbed by the glazing.

$g \approx 0.75$: conventional clear double glazing
$g \approx 0.60$: modern low-e double glazing
$g \approx 0.50$: modern low-e triple glazing
$g \approx 0.35$: solar-control glazing
$g \approx 0.20 - 0.25$: strong solar-control façade glazing

Exercise: identify the type of wave length of each beam



Explain why interior blinds does not significantly protect from the heat of solar radiations.

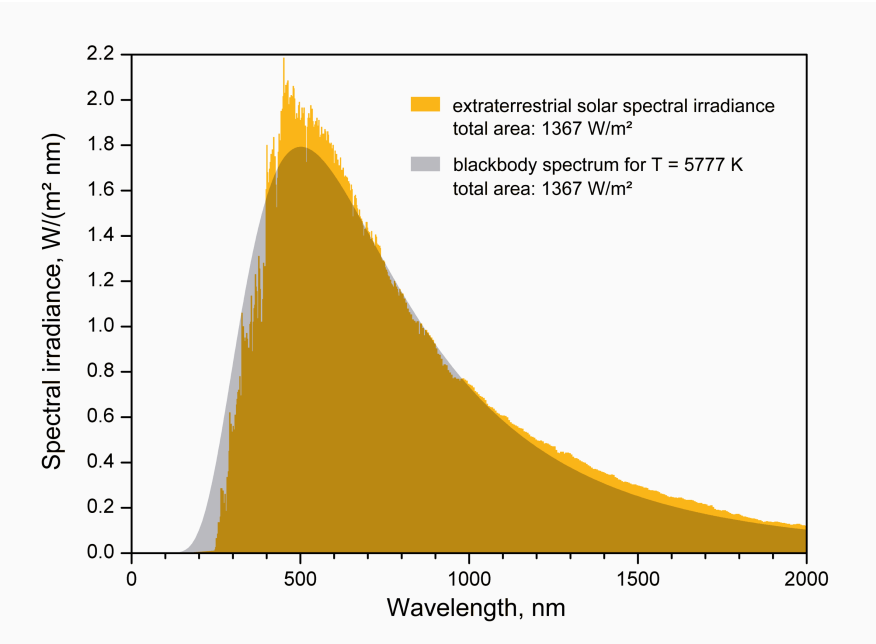
4. Radiation sources and bodies

4.1 Black body

A **black body** is an **ideal emitter** who absorbs all the incident beams.

This is equivalent to:

- no reflection
- no transmission



Spectrum comparison of a theoretical black body at 5777K and the solar radiance

Note: the sun temperature is about at 5772K

Note: In reality, no object is a pure black body although most have a close behavior.

4.1.1 Energy emission of a black body

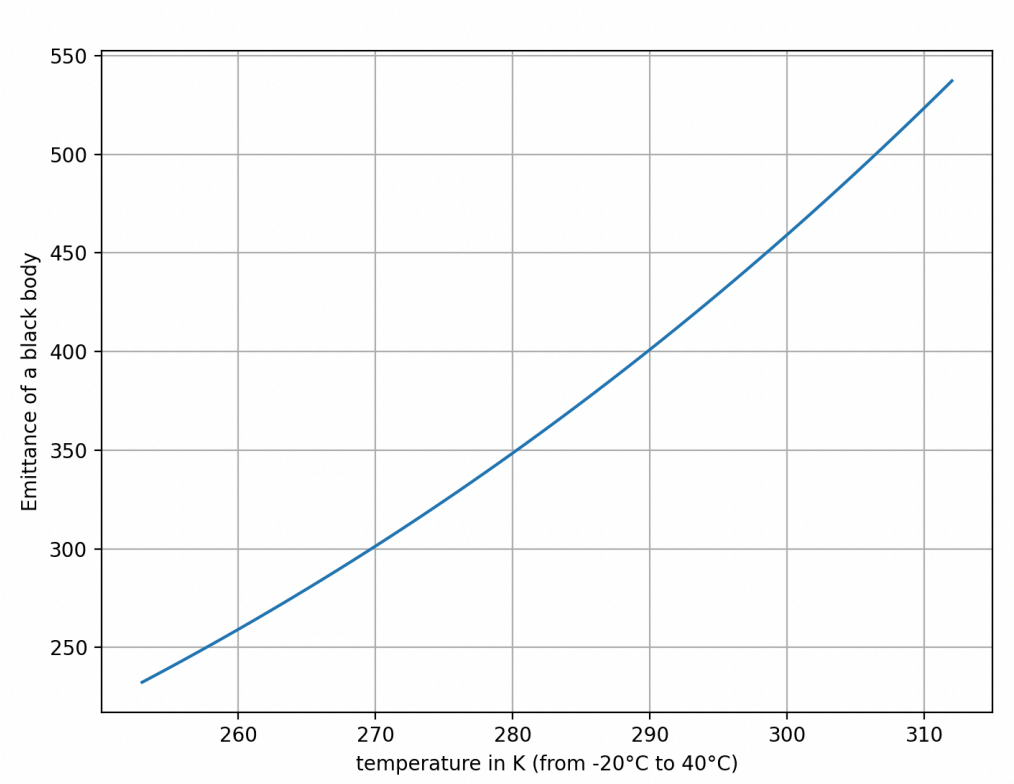
A black body emits:

$$E = \sigma T_{Kelvin}^4 [W/m^2]$$

where

- $T_{Kelvin} = (T_{celsius} + 273.15^\circ C)$
- σ is the Stefan-Boltzmann constant $\sigma = 5.670374419 \times 10^{-8} W/m^2/K^4$.

4.1.2 Black body energy emission in function of temperature



Linearized version of the black body energy emission in function of temperature.

All the incident radiations are aborbed.

4.2 Grey body

A **grey body** absorbs a part of the incident beams.

The amount of energy absorbed *depends on the temperature*.

The amount of energy absorbed *does not depend on the wave length*.

Note: Most materials are considered as piece-wise grey bodies__ where pieces depends of wave length intervals.

4.2.1 Energy emission of a grey body

Grey bodies emit only a fraction ϵ of black body radiation, where ϵ is the emissivity:

$$E = \epsilon \sigma T_{\text{Kelvin}}^4$$

with the emissivity $\epsilon \in [0, 1]$.

Note: On the right, we can see the emissivity of some materials. The file propertiesDB.xlsx provide much more values. Note that most values are closed to $\approx$ 0.9 except for metals

Materials	Nominal	Polished	Oxidized
Aluminum		0.05	0.15
Brass		0.09	0.60
Cast Iron		0.21	0.70
Copper		0.02	0.60
Galvanized		0.02	0.60
Glass	0.94		
Stainless Steel		0.17	0.85
Steel		0.11	0.75
Rubber	0.86-0.95		
Wood	0.95		

Infrared thermo-meters assume an emissivity of 0.9: the measured temperature is true for most materials but fake for metallic surfaces and low emissivity materials.

4.2.2 Grey body energy emission in function of the wave length - Planck's law

Plank's law gives the spectral radiance of a grey body in function of the wave length:

$$B(\lambda, T_{Kelvin}) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T_{Kelvin}}} - 1}$$

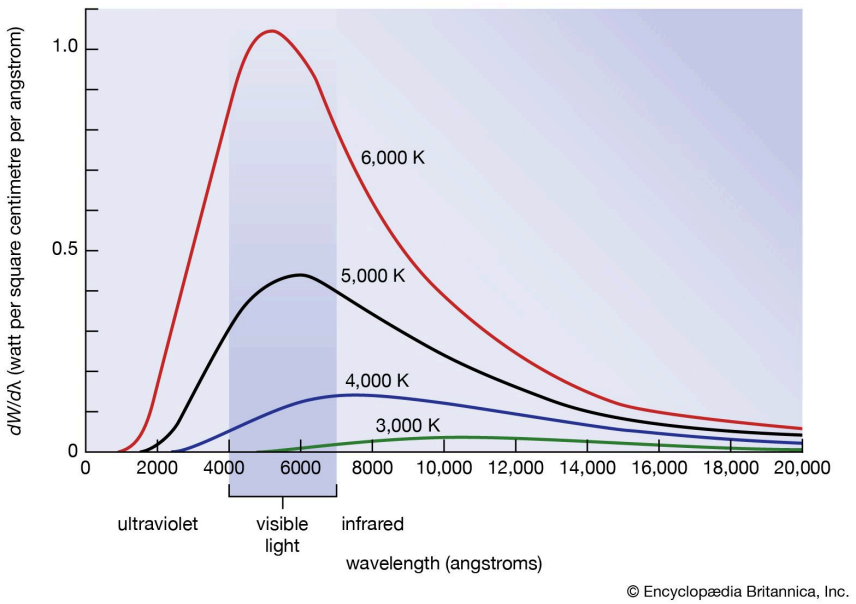
where

- k_B is the Boltzmann constant ($k_B = 1.38 \times 10^{-23} J/K$), which is different from the Stefan-Boltzmann constant,
- $B(\lambda, T_{Kelvin})$ is the spectral radiance expressed in $W/m^2/sr/Hz$ and other notations have been introduced previously.

4.2.3 Grey body energy emission in function of the wave length

Note: The emitted energy in function of the wave length is $E = \int_\lambda E(\lambda) d\lambda$

Note: As the temperature of a black body decreases, the emitted thermal radiation decreases in intensity and its maximum moves to longer wavelengths.



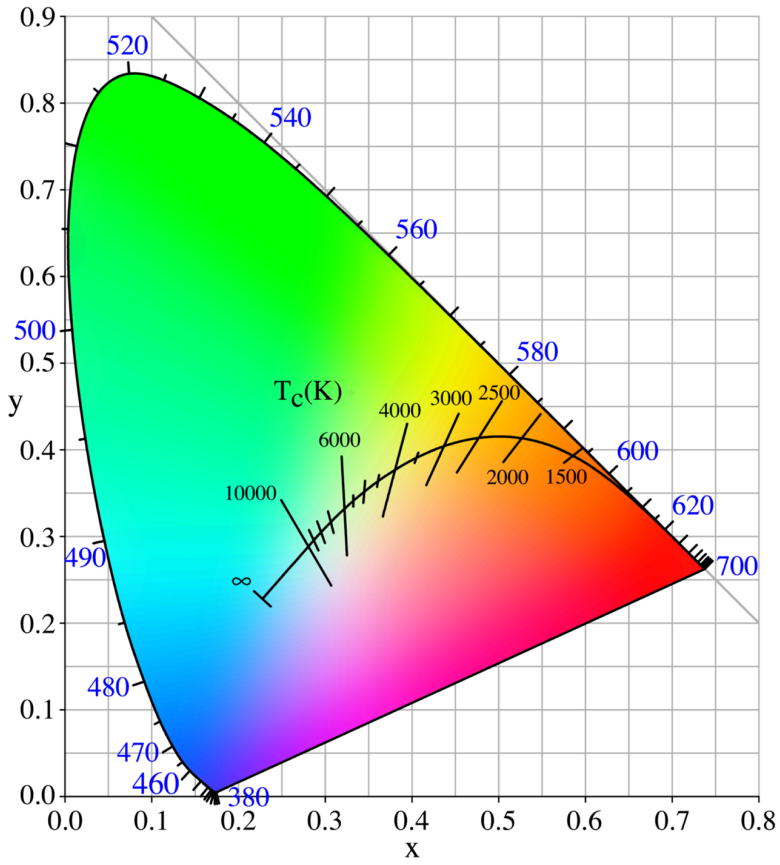
Frequency spectrum of the black body radiation, in function of the wave length and the temperature

4.2.4 Grey body energy emission in function of the wave length - Wien displacement law

The color of blackbody radiation scales inversely with the temperature of the black body.

These colors are often used as a color scale (in photography lighting for instance): it can be obtained by the approximate Wien displacement law that shows that the maximum spectral radiance depends only on the temperature of a black body:

$$\lambda_{max} = \frac{hc}{5k_B T} \approx \frac{2.898 \times 10^{-3}}{T_{Kelvin}} \text{ in } \mu m$$



4.2.5 Color temperature

Note: Color temperature is a parameter describing the color of a visible light source by comparing it to the color of light emitted by an idealized opaque, non-reflective body called black body.

The temperature of the ideal emitter that matches the color most closely is defined as the color temperature of the original visible light source. Color temperature is usually measured in Kelvins. The color temperature scale describes only the color of light emitted by a light source, which may actually be at a different (and often much lower) temperature.

4.3 White body

A **white body** reflects all the incident radiations falling on it.

It is an **ideal reflector**.

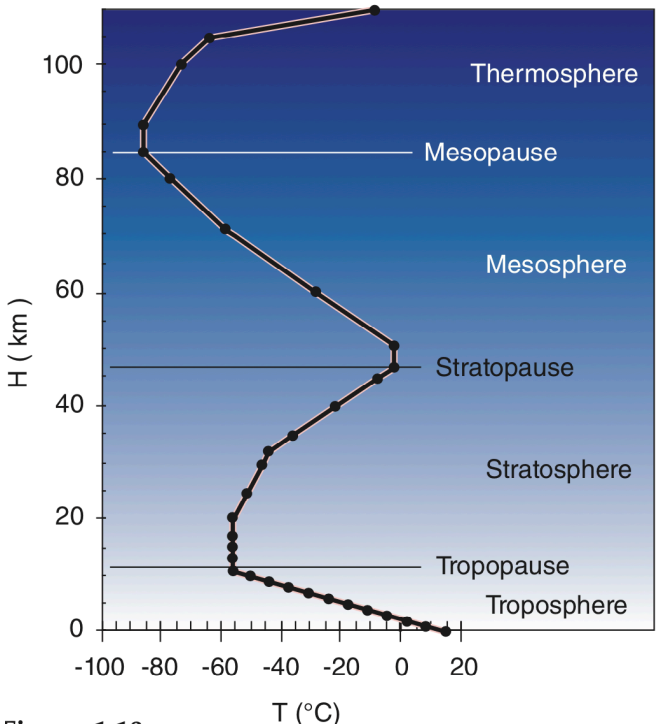
4.4 Example: sun as a radiative source

Sun's characteristics

The Sun behaves like a **black body** at 5772K.

The atmosphere is not transparent to all wavelengths, as absorption and scattering mechanisms modify the spectrum.

Note: The thicker the atmosphere, the greater the attenuation.



4.4.1 Earth's orbit

Earth's orbit is elliptical.

- Minimum Sun-Earth distance considering planet apexes (Perihelion): $\check{D} = 147.1 \times 10^9$ m
- Mean Sun-Earth distance: $D = 149.6 \times 10^9$ m
- Maximum Sun-Earth distance (Aphelion): $\hat{D} = 152.1 \times 10^9$ m
- Sun equatorial radius: $R = 696 \times 10^6$ m
- Temperature of the photosphere: $T = 5772$ K
- Earth radius: $r = 6.38 \times 10^6$ m

4.4.2 Atmospheric effects

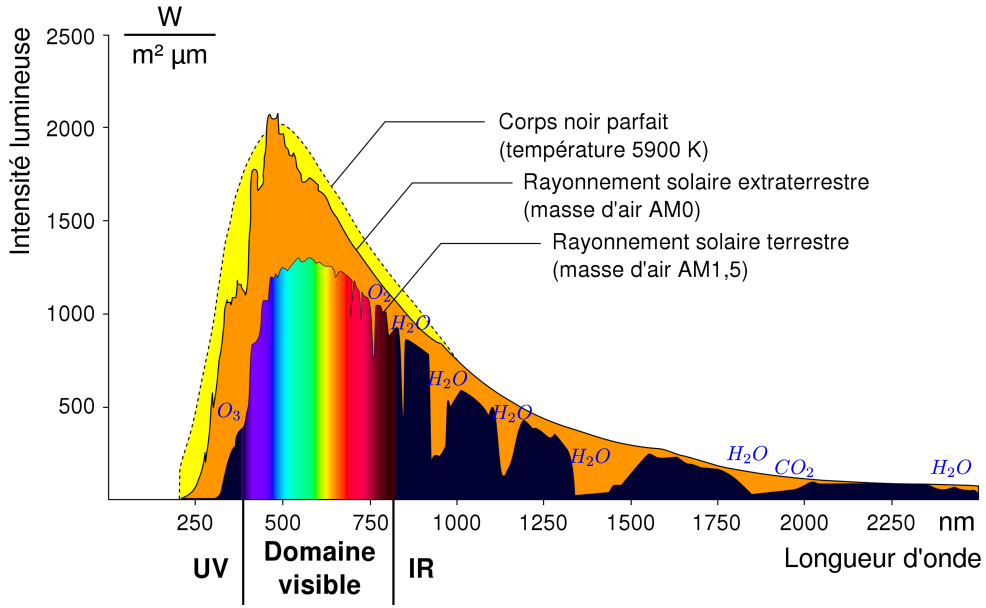
The Earth behaves like a **grey body**.

Certain gases in the atmosphere absorb specific wavelengths, which are very strongly attenuated in the solar radiation reaching the ground: this is the case of water vapour, which absorbs several wavelengths, as well as nitrogen, oxygen and carbon dioxide.

The solar spectrum at ground level is therefore narrowly restricted to wavelengths between the far infrared and the near ultraviolet.

We can see the effects of different greenhouse effect gas. Note that the water is the main cause of modification of the ideal black body spectrum.

Most of the UVs do not pass through the atmosphere.

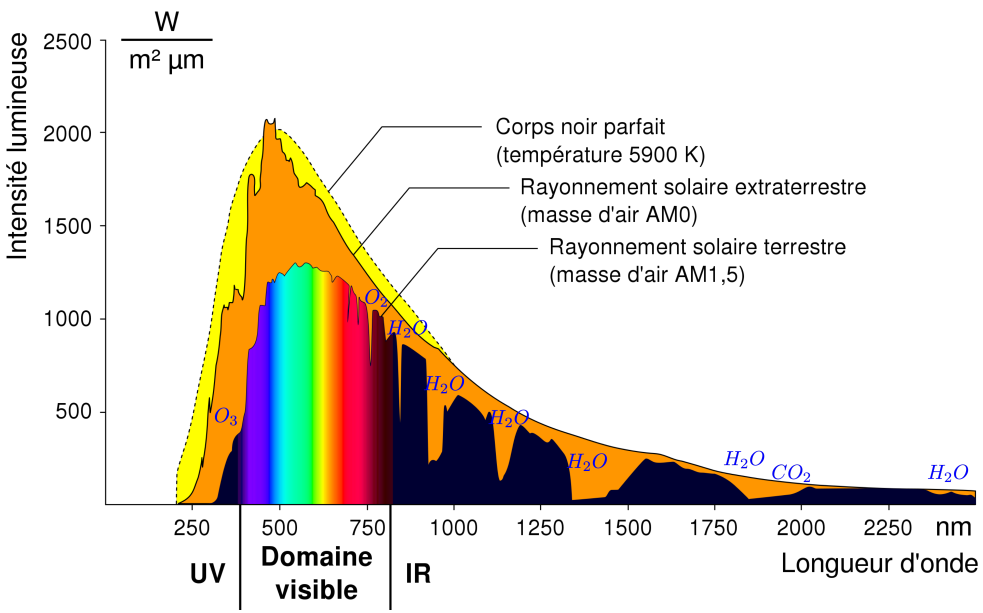


Most of the energy is located in the visible range of the spectrum but still there is energy in the infrared range.

4.4.2 Atmospheric effects

AM_0 and $AM1.5$ represent the optical paths, depending on the Sun altitude and followed by the solar beams with (L_0 is the atmosphere thickness):

$$AM_x = \frac{L}{L_0}$$



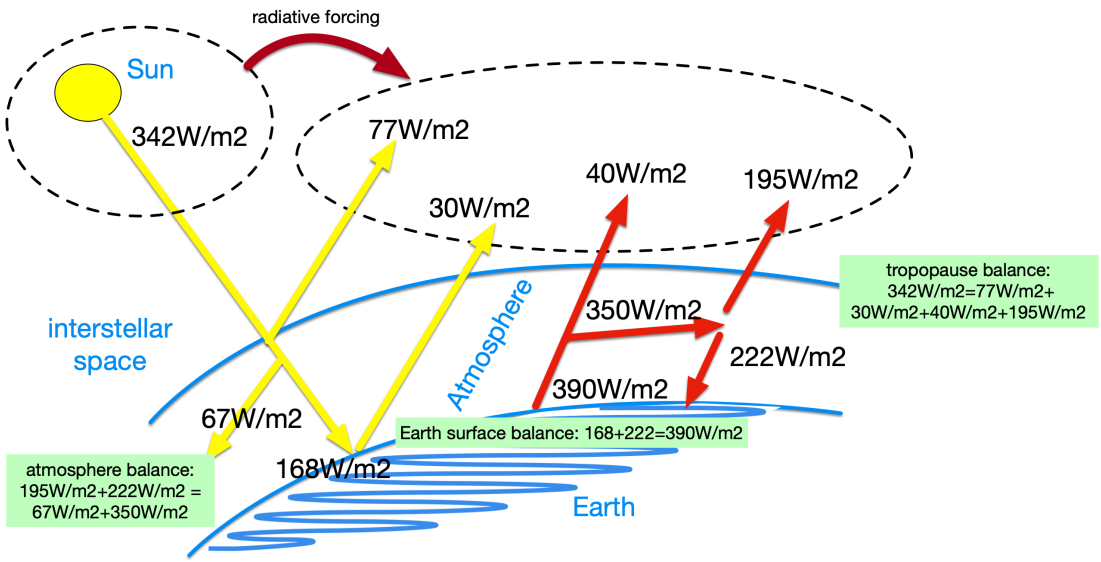
4.4.3 Energy balances

The yellow arrows represent short wavelength visible radiations, while the red arrows represent long wavelength infrared radiations.

The values shown are daily average power measurements.

Three energy balances that must be satisfied in the Earth's climate system:

1. At the Earth's surface
2. Within the atmosphere
3. At the tropopause (boundary between troposphere and stratosphere)



Different radiations appearing in the Solar-Earth system.

4.4.4 Energy calculations

Considering Sun as a black body, the solar radiant exitance is given:

$$M_{Sun} = \sigma T^4 \text{ in } [W/m^2]$$

where $\sigma = 5.670 \times 10^{-8}$ is the Stefan-Boltzmann constant

The total emitted solar power is:

$$L_{Sun} = 4\pi R^2 M_{Sun} = 4\sigma\pi R^2 T^4 [W]$$

4.4.5 Energy calculations

The total solar power emitted by Sun per surface unit at the distance of Earth is given by:

$$P_D = \frac{L_{Sun}}{4\pi D^2} = \sigma \frac{R^2}{D^2} T^4 [W/m^2]$$

The total power received by Earth is:

$$P_{Earth} = \pi r^2 P_D = \frac{\sigma \pi r^2 R^2 T^4}{D^2} [W]$$

4.4.6 Results

Solar irradiance of Earth at:

- shortest distance (Perihelion): 180 petaW (1 peta Watt =$10^{15} W$)
- mean distance: 174 petaW
- longest distance (Aphelion): 169 petaW

Notes:

- *The variation of the energy received from Sun of* $\pm 3\%$.
- *The declination of the Earth rotation axis has a much bigger influence and yields seasons.*

5. Radiometry and photometry

5.1 Basic concepts

Photometry is used to evaluate light radiation as perceived by human vision, and to study the transmission of this radiation from a quantitative point of view.

Radiometry measures all electromagnetic waves (from γ-rays to radio waves).

Radiometry concepts are usefull when working with energies.

Radiometry is focused on the visible and infrared band-passes: they correspond to the 2 ranges of temperatures met in the Sun-Earth system.

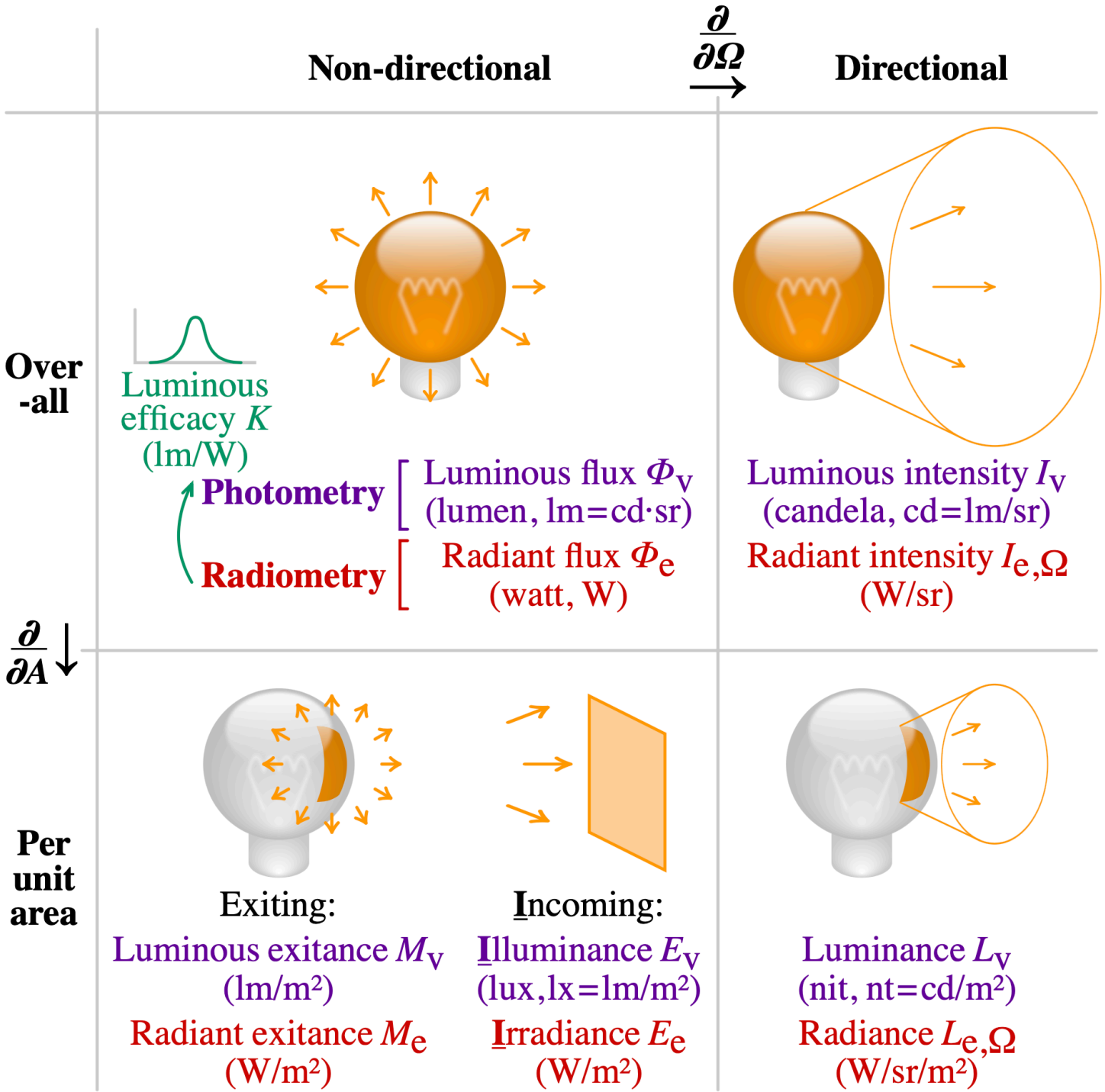
Note: For both perspectives, different names are sometimes used dealing with the same concept: units are useful to clarify confusions.

5.1 Basic concepts

Primary sources - objects/bodies **emitting** radiations

Secondary sources - objects/bodies **reflecting** radiations

Receptors - objects/bodies **absorbing** radiations. (secondary sources are also receptors); A black body is a *pure receptor*.



Non-directional
$\frac{\partial}{\partial \Omega}$
Directional
Over -all
Luminous efficacy K (lm/W)
Photometry
Luminous flux Φ_v (lumen, lm=cd·sr)
Luminous intensity I_v (candela, cd=lm/sr)
Radiometry
Radiant flux Φ_e (watt, W)
Radiant intensity $I_{e,\Omega}$ (W/sr)
$\frac{\partial}{\partial A}$
Per unit area
Exiting:
Luminous exitance M_v (lm/m²)
Radiant exitance M_e (W/m²)
Incoming:
Illuminance E_v (lux,lx=lm/m²)
Irradiance E_e (W/m²)
Luminance L_v (nit, nt=cd/m²)
Radiance $L_{e,\Omega}$ (W/sr/m²)

5.1 Basic concepts

photometry / radiometry	photometry	radiometry	comment	symbols (radiometry)
luminous flux / radiant flux	lumen (lm) or candela.steradian (cd.sr)	Watt	total power emitted by a radiative source in all directions	φ or Φ
luminous / radiant intensity (in a specific direction)	candela (cd)	Watt/sr	power intensity of a radiative source emitted in a solid angle directed by $\overrightarrow{u}$	I or Ω
luminance / radiance	candela/m^2 (cd/m^2)	Watt/sr/m^2	power intensity in a specific direction $\overrightarrow{u}$ emitted by a unit surface	L
exitance / emittance	$W.m^2/Hz$	Watt/m^2	light radiated per unit of surface in all directions	M (or E)
illuminance / irradiance	lux (1lm/m^2)	Watt/m^2	light received by a surface from all directions	E

5.2 Notations

Radiometric variables - denoted X

Photometric variables - denoted X_ν when related to the visible light spectrum and X_λ when related to a specific wavelength λ

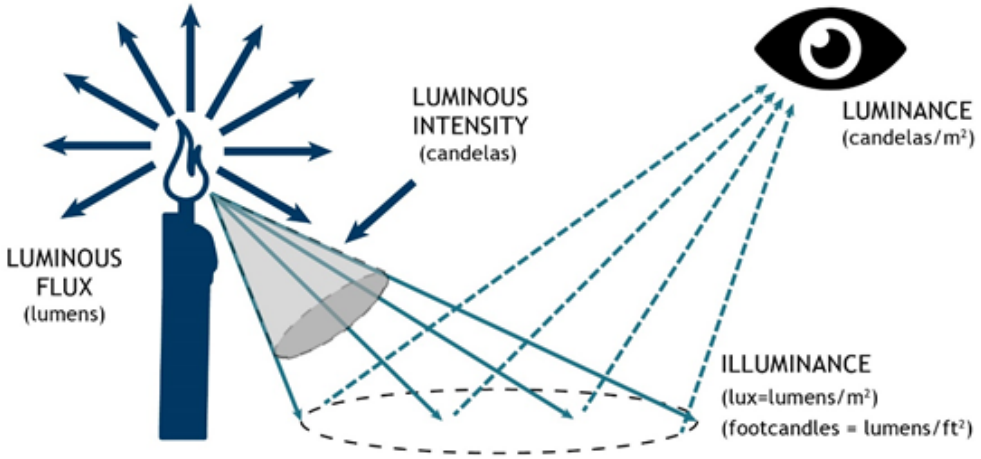
It satisfies: $X = \int X_\lambda d\lambda$ and $X_\nu = \int_\nu X_\lambda d\lambda$.

Irradiance expressed in W/m^2 becomes irradiations corresponding to Wh/m^2 when it deals with energy.

5.3 Luminous flux

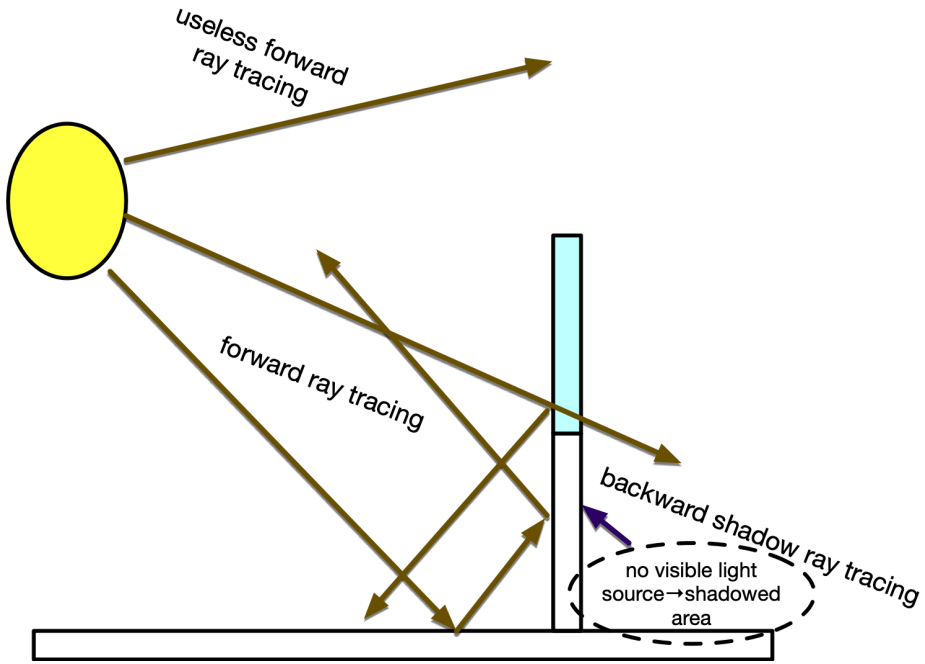
Luminous flux (radiometry)/radiant flux (photometry) - φ or Φ [W]

- represents the **total power** emitted by a source of surface S in all directions.



5.4 Radiant intensity

The **radiant intensity** (I) of a radiation corresponds to the quantity of energy emitted by a surface per unit of solid angle directed by a direction $\overrightarrow{u}$ and per unit of emitting surface perpendicular to $\overrightarrow{u}$.



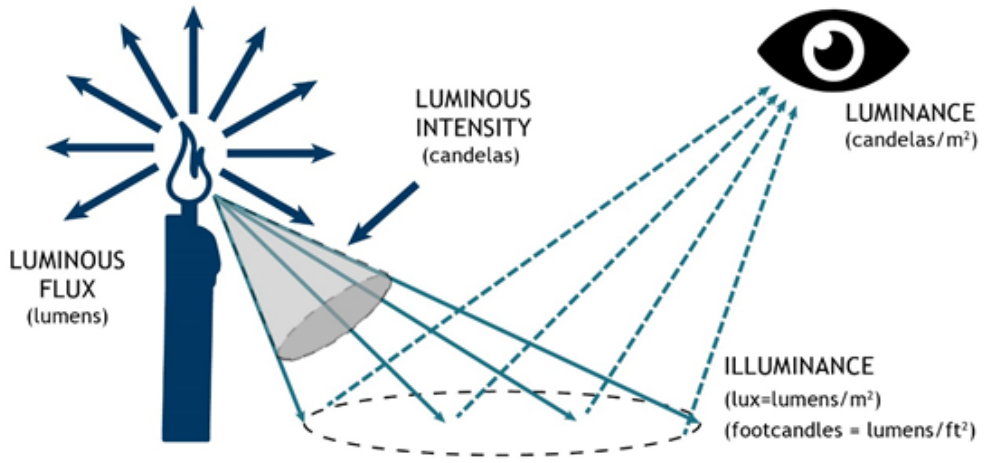
*Note: Software like **Radiance** can model the impact of radiations in a complex scene using a **ray tracing mechanism** lying on a forward propagation mechanism, starting from a radiation source and a backward mechanism following the enlightened object.*

5.4 Radiant intensity

Radiant intensity (radiometry)/luminous intensity (photometry) - I or Ω [W/sr]

The candela (cd) is the light intensity, in a given direction, of a source emitting monochromatic radiation of frequency $540.10^{12} Hz$ (555nm, $\lambda = c/f$ where $c = 299792458 m/s$) and whose energy intensity in this direction is 1⁄683 Watt/steradian.

The frequency chosen corresponds to a wavelength of 555 nm, in the green-yellow part of the visible spectrum, which is close to the human eye's maximum sensitivity to daylight.



1 lumen (lm) corresponds to the luminous flux emitted within a solid angle of 1 steradian (sr) by an isotropic spot light source located at the summit of a solid angle, with a luminous intensity of 1 candela. We have 1 ln = 1 cd.sr.

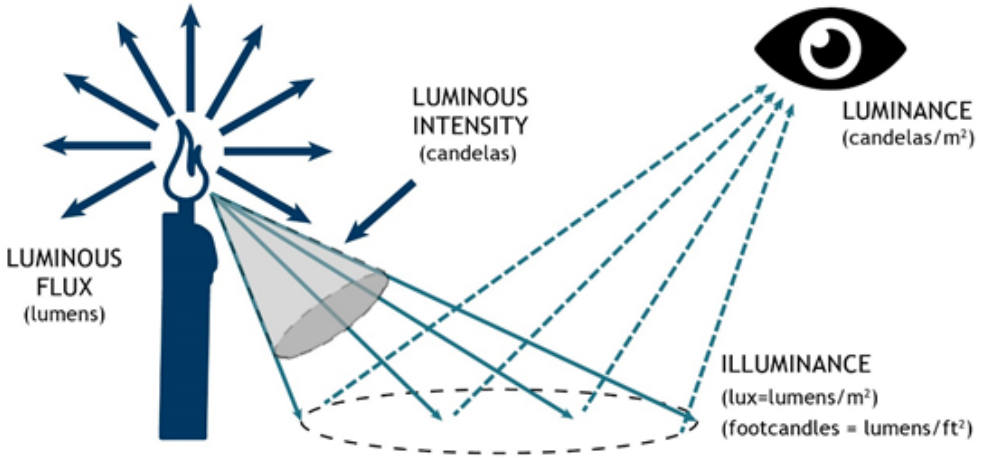
5.4 Radiant intensity

The power emitted by a surface S in the direction of a solid angle $d\Omega$ directed by $\overrightarrow{u}$ is denoted $d\varphi_{\overrightarrow{u}}$

The power intensity in a direction $\overrightarrow{u}$ is defined as the power emitted by a surface S through a solid angle directed by $\overrightarrow{u}$:

$$L_{\overrightarrow{u}} = \frac{d\varphi_{\overrightarrow{u}}}{d\Omega}$$

We have $\varphi = \int_S d\varphi = \int_\Omega d\varphi_{\overrightarrow{u}}$



Examples

Items	Lumens
Common Candle	14 lumens or so
sunset	400 lumens
Office Florescent light	400 lumens
Lighting on a movie set	1,000 lumens
100W Lightbulb	1,600 lumens
Sunny day	100,000 lumens

5.4.1 Radiance, emittance and irradiance

In radiometry, we distinguish between several key quantities:

- Radiance is the power emitted or *received* by a surface per unit solid angle per unit projected area. It is directional and depends on the angle of emission or reception.
- Emittance (or exitance) is the total power *emitted* by a surface per unit area, integrated over all directions. It is a scalar quantity.
- Irradiance is the total power *received* by a surface per unit area, integrated *over all directions*. It is also a scalar quantity.

Note: While radiance is directional, emittance and irradiance are integrated quantities (scalars). Emittance describes the total power leaving a surface, while irradiance describes the total power arriving at a surface. They are symmetric in the sense that, in any energy balance, the same amount of energy is emitted as received.

5.4.2 Radiance

Radiance (radiometry) / Luminance (photometry) - L or Ω [W/sr/m^2 or Candela/m^2]

- represents the flux emitted, reflected, transmitted or received by a given surface, per unit solid angle per unit projected area.
- it is a quantity corresponding to the visual sensation of brightness of a surface.
- luminance is the power of visible light passing through or being emitted from a surface element in a given direction, per unit area and per unit solid angle.

5.4.2 Radiance

The total power emitted by a surface dS directed by $\overrightarrow{n}$ through a solid angle $d\Omega$ directed by $\overrightarrow{u}$ is denoted $d^2\varphi$ or $d^2\varphi_{\overrightarrow{n},\overrightarrow{u}}$

The power emitted by a surface dS in all directions is:

$$d\varphi = \iint_{\Omega} d^2\varphi dS$$

5.4.2 Radiance

The power radiance is defined as the power intensity emitted by the apparent surface dS_1 in the direction $\overrightarrow{u}$:

$$L_{\overrightarrow{u}} = \frac{dL_{\overrightarrow{u}}}{dS_{\overrightarrow{u}}} = \frac{dL_{\overrightarrow{u}}}{dS\cos(\alpha)} = \frac{d^2\varphi_{:\overrightarrow{u}}}{d\Omega dS\cos(\alpha)}$$

A source is isotrope if its luminance is independent of the direction: $L_{\overrightarrow{u}} = L$.

5.4.3 Irradiance

- Irradiance (radiometry) / illuminance I (photometry) [lux or Watt/m^2]
- Illuminance (symbol I) is the total flux incident received by a surface, per unit area.
- It is a measure of how much the incident light illuminates the surface, wavelength-weighted by the luminosity function to correlate with human brightness perception in photometry.
- Irradiance is also named *Intensity of light* or *Power density* at a specific surface element dS:

$$I = \int_{2\pi} L_{\vec{u}} cos(\theta)$$

Examples

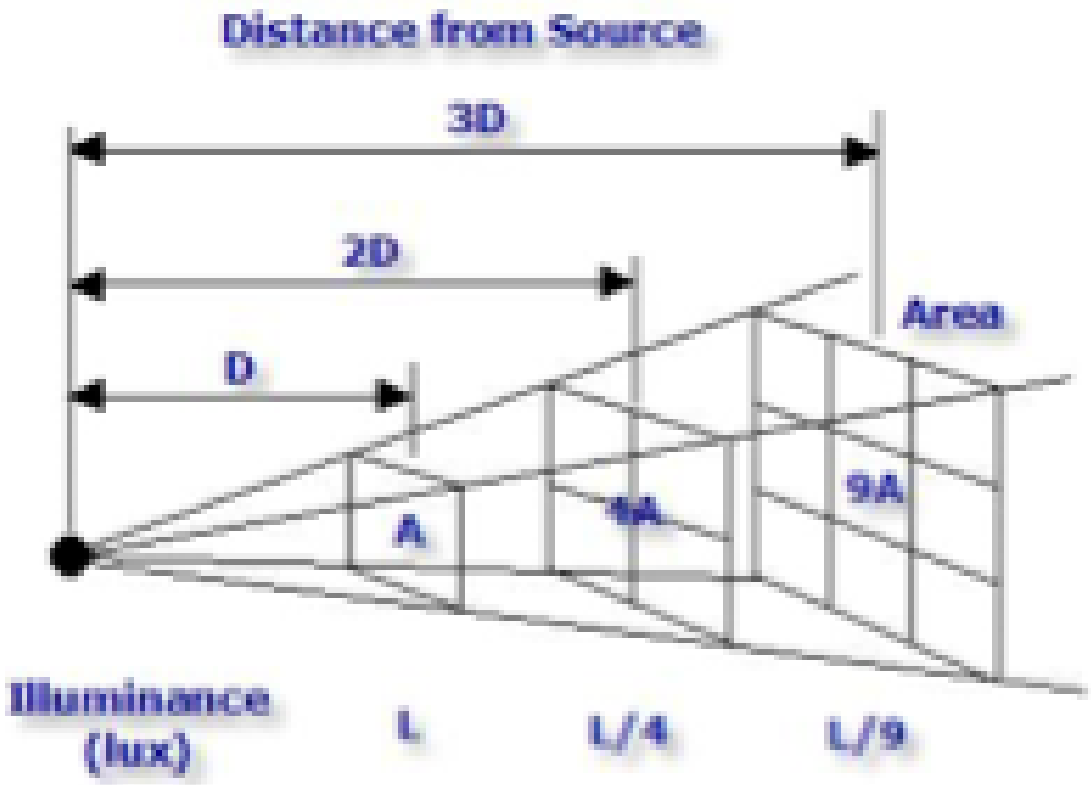
Lighting condition	Foot-candles	Lux
Sunlight	10,000 [6]	107,527
Shade on a sunny day	1,000	10,752
Overcast day	100	1,075
Very dark day	10	107
Twilight	1	10.8
Deep twilight	0.1	1.08
Full moon	0.01	0.108
Quarter moon	0.001	0.0108
Starlight	0.0001	0.0011
Overcast night	0.00001	0.0001

5.5 Solid angles

Radiations propagate along 3D angles.

As shown on the right, the quantity of radiation passing through the surface S at distance d, is the same than the quantity of radiation passing through the surface $4S$ at distance $2d$ and similarly the same than the quantity of radiations passing through a surface $9S$ at distance $3d$.

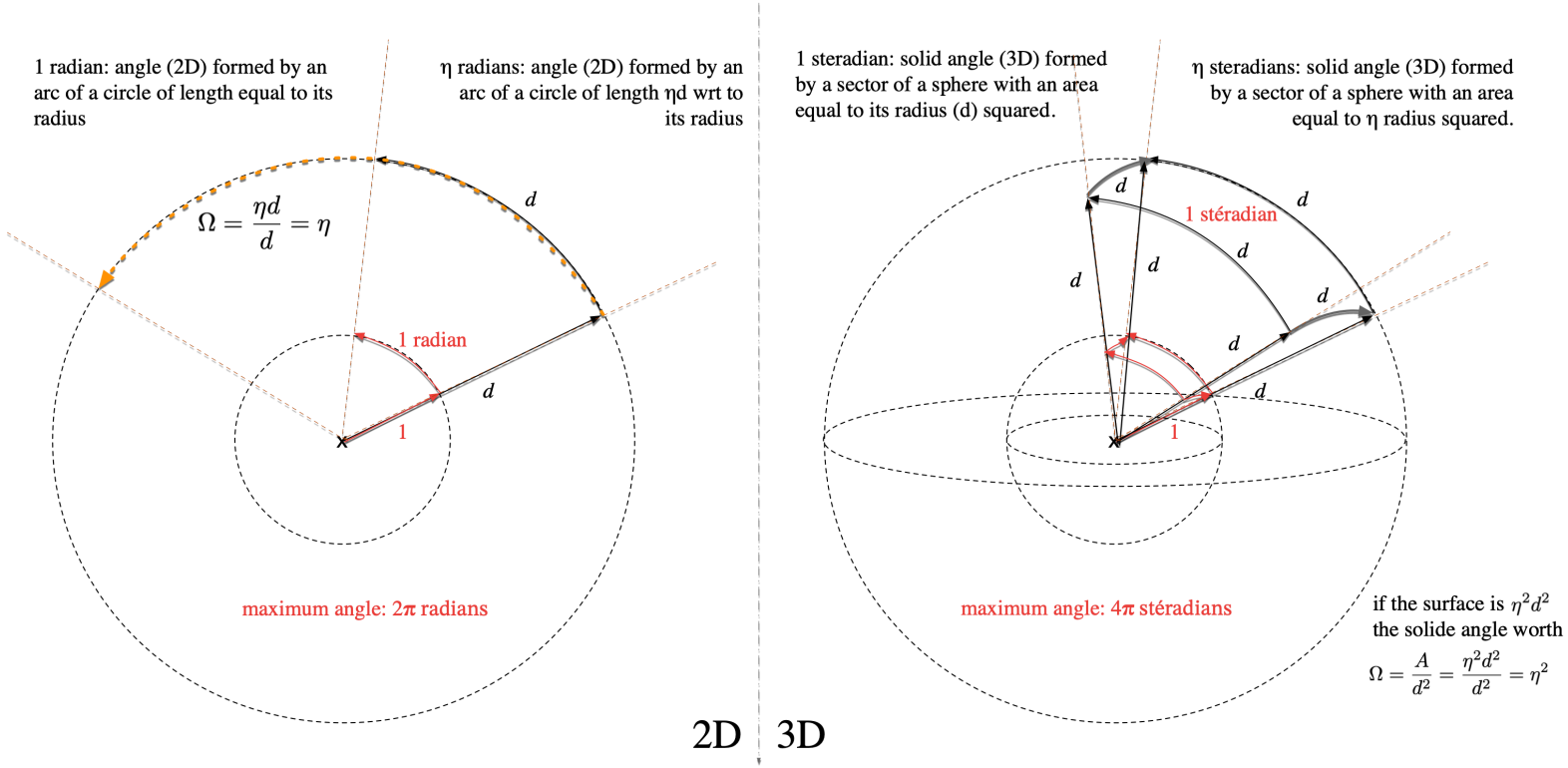
Angles in 3D space are crucial when analysing the energy transfer.



5.5.1 Basic concepts

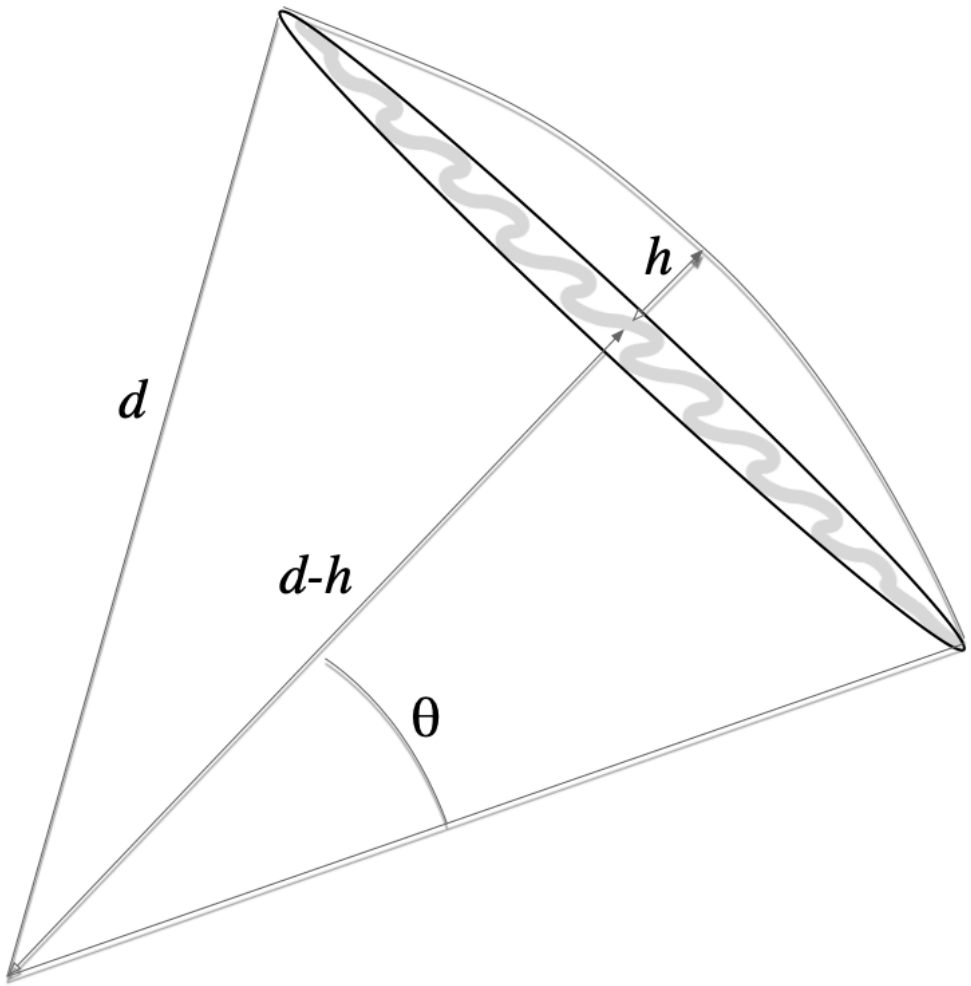
A solid angle is a measure of the amount of space in a 3D space that is visible from a point. It is a measure of the size of a portion of a sphere.

- In 2D, angles are measured in radians.
- In 3D, angles are measured in steradians.



5.5.1 Basic concepts

- A solid angle can be seen as the apparent surface of remote objects.
- From a global point of view, radiations rather propagates according to a cone



5.5.1 Basic concepts

The surface of the cap cut in the sphere by the cone is given by:

$$S = 2\pi dh$$

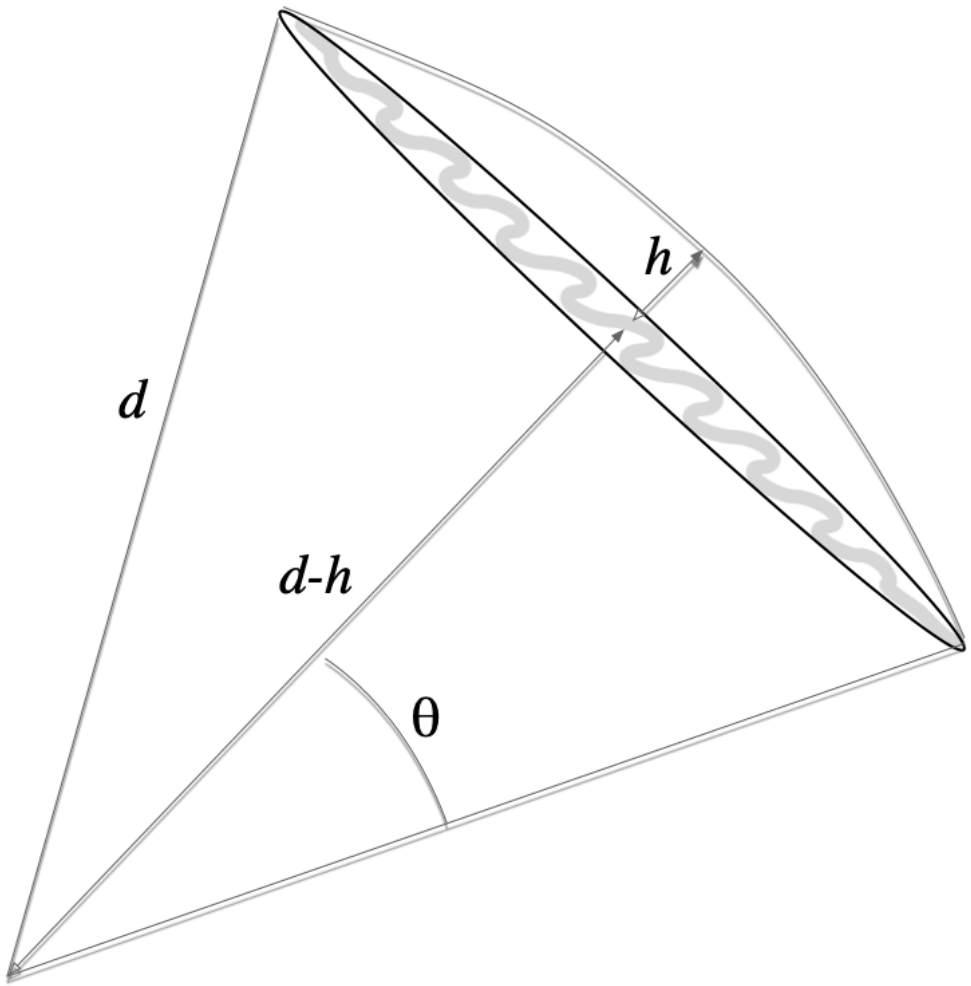
and the resulting solid angle is

$$\Omega = \frac{2\pi h}{R}$$

Introducing the angle θ thanks to $cos(\theta) = \frac{d-h}{d}$, which is equivalent to $\frac{h}{d} = 1 - cos(\theta)$.

It leads to:

$$\Omega = 2\pi(1 - cos(\theta))$$

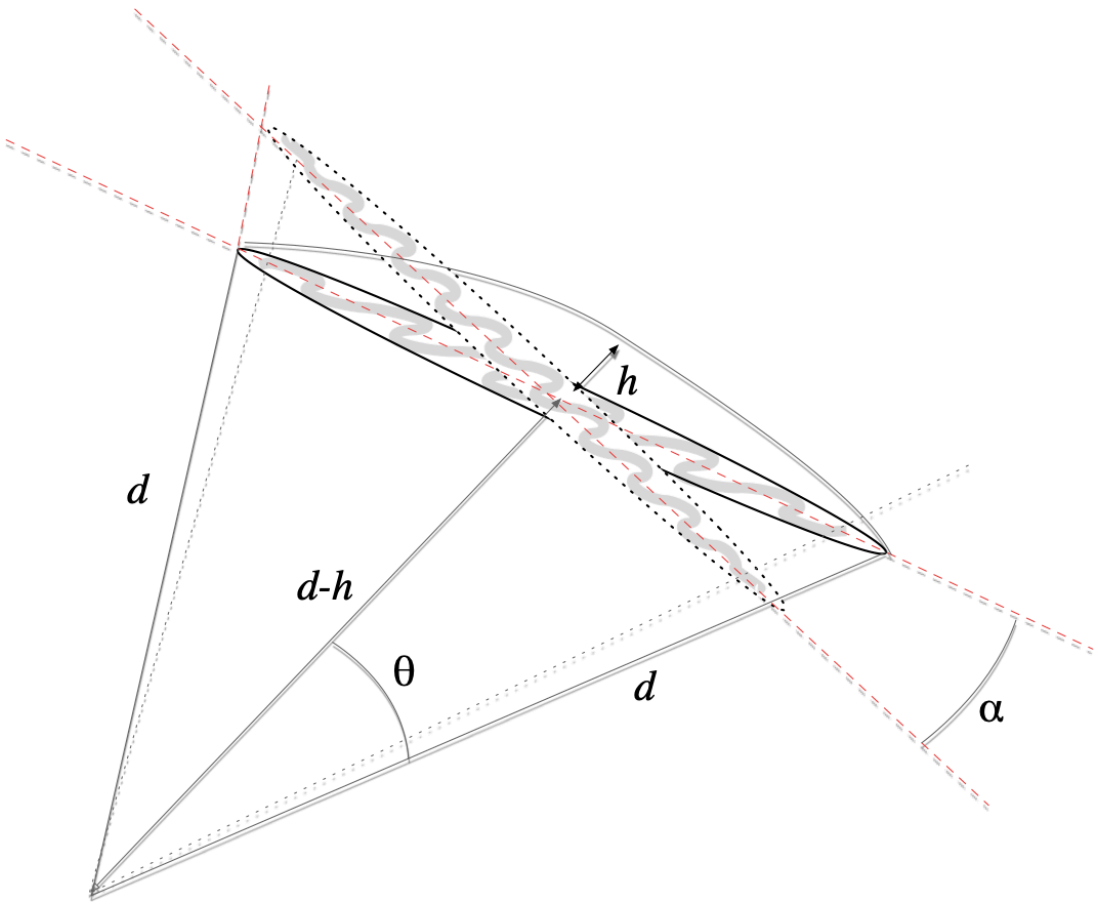


5.5.1 Basic concepts

Consider now that radiations hit a surface, which is not perpendicular to the propagation direction:

The solid angle is given by:

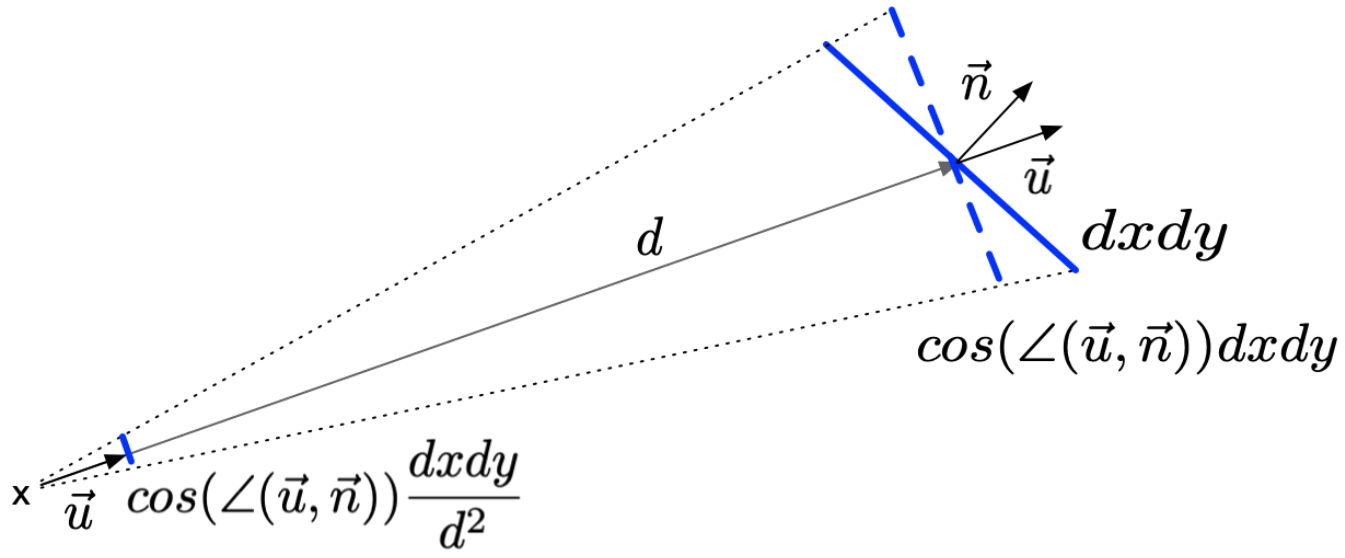
$$\Omega = \iint_S sin(\alpha) d\theta d\alpha$$



5.5.1 Basic concepts

A solid angle Ω is the apparent surface directed by $\overrightarrow{n}$ at a unary distance of a surface $dxdy$ located at a distance d along the direction $\overrightarrow{u}$:

$$d\Omega = \frac{dxdy}{d^2}\overrightarrow{u}.\overrightarrow{n} = \frac{dxdy}{d^2}cos(angle(\overrightarrow{u},\overrightarrow{n}))$$



5.5.1 Basic concepts

A solid angle $\Omega \in [0, 4\pi]$ is expressed in steradians (it is equal to 2π for an canonical hemisphere with a radius equal to 1 because the surface of a sphere is given by $4\pi r^2$).

For a cone starting from the origine of $\overrightarrow{u}$, whose half angle at the vertex if α, it leads to:

$$\Omega = 2\pi(1 - \cos(\alpha))$$

6. Solar System and Earth

Sun is our only power source

6.1 Sun irradiance breakdown

We start by defining the different irradiances.

The total solar irradiance (TSI) is a measure of the solar power over all wavelengths per unit area incident on the Earth's upper atmosphere.

- It is measured perpendicular to the incoming sunlight.
- In section (I.2) named Aphelion and Perihelion, the solar radiation reaching Earth is: $1367 W/m^2$.

6.1.1 Direct normal irradiance

The direct normal irradiance (DNI), or beam radiation, is measured at the surface of the Earth at a given location with a surface element perpendicular to the Sun direction.

- It excludes diffuse solar radiation (radiation that is scattered or reflected by atmospheric components).
- Direct irradiance is equal to the extraterrestrial irradiance above the atmosphere minus the atmospheric losses due to absorption and scattering.
- Losses depend on time of day (length of light's path through the atmosphere depending on the solar elevation angle), cloud cover, moisture content and other contents.
- The irradiance above the atmosphere also varies with time of year because the distance to the Sun varies, although this effect is generally less significant compared to the effect of losses on DNI.

6.1.2 Diffuse horizontal irradiance

The diffuse horizontal irradiance (DHI), or diffuse sky radiation is the radiation at the Earth's surface from light scattered by the atmosphere.

- It is measured on a horizontal surface with radiation coming from all points in the sky excluding circumsolar radiation i.e. radiation coming from the sun disk.
- There would be almost no DHI in the absence of atmosphere.

6.1.3 Global horizontal irradiance

The global horizontal irradiance (GHI) is the total irradiance from the Sun on a horizontal surface on Earth.

- It is the sum of direct irradiance (after accounting for the solar zenith angle of the Sun z) and diffuse horizontal irradiance: $GHI = DHI + DNI \times \cos(z)$

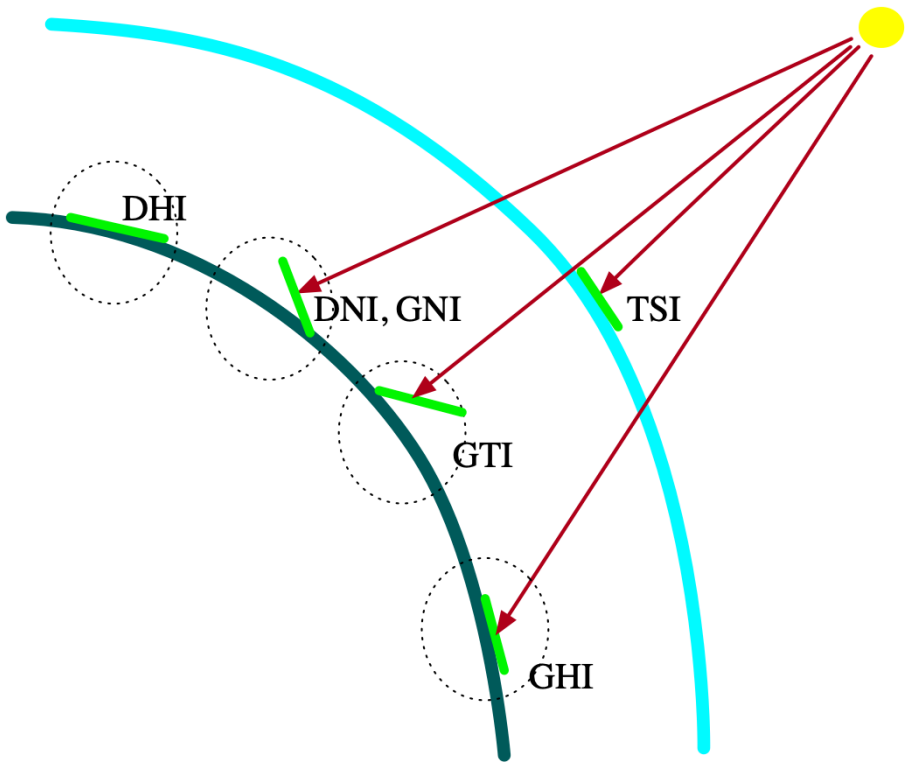
6.1.4 Global tilted irradiance

The global tilted irradiance (GTI) is the total radiation received on a surface with defined tilt and azimuth, fixed or sun-tracking. GTI can be measured or modeled from GHI, DNI and DHI.

- It is often a reference for photovoltaic power plants, while photovoltaic modules are mounted on the fixed or tracking constructions.

6.1.5 Global normal irradiance

The global normal irradiance (GNI) is the total irradiance from the Sun at the surface of Earth at a given location with a surface element perpendicular to the Sun.

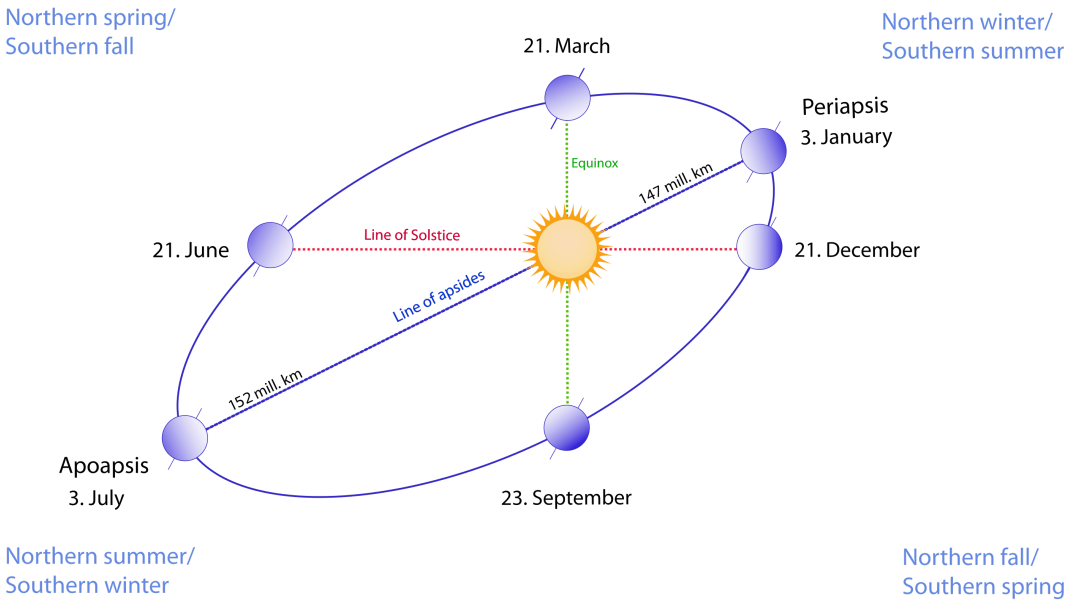


6.1.6 Total solar irradiance

- Every day, the earth receives a large amount of solar energy.
- The strength of this radiation reaching Earth depends on a number of factors, including weather conditions and atmospheric scattering (dispersion, reflection and absorption).
- At the sun's average distance from the earth, a surface normal to solar radiation (perpendicular to it) outside the atmosphere receives around $TSI = 1367 W/m^2$.
- This irradiance is called the solar constant: the solar constant expresses the amount of solar energy that a surface of $1m^2$ at the average distance from the Earth to the Sun, exposed perpendicular to the sun beams, would receive in the absence of an atmosphere.
- For the Earth, this is the energy flux density at the top of the atmosphere.

6.2 Solar angles

- Earth follows an elliptical orbit around the Sun
- Maximum Earth-Sun distance (aphelion): ~ 153×10^6 km (occurs around July 3rd)
- Minimum Earth-Sun distance (perihelion): ~ 147×10^6 km (occurs around January 3rd)
- This orbital variation causes **solar irradiance to fluctuate** by approximately $\pm 3.4\%$ annually
  - Maximum solar irradiance: ~1413 W/m² (January)
  - Minimum solar irradiance: ~1321 W/m² (June)



6.2.1 Azimuth and altitude

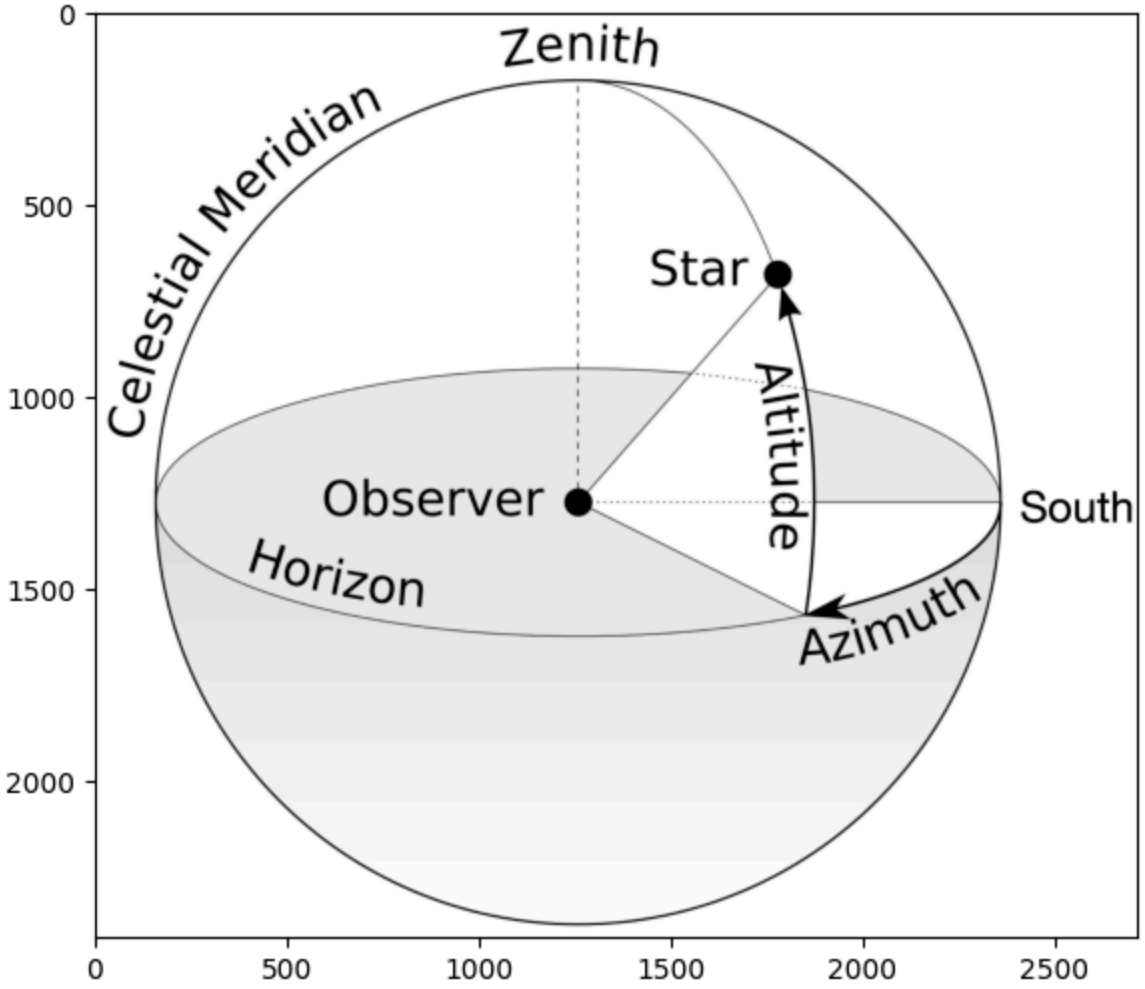
The position of the sun is represented by 2 angles:

- the azimuth, the angle formed by a vertical plan directed to the south and the vertical plan where the sun is i.e. the azimuth angle
- the altitude (or elevation sometimes) of the sun formed by the horizontal plan tangent to earth and the horizontal where the sun is with __0° means directed to the south, according to the buildingenergy convention.

Notes on azimuth:

- *East is negative and west positive.*
- *Altitudes 0° and 90° stand respectively for horizontal and vertical positions.*

6.2.1 Azimuth and altitude



6.2.1.1 Altitude

- The **altitude** of the Sun above the horizon is given by the solar altitude.
- The solar altitude changes continuously, its value is 0 at sunrise and sunset while its maximum is reached at **culmination height**.
- The culmination height is also different in every season:

```
altitude_rad = max(0, asin(sin(declination_rad) * sin(latitude_rad)
    + cos(declination_rad) * cos(latitude_rad) * cos(solar_hour_angle_rad)))
```

6.2.1.2 Azimuth angle

The **azimuth angle** of the Sun is the angle between solar rays and the North–South direction of the earth.

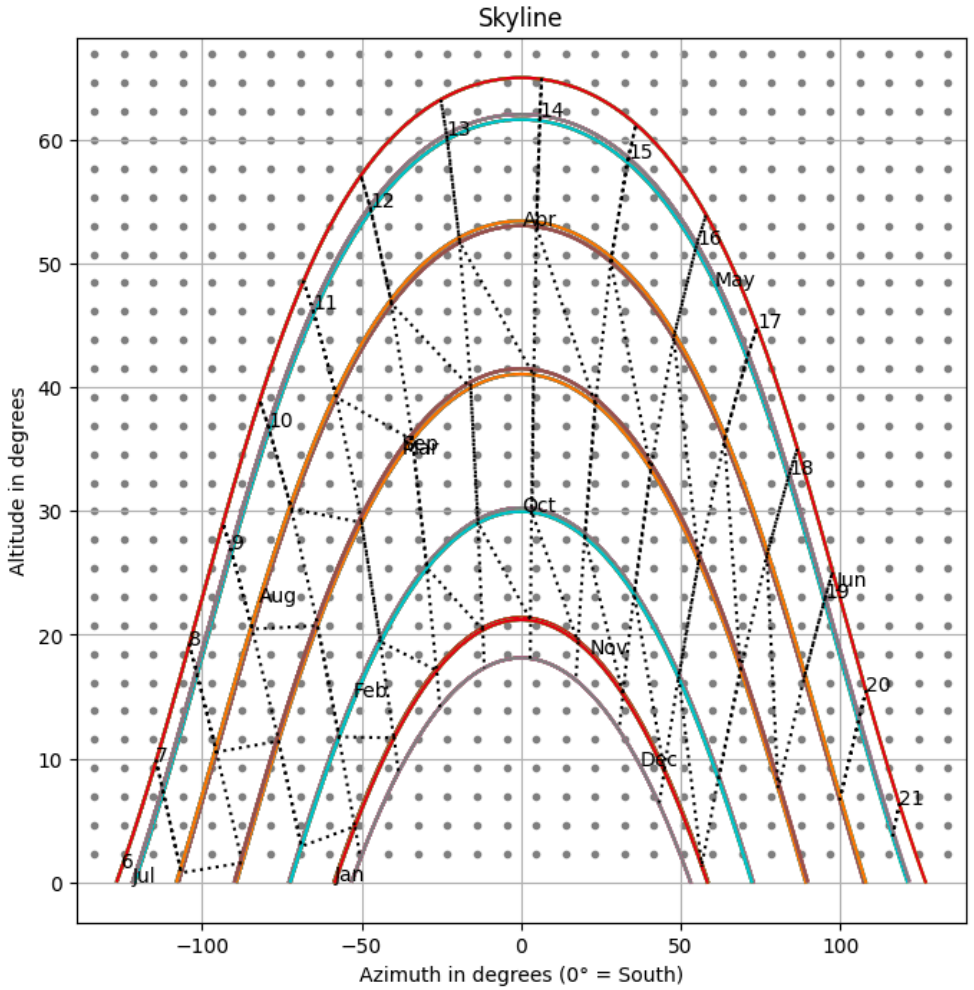
It is calculated based on the following equation for which we have to know:

- geographical latitude
- declination angle
- hour angle
- solar altitude

```
cos_azimuth = (cos(declination_rad) * cos(solar_hour_angle_rad) * sin(latitude_rad) - sin(declination_rad) * cos(latitude_rad))
     / cos(altitude_rad)
sin_azimuth = cos(declination_rad) * sin(solar_hour_angle_rad) / cos(altitude_rad)
azimuth_rad = atan2(sin_azimuth, cos_azimuth)
```

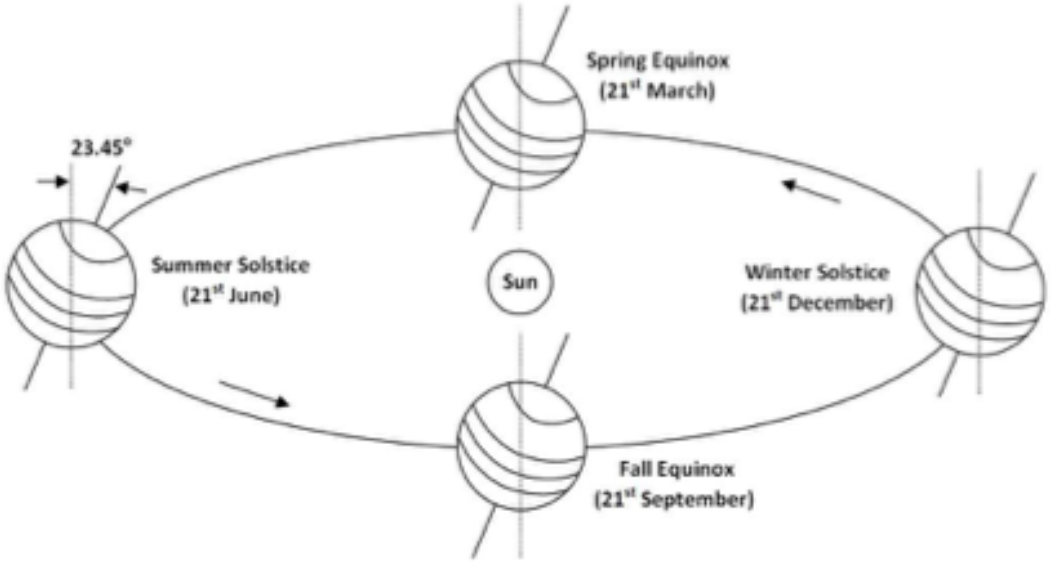
6.2.2 Plotting the azimuth and altitude angles in the heliodon

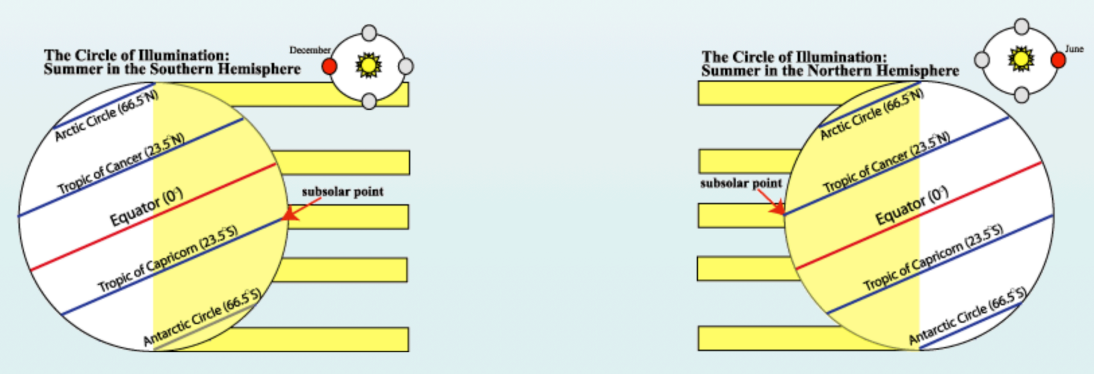
The **heliodon** is the the plotting of the azimuth and altitude angles in function of the local position (latitude and longitude) of the observer and the 21st of each month.



6.2.3 Declination

- The axis of Earth's rotation is inclined by a declination of 23.45° to the azimuthal axis
- *Azimuthal plane* - plane that intersects the centers of the Sun and Earth
- *Azimuthal axis* - perpendicular to the plane that intersects the centers of the Sun and Earth
- This inclination is responsible for the occurrence of seasons on Earth

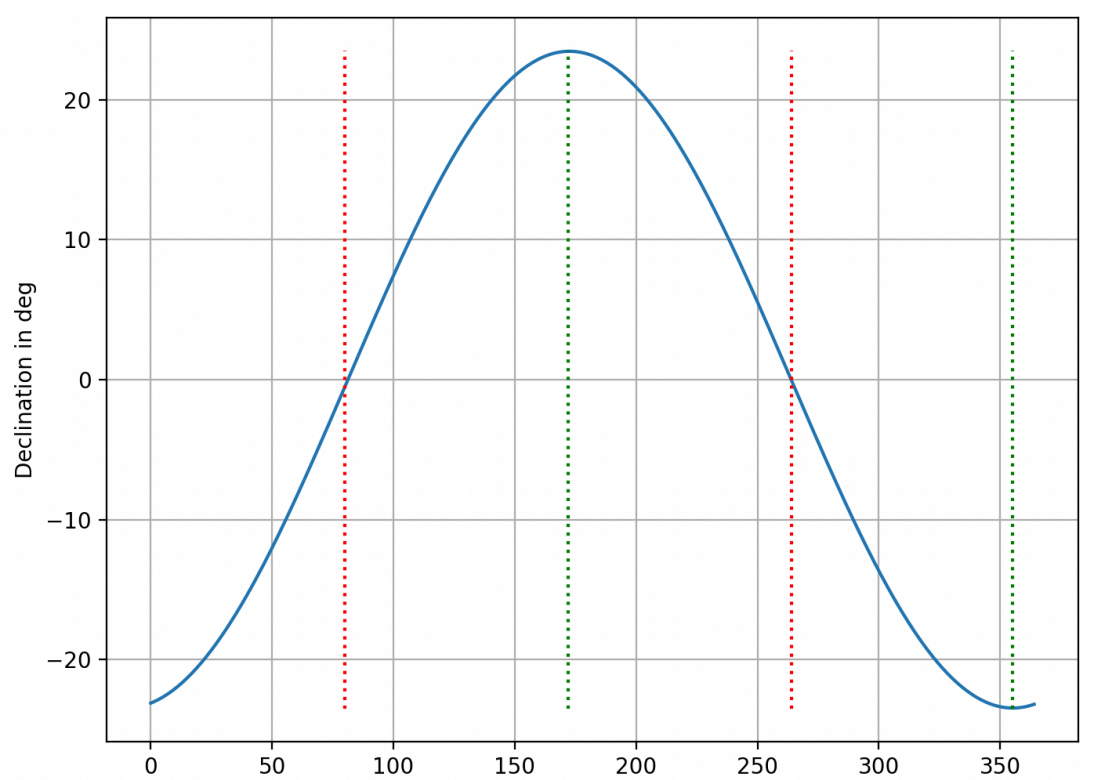


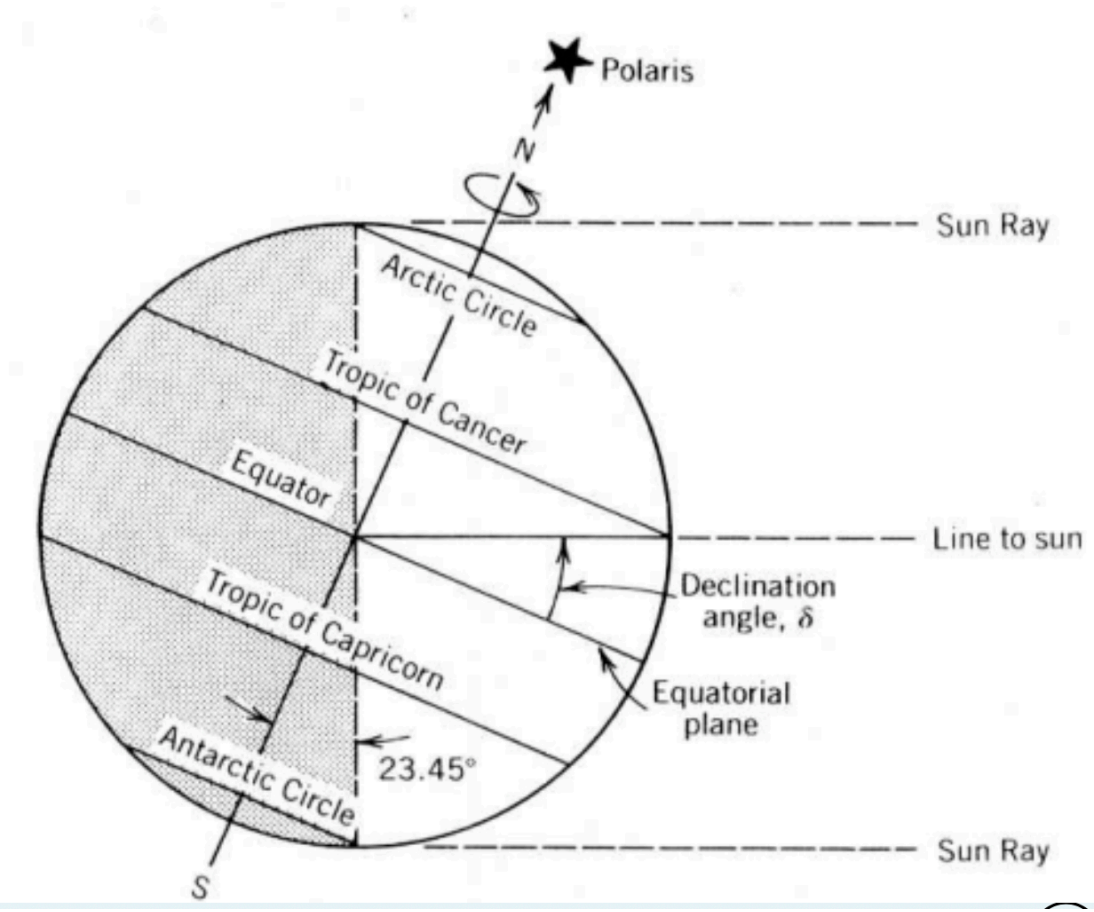


6.2.3 Declination

The solar declination angle is the latitude at which the Sun is directly overhead (Zenith) at solar noon.

```
declination_rad = 23.45/180*pi*sin(2*pi*(day_in_year+284) / 365.25)
```





6.2.4 Hour angle

- The **hour angle** describes the seeming movement of the Sun around Earth.
- Its value shows the seeming place of the Sun towards east or west compared to the local meridian.
- Its value is:
  - negative in the morning
  - positive in the afternoon
  - 0 when the Sun is right on the meridian

```
solar_hour_angle_rad = 2*pi * (solartime_secondes / 3600 - 12) / 24
```

6.3 Solar time and administrative time

Solar time is the time measured by the position of the sun in the sky.

Administrative time is measured by the rotation of the earth around the sun.

Solar time differs from **administrative time**: let's pass from one to another.

Let's first convert **administrative time in the day to the universal time coordinated (UTC) expressed in seconds_** i.e. the world reference time at Greenwich meridian.

```
utc_datetime = administrative_datetime.astimezone(utc)
utc_timetuple = utc_datetime.timetuple()
day_in_year = utc_timetuple.tm_yday

UTC_time_in_seconds = utc_timetuple.tm_hour * 3600 + utc_timetuple.tm_min * 60 + utc_timetuple.tm_sec
```

6.3.1 Local solar time

Then, the **_local solar time** is deduced :

```
local_solar_time_seconds = UTC_time_in_seconds + longitude_rad / (2*pi) * 24 * 3600
```

Examples:

- If you're in New York (approximately -74° longitude):
  - The adjustment would be negative (subtracting time)
  - Because the sun reaches New York before it reaches Greenwich
- If you're in Paris (approximately 2° longitude):
  - The adjustment would be positive (adding time)
  - Because the sun reaches Paris after it reaches Greenwich

6.3.2 True solar day

Time correction are applied to take into account **seasonal variation of sun speed**, **eccentricity of sun trajectory** and the **inclination of the earth axis**.

The **true solar day** is the time that elapses between two consecutive passages of the center of the solar disk in the plane of the same terrestrial meridian.

The corresponding rotation of the Earth is called a **synodic revolution**.

*Note: these **true solar days are unequal** for several reasons*:

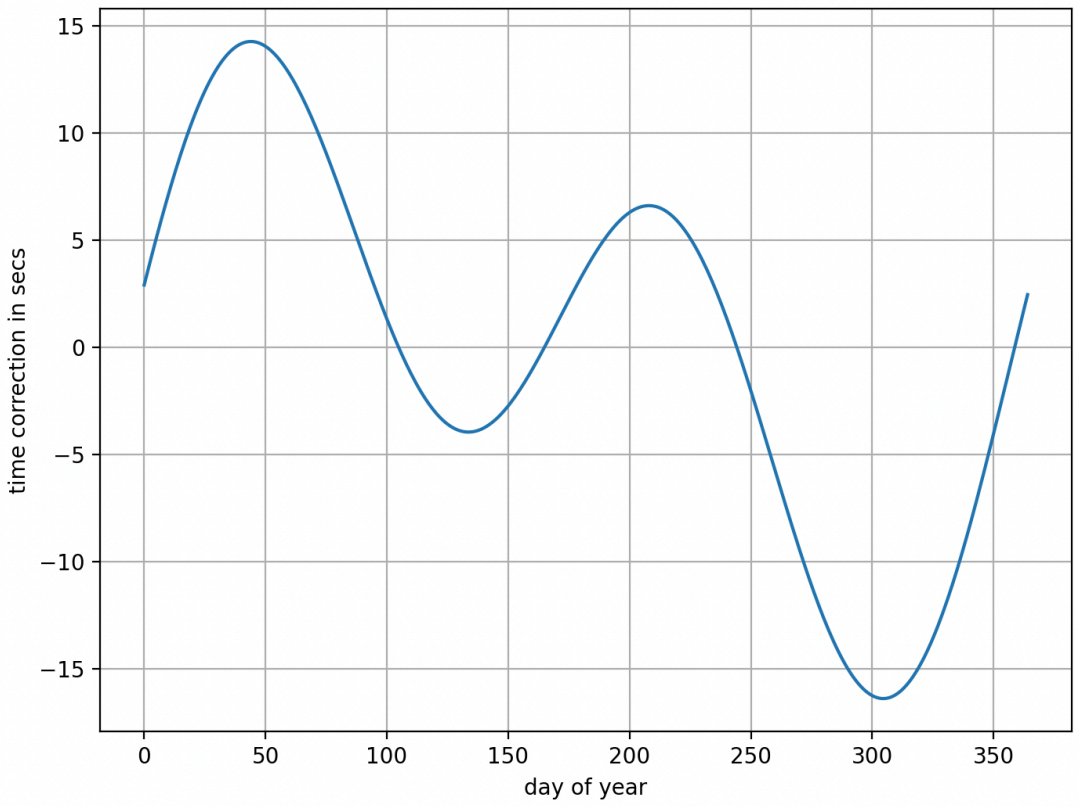
- *because of the **variation of the Earth's speed on its orbit** (2nd Kepler's Law)*
- *because of the **inclination of the ecliptic on the equator**.*

6.3.3 Average solar day

- The **average solar day** has a duration equal to the annual average of the duration of the **true solar day**.
- This duration has been fixed at 24 hours and is used to define the *duration of the second*:

$$1 \text{ second} = \frac{1}{86400} \text{ mean solar day}$$

```
time_correction_equation_seconds = 229.2 * (0.000075
+ 0.001868*cos(2*pi*day_in_year/365)
- 0.032077*sin(2*pi*day_in_year/365)
- 0.014615*cos(4*pi*day_in_year/365)
- 0.04089*sin(4*pi*day_in_year/365))
```



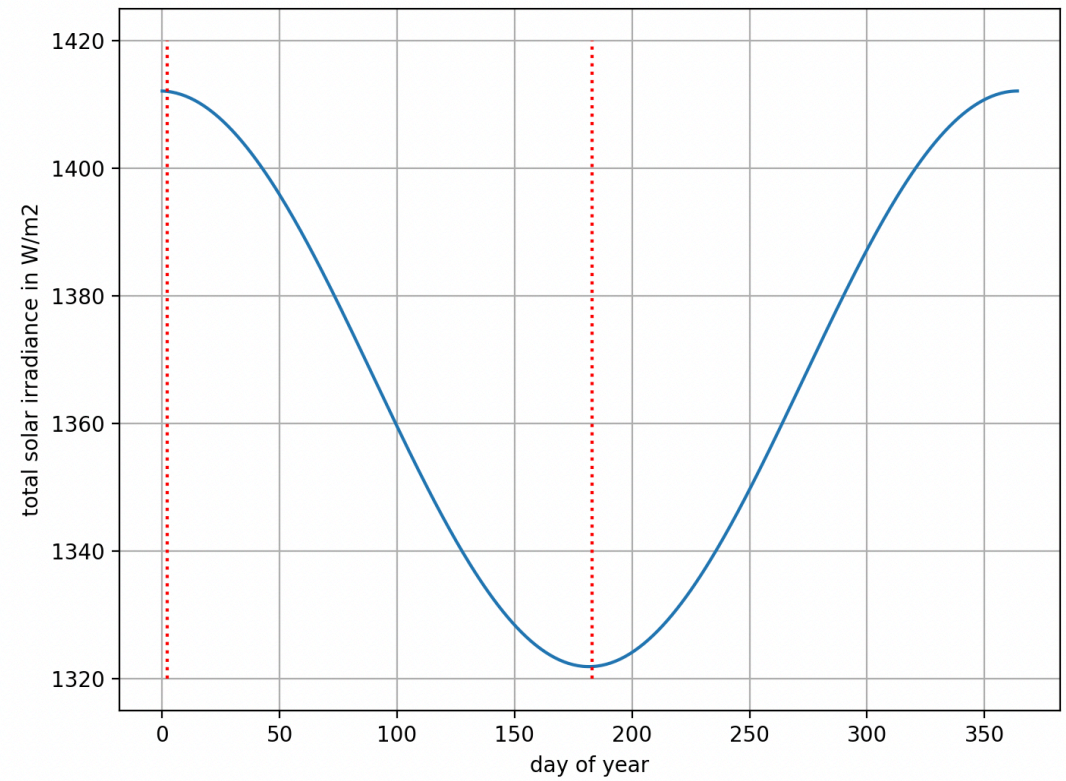

```
actual_solartime_in_secondes = local_solar_time_seconds - time_correction_equation_seconds
```

6.4 Total solar irradiance model

The total solar irradiance at the entrance of the atmosphere is given by:

$$TSI = \bar{L}_{sun}\left(1 + 0.033\cos\left(\frac{2\pi(1 + \text{day in year})}{365.25}\right)\right)$$

where $\bar{L}_{sun}$ stands for the average yearly radiance of the sun at the atmosphere external surface and day in year is the day of the year.



6.4.1 Average yearly radiance of the sun at the atmosphere external surface

The **average yearly radiance of the sun at the atmosphere external surface** is given by:

$$\bar{L}_{sun} = \sigma T^4 \left(\frac{4\pi R^2}{4\pi d} \right)^2 = 1367 W/m^2$$

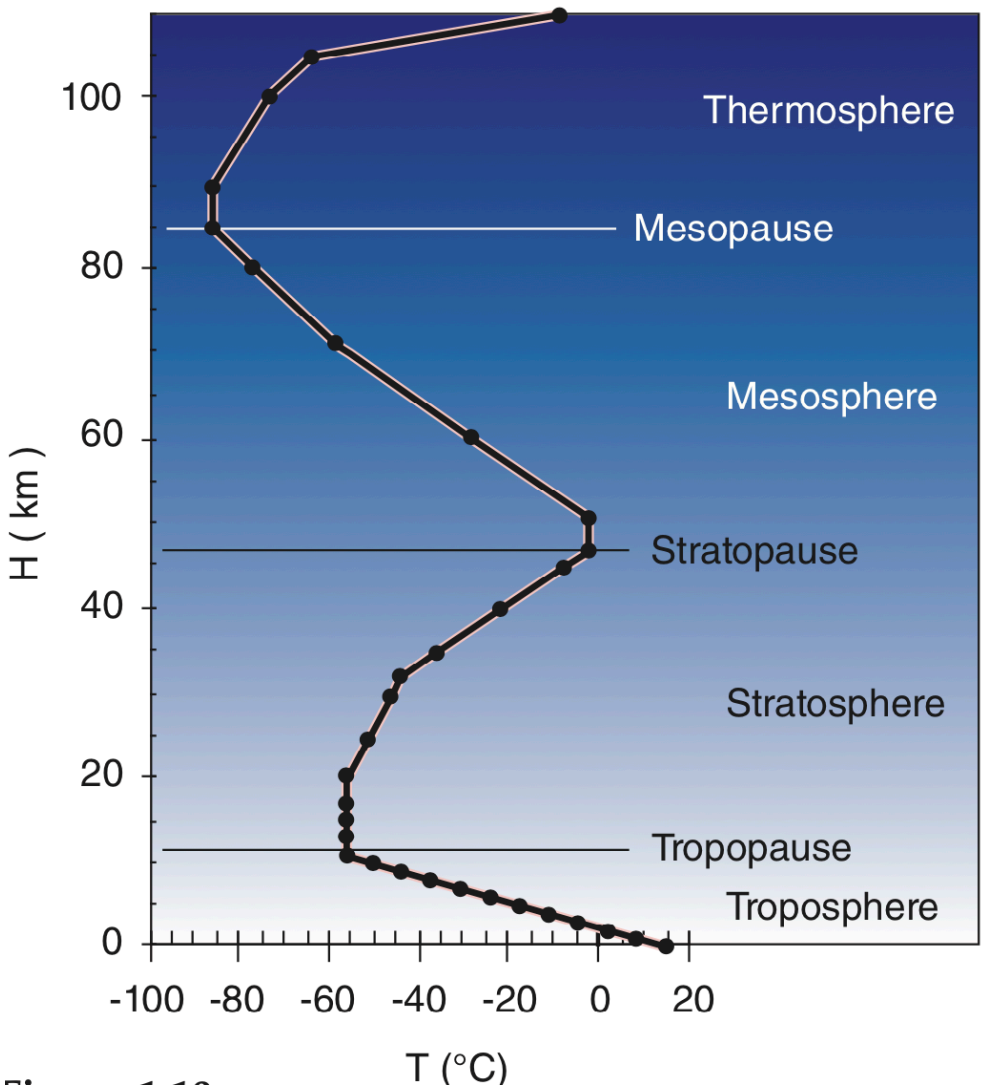
where

- $\sigma = 5.67\ 10^{-8} W/m^2/K^4$ is the Stefan-Boltzmann constant,
- $R = 696.10^6 m$, the sun radius,
- $d = 149.6\ 10^9 m$ the average distance between the sun and Earth.

7. Atmospheric effects

7.1 Layers

- If the sun were not hidden by Earth, its altitude would belong to [0,180°].
- The total solar irradiance reaching Earth's surface decreases as solar radiation travels through the atmosphere due to scattering and absorption effects.
- The thermosphere (upper atmosphere) reaches temperatures exceeding 2000°C due to absorption of high-energy solar radiation.
    - This is due to the absorption of extreme ultraviolet radiation from the Sun.
- The thermosphere's low density allows extreme ultraviolet radiation to penetrate this layer without being completely absorbed, unlike in the lower atmosphere.



7.2 The thermosphere

- This high-energy radiation gets absorbed by the thermosphere's molecules, causing them to vibrate more intensely, translating to a significant rise in temperature. The thermosphere is the least dense layer of the atmosphere.
- While the temperature reading might seem very high, it's important to remember that temperature is a measure of the average kinetic energy of gas molecules.
- With so few molecules in the thermosphere, even a small amount of energy absorbed can result in a large increase in temperature per molecule.

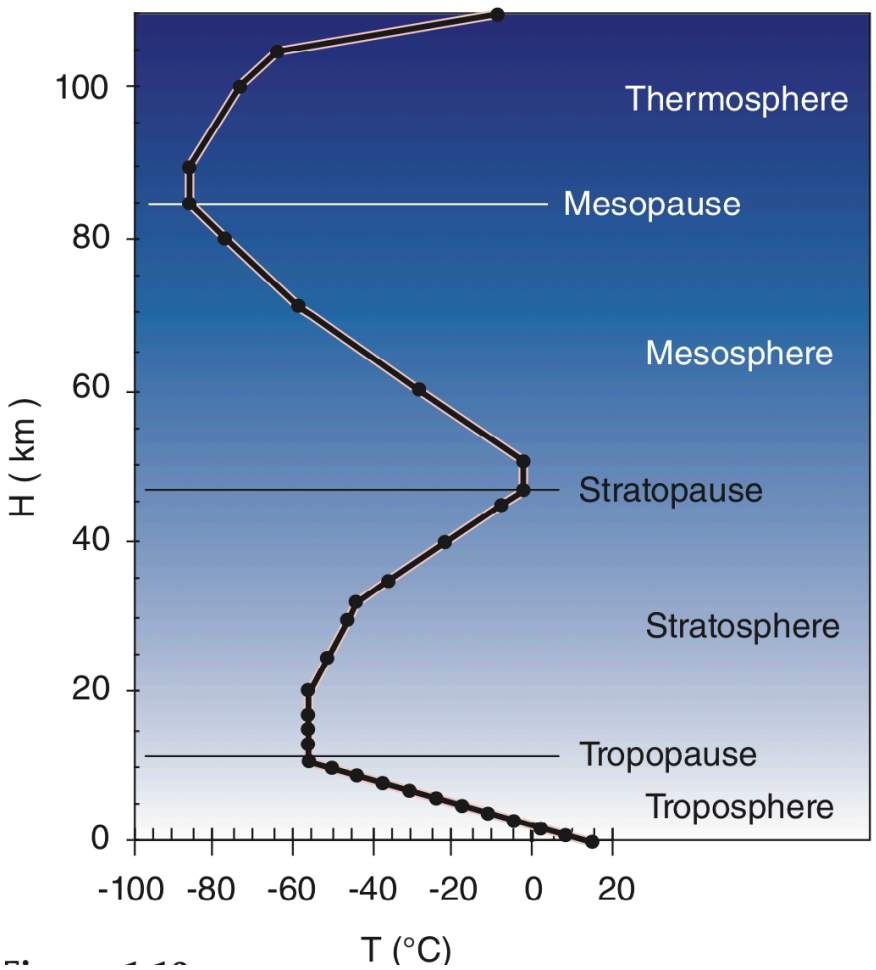
7.2 The thermosphere

- The temperature in the **thermosphere** isn't constant.
- It's heavily influenced by solar activity (see Milankovitch cycles).
- During periods of high solar activity, with more extreme ultraviolet radiation being emitted, the thermosphere can become significantly hotter.
- This can cause the layer to expand upwards.
- In essence, the **thermosphere acts like a super-sparse gas that gets energized by absorbing the Sun's high-energy rays**.
- This unique combination leads to the seemingly counterintuitive phenomenon of the hottest layer being at the very top of the atmosphere.

7.3 The stratosphere

Stratosphere includes **the ozone layer** (19-30 km): the temperature increases due to **UV-absorption**.

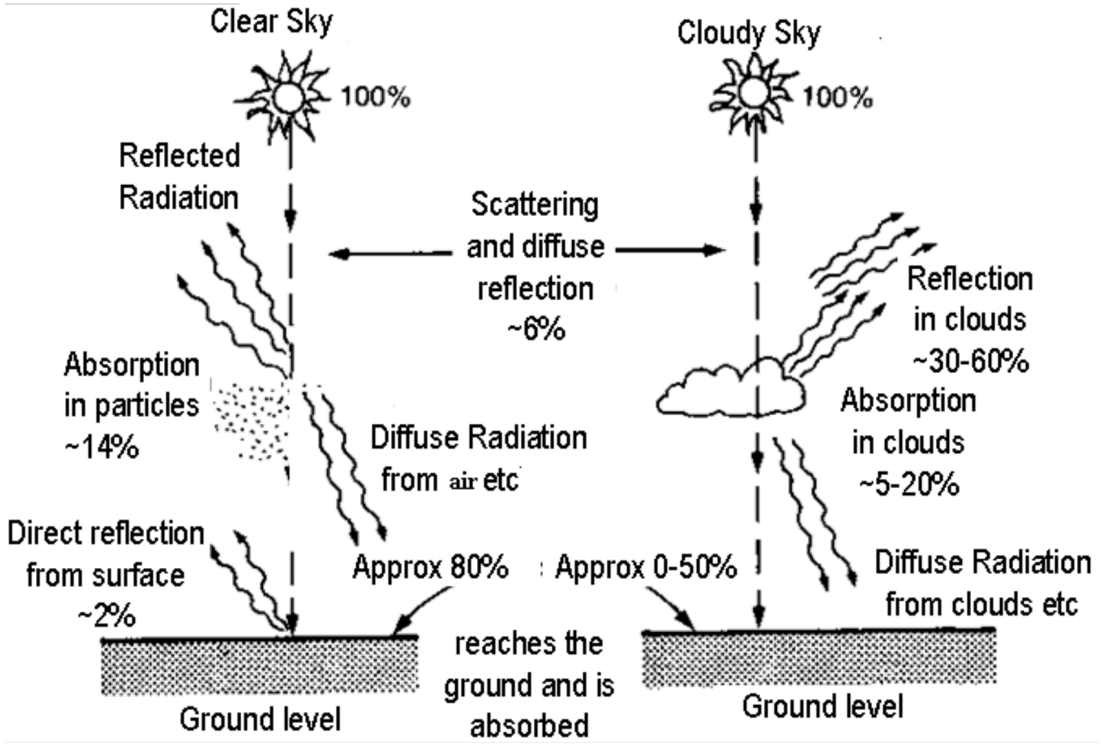
- Ozone acts like **a shield, absorbing most of the sun's harmful ultraviolet B and ultraviolet C radiation**.
- Molecules and particles in the atmosphere, primarily in the stratosphere and mesosphere, can **scatter sunlight**.
- This scattering is what **causes the sky to appear blue**.
- Blue wavelengths are more easily scattered by these particles than other colors, which is why the sky appears predominantly blue.



7.4 Solar radiation breakdown in the atmosphere

Passing through the atmosphere:

- ~30% solar radiation is reflected,
- ~17% is absorbed,
- Only 53% reaches the earth surface as direct radiation:
  - ~31% direct radiation
  - ~22% diffuse radiation



7.4.1 Rayleigh length

The **Rayleigh length** is a distance used to characterize this phenomenon. It's **the distance a beam travels** from its narrowest point (beam waist) to the point where the beam's cross-sectional area has doubled:

```
rayleigh_length = 1 / (0.9 * relative_optical_air_mass + 9.4)
```

where the **relative optical air mass** is defined as **the ratio of the air mass actually flown through to the air mass at vertical incidence**.

Note that:

- *it depends on the solar altitude and on the ground pressure.*
- *If not available, the ground atmospheric pressure can be estimated with*:

$$P_{atm} = 101325(1 - 2.26\ 10^{-5}h)^{5.26}$$

  where h stands for the local ground elevation.

7.4.2 Relative optical air mass

The relative optical air mass is given by:

```
relative_optical_air_mass = ground_atmospheric_pressure
    / (101325 * sin(altitude) + 15198.75 * (3.885 + 57.296 * altitude)(-1.253))
```

7.4.3 Transmitivity coefficient

The **transmitivity coefficient** represents **the fraction of the incident radiation at the surface of the atmosphere that reaches the ground along a vertical path i.e. that is not scattered**.

```
transmitivity_coefficient = 0.6Mh
```

where M_h reflects the variation in atmospheric thickness as a function of the time of day.

7.4.4 Relative optical air mass

To determine the relative optical air mass, the Piedallu & Gégout formula is used:

```
M0 = sqrt(1229 + (614*sin(altitude))2) - 614*sin(altitude_rad)
Mh: float = (1-0.0065/288*elevation)5.256 * M0
```

Mh is a correction factor linked to atmospheric pressure, which depends on elevation of the ground (particularly useful in mountainous areas).

7.4.5 Linke factor

The **Linke factor** models the disturbances due to steam, fog or dust.

```
l_Linke = 2.4 + 14.6 * pollution + 0.4 * (1 + 2 * pollution) * log(partial_steam_pressure)
```

where Pollution = 0.002 in mountains, 0.05 in countryside, 0.1 in urban area and 0.2 in highly polluted area.

The fog is deduced from the partial steam pressure:

```
partial_steam_pressure = 2.165 * (1.098 + temperature/100)8.02 * humidity
```

where humidity and temperature can be obtained from the local meteorology.

7.4.6 Direct normal irradiance

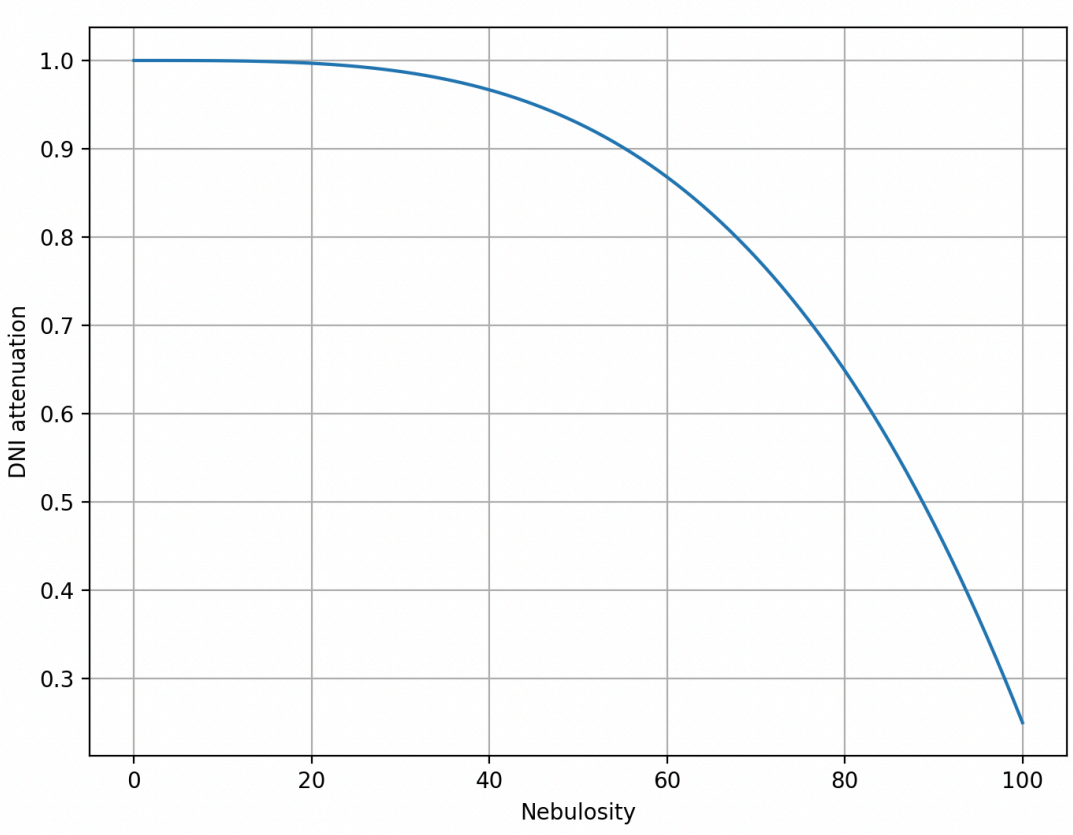
Eventually, the **direct normal irradiance** is given by:

```
direct_normal_irradiance = transmitivity_coefficient * normal_atmospheric_irradiance_with_clouds *
    exp(-relative_optical_air_mass * rayleigh_length * l_Linke)
```

which is affected by clouds:

```
normal_atmospheric_irradiance_with_clouds = (1 - 0.75 * (nebulosity_percent/100)  3.4)
    * total_solar_irradiance
```

Nebulosity is the percentage of the sky covered by clouds.



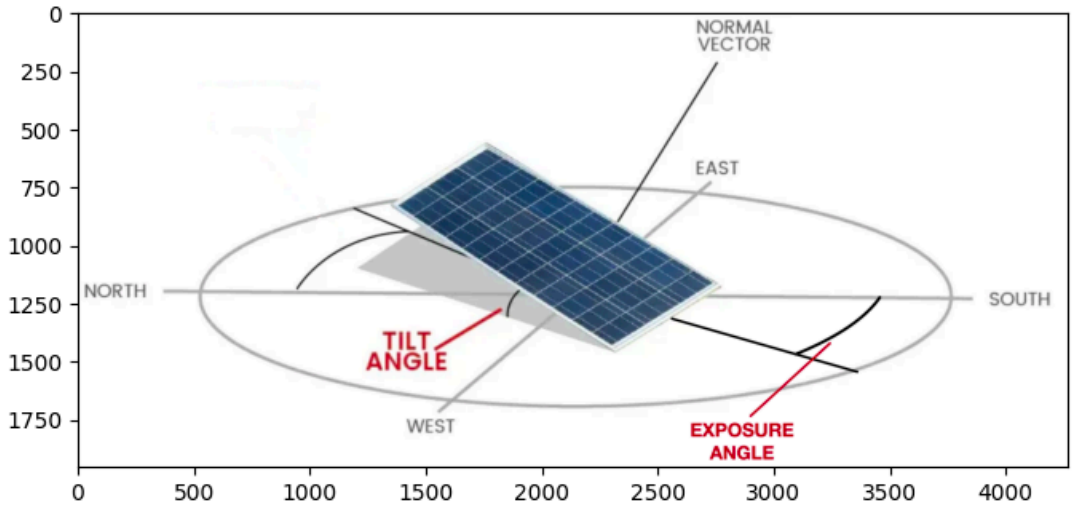
8. Building applications

8.1 Incident surface

Let's consider a flat solar collector lying on Earth.

Let's introduce 2 new angles to model the direction of a flat solar collector lying on Earth:

- the exposure angle
- the tilt angle



8.1.1 Incidence angle

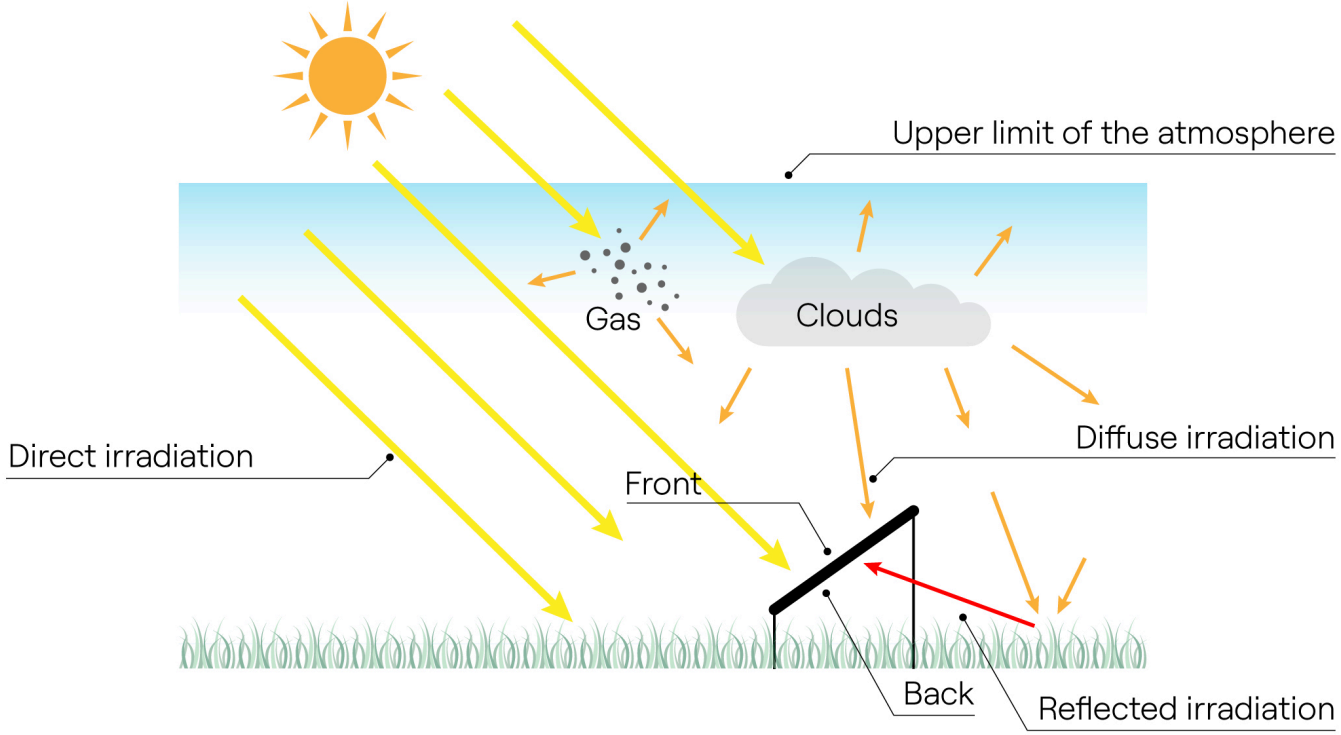
The incidence angle between the solar angle and an inclined surface, defined by an exposure angle and a tilt angle, can be calculated with:

```
incidence_rad = acos(cos(pi/2-altitude_rad) * cos(slope_rad) +
     sin(pi/2-altitude_rad) *  sin(slope_rad) * cos(azimuth_rad - exposure_rad))
```

8.1.2 Direct tilt irradiance

When the collector is oriented in the direction of the sun, we get:

```
direct_tilt_irradiance = cos(incidence_rad-pi) * direct_normal_irradiance
```



8.1.3 Diffuse tilt irradiance

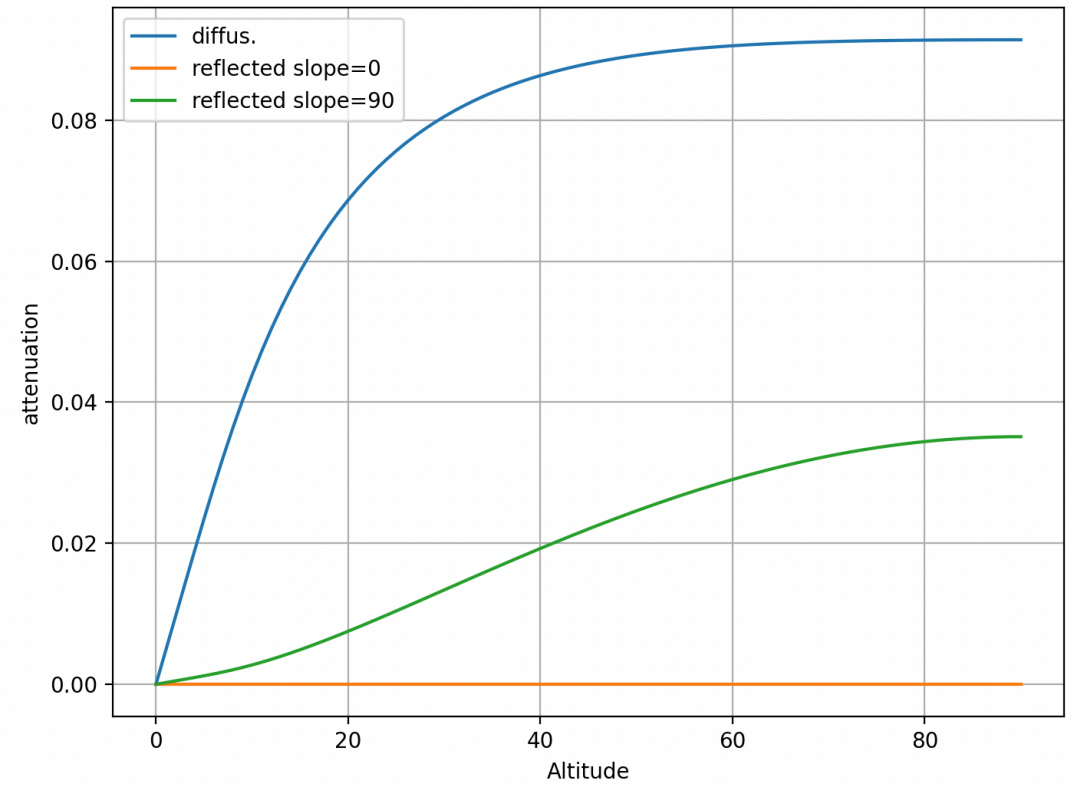
Clouds diffuse a part of the solar beams in all directions:

```
diffuse_tilt_irradiance = max(0, normal_atmospheric_irradiance_with_clouds *
     (0.271 - 0.294 * transmitivity_coefficientMh) * sin(altitude))
```

8.1.4 Reflected tilt irradiance

The reflected tilt irradiance is given by:

```
reflected_tilt_irradiance = max(0, albedo * normal_atmospheric_irradiance_with_clouds * (0.271 + 0.706 * transmitivity_coefficientMh) * sin(altitude) * sin(slope_rad / 2)  2)
```



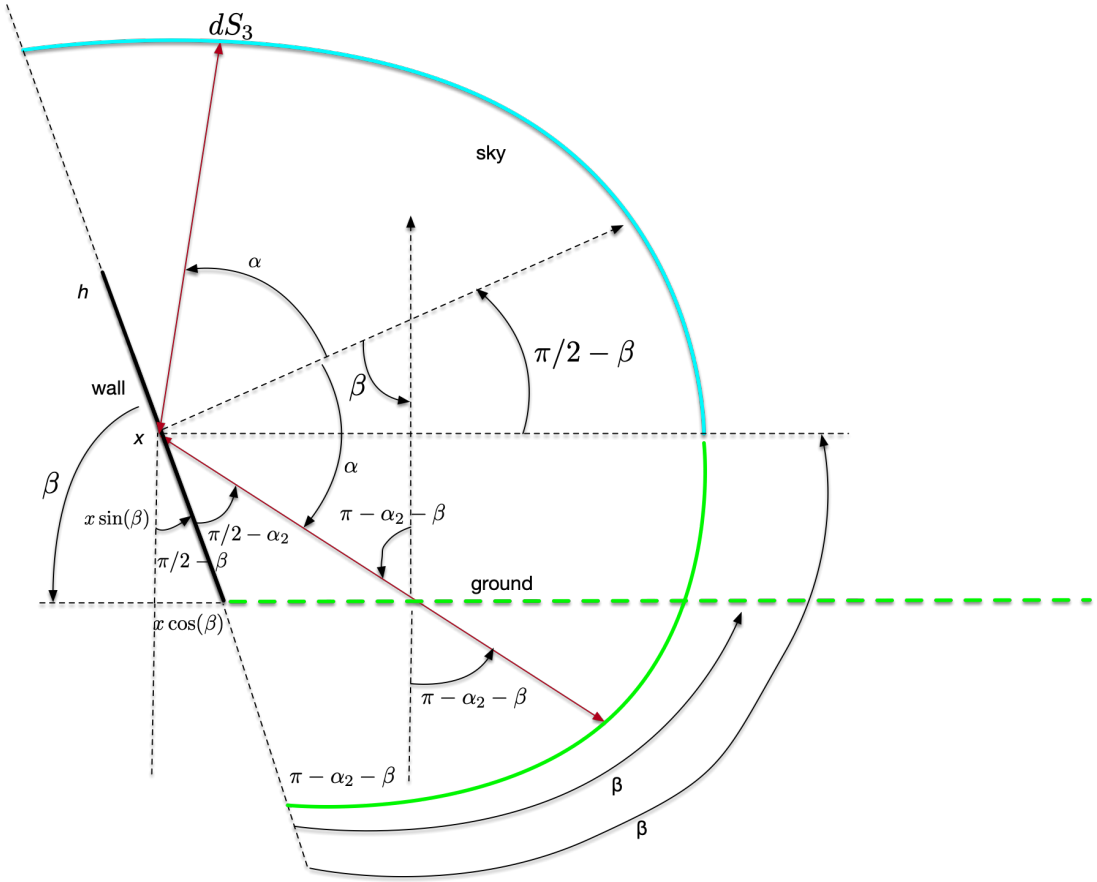
8.2 Long wave radiation due to celestial vault (atmosphere)

We consider a wall facing the sky.

We want to calculate the heat transfer between the wall and the sky.

We need to calculate the solid angle of the sky visible from the wall.

Let's denote: h, l, s_{wall} and β are respectively the height, the width, the surface and the slope of the wall.



8.2.1 Solid angles practical examples

For a vertical wall ($\beta = 90^\circ$):

- It can "see" half the sky (solid angle = π)
- It can "see" half the ground (solid angle = π)

For a horizontal surface ($\beta = 0^\circ$):

- It can "see" all of the sky (solid angle = 2π)
- It can "see" all of the ground (solid angle = 2π)

8.2.2 Heat transfer between the wall and the sky

The heat transfer between the wall and the sky is given by:

$$d\varphi_{wall,sky} = (L_{wall} - L_{sky})\cos(\alpha)d\alpha d\omega dxdy$$

with the bounds:

$$x \in [0, h], y \in [0, l]\alpha \in [\beta - \pi/2, \pi/2]\omega = [0, \pi]$$

Where:

- L_{wall} and L_{sky} are the long wave radiation of the wall and the sky respectively.
- α is the angle between the normal to the wall and the sun.
- ω is the solid angle of the sky visible from the wall.

8.2.3 Heat transfer between the wall and the sky

Therefore, the solution is:

$$\varphi_{wall,sky} = (L_{wall} - L_{sky})\pi l \int_0^h \int_{\beta-\pi/2}^{\pi/2} cos(\alpha) d\alpha dx \rightarrow \varphi_{wall,sky} = (E_{wall} - E_{sky}) \frac{1+\cos(\beta)}{2} s_{wall}$$

where:

- E_{wall} and E_{sky} represent the emissive power of the wall and the sky respectively.
- $(1+\cos(\beta))/2$ is the solid angle of the sky visible from the wall.

Note: the heat transfer therefore depends on the temperature difference (through emissive power), the geometry and positioning of the wall.

8.2.4 Heat transfer between the wall and the ground

Let's calculate the transfer between the wall and the ground:

$$d\varphi_{wall,ground} = (L_{wall} - L_{ground})\cos(\alpha)d\alpha d\omega dxdy$$

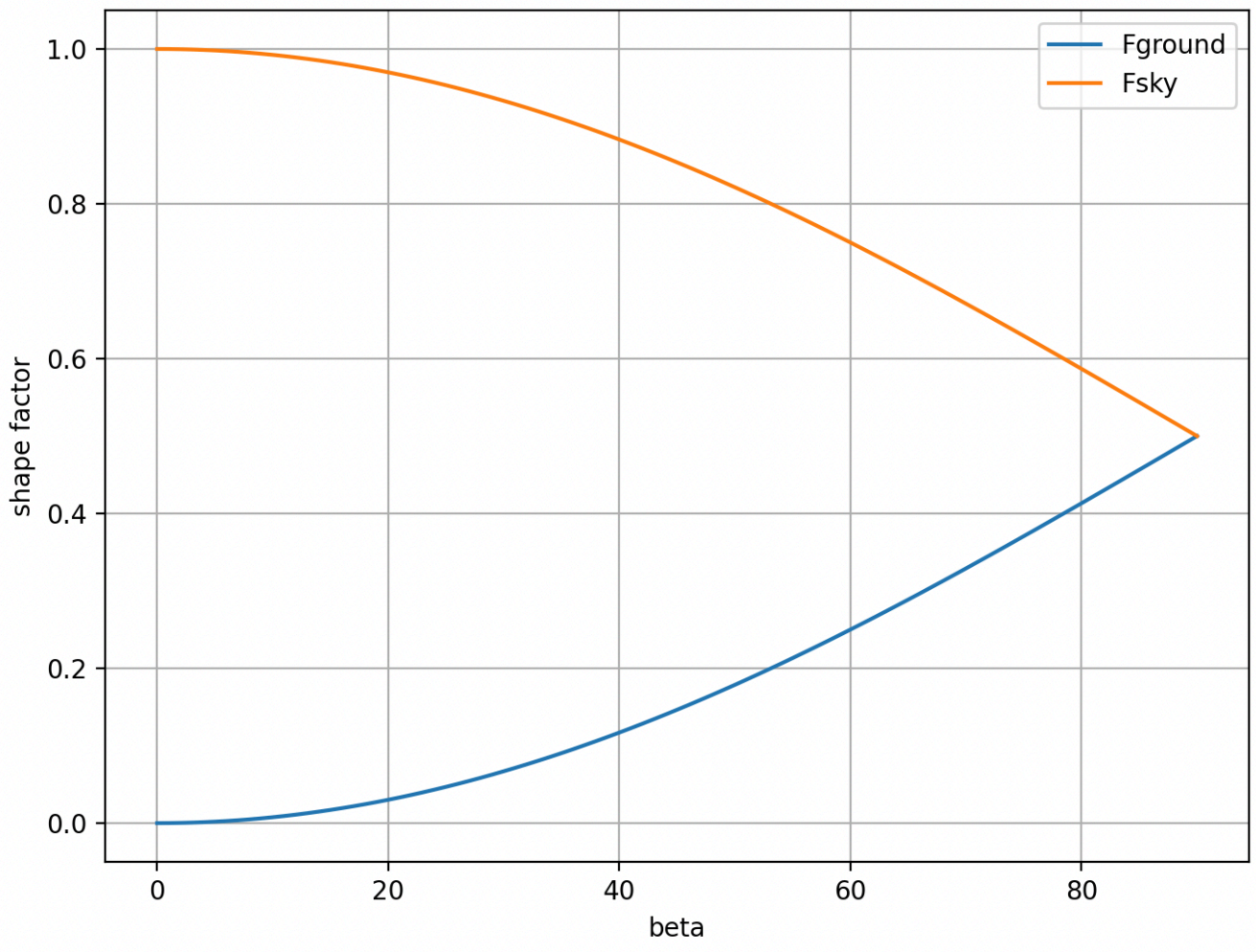
with the bounds:

$$x \in [0, h], y \in [0, l]\alpha \in [-\pi/2, \beta - \pi/2]\omega = [0, \pi]$$

Therefore,

$$\varphi_{wall,ground} = (L_{wall} - L_{ground})\pi l \int_0^h \int_{-\pi/2}^{\beta-\pi/2} cos(\alpha)d\alpha dx \rightarrow \varphi_{w,g} = (E_{wall} - E_{ground})\frac{1-\cos(\beta)}{2}s_{wall}$$

8.2.5 Heat transfer between the wall and the ground



8.2.6 Emissive power (exittance in W/m^2) of the sky

We want to calculate the emissive power (or the emittance) of the sky.

$$P_{sky} = (1-\zeta)P_{clear} + \zeta P_{cloud}$$

where:

- P_{clear} is the emissive power of the clear sky.
- P_{cloud} is the emissive power of the cloudy sky.
- ζ is the cloud cover (or the cloudiness)

Regarding the celestial vault, we know its emissivity thanks to the Berdahl model (Bertin and Martin, 1984):

$$\epsilon_{clear} = 0.711 + 0.56\frac{T_{dp}}{100} + 0.73\left(\frac{T_{dp}}{100}\right)^2$$

$$\epsilon_{cloud} = 0.96$$

where $T_{weather}$ the air temperature in Kelvin at ground level and T_{dp} the dew pressure temperature in Celsius at ground level (which represents the amount of water vapour present in the air, and therefore its ability to absorb direct sunlight)

The Swinbank model gives the equivalent temperature of the clear sky:

$$T_{clear} = 0.0552T_{weather}^{1.5} \approx T_{weather} - 20$$

Consequently,

$$P_{clear} = \epsilon_{clear}\sigma T_{clear}^4$$

$$P_{cloud} = \epsilon_{cloud}\sigma T_{cloud}^4$$

where: $\epsilon_{cloud} = 0.96$ and $T_{cloud} = T_{weather} - 5$.

8.2.7 Emissive power (exittance in W/m^2) of the wall and the ground

Regarding the ground and the wall, we have:

$$P_{wall} = \epsilon_{wall} \sigma T_{wall}^4$$
$$P_{ground} = \epsilon_{ground} \sigma T_{ground}^4$$

where:

- ϵ is the emissivity.
- T is the temperature in Kelvin.
- σ is the Stefan-Boltzmann constant.

8.2.8 Heat transfer between the wall and the sky

Let's consider that $\epsilon_{wall} = \epsilon_{ground}$ and $T_{wall} = T_{ground}$ i.e. $\varphi_{wall,ground} = 0$.
All temperatures are in Kelvin except T_{dp} in Celsius.

This assumption means there is no heat transfer between the wall and the ground. It yields:

$$\varphi_{wall \to sky} = (P_{wall} - P_{sky}) \frac{1 + \cos(\beta)}{2} s_{wall}$$

with:

$$P_{wall} = \epsilon_{wall} \sigma T_{wall}^4$$

$$P_{sky} = (1 - \zeta) \left(0.711 + 0.56 \frac{T_{dp}}{100} + 0.73 \left(\frac{T_{dp}}{100} \right)^2 \right) \sigma T_{clear}^4 + \varepsilon_{cloud} \zeta \sigma (T_{weather} - 5)^4$$

8.2.9 Particular cases

- Particular case: wall facing the sky ($\beta = 0$)

$$\varphi_{wall,sky} = (P_{wall} - P_{sky})s_{wall}$$
$$\varphi_{wall,ground} = 0$$

- Particular case : vertical wall ($\beta = \pi/2$)

$$\varphi_{wall,sky} = \frac{P_{wall} - P_{sky}}{2} s_{wall}$$
$$\varphi_{wall,ground} = \frac{P_{wall} - P_{ground}}{2} s_{wall}$$

8.3 The underground temperature

- Buildings are often built on a slab.
- The slab is often buried in the ground.
- The temperature of the slab is influenced by the temperature of the ground.
  -At around 10m deep, the underground temperature is worth about the average air outdoor temperature ($\approx 13^\circ$).
- For the sake of simplicity, we will assume that the other side of the slab is at the average air outdoor temperature.

8.3 The underground temperature

Note: [Marchio et Reboux 2003] proposed a simple model based on year temperature profile (other approaches can be found in Stephane Thiers's PhD thesis, 2008):

$$T_{\text{ground}}(t,z) = \bar{T} - \frac{\Delta_T}{2} e^{-\frac{z}{\delta}} \cos\left(2\pi \frac{t-\hat{t}}{D_{\text{year}}} - \frac{z}{\delta}\right)$$

where time and durations are in seconds.

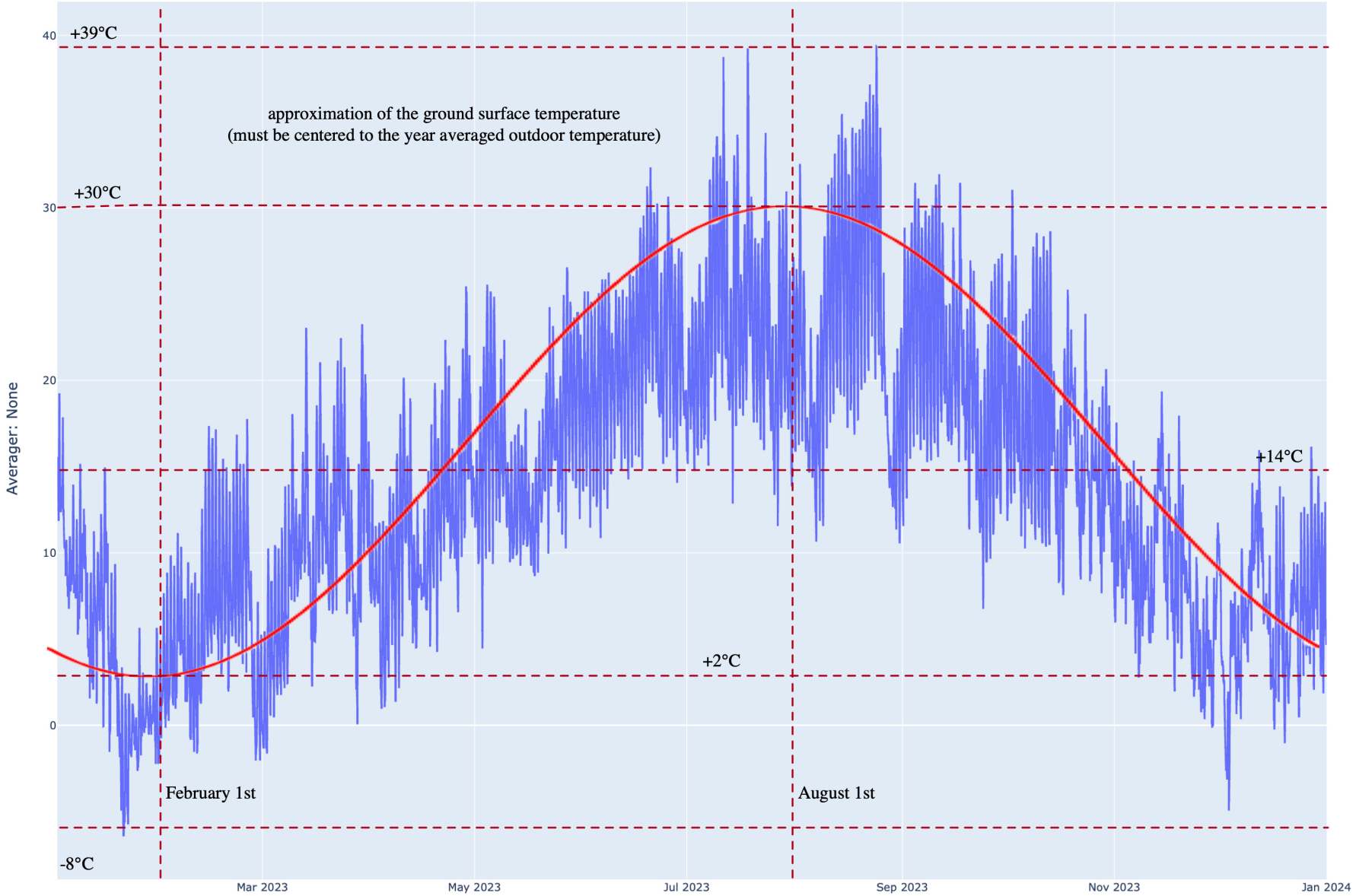
- $\delta = \sqrt{\frac{\lambda_{ground} D_{\text{year}}}{\rho_{ground} c_{ground} \pi}}$, the depth of penetration of the surface year wave temperature with λ_{ground}, the thermal conductivity of the ground, ρ_{ground}, the ground density and c_{ground}, the specific heat of the ground
- $\bar{T}$, the average year temperature, and Δ_T, the maximum temperature amplitude when the year temperature is assimilated to a wave
- $D_{year} = 3600 \times 24 \times 365$ secondes (31,536,000 secondes)
- t, the time in seconds starting at January 1st
- $\hat{t}$, the time in seconds of the surface wave temperature peak time

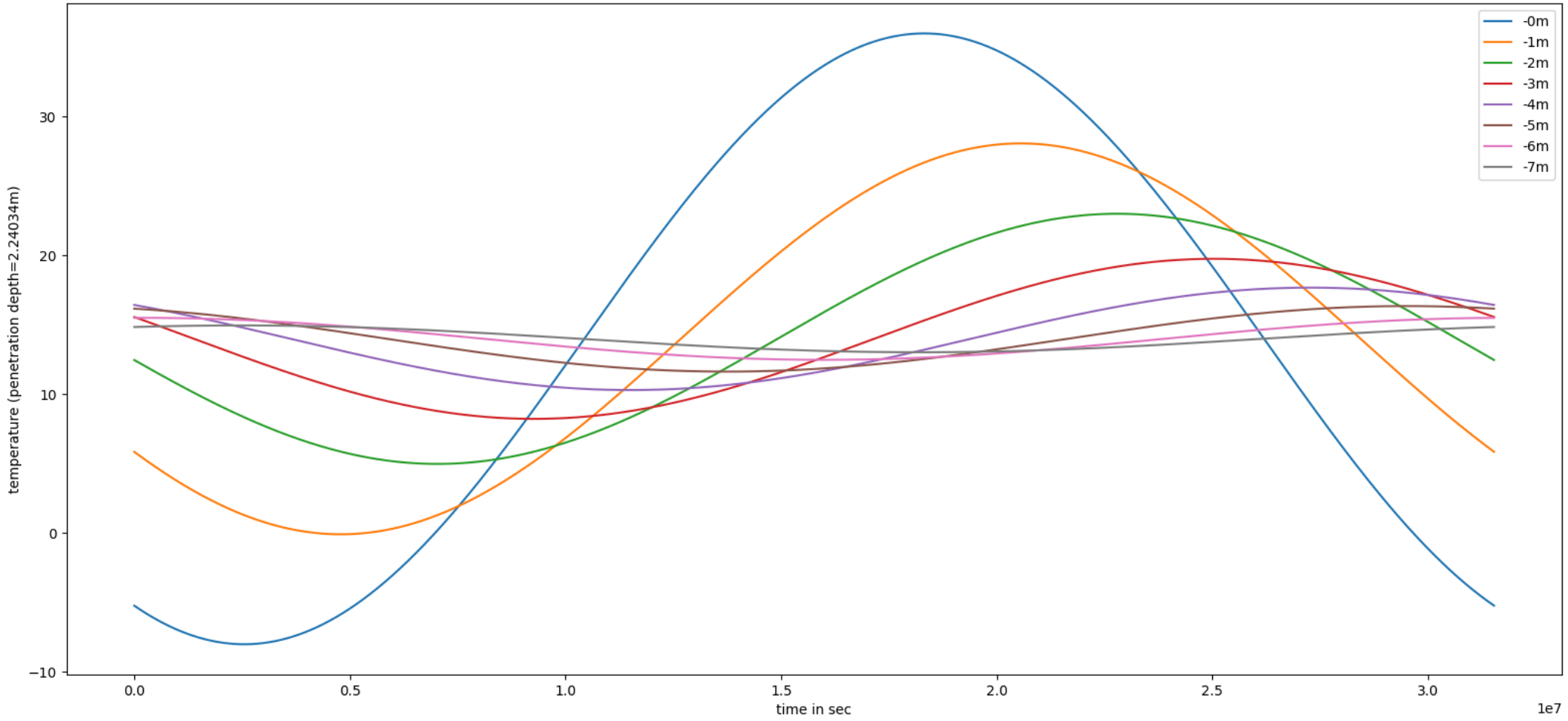
8.3.1 Example: ground temperature profile

Let's describe the SideMask Parameters Schema. The SideMask class represents a rectangular flat surface, named a side, that can obstruct solar radiation. This document provides a comprehensive explanation of all parameters and their geometric relationships.

Consider for instance a ground characterized by: $\rho_{\text{gound}} = 1200 kg/m^3$, $c_{\text{ground}} = 1000 J/kg.K$ and $\lambda = 0.6 W/m.K$ located around Grenoble.

Let's first fit a surface year wave temperature centered on the average year temperature (+14°C) and with the minimum and maximum temperature respectively at February 1st and August first (6 months must seperate these dates).


+39°C
+30°C
+14°C
+2°C
-8°C
approximation of the ground surface temperature
(must be centered to the year averaged outdoor temperature)
February 1st
August 1st
Averager: None
40
30
20
10
0
Mar 2023
May 2023
Jul 2023
Sep 2023
Nov 2023
Jan 2024


-0m
-1m
-2m
-3m
-4m
-5m
-6m
-7m
temperature (penetration depth=2.24034m)
30
20
10
0
−10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1e7
time in sec

Solar Masks

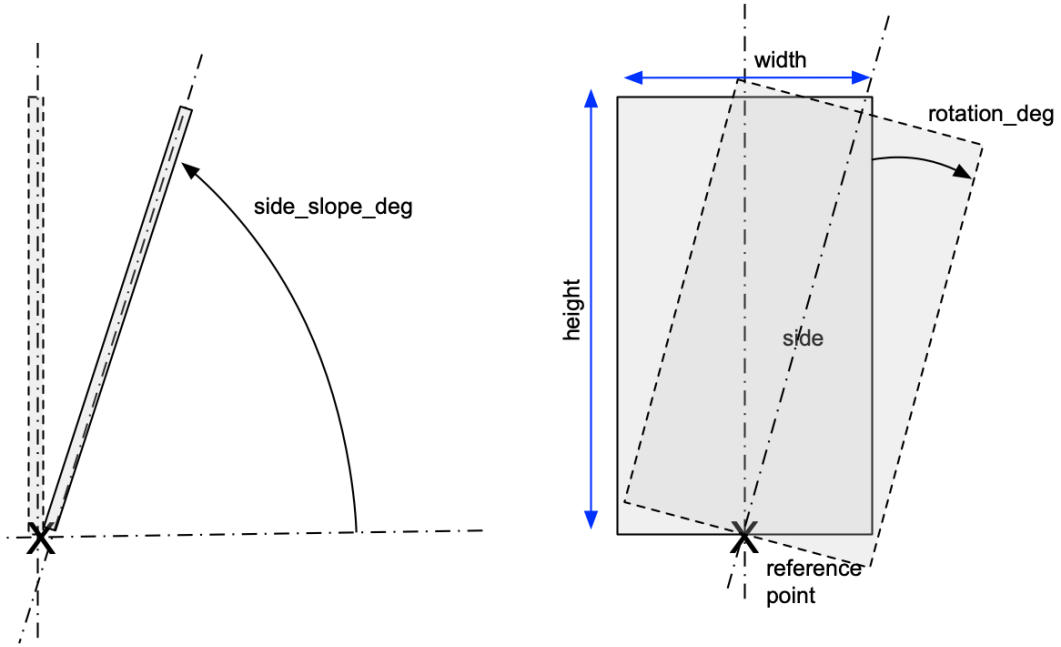
At a given location, the direct solar radiations can be blocked by different obstacles. The obstacle can be a building, a tree, a mountain, etc.
Different types of masks can be used:

- the horizon mask: this mask is loaded automatically with the solar system and the solar positions are blocked if they are below the horizon. It is defined by a series of points with increasing azimuths. The intermediate points are interpolated.

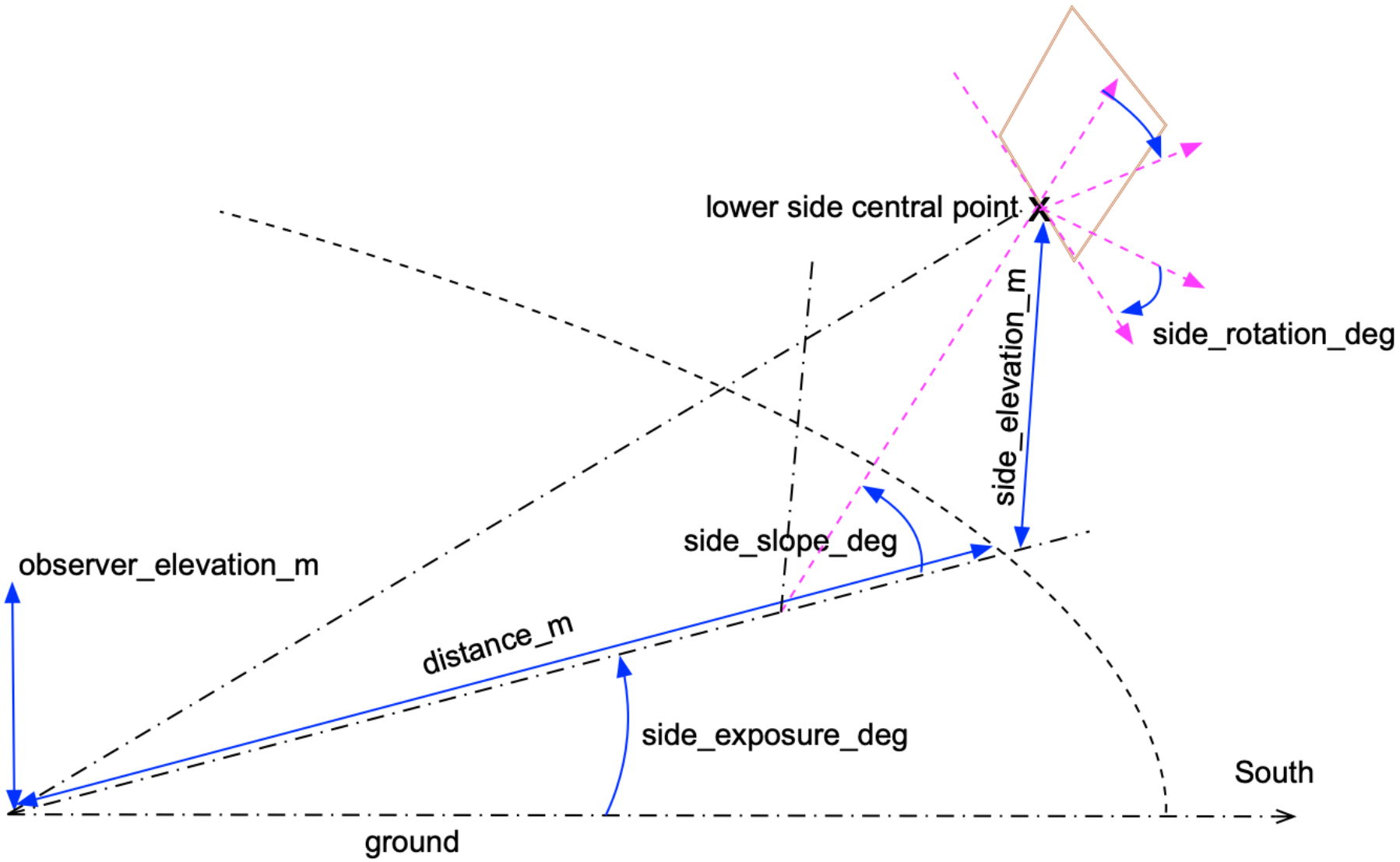
- the rectangular mask: the solar positions are blocked if they are inside a rectangle or if the inverted flag is used, the solar positions are blocked if they are outside the rectangle
- the ellipsoid mask: the solar positions are blocked if they are inside a ellipse or if the inverted flag is used, the solar positions are blocked if they are outside the ellipse
- the side mask: the solar positions are blocked if they are on the side of the obstacle.

Let's describe the SideMask parameters Schema. The SideMask class represents a rectangular flat surface, named a side, that can obstruct solar radiation. This document provides a comprehensive explanation of all parameters and their geometric relationships.

Fist of all, the side is referred by the position of the lower central point of the side:



The different parameters are represented in the following figure:


lower side central point
side_rotation_deg
side_elevation_m
side_slope_deg
observer_elevation_m
distance_m
side_exposure_deg
South
ground

1. **distance_m (float)**
   - Definition: Distance from the observer to the center of the rectangular flat surface
   - Units: Meters
   - Range: Positive values (typically 0.1 to 100+ meters)
   - Geometric Meaning: The perpendicular distance from the observer's position to the center of the side mask
   - Reference Point: All transformations refer to the center of the surface (symmetric axis reference point)

2. **exposure_deg (float)**
   - Definition: The angle between the normal of the surface and the south direction
   - Units: Degrees
   - Range: -180° to +180°
   - Convention:
     - 0° = South-facing
     - 90° = West-facing
     - 180° = North-facing
     - -90° = East-facing
   - Geometric Meaning: The azimuthal orientation of the surface normal vector

3. observer_elevation_m (float)

- Definition: Height of the observer above ground level
- Units: Meters
- Range: Any real value (typically 0 to 50+ meters)
- Default: 0.0
- Geometric Meaning: The vertical offset of the observer's position from the ground plane

4. **slope_deg (float)**
   - Definition: The orientation of the surface normal relative to the ground and sky
   - Units: Degrees
   - Range: 0° to 180°
   - Convention:
     - 0° = Surface facing ground (normal points downward)
     - 90° = Vertical surface (normal points horizontally)
     - 180° = Surface facing sky (normal points upward)
     - 45° = Surface tilted 45° from ground-facing
     - 135° = Surface tilted 45° from sky-facing
   - Geometric Meaning: The slope of the surface from ground-facing to sky-facing

5. **side_rotation_deg (float)**

- Definition: The angle of rotation around the surface normal axis (passing through the center of the surface)
- Units: Degrees
- Range: Any value (0° to 360°+)
- Convention:
  - 0° = original orientation
  - 90° = 90° counter-clockwise rotation around the normal axis
  - 180° = 180° rotation around the normal axis (width and height vectors swapped)
- Geometric Meaning: Rotates the width and height vectors around the surface normal axis (symmetric axis), preserving the angle between the normal and South direction and keeping the surface center fixed

6. **side_elevation_m (float)**

- Definition: Height of the lower edge of the side mask above ground level
- Units: Meters
- Range: Any real value
- Default: 0.0
- Geometric Meaning: The vertical offset of the lower edge of the side mask from the ground plane
- Note: Can be negative for underground structures. The entire surface extends from this elevation to (elevation + height_m)

Additional Parameters

7. width_m (float)

- Definition: Width of the rectangular flat surface
- Units: Meters
- Range: Positive values

8. height_m (float)

- Definition: Height of the rectangular flat surface
- Units: Meters
- Range: Positive values

9. **margin_m (float)**

- Definition: Margin around the rectangular flat surface
- Units: Meters
- Range: Non-negative values
- Default: 0.0

The Coordinate System is the following:

Global Coordinate System

- X-axis: East direction (positive X = West)
- Y-axis: North direction (positive Y = South)
- Z-axis: Up direction (positive Z = Up)

Surface Coordinate System

- Normal vector: Perpendicular to the surface, pointing outward
- Width vector: Along the width of the rectangle
- Height vector: Along the height of the rectangle

Geometric Relationships

1. Surface Normal Calculation

The surface normal is calculated through a series of rotations:

1. Base normal: Start with a vertical surface facing the exposure direction
2. Slope: Rotate around horizontal axis perpendicular to exposure by `slope_deg`
3. Side rotation: Rotate width and height vectors around the surface normal axis by `side_rotation_deg`

2. Position Calculation

The lower left corner position is calculated as:

```
position = distance_m * normal_vector - observer_elevation_m + side_elevation_m
```

3. Shape Orientation

The rectangular shape is oriented using:

- Width vector: Perpendicular to normal and height vectors
- Height vector: Perpendicular to normal and width vectors
- Shape rotation: Applied within the surface plane

Common Use Cases

1. Vertical Building Facade

```
SideMask(
    width_m=10.0,
    height_m=6.0,
    distance_m=15.0,
    exposure_deg=0.0,      # South-facing
    slope_deg=90.0,  # Vertical
    side_rotation_deg=0.0, # No rotation
    side_elevation_m=0.0    # At ground level
)
```

2. Tilted Solar Panel

```
SideMask(
    width_m=2.0,
    height_m=1.0,
    distance_m=5.0,
    exposure_deg=0.0,      # South-facing
    slope_deg=120.0, # 30° from sky-facing (150° - 30°)
    side_rotation_deg=0.0, # No rotation
    side_elevation_m=2.0    # 2m above ground
)
```

3. Underground Wall

```
SideMask(
    width_m=5.0,
    height_m=3.0,
    distance_m=8.0,
    exposure_deg=180.0,    # North-facing
    slope_deg=90.0,  # Vertical
    side_rotation_deg=0.0, # No rotation
    side_elevation_m=-1.0   # 1m below ground
)
```

Parameter Interactions

1. Exposure and Tilt Relationship

- Exposure determines the azimuthal direction of the surface
- Vertical tilt determines how much the surface deviates from vertical
- Horizontal tilt provides additional rotation around the vertical axis

2. Elevation Parameters

- Observer elevation: Affects the relative position of the observer
- Side elevation: Affects the absolute position of the side mask
- Both parameters work together to determine the final vertical positioning

3. Distance and Positioning

- Distance is measured along the surface normal direction
- The actual 3D position depends on the surface orientation and elevation parameters

Validation and Constraints

1. Angle Constraints

- `slope_deg` : Should be between 0° and 180°
- `side_rotation_deg` : Can be any value (represents rotation around normal axis)
- `exposure_deg` : Automatically normalized to -180° to +180° range

2. Physical Constraints

- `distance_m`: Must be positive
- `width_m`, `height_m`: Must be positive
- `margin_m`: Should be non-negative

3. Warnings

The system issues warnings for:

- Tilt angles outside the recommended range
- Unusual parameter combinations

Mathematical Formulation

Surface Normal Vector

```
base_normal = [cos(exposure), sin(exposure), 0]
horizontal_rotated = rotate_around_z(base_normal, tilt_horizontal)
final_normal = rotate_around_horizontal_axis(horizontal_rotated, tilt_vertical)
```

Position Vector

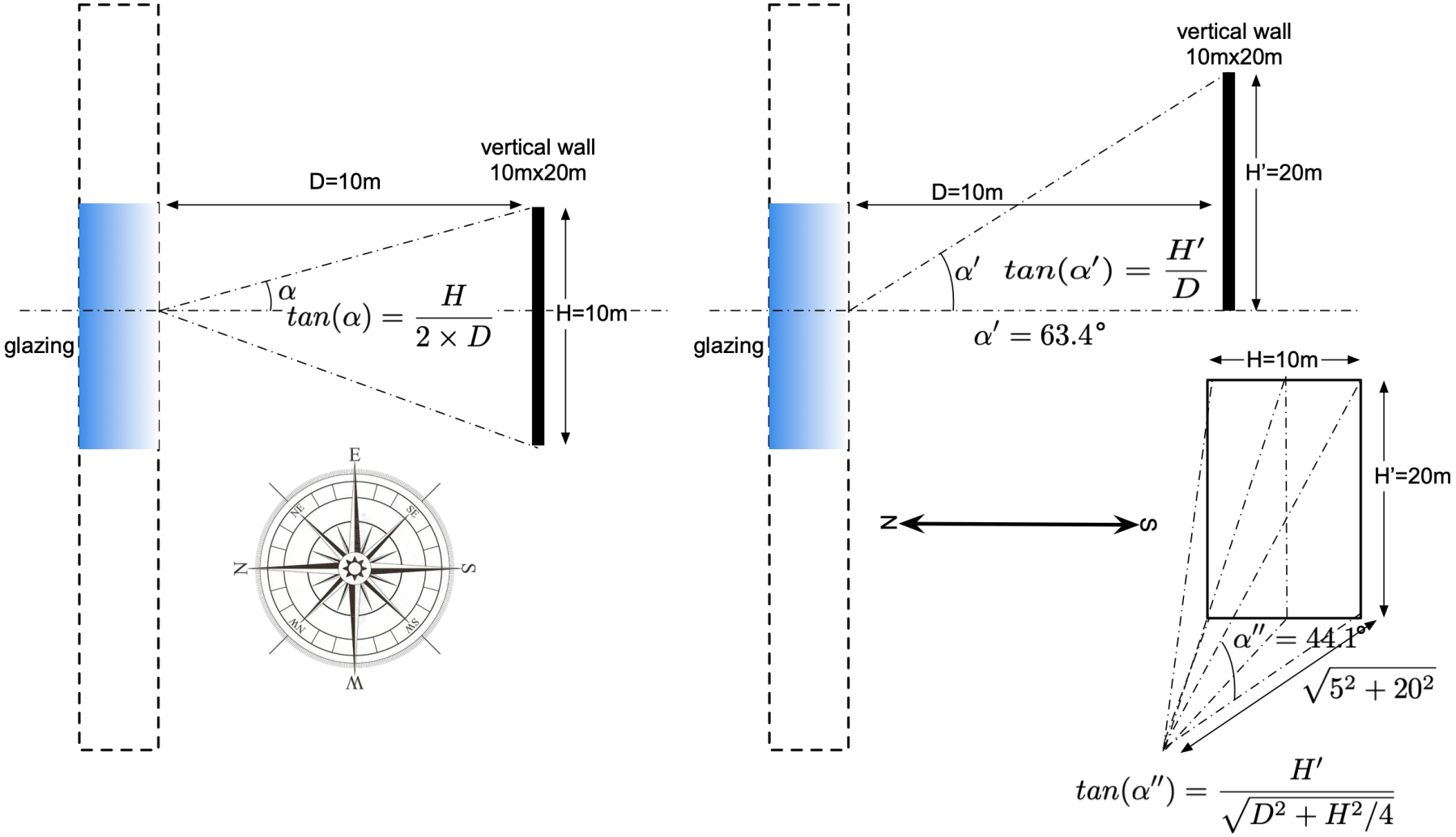
```
lower_central_reference = distance_m * final_normal + [0, 0, side_elevation_m - observer_elevation_m]
lower_left_corner = lower_central_reference - 0.5 * width_m * width_vector
surface_center = lower_central_reference + 0.5 * height_m * height_vector
```

Coordinate Vectors

```
height_vector = [0, 0, 1] (after rotations)
width_vector = cross_product(height_vector, normal_vector)
```

This schema provides a complete understanding of how all SideMask parameters work together to define a rectangular obstruction surface in 3D space for solar analysis.

Why sides are deformed in the polar heliodon plan? An example



Self-check questionnaire

12 questions — Sun, Earth, and radiation

Choose the best answer each time.

(In class: discuss in pairs; answers on the following slides.)

Q1 — Wavelength and frequency

For electromagnetic waves in vacuum, $c = \lambda\nu$. If the wavelength is doubled, the frequency:

- A. is halved
- B. is doubled
- C. stays exactly the same
- D. becomes zero unless the medium is air

Q2 — Sun as a blackbody (order of magnitude)

To first approximation, the solar spectrum is close to a Planck blackbody at about:

- A. $T_\odot \approx 5778\ \mathrm{K}$
- B. $T \approx 288\ \mathrm{K}$
- C. $T \approx 100\ \mathrm{K}$
- D. $T \approx 10\,000\ \mathrm{K}$

Q3 — Wien's law (B_λ)

For the Sun treated as $\sim 5778\ \mathrm{K}$, the Wien maximum of B_λ lies roughly in the:

- A. radio band
- B. green–visible range (order $500\ \mathrm{nm}$)
- C. mid-infrared around $10\ \mu\mathrm{m}$
- D. hard X-ray range

Q4 — Solar vs terrestrial spectra

Which pairing best matches the lecture's shortwave vs longwave picture?

- A. Sun mainly shortwave (~ 0.3–$3\ \mu\mathrm{m}$); Earth (system) emission is mainly longwave infrared (~ 4–$100\ \mu\mathrm{m}$)
- B. Both are dominated by the same wavelength band near $10\ \mathrm{nm}$
- C. The Sun emits mainly longwave; the Earth emits mainly UV-C
- D. Terrestrial emission peaks in the visible; solar emission peaks in the far infrared

Q5 — Clear window glass

For ordinary clear soda-lime glass, the lecture's qualitative summary is closest to:

- A. Opaque to visible light; fully transparent to terrestrial longwave IR
- B. Transmits much visible (and part of near-IR); blocks much longwave thermal IR
- C. Opaque to all solar wavelengths
- D. Transparent to UV-C and opaque to visible light

Q6 — Irradiance

Irradiance at a surface (radiometric usage on the slides) is best described as:

- A. Radiant power received per unit area of the surface ($\mathrm{W/m^2}$)
- B. Luminous flux in lumens only, with no area
- C. Energy stored in the material (J) regardless of area
- D. Solid angle in steradians

Q7 — Radiance

Radiance (as used in radiometry) typically carries the physical dimension:

- A. W/m^2 with no directional dependence
- B. $\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{sr})$ in a specified direction (power per projected area per solid angle)
- C. candela only, with no area or angle
- D. J/K

Q8 — Total solar irradiance (TSI)

Outside the atmosphere, at 1 AU, a surface normal to the sun rays receives a total solar irradiance of order:

- A. $\sim 14\ \mathrm{W/m^2}$
- B. $\sim 1367\ \mathrm{W/m^2}$ (value cited in the lecture)
- C. $\sim 10^5\ \mathrm{W/m^2}$
- D. $\sim 1\ \mathrm{W/m^2}$

Q9 — Solar zenith angle

The solar zenith angle is:

- A. The angle between the local vertical and the direction to the Sun
- B. The latitude of the site only
- C. The difference between civil time and UTC
- D. The azimuth measured from North only

Q10 — Grey body

A grey body (relative to a blackbody at the same temperature) is characterized by a directional emissivity ε with:

- A. ε strictly greater than 1 for all directions and wavelengths
- B. $0 \leq \varepsilon \leq 1$ (often taken wavelength-independent in simple models)
- C. $\varepsilon = 0$ always
- D. ε undefined unless the surface is a perfect conductor

Q11 — Stefan–Boltzmann (blackbody hemispherical emission)

For a blackbody surface of temperature T, the hemispherical emissive power (integrated over all outgoing directions and wavelengths) scales as:

- A. σT^4
- B. σT
- C. σT^2
- D. $k_B T$ only, with no dependence on σ

Q12 — Ozone and UV

The stratospheric ozone layer is important for surface UV mainly because it:

- A. Strongly attenuates much UV-B and essentially all UV-C before it reaches the ground (together with other atmospheric constituents)
- B. Eliminates all visible sunlight
- C. Increases UV-C at the surface by fluorescence
- D. Has no measurable effect on the surface UV spectrum

Answer key (1/2)

Q	Answer	Short rationale
1	A	$\nu = c/\lambda$: longer λ implies lower ν.
2	A	$T_\odot \approx 5778\ \mathrm{K}$ blackbody approximation.
3	B	B_λ peak near $\sim 500\ \mathrm{nm}$ for the Sun.
4	A	Shortwave solar vs longwave terrestrial emission bands.
5	B	Clear glass: transmits much visible/NIR; blocks much LWIR.
6	A	Irradiance: $\mathrm{W/m^2}$ onto a receiving surface.

Answer key (2/2)

Q	Answer	Short rationale
7	B	Radiance: $\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{sr})$ in a direction.
8	B	TSI $\approx 1367\ \mathrm{W}/\mathrm{m}^2$ at 1 AU (lecture figure).
9	A	Zenith angle: vertical vs sun direction.
10	B	Grey body: emissivity between 0 and 1.
11	A	Stefan–Boltzmann: σT^4 for blackbody hemispherical power.
12	A	Ozone strongly cuts UV-B/UV-C reaching the surface.

Open and explore the workshop01_sun.ipynb