

Runir's Description Logic Feature Syntax and Semantics

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1 Preliminaries

Let \mathcal{I} be an interpretation with non-empty domain $\Delta^{\mathcal{I}}$. Each atomic concept A is interpreted as a set $(A)^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, each atomic role R as a binary relation $(R)^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and each individual name a as an element $(a)^{\mathcal{I}} \in \Delta^{\mathcal{I}}$.

Concept descriptions denote subsets of $\Delta^{\mathcal{I}}$. Role descriptions denote binary relations over $\Delta^{\mathcal{I}}$. For planning states, let s denote the set of atoms true in the current state, and let γ denote the set of goal literals.

2 Concept Constructors

Name	Syntax	Semantics
Top	\top	$(\top)^{\mathcal{I}} = \Delta^{\mathcal{I}}$
Bottom	\perp	$(\perp)^{\mathcal{I}} = \emptyset$
Atomic state concept	P	$(P)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid P(a) \in s\}$
Positive atomic goal concept	G^+	$(G^+)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid G(a) \in \gamma\}$
Negative atomic goal concept	G^-	$(G^-)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \neg G(a) \in \gamma\}$
Intersection	$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}} = (C)^{\mathcal{I}} \cap (D)^{\mathcal{I}}$
Union	$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}} = (C)^{\mathcal{I}} \cup (D)^{\mathcal{I}}$
Negation	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus (C)^{\mathcal{I}}$
Value restriction	$\forall R.C$	$(\forall R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b. (a, b) \in (R)^{\mathcal{I}} \Rightarrow b \in (C)^{\mathcal{I}}\}$
Existential quantification	$\exists R.C$	$(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b. (a, b) \in (R)^{\mathcal{I}} \wedge b \in (C)^{\mathcal{I}}\}$
At-least number restriction	$\geq n R$	$(\geq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}}\} \geq n\}$
At-most number restriction	$\leq n R$	$(\leq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}}\} \leq n\}$
Exact number restriction	$= n R$	$(= n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}}\} = n\}$
Qualified at-least number restriction	$\geq n R.C$	$(\geq n R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}} \wedge b \in (C)^{\mathcal{I}}\} \geq n\}$
Qualified at-most number restriction	$\leq n R.C$	$(\leq n R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}} \wedge b \in (C)^{\mathcal{I}}\} \leq n\}$

Name	Syntax	Semantics
Qualified exact number restriction	$= n R.C$	$(= n R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}} \wedge b \in (C)^{\mathcal{I}}\} = n\}$
Role-value map inclusion	$R \subseteq S$	$(R \subseteq S)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b. (a, b) \in (R)^{\mathcal{I}} \Rightarrow (a, b) \in (S)^{\mathcal{I}}\}$
Agreement	$R \doteq S$	$(R \doteq S)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b. (a, b) \in (R)^{\mathcal{I}} \Leftrightarrow (a, b) \in (S)^{\mathcal{I}}\}$
Role fillers	$R : \{a_1, \dots, a_k\}$	$(R : \{a_1, \dots, a_k\})^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \{(a_1)^{\mathcal{I}}, \dots, (a_k)^{\mathcal{I}}\} \subseteq \{b \in \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}}\}\}$
One-of	$\{a_1, \dots, a_k\}$	$(\{a_1, \dots, a_k\})^{\mathcal{I}} = \{(a_1)^{\mathcal{I}}, \dots, (a_k)^{\mathcal{I}}\}$
Nominal	$\{a\}$	$(\{a\})^{\mathcal{I}} = \{(a)^{\mathcal{I}}\}$

Here $n \in \mathbb{N}$. In role fillers and one-of constructors, the listed object names are interpreted as individual names in \mathcal{I} .

3 Concrete Syntax for Concept Constructors

Name	Concrete syntax	Abstract syntax
Top	<code>c_top</code>	\top
Bottom	<code>c_bot</code>	\perp
Atomic state concept	<code>(c_atomic_state "p")</code>	P
Positive atomic goal concept	<code>(c_atomic_goal "p" true)</code>	G^+
Negative atomic goal concept	<code>(c_atomic_goal "p" false)</code>	G^-
Intersection	<code>(c_and C1 ... Cn)</code>	$C_1 \sqcap \dots \sqcap C_n$
Union	<code>(c_or C1 ... Cn)</code>	$C_1 \sqcup \dots \sqcup C_n$
Negation	<code>(c_not C)</code>	$\neg C$
Value restriction	<code>(c_all R C)</code>	$\forall R.C$
Existential quantification	<code>(c_some R C)</code>	$\exists R.C$
At-least number restriction	<code>(c_at_least n R)</code>	$\geq n R$
At-most number restriction	<code>(c_at_most n R)</code>	$\leq n R$
Exact number restriction	<code>(c_exactly n R)</code>	$= n R$
Qualified at-least restriction	<code>(c_at_least n R C)</code>	$\geq n R.C$
Qualified at-most restriction	<code>(c_at_most n R C)</code>	$\leq n R.C$
Qualified exact restriction	<code>(c_exactly n R C)</code>	$= n R.C$
Same-as / agreement	<code>(c_same_as R1 R2)</code>	$R_1 \doteq R_2$
Role-value map	<code>(c_subset R1 R2)</code>	$R_1 \subseteq R_2$
Role fillers	<code>(c_fillers R a1 ... an)</code>	$R : \{a_1, \dots, a_n\}$
One-of	<code>(c_one_of a1 ... an)</code>	$\{a_1, \dots, a_n\}$
Nominal	<code>(c_nominal a)</code>	$\{a\}$

Polarity of atomic state vs. goal constructors. Atomic *state* concepts and roles take no polarity argument: the concrete syntax is (`c_atomic_state "p"`) and (`r_atomic_state "p"`), and a negated state concept/role is built compositionally with `c_not/r_complement`. Atomic *goal* concepts and roles carry an explicit polarity, (`c_atomic_goal "p" true`) / (`c_atomic_goal "p" false`) (and likewise for `r_atomic_goal`), because the goal condition may assert a literal positively or negatively. The boolean atomic constructors below also carry a polarity argument and require a nullary (0-arity) predicate.

4 Restrictions on Role Interpretations

Functional roles. A feature f is interpreted as a functional binary relation:

$$\forall a, b, c. (a, b) \in (f)^{\mathcal{I}} \wedge (a, c) \in (f)^{\mathcal{I}} \Rightarrow b = c.$$

Equivalently, features may be viewed as partial functions.

Transitive roles. A transitive role R is interpreted as a transitive binary relation:

$$\forall a, b, c. (a, b) \in (R)^{\mathcal{I}} \wedge (b, c) \in (R)^{\mathcal{I}} \Rightarrow (a, c) \in (R)^{\mathcal{I}}.$$

5 Role Constructors

Name	Syntax	Semantics
Universal role	U	$(U)^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Atomic state role	P	$(P)^{\mathcal{I}} = \{(a, b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid P(a, b) \in s\}$
Positive atomic goal role	G^+	$(G^+)^{\mathcal{I}} = \{(a, b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid G(a, b) \in \gamma\}$
Negative atomic goal role	G^-	$(G^-)^{\mathcal{I}} = \{(a, b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid \neg G(a, b) \in \gamma\}$
Intersection	$R \sqcap S$	$(R \sqcap S)^{\mathcal{I}} = (R)^{\mathcal{I}} \cap (S)^{\mathcal{I}}$
Union	$R \sqcup S$	$(R \sqcup S)^{\mathcal{I}} = (R)^{\mathcal{I}} \cup (S)^{\mathcal{I}}$
Complement	$\neg R$	$(\neg R)^{\mathcal{I}} = (\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \setminus (R)^{\mathcal{I}}$
Inverse	R^-	$(R^-)^{\mathcal{I}} = \{(b, a) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid (a, b) \in (R)^{\mathcal{I}}\}$
Composition	$R \circ S$	$(R \circ S)^{\mathcal{I}} = \{(a, c) \mid \exists b. (a, b) \in (R)^{\mathcal{I}} \wedge (b, c) \in (S)^{\mathcal{I}}\}$
Transitive closure	R^+	$(R^+)^{\mathcal{I}} = \bigcup_{n \geq 1} ((R)^{\mathcal{I}})^n$
Reflexive-transitive closure	R^*	$(R^*)^{\mathcal{I}} = \bigcup_{n \geq 0} ((R)^{\mathcal{I}})^n$
Role restriction	$R C$	$(R C)^{\mathcal{I}} = (R)^{\mathcal{I}} \cap (\Delta^{\mathcal{I}} \times (C)^{\mathcal{I}})$
Identity	$\text{id}(C)$	$(\text{id}(C))^{\mathcal{I}} = \{(d, d) \mid d \in (C)^{\mathcal{I}}\}$

The iterated composition $((R)^{\mathcal{I}})^n$ is defined by

$$((R)^{\mathcal{I}})^0 = \{(d, d) \mid d \in \Delta^{\mathcal{I}}\}, \quad ((R)^{\mathcal{I}})^{n+1} = ((R)^{\mathcal{I}})^n \circ (R)^{\mathcal{I}}.$$

6 Concrete Syntax for Role Constructors

Name	Concrete syntax	Abstract syntax
Universal role	<code>r_universal</code>	U
Atomic state role	<code>(r_atomic_state "p")</code>	P
Positive atomic goal role	<code>(r_atomic_goal "p" true)</code>	G^+
Negative atomic goal role	<code>(r_atomic_goal "p" false)</code>	G^-
Intersection	<code>(r_and R1 ... Rn)</code>	$R_1 \sqcap \dots \sqcap R_n$
Union	<code>(r_or R1 ... Rn)</code>	$R_1 \sqcup \dots \sqcup R_n$
Complement	<code>(r_complement R)</code>	$\neg R$
Inverse	<code>(r_inverse R)</code>	R^-
Composition	<code>(r_composition R1 ... Rn)</code>	$R_1 \circ \dots \circ R_n$
Transitive closure	<code>(r_transitive_closure R)</code>	R^+
Reflexive-transitive closure	<code>(r_reflexive_transitive_closure R)</code>	R^*
Role restriction	<code>(r_restriction R C)</code>	$R C$
Identity	<code>(r_identity C)</code>	$\text{id}(C)$

7 Boolean Constructors

Name	Syntax	Semantics
Positive atomic state predicate	<code>holds(p)</code>	true iff there exists an atom $p() \in s$
Negative atomic state predicate	<code>\negholds(p)</code>	true iff no current atom with predicate p is true in the state
Positive atomic goal predicate	<code>goal(p)</code>	true iff $p() \in \gamma$
Negative atomic goal predicate	<code>\neggoal(p)</code>	true iff $\neg p() \in \gamma$
Non-empty concept	<code>$C \neq \emptyset$</code>	true iff $(C)^{\mathcal{I}} \neq \emptyset$
Non-empty role	<code>$R \neq \emptyset$</code>	true iff $(R)^{\mathcal{I}} \neq \emptyset$

8 Concrete Syntax for Boolean Constructors

Name	Concrete syntax	Abstract syntax
Positive atomic state predicate	<code>(b_atomic_state "p" true)</code>	<code>holds(p)</code>

Name	Concrete syntax	Abstract syntax
Negative atomic state predicate	(b_atomic_state "p" false)	$\neg \text{holds}(p)$
Positive atomic goal predicate	(b_atomic_goal "p" true)	$\text{goal}(p)$
Negative atomic goal predicate	(b_atomic_goal "p" false)	$\neg \text{goal}(p)$
Non-empty concept	(b_nonempty C)	$C \neq \emptyset$
Non-empty role	(b_nonempty R)	$R \neq \emptyset$

9 Numerical Constructors

Name	Syntax	Semantics
Concept count	$ C $	$ (C)^{\mathcal{I}} $
Role count	$ R $	$ (R)^{\mathcal{I}} $
Role distance	$d_R(C, D)$	$\min\{n \in \mathbb{N} \mid (C)^{\mathcal{I}} \times (D)^{\mathcal{I}} \cap ((R)^{\mathcal{I}})^n \neq \emptyset\}$, or ∞ if empty

10 Concrete Syntax for Numerical Constructors

Name	Concrete syntax	Abstract syntax
Concept count	(n_count C)	$ C $
Role count	(n_count R)	$ R $
Role distance	(n_distance C R D)	$d_R(C, D)$

11 Unsolvability Family Constructors

The unsolvability family (**uns**) extends the base family with comparison operators and constants. A comparison takes two operands of the same category—either two Booleans or two Numericals—and yields a Boolean. Boolean operands are interpreted numerically as 0 (false) or 1 (true), so the same relational operators apply uniformly to both operand categories. We write $(e)^{\mathcal{I}}$ for the numeric value of an operand e (a count/distance for Numericals, or 0/1 for Booleans).

Name	Syntax	Semantics
Equal	$e_1 = e_2$	true iff $(e_1)^{\mathcal{I}} = (e_2)^{\mathcal{I}}$
Not equal	$e_1 \neq e_2$	true iff $(e_1)^{\mathcal{I}} \neq (e_2)^{\mathcal{I}}$

Name	Syntax	Semantics
Less than	$e_1 < e_2$	true iff $(e_1)^{\mathcal{I}} < (e_2)^{\mathcal{I}}$
Less or equal	$e_1 \leq e_2$	true iff $(e_1)^{\mathcal{I}} \leq (e_2)^{\mathcal{I}}$
Greater than	$e_1 > e_2$	true iff $(e_1)^{\mathcal{I}} > (e_2)^{\mathcal{I}}$
Greater or equal	$e_1 \geq e_2$	true iff $(e_1)^{\mathcal{I}} \geq (e_2)^{\mathcal{I}}$
Boolean constant	$b \in \{\text{true}, \text{false}\}$	the constant truth value b
Numerical constant	$k \in \mathbb{N}$	the constant nonnegative integer k

12 Concrete Syntax for Unsolvability Family Constructors

Name	Concrete syntax	Abstract syntax
Boolean equal	(b_eq B1 B2)	$B_1 = B_2$
Boolean not equal	(b_neq B1 B2)	$B_1 \neq B_2$
Boolean less than	(b_lt B1 B2)	$B_1 < B_2$
Boolean less or equal	(b_le B1 B2)	$B_1 \leq B_2$
Boolean greater than	(b_gt B1 B2)	$B_1 > B_2$
Boolean greater or equal	(b_ge B1 B2)	$B_1 \geq B_2$
Numerical equal	(n_eq N1 N2)	$N_1 = N_2$
Numerical not equal	(n_neq N1 N2)	$N_1 \neq N_2$
Numerical less than	(n_lt N1 N2)	$N_1 < N_2$
Numerical less or equal	(n_le N1 N2)	$N_1 \leq N_2$
Numerical greater than	(n_gt N1 N2)	$N_1 > N_2$
Numerical greater or equal	(n_ge N1 N2)	$N_1 \geq N_2$
Boolean constant	(b_const true) / (b_const false)	true / false
Numerical constant	(n_const k)	k

13 Module Programs

A module program state is an extended state

$$x = (\bar{s}, m, \mu, \rho_C, \rho_R)$$

where \bar{s} is an annotated planning state, m is the current module, μ is a typed memory state, and ρ_C, ρ_R are the concept and role register valuations. The implementation keeps memory as $\mu \in M_{int} \uplus M_{ext}$ so internal control states and externally visible module states are not conflated.

A proof edge records the transition that changed the planning state and the rule that justified the transition. For sketch proofs the edge label is (e, r) with a state-graph edge e and a sketch rule r . For module-program proofs the corresponding label is (e, r_v) where r_v is a rule variant (load, do, sketch, or call). Edges that only expose control failure or an open state may omit e or r_v until the parser/executor rejects the corresponding ill-formed program earlier.

The intended small-step relation has the form

$$(\bar{s}, m, \mu, \rho_C, \rho_R) \xrightarrow{r_v, e} (\bar{s}', m', \mu', \rho'_C, \rho'_R).$$

Load rules update registers and internal memory, do and sketch rules may change \bar{s} through a planning transition, and call rules push the caller frame before entering the callee module. A proof succeeds when the initial extended state reaches a goal-annotated state and the constructed proof graph has no open states and no directed cycle.

Source

This document summarizes Appendix 1, Section A1.2 of *The Description Logic Handbook*, edited by Franz Baader, Deborah L. McGuinness, Daniele Nardi, and Peter F. Patel-Schneider.