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Radiant floor Type 1792

Documentation

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Type 1792 provides a simple radiant floor model. The dynamics of the radiator fluid as well as the heat transfer in the floor are modeled using a finite volume approach. In the floor model, an uniform temperature distribution is assumed in the horizontal x-y-plane. In this documentation, the Parameters and the INPUT/OUTPUT variables are specified and a validation of the model is given.

Contents

1. Input/Output parameters

Number	Variable name	Unit	Description
Parameters			
1	A_{floor}	$[m^2]$	Total area of the floor
2	d_x	$[m]$	average distance between pipes
3	d_{pipe}	$[m]$	diameter of pipe
4	$n_{parallel}$	$[-]$	Number of parallel pipes
5	i_{pipe}	$[-]$	Control volume where the pipe is embedded
6	$d_{pipe,wall}$	$[m]$	thickness of pipe wall
7	$\lambda_{pipe,wall}$	$[W/m/K]$	heat conductivity pipe wall
8	α_{mode}	$[-]$	$-1 \rightarrow$ compute exact α, $0 \rightarrow$ use temperature invariant α, $>0 \rightarrow$ use constant α
9	$accelerator$	$[-]$	1 accelerator mode is on, 0 do normal iterations
10	$c_{p,w}$	$\frac{kJ}{kgK}$	specific heat of the fluid under standard conditions
11	n_{fluid}	$[-]$	Number of control volumes fluid
12	T_{ini}	$[°C]$	Initial floor temperature
13	ϵ_{floor}	$[-]$	Emissivity of the floor layer at the interface to the room
14	$n_{materials}$	$[-]$	Number of different materials
14 + 5*j	x_{layer}	$[m]$	Thickness of layer
15 + 5*j	λ_{layer}	$[W/m/K]$	Heat conduction coefficient of layer
16 + 5*j	$c_{p,layer}$	$[kJ/K/kg]$	Specific heat of layer material
17 + 5*j	ρ_{layer}	$[-]$	Density of layer material
18 + 5*j	n_{layer}	$[-]$	number of control volumes in the floor
Input variables			
1	T_{sup}	$°C$	Supply (inlet) water temperature provided to the radiator
2	$\dot{m}$	$\frac{kg}{h}$	Mass flow rate
3	$T_{room,up}$	$°C$	Room (air) temperature in the building above the radiant floor
4	$T_{room,down}$	$°C$	Room (air) temperature in the building below the radiant floor
5	$T_{view,up}$	$°C$	Room (field) temperature in the building above the radiant floor
6	$T_{view,down}$	$°C$	Room (field) temperature in the building below the radiant floor
Output variables			
1	T_{out}	$°C$	Return temperature of the radiator
2	$\dot{m}$	$\frac{kg}{h}$	Mass flow rate through the radiator
3	P^{up}_{tot}	$\frac{kJ}{h}$	Total power emitted to the room above by the radiant floor
4	P^{in}_{rad}	$\frac{kJ}{h}$	Total power injected in the radiator
5	P^{acum}_{rad}	$\frac{kJ}{h}$	Total power to the radiator capacity
6	P^{down}_{tot}	$\frac{kJ}{h}$	Total power emitted to the room below by the radiant floor
7	T_{floor}	$°C$	Temperature of the floor surface
9	P^{up}_{rad}	$\frac{kJ}{h}$	Radiative power emitted to the upper room
10	P^{up}_{conv}	$\frac{kJ}{h}$	Convective power emitted to the room below by the radiant floor
11	P^{down}_{rad}	$\frac{kJ}{h}$	Radiative power emitted to the room below the radiant floor
12	P^{down}_{conv}	$\frac{kJ}{h}$	Convective power emitted to the room below by the radiant floor

2. Guideline how to use Type 1972

- Multiple instances of type 1972 can be used in the same deck.
- Additional information for selected parameters:
- Parameter 1: Use total heated area that uses fluid from one mass flow controller
- Parameter 2,4: Length of one Pipe $\rightarrow A_{floor}/d_x/n_{parallel}$
- Parameter 4: Number of pipes with parallel flow that use water from one mass flow controller.
- Parameter 5: Floor control volume in which the pipe is embedded
- Parameter 8: Use 0 for fast computation, uniform forced heat transfer coefficient in the pipe. Use 1 for exact computation, heat transfer coefficient is computed for each control volume separately.
- Parameter 9: Maximal mass flow in this heating area (sum of all parallel pipes).
- Parameter 11: Use ~ 100 for good trade-off between speed and accuracy.
- Parameter 14: Needs to be consistent with the following parameters:
- Parameters $14-18+5*j$: Fill in layer thickness, heat transfer coefficient, specific heat, density and number of control volumes for each material in the floor starting from the bottom.

3. Mathematical reference

3.1. Model

3.1.1 Fluid motion

The model is based on the heat exchanger equation

$$\int_{V_i} \rho_{fl} c_p \frac{\partial T(t,x)}{\partial t} + \int_{V_i} \rho_{fl} c_p v_{fl} \frac{\partial T(t,x)}{\partial s} + \int_{V_i} \frac{A}{V} U(T(x) - T_{ext}) = 0 \tag{1}$$

where ρ_{fl} and c_p are the density and the specific heat of the radiator fluid respectively, S/V denotes the surface volume ratio of the pipe and U the overall heat transfer coefficient. The integrals go over arbitrary control volumes. After implicit discretization of the temporal derivative the integration leads to

$$V_i \rho_{fl} c_p \frac{T_i(t) - T_i(t-1)}{\Delta t} + A_{pipe} \rho_{fl} c_p v_{fl} (T_i - T_{i-1}) + UA(T_i - T_{ext}) = 0 \tag{2}$$

This equation can be solved explicitly for all cells since it is linear in T_i and the equation of volume i depends only on the previous volume $i-1$.

The overall heat transfer coefficient between fluid and floor element is built of three different parts: The heat transfer coefficient of forced convection $\alpha(T)$, heat conductivity of the pipe wall d_{Pipe}/λ_{Pipe} as well as an additional resistance R_x that takes into account the pipe geometry. All together, the overall UA value can be expressed as

$$\frac{1}{UA} = \frac{1}{\alpha A} + \frac{d_{Pipe,wall}}{\lambda_{Pipe} A} + R_x \tag{3}$$

The heat transfer coefficient α is determined through the experimental equation

$$Nu = \frac{\frac{f}{8}(Re - 1000)Pr}{1 + 12.7\left(\frac{f}{8}\right)^{1/2}\left(Pr^{2/3} - 1\right)} \tag{4}$$

where $Re = v_{fluid} * d_{Pipe}/\nu_{fluid}$ is the Reynolds number and Pr the Prandtl number of the fluid. The heat transfer coefficient can then be determined from the definition of the Nusselt number $Nu = \alpha d_{Pipe}/k$ (k denotes the themral conductivity of the fluid).

The coefficient of forced convection α has to be computed at each timestep for each control volume seperately due to its dependence on the fluid temperature (see Figure 1). To reduce computation time, Type 1972 offers the opportunity to use a constant value for α that uses the inlet temperature as an approximation for the temperature. In Figure 1 it can be seen that α depends mainly on d_{Pipe} and on v_{flow} while the temperature dependence has a minor influence. The heat transfer coefficient computation is then done only once for each call of the Type's subroutine.

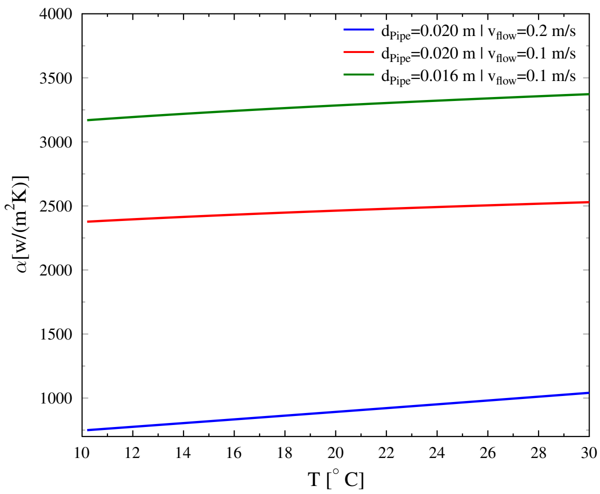


Figure 1: Dependence of the heat transfer coefficient of forced convection α on the temperature of the fluid for different pipe diameters and flow velocities.

3.1.2 Pipe geometry model

In [1] an analytical solution to the temperature distribution in a solid material with equally separated embedded pipes is given in terms of an additional resistance R_x that is added between the pipe temperature and the mean temperature of the floor control volume element. The additional resistance has a very simple form

$$R_x \approx \frac{d_x ln\left(\frac{d_x}{\pi\delta}\right)}{2 * pi * \lambda_{iPipe}} \tag{5}$$

However, this is only valid for parallel pipes with equal inlet temperatures. Most of todays hydronic radiant floors are built in a serpentine layout that have a serial flow regime. Nevertheless this analysis is still used in the type since it provides a good way of account for the average pipe distance that can be changed in order to achieve a higher heating load. It is important to note that this is only a rough approximation.

3.1.3 Floor model

To keep the computation as simple as possible, Type 1972 uses a simplified one dimensional model of the radiant floor, where the temperature is assumed to be uniform in the horizontal planes. The used one-dimensional heat conduction equation is

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) \tag{6}$$

The fully implicit discretized form of the equation can again be derived by integrating over arbitrary control volumes

$$V_i \rho_{fl} c_p \frac{T_i(t) - T_i(t-1)}{\Delta t} = \lambda A_{floor}\left(\frac{T_{i+1}(t) - T_i}{\Delta x} - \frac{T_i(t) - T_{i-1}}{\Delta x}\right) + P_{in} - P_{out} \tag{7}$$

where additional source terms have been added. To complete the modeling an additional massless node at the upper surface is added that fulfills the reduced equation

$$0 = \lambda A_{floor}\left(\frac{T_{N+1}(t) - T_N}{\Delta x}\right) + P_{out} \tag{8}$$

The different source terms for the heat transfer into the rooms are

$$P^{conv}_{out,N+1} = 2 \mid T_{N+1} - T_{room,up} \mid^{0.31} \tag{9a}$$

$$P^{conv}_{out,1} = 0.45 \mid T_{N+1} - T_{room,up} \mid^{0.31} \tag{9b}$$

$$P^{rad}_{out,N+1} = 4\epsilon\sigma\left(\frac{T_{N+1} + T_{room,up} + 273.15}{2}\right)^3 \tag{9c}$$

3.2. Mathematical methods

The discretized equations of the fluid model can be solved explicitly. The floor model leads to a system of linear equations that take the form of a tridiagonal matrix. Such systems are most efficiently solved using a tridiagonal matrix solver which scales with $\mathcal{O}(n)$ instead of $\mathcal{O}(n^3)$ required by normal Gaussian elimination. The interaction between the fluid model and the floor model is then solved by successive substitution iterations. The iterations are stopped as soon as the relative error drops below 1%.

When the 'accelerator' parameter is set to 1 the Type does not do successive substition iterations but uses the last time values of T_{floor} in the computation of the fluid equations. Since the temperature dynamics of the floor are relatively slow compared to the fluid motion, this approximation will only lead to minor errors.

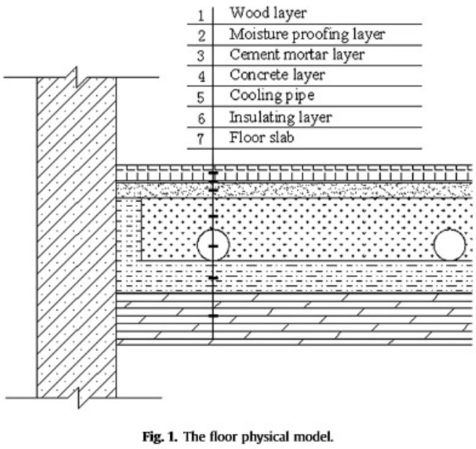
4. Evaluation

4.1. Steady state

The Type was evaluated using measured data from an floor cooling system [2]. The different layers of the floor can be seen in Figure 2. The data is presented in Table 1. The input parameters that are not specified by the physical model of the floor or the temperature table were:

- Emissivity of the floor surface $\epsilon = 0.8$
- Floor area: $A_{floor} = 12m^2$
- Average Distance between pipes: $d_x = 0.17m$ (calculated from pipe length)
- Number of control volumes in the fluid: n_{fluid}=30
- Thickness of pipe wall: $d_{pipe,wal} = 0.002m$
- Heat conductivity pipe wall: $\lambda_{pipe,wall} = 0.35[\frac{W}{m^2K}]$

Since the measured data gives only the mean of inlet and outlet temperature, this value was fitted by adjusting the inlet temperature. The results in Table 1 show that Type 1972 can model these experimental settings quite well. All absolute errors are smaller than $0.5K$. Since the heat transfer to the room is completely determined by the surface temperature of the floor, this results are also valid for the heat load of the radiant floor system. When α is set to be uniform through all fluid volume elements are small, the deviations compared to the standard mode ($\alpha - mode = 1$) are small ($< 0.1K$). The deviations increase for low mass flows which is explained by the steeper temperature gradient in this case.



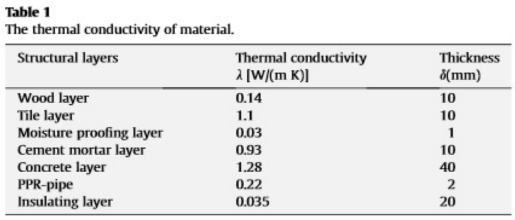


Table 1
The thermal conductivity of material.

Structural layers	Thermal conductivity λ [W/(m K)]	Thickness δ(mm)
Wood layer	0.14	10
Tile layer	1.1	10
Moisture proofing layer	0.03	1
Cement mortar layer	0.93	10
Concrete layer	1.28	40
PPR-pipe	0.22	2
Insulating layer	0.035	20

Figure 2: Physical model of the floor that was used in [2]. This floor was rebuilt in Type 1972 in order to evaluate the accuracy of the used models and approximations.

Average temperature of walls and ceiling $t_{os}(°C)$	Air temperature $t_a(°C)$	Average water temperature $t_w(°C)$	Water velocity $v_w(m/s)$	Floor surface temperature measured $t_f^m(°C)$	Floor surface temperature simulated $\alpha = const$ $t_f^s(°C)$		
					$\alpha_{mode} = -1$	$\alpha_{mode} = 0$	$\alpha_{mode} = 2000$
28.3	27.2	10.8 (10.79)	0.666	20.7	21.09	21.11	21.10
27.5	26.4	15.8 (15.79)	0.666	22.4	22.42	22.45	22.44
26.7	25.8	20.2 (20.16)	0.666	23.8	23.65	23.66	23.66
25.5	26.6	11.0 (10.99)	0.132	20.2	20.68	20.74	20.80
25.3	26.4	11.4 (11.37)	0.329	19.9	20.21	20.22	20.21
24.4	25.6	11.2 (11.17)	0.803	19.6	19.46	19.46	19.46

Table 1: Comparison of simulation result and measured floor surface data from [2]. All errors are below 0.5 K. In the last row, the effect of setting a constant forced heat transfer coefficient α is shown. The effects are generally below 0.1%. As expected the deviation is largest for low mass flow due to the higher temperature gradient.

4.2. Dynamics

An example plot of the energy balance of the radiant floor model can be seen in Figure 3. The mass flow was turned from 0 to 1 at $t = 1h$. The emitted power starts to increase slowly as the capacity of the floor is heated up. The proportion of the power that goes to the capacity decays in the same period. The energy imbalance stays to zero during the whole process. The return temperature of the fluid increases after a short delay in which the flow has to reach the end of the pipe.

The response time of the heating fluid is studied in detail in Figure 4. For 1000 control volumes in the fluid equation the model leads to the expected sharp step in the fluid outlet temperature after approximately 3 minutes which is consistent with the used pipe length of $\sim 192[m]$ and the fluid velocity of $\sim 0.98[\frac{m}{s}]$. When less control volumes are used the response gets smeared out due to the finite difference approximation. An additional inaccuracy is introduced by the one-dimensional floor approximation. All control volumes of the pipe interact with the same floor element, that is described by only one temperature node. Therefore, when the first control volume is filled with hot water, the last element gets heated up through the floor element temperature node. This effect can clearly be seen in the right hand side of Figure 4. When heat flow between pipe and floor is suppressed, the outlet temperature stays zero until the hot water flow reaches the end of the pipe (dotted line). When there is an interaction between pipe and floor element the return temperature of the fluid starts to increase slowly in advance (solid line).

Another error is introduced by the finite time step of the TRNSYS simulation. In Figure 5 it is shown that a simulation time step of 1 min leads to a significantly faster increase of T_{ret}. This feature can only be removed by decreasing the time step of TRNSYS which leads to a large increase in computation time since this equally affects all other Types in the simulation.

Finally, in Figure 5 a rough validation using a transient measurement of a radiant floor heating system is shown. The exact materials and structure of the floor layers was unknown. It was possible to approximately fit T_{ret} using realistic values. The overall behavior of Type 1792 can reproduce the measurement data. Note that the reaction time of the return temperature is faster than in reality due to the limitations mentioned above.

4.3. Computation time

The computation time of the Type has been evaluated using an example simulation with the duration of one month. The test were done on a 2.3 GHz CPU. The results are presented in Table 2. The time consumption of Type 1792 is 0.52 for a fast setting where only one control volume in the fluid is used, α is set constant and iterations are suppressed. Computation time increases to $\sim 3s$ for a higher degree of detail. Therefore, in a yearly simulation, the type will need approximately 6 to 30 seconds computation time depending on the settings.

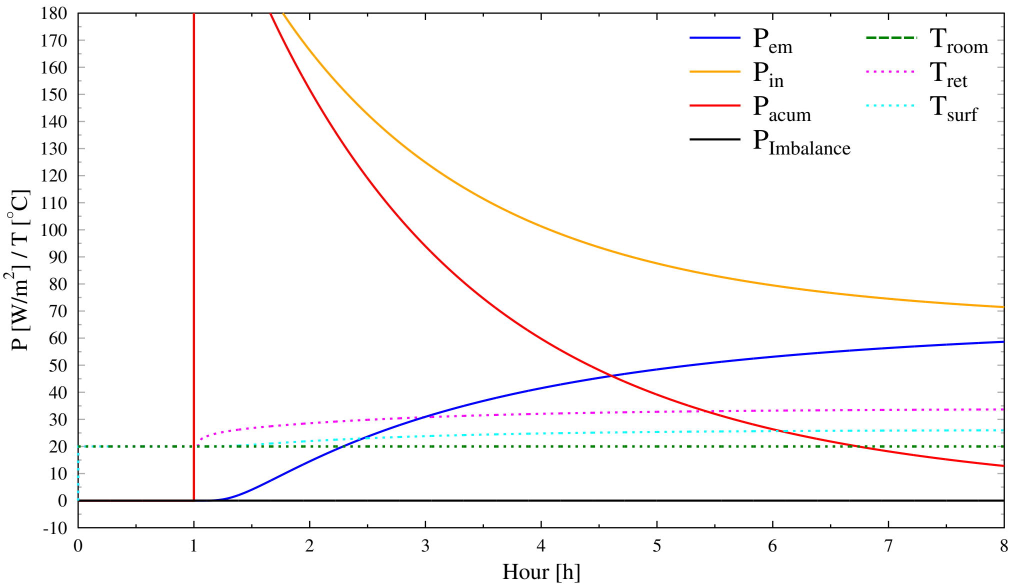


Figure 3: Energy balance example of the radiant floor Mittlere $T_{in} = 37.5°$ $d_x = 0.13m$.

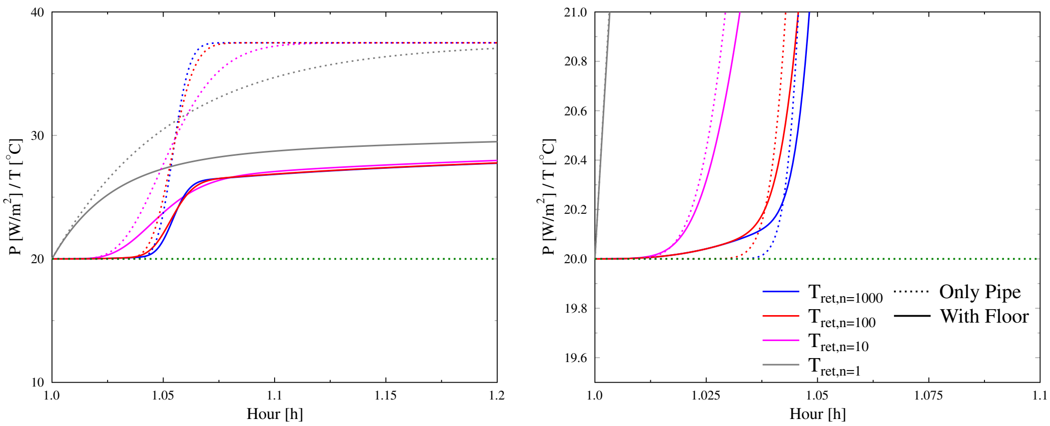


Figure 4: Dynamic response of T_{ret} for different number of control volumes in the fluid (n_{fluid}). As the number of control volumes in the fluid is increased the simulation approaches the expected asymptotic solution. This plot suggests that $n_{fluid} = 100$ is a good trade-off between accuracy and computation time.

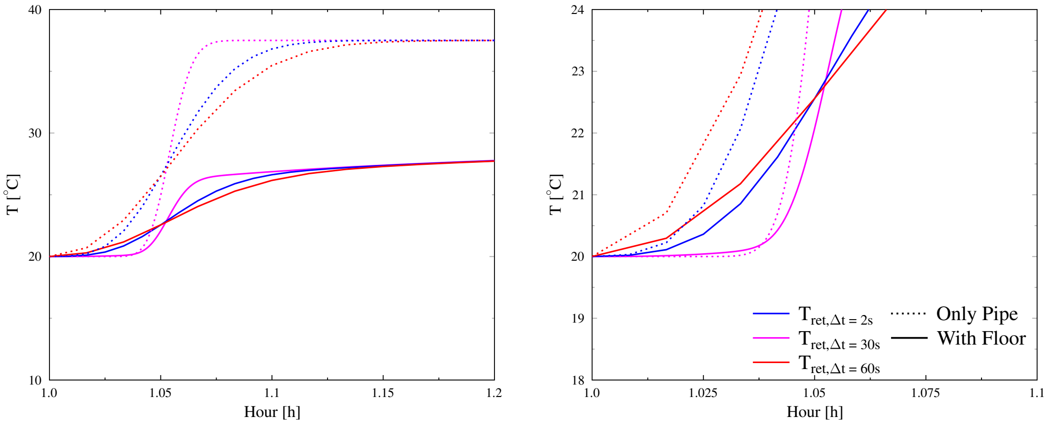


Figure 5: Dynamic response of T_{ret} for different simulation time steps Δt. As the simulation timestep is decreased the simulation approaches the expected asymptotic solution. For frequently used simulation timesteps in the order of one minute the response on a step change of mass flow broadens out significantly.

Mode of α-computation	n_{fluid}	n_{floor}	accelerator on	computation time [s]
$\alpha_{mode} = 2000$	1	4	1	0.52
$\alpha_{mode} = 2000$	10	4	1	0.65
$\alpha_{mode} = 0$	1	4	1	0.67
$\alpha_{mode} = -1$	10	4	1	0.83
$\alpha_{mode} = -1$	10	4	0	1.42
$\alpha_{mode} = -1$	50	4	0	3.13
$\alpha_{mode} = -1$	50	22	0	2.86

Table 2: Computation time of type 1972 for different number of control volumes and defferent settings.

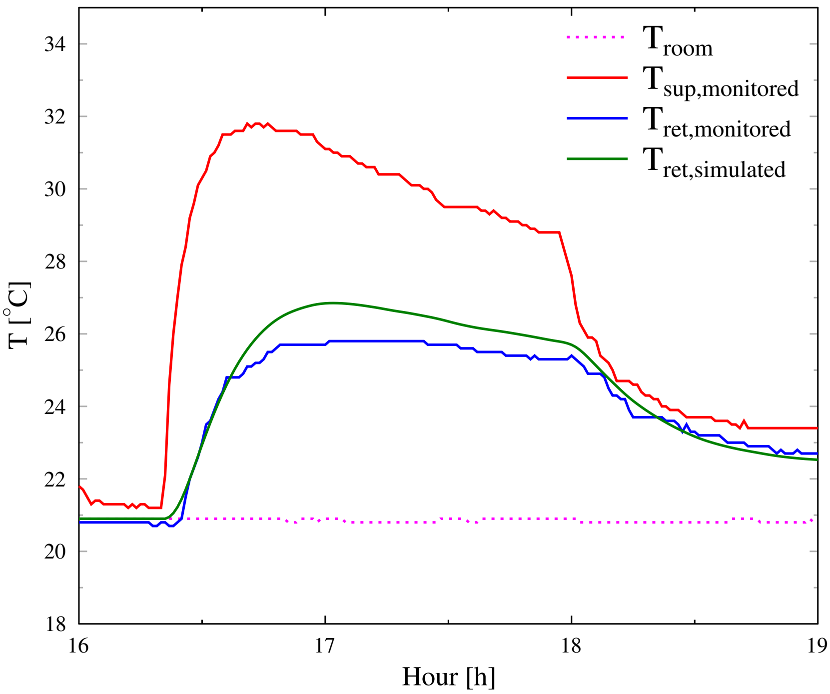


Figure 6: Evaluation using rough measurements from a radiant floor heating system. Heated area $150m^2$, $d_x = 0.13$, $n_parallel = 11$, $d_{pipe} = 0.012m$, $XMFR = 1455kg/h$. The properties of the floor were fitted to the measurement. The overall behavior of T_{ret} is in agreement with the measurement results. However, the reaction time of T_{ret} is significantly faster due to different simplifications in the model.

References

[1] M. Koschenz and B. Lehmann. *Thermoaktive Bauteilsysteme tabs*. EMPA, 2000.

[2] Xing Jin, Xiaosong Zhang, Yajun Luo, and Rongquan Cao. Numerical simulation of radiant floor cooling system: The effects of thermal resistance of pipe and water velocity on the performance. *Building and Environment*, 45(11):2545 – 2552, 2010.