Past Year: C1 Quadratics, C2 Functions, C3 Coordinate Geometry

May/June 2002

1 The line $x + 2y = 9$ intersects the curve $xy + 18 = 0$ at the points A and B. Find the coordinates of A and B. [4]

10 The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + 2, \qquad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{6}{2x+3}, \qquad x \in \mathbb{R},\ x \neq -1.5.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 3$. [3]

(ii) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [3]

(iii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x, and solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

Nov/Dec 2002

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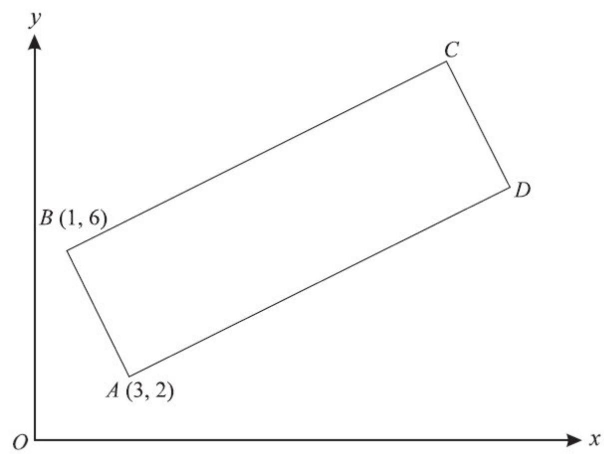

The diagram shows a rectangle $ABCD$, where A is $(3, 2)$ and B is $(1, 6)$.

(i) Find the equation of BC. [4]

Given that the equation of AC is $y = x - 1$, find

(ii) the coordinates of C, [2]

(iii) the perimeter of the rectangle $ABCD$. [3]

May/June 03

5 The function f is defined by $\mathrm{f} : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $\mathrm{f}(2) = 1$ and $\mathrm{f}(5) = 7$.

(i) Find the values of a and b. [2]

(ii) Solve the equation $\mathrm{ff}(x) = 0$. [3]

7 The line L_1 has equation $2x + y = 8$. The line L_2 passes through the point A $(7, 4)$ and is perpendicular to L_1.

(i) Find the equation of L_2. [4]

(ii) Given that the lines L_1 and L_2 intersect at the point B, find the length of AB. [4]

11 The equation of a curve is $y = 8x - x^2$.

(i) Express $8x - x^2$ in the form $a - (x + b)^2$, stating the numerical values of a and b. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. [2]

(iii) Find the set of values of x for which $y \geqslant -20$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 8x - x^2$, for $x \geqslant 4$.

(iv) State the domain and range of g^{-1}. [2]

(v) Find an expression, in terms of x, for $\mathrm{g}^{-1}(x)$. [3]

May/June 2004

6 The curve $y = 9 - \dfrac{6}{x}$ and the line $y + x = 8$ intersect at two points. Find

(i) the coordinates of the two points, [4]

(ii) the equation of the perpendicular bisector of the line joining the two points. [4]

10 The functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

(i) Find the set of values of x for which $\mathrm{f}(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $\mathrm{gf}(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$, making clear the relationship between the graphs. [2]

May/June 2005

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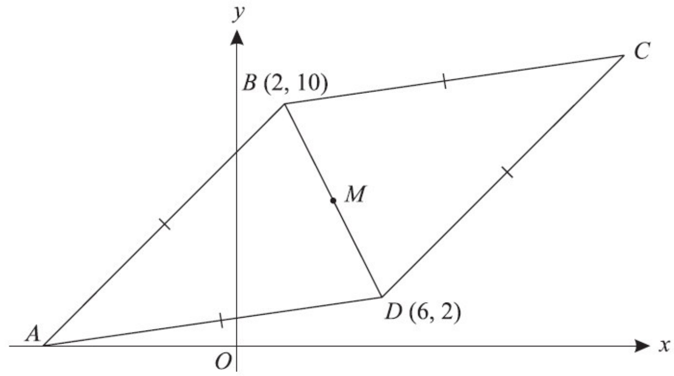

The diagram shows a rhombus $ABCD$. The points B and D have coordinates (2, 10) and (6, 2) respectively, and A lies on the x-axis. The mid-point of BD is M. Find, by calculation, the coordinates of each of M, A and C. [6]

May/June 2006

5 The curve $y^2 = 12x$ intersects the line $3y = 4x + 6$ at two points. Find the distance between the two points. [6]

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$
$$\mathrm{g} : x \mapsto \frac{9}{x+2} \quad \text{for } x \in \mathbb{R},\ x \neq -2.$$

(i) Find the values of k for which the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has two equal roots and solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$ in these cases. [6]

(ii) Solve the equation $\mathrm{fg}(x) = 5$ when $k = 6$. [3]

(iii) Express $\mathrm{g}^{-1}(x)$ in terms of x. [2]

May/June 2007

1 Find the value of the constant c for which the line $y = 2x + c$ is a tangent to the curve $y^2 = 4x$. [4]

4 Find the real roots of the equation $\dfrac{18}{x^4} + \dfrac{1}{x^2} = 4$. [4]

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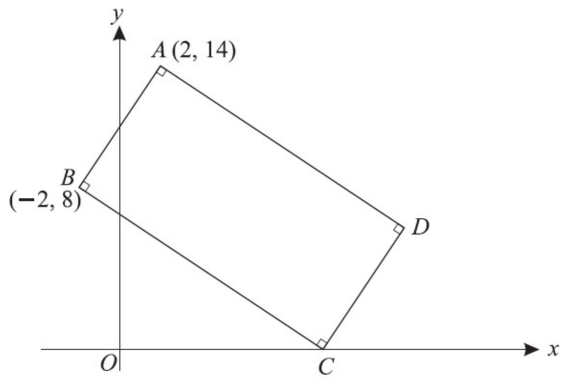

The diagram shows a rectangle $ABCD$. The point A is $(2, 14)$, B is $(-2, 8)$ and C lies on the x-axis. Find

(i) the equation of BC, [4]

(ii) the coordinates of C and D. [3]

May/June 2008

4 The equation of a curve C is $y = 2x^2 - 8x + 9$ and the equation of a line L is $x + y = 3$.

(i) Find the x-coordinates of the points of intersection of L and C. [4]

(ii) Show that one of these points is also the stationary point of C. [3]

8 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g} : x \mapsto \frac{9}{2 - x} \quad \text{for } x \in \mathbb{R}, x \neq 2.$$

(i) Find the values of k for which the equation $\mathrm{fg}(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $\mathrm{fg}(x) = x$ for the values of k found in part **(i)**. [3]

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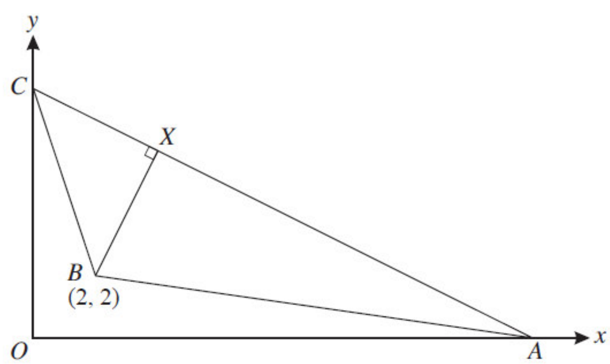

In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates $(2, 2)$. The perpendicular from B to AC meets AC at the point X.

(i) Find the coordinates of X. [4]

The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$. [3]

May/June 2009

2 Find the set of values of k for which the line $y = kx - 4$ intersects the curve $y = x^2 - 2x$ at two distinct points. [4]

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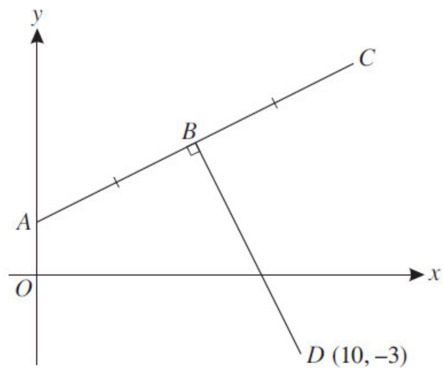

The diagram shows points A, B and C lying on the line $2y = x + 4$. The point A lies on the y-axis and $AB = BC$. The line from $D(10, -3)$ to B is perpendicular to AC. Calculate the coordinates of B and C. [7]

10 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 12x + 13$ for $0 \leqslant x \leqslant A$, where A is a constant.

(i) Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the value of A for which the graph of $y = \mathrm{f}(x)$ has a line of symmetry. [1]

(iii) When A has this value, find the range of f. [2]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 12x + 13$ for $x \geqslant 4$.

(iv) Explain why g has an inverse. [1]

(v) Obtain an expression, in terms of x, for $\mathrm{g}^{-1}(x)$. [3]

Oct/Nov 2001

1 Find the value of the constant k for which the line $y + 2x = k$ is a tangent to the curve $y = x^2 - 6x + 14$. [4]

2 **(i)** Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$. [3]

(ii) Given that $\mathrm{f} : x \mapsto 2x^2 - 12x + 11$, for the domain $x \geqslant 0$, find the range of f. [2]

6 Three points have coordinates $A(2, 5)$, $B(10, 9)$ and $C(6, 2)$. Line L_1 passes through A and B. Line L_2 passes through C and is perpendicular to L_1. Find the coordinates of the point of intersection of L_1 and L_2. [7]

Oct/Nov 2002

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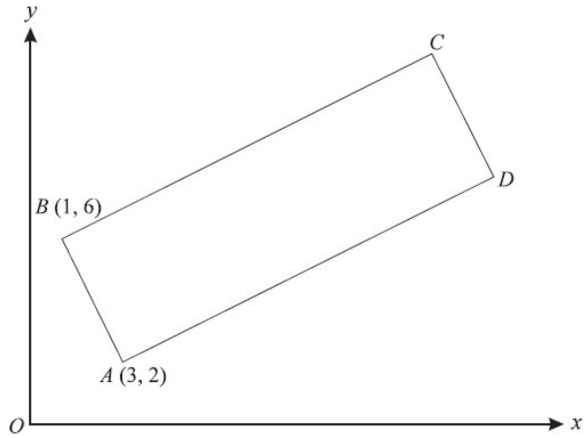

The diagram shows a rectangle $ABCD$, where A is $(3, 2)$ and B is $(1, 6)$.

(i) Find the equation of BC. [4]

Given that the equation of AC is $y = x - 1$, find

(ii) the coordinates of C, [2]

(iii) the perimeter of the rectangle $ABCD$. [3]

11 **(i)** Express $2x^2 + 8x - 10$ in the form $a(x + b)^2 + c$. [3]

(ii) For the curve $y = 2x^2 + 8x - 10$, state the least value of y and the corresponding value of x. [2]

(iii) Find the set of values of x for which $y \geqslant 14$. [3]

Given that $\mathrm{f} : x \mapsto 2x^2 + 8x - 10$ for the domain $x \geqslant k$,

(iv) find the least value of k for which f is one-one, [1]

(v) express $\mathrm{f}^{-1}(x)$ in terms of x in this case. [3]

Oct/Nov 2003

1 Find the coordinates of the points of intersection of the line $y + 2x = 11$ and the curve $xy = 12$. [4]

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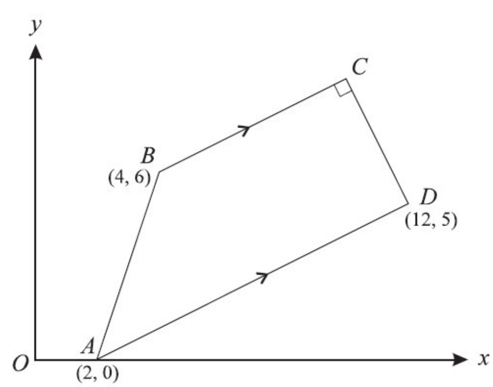

The diagram shows a trapezium $ABCD$ in which BC is parallel to AD and angle $BCD = 90°$. The coordinates of A, B and D are $(2, 0)$, $(4, 6)$ and $(12, 5)$ respectively.

(i) Find the equations of BC and CD. [5]

(ii) Calculate the coordinates of C. [2]

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{4}{2 - x}, \quad x \in \mathbb{R},\ x \neq 2.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 7$. [3]

(ii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [3]

(iii) Show that the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between these two graphs. [3]

Oct/Nov 2004

5 The equation of a curve is $y = x^2 - 4x + 7$ and the equation of a line is $y + 3x = 9$. The curve and the line intersect at the points A and B.

(i) The mid-point of AB is M. Show that the coordinates of M are $\left(\frac{1}{2}, 7\frac{1}{2}\right)$. [4]

(ii) Find the coordinates of the point Q on the curve at which the tangent is parallel to the line $y + 3x = 9$. [3]

(iii) Find the distance MQ. [1]

9 The function $\mathrm{f} : x \mapsto 2x - a$, where a is a constant, is defined for all real x.

(i) In the case where $a = 3$, solve the equation $\mathrm{ff}(x) = 11$. [3]

The function $\mathrm{g} : x \mapsto x^2 - 6x$ is defined for all real x.

(ii) Find the value of a for which the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has exactly one real solution. [3]

The function $\mathrm{h} : x \mapsto x^2 - 6x$ is defined for the domain $x \geqslant 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for $\mathrm{h}^{-1}(x)$ and state the domain of h^{-1}. [4]

Oct/Nov 2005

7 Three points have coordinates $A\,(2, 6)$, $B\,(8, 10)$ and $C\,(6, 0)$. The perpendicular bisector of AB meets the line BC at D. Find

(i) the equation of the perpendicular bisector of AB in the form $ax + by = c$, [4]

(ii) the coordinates of D. [4]

9 The equation of a curve is $xy = 12$ and the equation of a line l is $2x + y = k$, where k is a constant.

(i) In the case where $k = 11$, find the coordinates of the points of intersection of l and the curve. [3]

(ii) Find the set of values of k for which l does not intersect the curve. [4]

(iii) In the case where $k = 10$, one of the points of intersection is $P\,(2, 6)$. Find the angle, in degrees correct to 1 decimal place, between l and the tangent to the curve at P. [4]

Oct/Nov 2006

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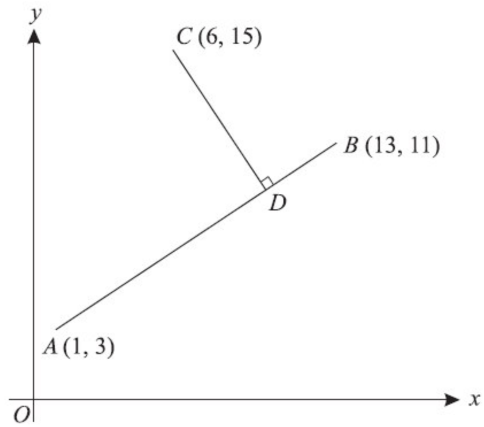


The three points A (1, 3), B (13, 11) and C (6, 15) are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

10 The function f is defined by $\mathrm{f} : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) > 4$. [3]

(ii) Express $\mathrm{f}(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

(iii) Write down the range of f. [1]

(iv) State, with a reason, whether f has an inverse. [1]

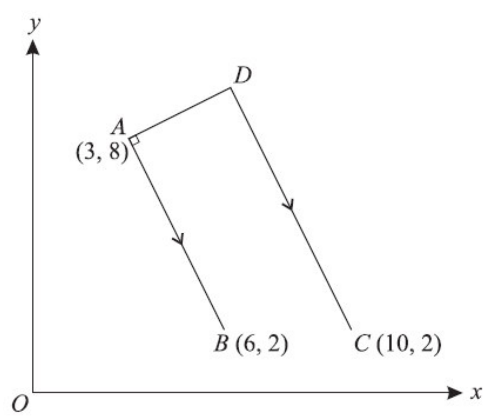
The function g is defined by $\mathrm{g} : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $\mathrm{g}(x) = 10$. [3]

Oct/Nov 2007

1 Determine the set of values of the constant k for which the line $y = 4x + k$ does not intersect the curve $y = x^2$. [3]

6



The three points A (3, 8), B (6, 2) and C (10, 2) are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. [7]

11 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $\mathrm{g}^{-1}(x)$ and state the range of g^{-1}. [4]

Oct/Nov 2008

8 The equation of a curve is $y = 5 - \dfrac{8}{x}$.

(i) Show that the equation of the normal to the curve at the point $P(2, 1)$ is $2y + x = 4$. [4]

This normal meets the curve again at the point Q.

(ii) Find the coordinates of Q. [3]

(iii) Find the length of PQ. [2]

10 The function f is defined by

$$\mathrm{f} : x \mapsto 3x - 2 \quad \text{for} \quad x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto 6x - x^2 \quad \text{for} \quad x \in \mathbb{R}.$$

(ii) Express $\mathrm{gf}(x)$ in terms of x, and hence show that the maximum value of $\mathrm{gf}(x)$ is 9. [5]

The function h is defined by

$$\mathrm{h} : x \mapsto 6x - x^2 \quad \text{for} \quad x \geqslant 3.$$

(iii) Express $6x - x^2$ in the form $a - (x-b)^2$, where a and b are positive constants. [2]

(iv) Express $\mathrm{h}^{-1}(x)$ in terms of x. [3]

Oct/Nov 2009/11

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 1, \quad x \in \mathbb{R},\ x > 0,$$

$$\mathrm{g} : x \mapsto \frac{2x-1}{x+3}, \quad x \in \mathbb{R},\ x \neq -3.$$

(i) Solve the equation $\mathrm{gf}(x) = x$. [3]

(ii) Express $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

(iii) Show that the equation $\mathrm{g}^{-1}(x) = x$ has no solutions. [3]

(iv) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

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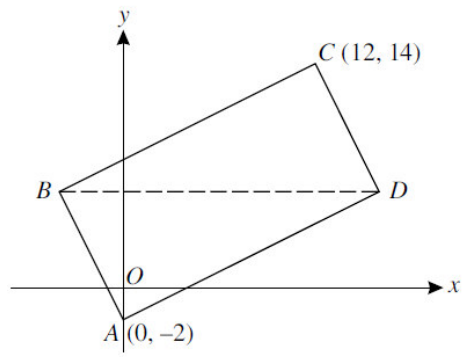

The diagram shows a rectangle $ABCD$. The point A is $(0, -2)$ and C is $(12, 14)$. The diagonal BD is parallel to the x-axis.

(i) Explain why the y-coordinate of D is 6. [1]

The x-coordinate of D is h.

(ii) Express the gradients of AD and CD in terms of h. [3]

(iii) Calculate the x-coordinates of D and B. [4]

(iv) Calculate the area of the rectangle $ABCD$. [3]

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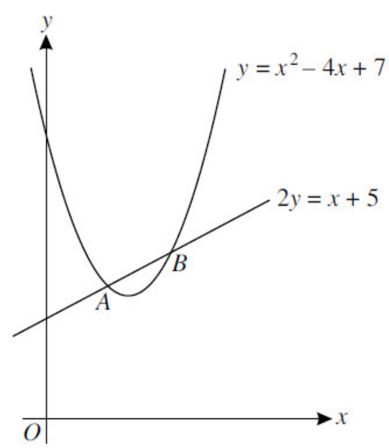

(i) The diagram shows the line $2y = x + 5$ and the curve $y = x^2 - 4x + 7$, which intersect at the points A and B. Find

(a) the x-coordinates of A and B, [3]

(b) the equation of the tangent to the curve at B, [3]

(c) the acute angle, in degrees correct to 1 decimal place, between this tangent and the line $2y = x + 5$. [3]

(ii) Determine the set of values of k for which the line $2y = x + k$ does not intersect the curve $y = x^2 - 4x + 7$. [4]

May/June 2010/11

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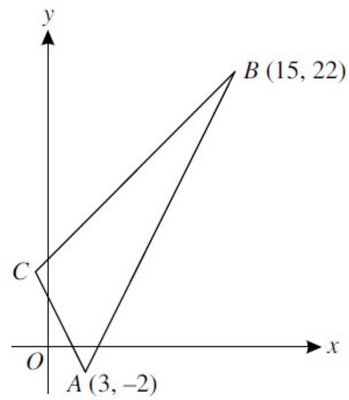


The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB, AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

(i) Find the gradient of AB and deduce the value of m. [2]

(ii) Find the coordinates of C. [4]

The perpendicular bisector of AB meets BC at D.

(iii) Find the coordinates of D. [4]

9 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $\mathrm{f}(x)$ in the form $a(x - b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $\mathrm{f}(x) < 21$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $\mathrm{gf}(x) = 0$ has two equal roots. [4]

May/June 2010/12

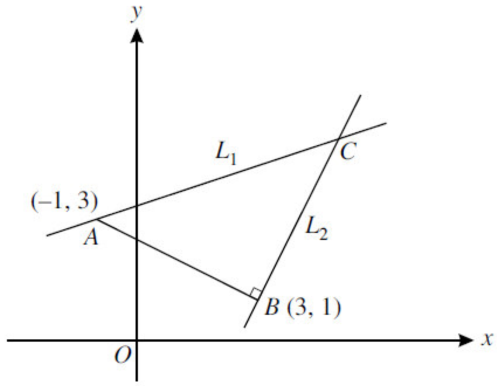
3 The functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 4x - 2x^2,$$
$$\mathrm{g} : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $\mathrm{gf}(x) = k$ has equal roots. [3]

4



In the diagram, A is the point $(-1, 3)$ and B is the point $(3, 1)$. The line L_1 passes through A and is parallel to OB. The line L_2 passes through B and is perpendicular to AB. The lines L_1 and L_2 meet at C. Find the coordinates of C. [6]

May/June 2010/13

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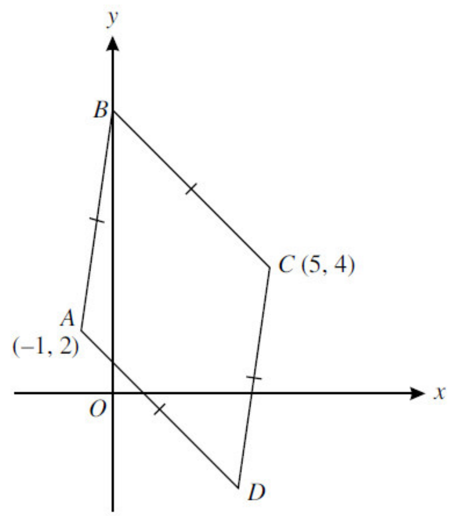

The diagram shows a rhombus $ABCD$ in which the point A is $(-1, 2)$, the point C is $(5, 4)$ and the point B lies on the y-axis. Find

(i) the equation of the perpendicular bisector of AC, [3]

(ii) the coordinates of B and D, [3]

(iii) the area of the rhombus. [3]

10 The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x. [3]

Oct/Nov 2010/11

3 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 3,$$
$$g : x \mapsto x^2 - 2x.$$

Express $gf(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [5]

Oct/Nov 2010/12

6 A curve has equation $y = kx^2 + 1$ and a line has equation $y = kx$, where k is a non-zero constant.

(i) Find the set of values of k for which the curve and the line have no common points. [3]

(ii) State the value of k for which the line is a tangent to the curve and, for this case, find the coordinates of the point where the line touches the curve. [4]

7 The function f is defined by

$$f(x) = x^2 - 4x + 7 \text{ for } x > 2.$$

(i) Express $f(x)$ in the form $(x-a)^2 + b$ and hence state the range of f. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by

$$g(x) = x - 2 \text{ for } x > 2.$$

The function h is such that f = hg and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]

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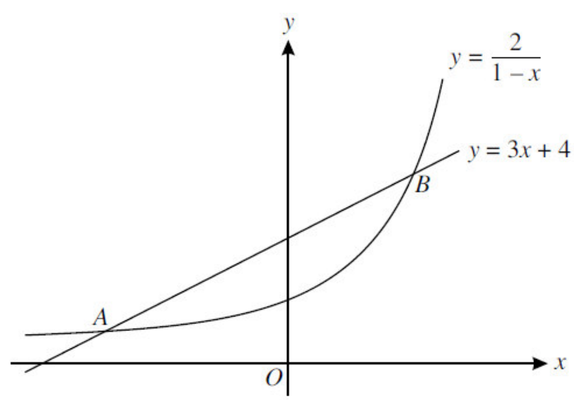


The diagram shows part of the curve $y = \frac{2}{1-x}$ and the line $y = 3x + 4$. The curve and the line meet at points A and B.

(i) Find the coordinates of A and B. [4]

(ii) Find the length of the line AB and the coordinates of the mid-point of AB. [3]

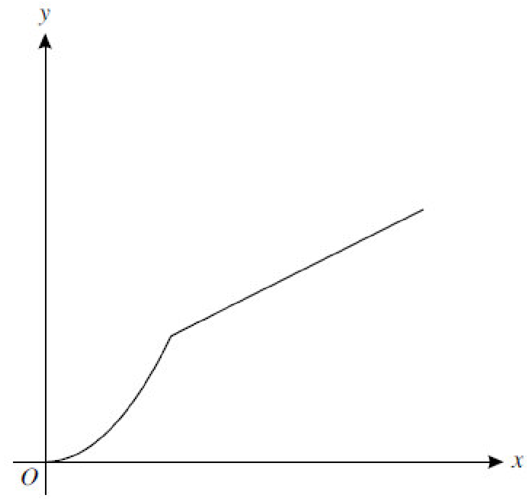
Oct/Nov 2010/13

2 Points A, B and C have coordinates (2, 5), (5, −1) and (8, 6) respectively.

(i) Find the coordinates of the mid-point of AB. [1]

(ii) Find the equation of the line through C perpendicular to AB. Give your answer in the form $ax + by + c = 0$. [3]

7



The diagram shows the function f defined for $0 \leqslant x \leqslant 6$ by

$$x \mapsto \tfrac{1}{2}x^2 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$x \mapsto \tfrac{1}{2}x + 1 \quad \text{for } 2 < x \leqslant 6.$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(iii) Obtain expressions to define $\mathrm{f}^{-1}(x)$, giving the set of values of x for which each expression is valid. [4]