Derivation of the one factor Clewlow and Strickland Spot Price Stochastic Differential Equation

The derived SDEs are solved numerically using a range of finite difference schemes

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Abstract. This documents details the derivation of the one factor spot price stochastic differential equation (SDE) by adding additional steps to the Clewlow and Strickland derivation (see Appendix A in reference [1]). Also discussed are several finite difference numerical schemes to solve the derived SDEs that are compared to the analytical solution of European call and put options; apologies in advance for grammatical errors, typos and lack of mathematical rigour.

1. Derivation

The derivation of the spot process starts from the one factor forward price SDE and adopting the notation from Clewlow and Strickland as detailed in reference [1] such that

$$\frac{dF(t,T)}{F(t,T)} = \sigma(t,T)dz(t). \tag{1}$$

Let $\eta = \ln(F(t,T))$ and applying Ito's lemma such that

$$d\eta = \frac{d\eta}{dF}dF + \frac{1}{2}\frac{d^2\eta}{dF^2}[dF]^2, \tag{2}$$

where $\frac{d\eta}{dF} = \frac{1}{F}$ and $\frac{d^2\eta}{dF^2} = -\frac{1}{F^2}$; using equation (1) for dF, the above equation can be rewritten as

$$\begin{aligned} d\ln(F(t,T)) &= \sigma(t,T)dz(t) - \frac{1}{2}\left[\sigma^2(t,T)dz^2(t)\right], \\ &= \sigma(t,T)dz(t) - \frac{1}{2}\sigma^2(t,T)dt. \end{aligned}$$

Assuming that $dz^2(t) = dt$ generally gives the correct formulae, see Wilmott for example (cf. reference [8]). Changing the short hand notation of the above SDE to the integral form to give

$$\ln(F(t,T)) - \ln(F(0,T)) = -\frac{1}{2}\int_0^t \sigma^2(u,T)du + \int_0^t \sigma(u,T)dz(u);$$

the first integral is generally referred to as a Lesbesgue integral and the second integral is of Ito type with respect to scalar Brownian motion process $z(t)$. Taking the exponential of both sides of the above equation such that

$$F(t,T) = F(0,T)\exp\left(-\frac{1}{2}\int_0^t \sigma^2(u,T)du + \int_0^t \sigma(u,T)dz(u)\right). \tag{3}$$

Setting $T = t$ gives the spot process as

$$S(t) = F(t,t) = F(0,t)\exp\left(-\frac{1}{2}\int_0^t \sigma^2(u,t)du + \int_0^t \sigma(u,t)dz(u)\right) \tag{4}$$

and taking logarithms using the above equation now gives

$$\ln(S(t)) = \ln(F(0,t)) - \frac{1}{2}\int_0^t \sigma^2(u,t)du + \int_0^t \sigma(u,t)dz(u). \tag{5}$$

The next step is to apply Ito's lemma to equation (5) for the change in spot price S as a function of time t and the Weiner process z such that

$$dS(t) = \frac{\partial S}{\partial t}dt + \frac{\partial S}{\partial z(t)}dz(t) + \frac{1}{2}\frac{\partial^2 S}{\partial z(t)^2}dt. \tag{6}$$

The last two terms in the above equation are given by

$$\frac{\partial S}{\partial z(t)} = \sigma(t,t)S(t) \quad \text{and} \quad \frac{\partial^2 S}{\partial z(t)^2} = \sigma^2(t,t)S(t) \tag{7}$$

from the partial differentiation of (5) using the last integral; noting that

$$\frac{\partial^2 S}{\partial z(t)^2} = \frac{\partial}{\partial z(t)}\left(\frac{\partial S}{\partial z(t)}\right) = \frac{\partial}{\partial z(t)}\left(\sigma(t,t)S(t)\right).$$

The next step is to derive the partial derivative of S with respect to time t from equation (5) to give

$$\frac{\partial S}{\partial t} = S(t)\left[\frac{\partial \ln(F(0,t))}{\partial t} - \frac{1}{2}\sigma^2(t,t) - \int_0^t \sigma(u,t)\frac{\partial \sigma}{\partial t}du + \int_0^t \frac{\partial \sigma}{\partial t}dz(u)\right]; \tag{8}$$

note that the additional term after the partially differentiated logarithm of the forward curve is from Leibniz's rule determined from the Lebesgue integral in equation (5). Substituting equations (8) and (7) into equation (6) gives

$$\begin{aligned}\frac{dS}{S(t)} = &\left[\frac{\partial \ln(F(0,t))}{\partial t} - \frac{1}{2}\sigma^2(t,t) - \int_0^t \sigma(u,t)\frac{\partial \sigma}{\partial t}du + \int_0^t \frac{\partial \sigma}{\partial t}dz(u)\right]dt \\ &+\sigma(t,t)dz(t) + \frac{1}{2}\sigma^2(t,t)dt.\end{aligned} \tag{9}$$

Simplifying the above equation now gives the spot price SDE as

$$\frac{dS}{S(t)} = \left[\frac{\partial \ln(F(0,t))}{\partial t} - \int_0^t \sigma(u,t)\frac{\partial \sigma}{\partial t}du + \int_0^t \frac{\partial \sigma}{\partial t}dz(u)\right]dt + \sigma(t,t)dz(t). \tag{10}$$

In order to derive a tractable spot price process, the next step is to apply a term structure. For the single factor model, the following relationships are applied:

$$\sigma(t,T) = \sigma\exp(-\alpha(T-t)), \tag{11}$$

$$\frac{\partial \sigma(t,T)}{\partial T} = -\alpha\sigma\exp(-\alpha(T-t)), \tag{12}$$

where σ is the cash volatility and α is the mean reversion rate. Substituting equations (11) and (12) into equation (10) with $T = t$ gives

$$\begin{aligned}\frac{dS}{S(t)} = &\left[\frac{\partial \ln(F(0,t))}{\partial t} + \int_0^t \sigma\exp(-\alpha(t-u))\alpha\sigma\exp(-\alpha(t-u))du - \right. \\ &\left.\int_0^t \alpha\sigma\exp(-\alpha(t-u))dz(u)\right]dt + \sigma dz(t).\end{aligned}$$

Simplifying such that

$$\frac{dS}{S(t)} = \left[\frac{\partial \ln(F(0,t))}{\partial t} + \alpha\sigma^2 \int_0^t \exp(-2\alpha(t-u))du - \right.$$
$$\left. \alpha\sigma \int_0^t \exp(-\alpha(t-u))dz(u) \right] dt + \sigma dz(t)$$

and evaluating the Lesbesgue integral to give

$$\frac{dS}{S(t)} = \left[\frac{\partial \ln(F(0,t))}{\partial t} + \alpha\sigma^2 \left[\frac{1}{2\alpha} \exp(-2\alpha(t-u)) \right]_{u=0}^{u=t} \right.$$
$$\left. - \alpha\sigma \int_0^t \exp(-\alpha(t-u))dz(u) \right] dt + \sigma dz(t).$$

Simplifying again to give

$$\frac{dS}{S(t)} = \left[\frac{\partial \ln(F(0,t))}{\partial t} + \frac{\sigma^2}{2} \left[1 - \exp(-2\alpha t)\right] - \alpha\sigma \int_0^t \exp(-\alpha(t-u))dz(u) \right] dt$$
$$+\sigma dz(t). \tag{13}$$

The next step is to determine the Ito integration term. This is done by substituting equations (11) and (12) into equation (5) to give

$$\ln(S(t)) = \ln(F(0,t)) - \frac{1}{2} \int_0^t \sigma^2 \exp(-2\alpha(t-u))du + \int_0^t \sigma \exp(-\alpha(t-u))dz(u). \tag{14}$$

Evaluating the Lesbesgue integral gives

$$\ln(S(t)) = \ln(F(0,t)) - \frac{1}{2}\sigma^2 \left[\frac{1}{2\alpha} \exp(-2\alpha(t-u)) \right]_{u=0}^{u=t} + \int_0^t \sigma \exp(-\alpha(t-u))dz(u) \tag{15}$$

and simplifying such that

$$\ln(S(t)) = \ln(F(0,t)) - \frac{\sigma^2}{4\alpha} \left[1 - \exp(-2\alpha t)\right] + \int_0^t \sigma \exp(-\alpha(t-u))dz(u). \tag{16}$$

Hence the relationship for the Ito integral is given as

$$\sigma \int_0^t \exp(-\alpha(t-u))dz(u) = \ln(S(t)) - \ln(F(0,t)) + \frac{\sigma^2}{4\alpha} \left[1 - \exp(-2\alpha t)\right]. \tag{17}$$

In order to complete the relationship for spot price SDE, equation (17) is substituted into equation (13) that gives

$$\frac{dS}{S(t)} = \left[\frac{\partial \ln(F(0,t))}{\partial t} + \frac{\sigma^2}{2} \left[1 - \exp(-2\alpha t)\right] - \right.$$
$$\left. \alpha \left(\ln(S(t)) - \ln(F(0,t)) + \frac{\sigma^2}{4\alpha} \left[1 - \exp(-2\alpha t)\right] \right) \right] dt + \sigma dz(t).$$

Simplifying the above equation gives the spot price SDE as

$$\frac{dS}{S(t)} = \left[\frac{\partial \ln(F(0,t))}{\partial t} + \alpha \left(\ln(F(0,t)) - \ln(S(t))\right) + \frac{\sigma^2}{4} \left[1 - \exp(-2\alpha t)\right] \right] dt$$
$$+\sigma dz(t). \tag{18}$$

This is the same spot price process given in Energy Derivatives by Clewlow and Strickland (see [2] in Section 8.5) and detailed further in the Valuing Energy options papers, see Appendix A in reference [1].

1.1. Logarithm coordinates of the Spot Price

Generally, the logarithm change in spot price is applied in the numerical approximation to simulate the spot process as discussed below. In order to derive the logarithm change in spot price, the derivation starts from equation (5) and using Ito's lemma such that $V = \ln(S(t))$ to give

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial z(t)}dz(t) + \frac{1}{2}\frac{\partial^2 V}{\partial z(t)^2}dt. \tag{19}$$

The last two terms in the above equation are now given by

$$\frac{\partial V}{\partial z(t)} = \sigma(t,t) \quad \text{and} \quad \frac{\partial^2 V}{\partial z(t)^2} = 0. \tag{20}$$

The next step is to derive the partial derivative of V with respect to time t from equation (5) to give

$$\frac{\partial V}{\partial t} = \left[\frac{\partial \ln(F(0,t))}{\partial t} - \frac{1}{2}\sigma^2(t,t) - \int_0^t \sigma(u,t)\frac{\partial \sigma}{\partial t}du + \int_0^t \frac{\partial \sigma}{\partial t}dz(u)\right]; \tag{21}$$

simplifying now gives the spot price SDE as

$$d\ln(S(t)) = \left[\frac{\partial \ln(F(0,t))}{\partial t} - \frac{1}{2}\sigma(t,t) - \int_0^t \sigma(u,t)\frac{\partial \sigma}{\partial t}du + \int_0^t \frac{\partial \sigma}{\partial t}dz(u)\right]dt \\ +\sigma(t,t)dz(t). \tag{22}$$

It is noted that the logarithm form of the spot price has an additional σ^2 term when compared to equation (10). Including the above term structure and following the above integrations gives the spot price relationship as

$$d\ln S = \left[\frac{\partial \ln(F(0,t))}{\partial t} + \alpha\left(\ln(F(0,t)) - \ln(S(t))\right) - \frac{\sigma^2}{4}\left[1+\exp(-2\alpha t)\right]\right]dt \\ +\sigma dz(t) \tag{23}$$

and is applied in the discretized form discussed in the following section. The logarithm transformation of spot price is applied in the tree building procedure in Clewlow and Strickland, see reference [1] that details the same equation.

2. Discretized Spot Price

This section discusses a range of numerical schemes that are applied to solve the above derived SDEs; see reference [3] that details the first four schemes implemented below.

2.1. Case 1: Transformed Explicit Euler Scheme

The logarithm transformation of the spot price is a good starting point since the convergence is generally more stable as discussed in Jackel (see reference [5]) and is the initial scheme investigated.

The discretization applies the Euler approach in equal time steps for the spot price such that $S(t) = S(n\Delta t)$ defined as the subscript notation S_n and similarly for the forward curve, hence

$$\ln S_n - \ln S_{n-1} = \left[\frac{(\ln(F_n) - \ln(F_{n-1}))}{\Delta t} + \alpha\left(\ln(F_{n-1}) - \ln(S_{n-1})\right) - \frac{\sigma^2}{4}\left[1 + \exp(-2\alpha(n-1)\Delta t)\right]\right]\Delta t + \sigma\epsilon\sqrt{\Delta t}, \tag{24}$$

where ϵ is the standard normally distributed random variable. The model is based on daily data such that $\Delta t = 1$ and simplifying the above relationship by taking exponentials to give

$$S_n = S_{n-1}^{1-\alpha} F_{n-1}^{\alpha-1} F_n \exp\left[-\frac{\sigma^2}{4}\left[1 + \exp(-2\alpha(n-1))\right] + \sigma\epsilon\right]. \tag{25}$$

The above discretized equation requires the initial condition $S_0 = F_0$. Extending the model to smaller time steps is discussed below.

2.2. Case 2: Explicit Euler Scheme

For comparison purposes, the discretized form of the non logarithmic spot price given by equation (18) using the Euler approach is

$$S_n = S_{n-1}\left[1 + \left((\ln(F_n/Fn-1)/\Delta t - \alpha\ln(F_{n-1}/S_{n-1}) + \frac{\sigma^2}{4}\left[1 - \exp(-2\alpha(n-1))\right]\right)\Delta t + \sigma\epsilon\sqrt{\Delta t}\right)\right]. \tag{26}$$

Simplifying for the daily discretization such that $\Delta t = 1$ gives

$$S_n = S_{n-1}\left[1 + \left(\ln(F_n/Fn-1) - \alpha\ln(F_{n-1}/S_{n-1}) + \frac{\sigma^2}{4}\left[1 - \exp(-2\alpha(n-1))\right] + \sigma\epsilon\right)\right] \tag{27}$$

and is applied in the validation section.

2.3. Case 3: Transformed semi implicit Euler Scheme

This scheme is similar to Case 1 accept that the logarithms of $F(0,t)$ and $S(t)$ are made implicit such that

$$S_n^{1+\alpha\Delta t} = S_{n-1}\exp\left(\left[\frac{\ln(F_n/F_{n-1})}{\Delta t} + \alpha\ln(F_n) - \frac{\sigma^2}{4}\left(1 + \exp(-2\alpha(n-1)\Delta t)\right)\right]\Delta t + \sigma\epsilon\sqrt{\Delta t}\right). \tag{28}$$

Again, the simplified daily scheme is given by

$$S_n^{1+\alpha} = S_{n-1}F_{n-1}^{-1}F_n^{1+\alpha}\exp\left[-\frac{\sigma^2}{4}\left[1 + \exp(-2\alpha(n-1))\right] + \sigma\epsilon\right]. \tag{29}$$

The same initial conditions are applied

2.4. Case 4: Weak predictor-corrector scheme

The next discretization details the a weak predictor-corrector scheme such that the spot process is only updated in the drift term. The predictor and corrector scheme is given by

$$\widehat{S_n} = S_{n-1} \exp\left(\left[\frac{\ln(F_n/F_{n-1})}{\Delta t} + \alpha \ln(F_{n-1}/S_{n-1}) - \frac{\sigma^2}{4}\left(1+\exp(-2\alpha(n-1)\Delta t)\right)\right]\Delta t \right.$$
$$\left. +\sigma\epsilon\sqrt{\Delta t}\right).$$
$$S_n = S_{n-1} \exp\left(\left[\frac{\ln(F_n/F_{n-1})}{\Delta t} + \alpha \ln(F_{n-1}/\widehat{S_n}) - \frac{\sigma^2}{4}\left(1+\exp(-2\alpha(n-1)\Delta t)\right)\right]\Delta t \right.$$
$$\left. +\sigma\epsilon\sqrt{\Delta t}\right). \quad (30)$$

Setting Δt to unity gives

$$\widehat{S_n} = S_{n-1}^{1-\alpha} F_{n-1}^{\alpha-1} F_n \exp\left[-\frac{\sigma^2}{4}\left[1+\exp(-2\alpha(n-1))\right] + \sigma\epsilon_n\right],$$
$$S_n = S_{n-1}\widehat{S_n}^{-\alpha} F_{n-1}^{\alpha-1} F_n \exp\left[-\frac{\sigma^2}{4}\left[1+\exp(-2\alpha(n-1))\right] + \sigma\epsilon_n\right]. \quad (31)$$

Note that the same random variable is used in each step.

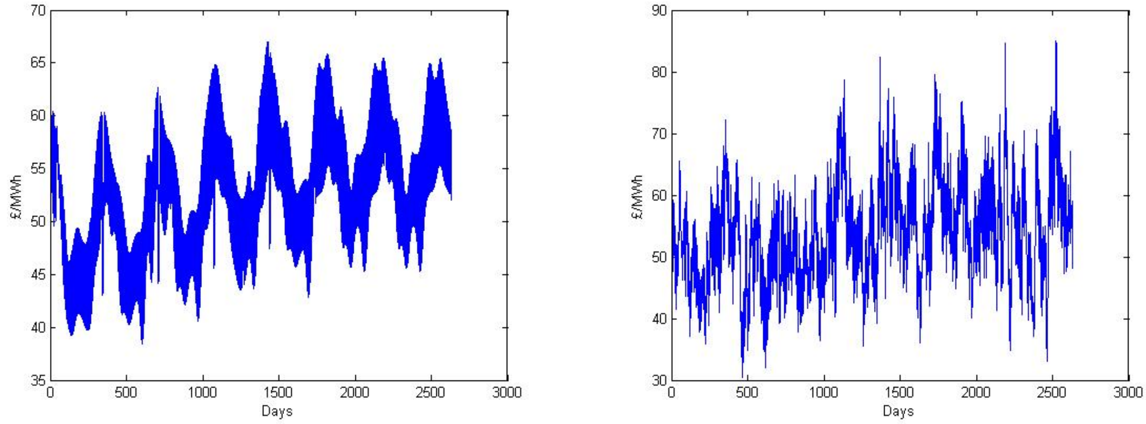
2.5. Interpolation

In order to achieve convergence and stability of any finite difference scheme, the discretized time step is an important component; refer to standard finite difference text books in reference [7] or [6]. Stochastic differential equations are no different, as observed in the case study in reference [3] when solving the Heston stochastic volatility model; for example they show that decreasing the time step generally increases the accuracy of the numerical scheme. Note that just increasing the number of simulations will not reduce the discretization bias.

The main problem when dealing with forward curve data is that the granularity is often daily and what is actually required is data that corresponds to the discretized time step. The solution adopted in the examples below is to simply linearly interpolate the forward curve at the required time step. Note that the spot price at the daily granularity will still be based on the actual forward curve and not from an interpolated point.

3. Monte Carlo Example

The following example details a spot price path that has applied the mean reversion value $\alpha = 0.06$ and cash volatility $\sigma = 0.04$; the spot process applies the daily power forward curve profiled below into the discretized logarithm coordinates spot equation (25) from the Euler scheme.



(a) Daily Power Forward Curve. (b) Selected daily spot price path.

Figure 1. $\sigma = 0.04$ and $\alpha = 0.06$: example of Power forward curve converted to spot price.

The simulation was implemented in Matlab using the Mersenne Twister random number generator.

4. Validation

The spot price process can be validated by pricing European options using the analytical formula derived by Black and Scholes and comparing the result to the Monte Carlo valuation.

The analytical formula for a standard European call option is given by:

$$C(t, S(t); K, T) = P(t, T)[F(t, T)N(h) - KN(h - \sqrt{w})], \tag{32}$$

where $P(t, T)$ is the T-maturity discount factor, $F(t, T)$ is the forward curve at maturity, $N()$ is the standardised normal distribution, K is the strike price,

$$h = \frac{\ln(F(t, T)/K) + \frac{1}{2}w}{\sqrt{w}} \quad \text{and} \quad w = \frac{\sigma^2}{2\alpha}\left[1 - \exp(-2\alpha(T - t)\right]. \tag{33}$$

The analytical formula for a standard European put option is given by:

$$P(t, S(t); K, T) = P(t, T)[KN(-h + \sqrt{w}) - F(t, T)N(-h)], \tag{34}$$

where h and w are defined by equations (33). The details of the derivation of the call and put analytical forms are discussed in reference [2].

The following valuations of the call and put options in the numerical results section are based on a strike price of £53.51 per MWh which is the first forward curve price such that the options are at the money; the maturities are set from 1 to 730 days with $\sigma = 0.04$ and $\alpha = 0.06$. Note that the discount factor is set to unity for simplicity.

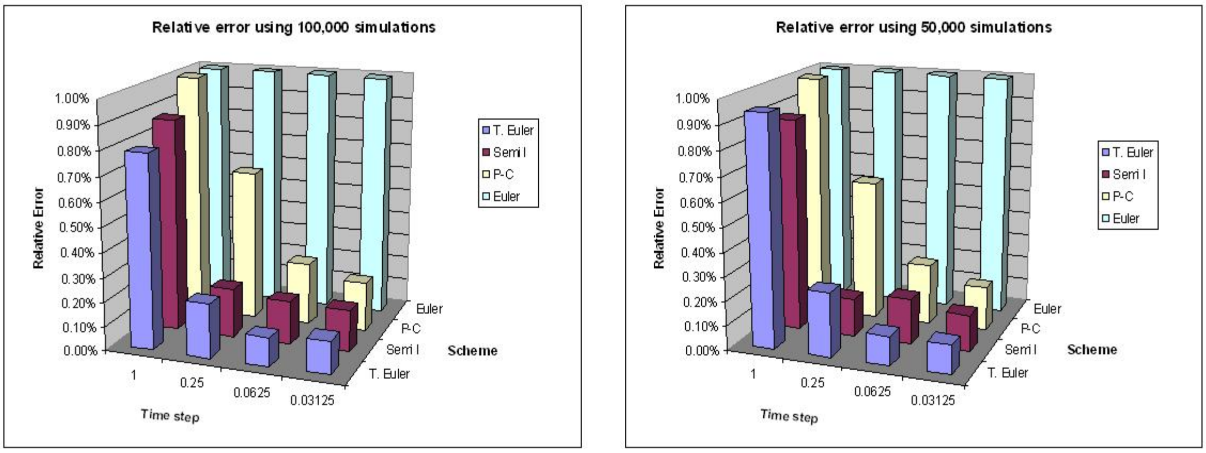
5. Numerical Results

In order to determine the accuracy of the Monte Carlo results based on the different finite difference schemes, the value of the option prices are averaged over the maturities 2 to 730 days (effectively valuing an Asian option maturing in 730 days averaged over 729 days) and using the above volatility parameters. The number of simulations is increased and the time step is decreased to highlight the convergence as summarised below. Note that all the results are listed in the appendix in greater detail. Note that the relative error is the basis of the validation which is defined as the absolute difference between the Monte Carlo simulate price and the analytical price that is referenced to the analytical price to obtain the percentage.

The first set of results detail the Euler scheme based on the logarithm transformation from equation (24) and comparing these results with the Euler scheme based on the non-transformed scheme from equation (26) highlight how the transformation improves the accuracy of valuing the options. It is noted that the logarithm transformation would give a similar order of convergence if the Milstein scheme involving higher orders of the Ito-Taylor expansion were applied to equation (18), the initial derived spot price process. The transformation and the Milstein scheme effectively make the discretization equations exact to order $O(\Delta t)$; see discussion in Jackel, reference [5].

It is noted that the Euler scheme based on the non-transformed spot equation can be improved considerably by reducing the time step; however the improvement is at a cost to increasing the number of iterations that will obviously increase the run time.

The following plots show the relative error to the analytical solution where the call and put options have been averaged to simplify the comparison of the schemes. The labelling on the figure refer to: Euler scheme on equation (18) (Euler); Euler scheme on the logarithm transformed coordinates SDE (T. Euler); Semi implicit scheme on the logarithm transformation SDE (Semi I) and the weak predictor-corrector scheme (P-C).



(a) 100,000 Simulations. (b) 50,000 Simulations.

Figure 2. Numerical relative errors of the different schemes.

Note that the above plots have been truncated at 1.0% such that the Euler results have relative errors greater than 1.0%. The Transformed Euler scheme appears to give the best results closely followed by semi implicit scheme based on the above simulations; note that having a time step of unit clearly leads to discrtisation bias; see discussion of bias errors in Glasserman (reference [4]) and requires a smaller time step to reduce this bias.

6. Conclusions

This document has detailed the derivation of the one factor Clewlow and Strickland spot price model and has lead to several numerical Monte Carlo models. The simulated spot prices have been validated by the Black and Scholes analytical formulae for European call and put options for a range of numerical schemes. Of the schemes implemented, the Euler logarithm transformation gave the most favourable results of the finite difference schemes. It is noted that further improvements to all the implemented schemes could be made by decreasing the time step and is dependent on the level of accuracy required and the time requirements since decreasing the time step will clearly lead to an increase in computational time.

Note that further variance reduction techniques can be applied such as implementing a Sobol sequence for example. It is also noted that other numerical approaches can be implemented, however the document aim is to investigate finite difference schemes.

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Appendix

A. Numerical Results

The following tables list the results of each scheme having time steps 1, 0.25, 0.0625 and 0.03125 respectively for each scheme.

A.1. Transformed Explicit Euler Scheme

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0588	0.86%	5.1363	5.1593	0.45%
4,000	2.0413	2.0686	1.34%	5.1363	5.1542	0.35%
10,000	2.0413	2.0600	0.92%	5.1363	5.1713	0.68%
15,000	2.0413	2.0555	0.70%	5.1363	5.1651	0.56%
20,000	2.0413	2.0754	1.67%	5.1363	5.1507	0.28%
50,000	2.0413	2.0742	1.61%	5.1363	5.1506	0.28%
100,000	2.0413	2.0662	1.22%	5.1363	5.1548	0.36%

Figure 3. Case 1: $\Delta t = 1$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0516	0.50%	5.1363	5.1323	0.08%
4,000	2.0413	2.0577	0.80%	5.1363	5.1361	0.00%
10,000	2.0413	2.0527	0.56%	5.1363	5.1359	0.01%
15,000	2.0413	2.0546	0.65%	5.1363	5.1358	0.01%
20,000	2.0413	2.0539	0.62%	5.1363	5.1348	0.03%
50,000	2.0413	2.0497	0.41%	5.1363	5.1424	0.12%
100,000	2.0413	2.0476	0.31%	5.1363	5.1432	0.13%

Figure 4. Case 1: $\Delta t = 0.25$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0318	0.47%	5.1363	5.1261	0.08%
4,000	2.0413	2.0368	0.22%	5.1363	5.1318	0.00%
10,000	2.0413	2.0392	0.10%	5.1363	5.1366	0.01%
15,000	2.0413	2.0546	0.65%	5.1363	5.1358	0.01%
20,000	2.0413	2.0539	0.62%	5.1363	5.1348	0.03%
50,000	2.0413	2.0381	0.16%	5.1363	5.1407	0.09%
100,000	2.0413	2.0380	0.16%	5.1399	5.1432	0.07%

Figure 5. Case 1: $\Delta t = 0.0625$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0318	0.47%	5.1363	5.1261	0.08%
4,000	2.0413	2.0368	0.22%	5.1363	5.1318	0.00%
10,000	2.0413	2.0392	0.10%	5.1363	5.1366	0.01%
15,000	2.0413	2.0546	0.65%	5.1363	5.1358	0.01%
20,000	2.0413	2.0539	0.62%	5.1363	5.1348	0.03%
50,000	2.0413	2.0381	0.16%	5.1363	5.1407	0.09%
100,000	2.0413	2.0380	0.16%	5.1399	5.1432	0.07%

Figure 6. Case 1: $\Delta t = 0.03125$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

A.2. Explicit Euler Scheme with no Transformation

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.2441	39.05%	5.1363	6.7664	31.74%
4,000	2.0413	1.2349	39.50%	5.1363	6.7789	31.98%
10,000	2.0413	1.2332	39.59%	5.1363	6.7708	31.82%
15,000	2.0413	1.2356	39.47%	5.1363	6.7703	31.81%
20,000	2.0413	1.2365	39.43%	5.1363	6.7655	31.72%
50,000	2.0413	1.2347	39.51%	5.1363	6.7711	31.83%
100,000	2.0413	1.2337	39.56%	5.1363	6.7722	31.85%

Figure 7. Case 2: $\Delta t = 1$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.7977	11.93%	5.1363	5.5251	7.57%
4,000	2.0413	1.8040	11.62%	5.1363	5.5291	7.65%
10,000	2.0413	1.7994	11.85%	5.1363	5.5287	7.64%
15,000	2.0413	1.8011	11.77%	5.1363	5.5286	7.64%
20,000	2.0413	1.8003	11.81%	5.1363	5.5276	7.62%
50,000	2.0413	1.7968	11.98%	5.1363	5.5355	7.77%
100,000	2.0413	1.7950	12.07%	5.1363	5.5364	7.79%

Figure 8. Case 2: $\Delta t = .25$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.9653	3.72%	5.1363	5.2237	1.70%
4,000	2.0413	1.9703	3.48%	5.1363	5.2294	1.81%
10,000	2.0413	1.9728	3.36%	5.1363	5.2341	1.90%
15,000	2.0413	1.9698	3.50%	5.1363	5.2334	1.89%
20,000	2.0413	1.9716	3.41%	5.1363	5.2362	1.94%
50,000	2.0413	1.9717	3.41%	5.1363	5.2383	1.99%
100,000	2.0413	1.9717	3.41%	5.1363	5.2374	1.97%

Figure 9. Case 2: $\Delta t = 0.0625$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0153	1.27%	5.1363	5.1857	0.96%
4,000	2.0413	2.0119	1.44%	5.1363	5.1812	0.87%
10,000	2.0413	2.0112	1.47%	5.1363	5.1902	1.05%
15,000	2.0413	2.0058	1.74%	5.1363	5.1952	1.15%
20,000	2.0413	2.0054	1.76%	5.1363	5.1978	1.20%
50,000	2.0413	2.0059	1.73%	5.1363	5.1922	1.09%
100,000	2.0413	2.0043	1.81%	5.1363	5.1903	1.05%

Figure 10. Case 2: $\Delta t = 0.03125$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

A.3. Transformed semi implicit Euler Scheme

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.9880	2.61%	5.1363	5.1456	0.18%
4,000	2.0413	2.0042	1.82%	5.1363	5.1322	0.08%
10,000	2.0413	2.0160	1.24%	5.1363	5.1119	0.48%
15,000	2.0413	2.0111	1.48%	5.1363	5.1157	0.40%
20,000	2.0413	2.0167	1.21%	5.1363	5.1170	0.38%
50,000	2.0413	2.0141	1.33%	5.1363	5.1151	0.41%
100,000	2.0413	2.0136	1.36%	5.1363	5.1166	0.38%

Figure 11. Case 3: $\Delta t = 1$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0379	0.17%	5.1363	5.1236	0.25%
4,000	2.0413	2.0441	0.14%	5.1363	5.1274	0.17%
10,000	2.0413	2.0391	0.11%	5.1363	5.1272	0.18%
15,000	2.0413	2.0409	0.02%	5.1363	5.1270	0.18%
20,000	2.0413	2.0360	0.05%	5.1363	5.1261	0.20%
50,000	2.0413	2.0141	0.26%	5.1363	5.1337	0.05%
100,000	2.0413	2.0340	0.36%	5.1363	5.1345	0.04%

Figure 12. Case 3: $\Delta t = 0.25$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0284	0.63%	5.1363	5.1239	0.24%
4,000	2.0413	2.0334	0.39%	5.1363	5.1296	0.13%
10,000	2.0413	2.0358	0.27%	5.1363	5.1344	0.04%
15,000	2.0413	2.0328	0.42%	5.1363	5.1337	0.05%
20,000	2.0413	2.0346	0.33%	5.1363	5.1365	0.00%
50,000	2.0413	2.0346	0.33%	5.1363	5.1385	0.04%
100,000	2.0413	2.0346	0.33%	5.1363	5.1377	0.03%

Figure 13. Case 3: $\Delta t = 0.0625$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	2.0472	0.29%	5.1363	5.1359	0.01%
4,000	2.0413	2.0437	0.12%	5.1363	5.1315	0.09%
10,000	2.0413	2.0430	0.08%	5.1363	5.1405	0.08%
15,000	2.0413	2.0375	0.19%	5.1363	5.1453	0.18%
20,000	2.0413	2.0372	0.20%	5.1363	5.1479	0.23%
50,000	2.0413	2.0376	0.18%	5.1363	5.1424	0.12%
100,000	2.0413	2.0360	0.26%	5.1363	5.1405	0.08%

Figure 14. Case 3: $\Delta t = 0.03125$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

A.4. Weak Predictor-Corrector Scheme

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.9294	5.48%	5.1363	5.1103	0.51%
4,000	2.0413	1.9455	4.69%	5.1363	5.0968	0.77%
10,000	2.0413	1.9570	4.13%	5.1363	5.0764	1.17%
15,000	2.0413	1.9522	4.36%	5.1363	5.0802	1.09%
20,000	2.0413	1.9576	4.10%	5.1363	5.0815	1.07%
50,000	2.0413	1.9551	4.22%	5.1363	5.0796	1.10%
100,000	2.0413	1.9546	4.25%	5.1363	5.0811	1.07%

Figure 15. Case 4: $\Delta t = 1$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.9294	5.48%	5.1363	5.1103	0.51%
4,000	2.0413	1.9455	4.69%	5.1363	5.0968	0.77%
10,000	2.0413	1.9570	4.13%	5.1363	5.0764	1.17%
15,000	2.0413	1.9522	4.36%	5.1363	5.0802	1.09%
20,000	2.0413	1.9576	4.10%	5.1363	5.0815	1.07%
50,000	2.0413	1.9551	4.22%	5.1363	5.0796	1.10%
100,000	2.0413	1.9546	4.25%	5.1363	5.0811	1.07%

Figure 16. Case 4: $\Delta t = 0.25$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.9294	5.48%	5.1363	5.1103	0.51%
4,000	2.0413	1.9455	4.69%	5.1363	5.0968	0.77%
10,000	2.0413	1.9570	4.13%	5.1363	5.0764	1.17%
15,000	2.0413	1.9522	4.36%	5.1363	5.0802	1.09%
20,000	2.0413	1.9576	4.10%	5.1363	5.0815	1.07%
50,000	2.0413	1.9551	4.22%	5.1363	5.0796	1.10%
100,000	2.0413	1.9546	4.25%	5.1363	5.0811	1.07%

Figure 17. Case 4: $\Delta t = 1$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.

	Call £/MWh			Put £/MWh		
Sims	Analytical	Monte	Error	Analytical	Monte	Error
2,000	2.0413	1.9294	5.48%	5.1363	5.1103	0.51%
4,000	2.0413	1.9455	4.69%	5.1363	5.0968	0.77%
10,000	2.0413	1.9570	4.13%	5.1363	5.0764	1.17%
15,000	2.0413	1.9522	4.36%	5.1363	5.0802	1.09%
20,000	2.0413	1.9576	4.10%	5.1363	5.0815	1.07%
50,000	2.0413	1.9551	4.22%	5.1363	5.0796	1.10%
100,000	2.0413	1.9546	4.25%	5.1363	5.0811	1.07%

Figure 18. Case 4: $\Delta t = 0.03125$, $\sigma = 0.04$ and $\alpha = 0.06$: Average of option prices over maturities 2 to 730 days.