Personalized Spending Floors and the Retirement Efficient Frontier

April 2026

Martin-D. Lacasse

Corresponding author
8905 Ashford Gables Ct
Tampa, FL 33626
martin.d.lacasse@gmail.com

Martin-D. Lacasse, Ph.D., is a former Associate Director in the Department of Mathematics at Johns Hopkins University and a retired Distinguished Research Scientist from ExxonMobil Research & Engineering Company.

John D. Leonard II

2121 Stuart Ave
Richmond, VA 23220
Professor Emeritus, Virginia Commonwealth University

John D. Leonard, Ph.D., is Professor Emeritus in the Department of Computer Science in the College of Engineering at Virginia Commonwealth University. He previously served as Executive Dean of the College. Prior to VCU, Dr. Leonard served as Associate Dean for the College of Engineering and held several tenured faculty positions at the Georgia Institute of Technology.

Personalized Spending Floors and the Retirement Efficient Frontier

April 2026

Abstract

What made Bengen's four-percent rule so influential was its simplicity and its grounding in actual historical data: spending decisions required no distributional assumptions, only the historical record. We build on this methodological idea using a mixed-integer linear programming framework capable of modeling the full complexity of a realistic retirement plan — accounts, taxes, Social Security, Medicare, and longevity — rather than the simplified portfolio originally assumed. Run on the historical record, it yields the *Historical Spending Floor* (HSF): the highest real spending level that survives every historical return sequence, personalized to a given retiree's accounts, taxes, Social Security, and longevity. This allows us to frame the spending decision as a choice on a spending/shortfall efficient frontier — formally analogous to the Markowitz mean-variance frontier — revealing the full trade-off between committed spending and shortfall risk, with the HSF as its conservative endpoint. When historical data may not be representative of prospective conditions, the same framework stress-tests the plan under explicitly stated assumptions via a *Synthetic Spending Floor* (SSF), computed from a forward-calibrated Monte Carlo scenario set. We illustrate both with case studies using Owl, an open-source retirement optimizer.

Key Takeaways

- We introduce the concept of the Historical Spending Floor (HSF): the highest real spending that survives every historical sequence, personalized to each retiree's accounts, income, tax situation, longevity, and spending profile.
- This allows us to reframe the spending decision as a choice on a spending/shortfall efficient frontier — analogous to the Markowitz frontier — mapping the full trade-off between committed spending and shortfall risk, with the HSF as its conservative endpoint.
- When historical data may not reflect prospective conditions, the same framework stress-tests the plan via a Synthetic Spending Floor (SSF) computed from Monte Carlo scenarios under explicitly stated return assumptions, complementing the HSF with a forward-looking perspective.

Keywords: retirement income planning, safe withdrawal rate, spending floor, efficient frontier, probability of success, shortfall risk, mixed-integer linear programming

JEL Codes: C61 (Optimization Techniques; Programming Models), D91 (Intertemporal Household Choice; Life Cycle Models and Saving), G11 (Portfolio Choice; Investment Decisions)

The lasting impact of Bengen (1994) on retirement income planning rests on a methodological insight as much as on the number itself: by grounding the analysis in the historical record, it avoided the need for speculative return assumptions and anchored retirement planning in observed reality. The 4.15% safe withdrawal rate was not derived from a model of expected equity premia or bond yields; it emerged from asking a simple empirical question — what is the highest initial spending, as a constant real fraction of portfolio value, that would have survived every 30-year historical sequence in the US data? That question requires no forecast and no distributional assumption. It needs only the data.

Building on this methodological idea, this article proceeds as follows. First, we introduce the concept of the *Historical Spending Floor* (HSF): the highest real spending level that survives every historical scenario for a given retiree's specific financial situation. Unlike Bengen's fixed fractional rule applied to a simplified portfolio, the HSF is found by solving a mixed-integer linear program (MILP) that models the retiree's full financial picture — account types, tax brackets, Social Security benefits, Medicare surcharges, Roth conversion opportunities, required minimum distributions, longevity, and lifestyle through the spending profile. The HSF is therefore case-specific: it is not a single number applicable to all retirees, but a number tailored to a specific individual or couple, computed from the same historical data that motivated Bengen's original work.

This allows us to reframe the spending decision as a choice on a *spending/shortfall efficient frontier* that maps the entire trade-off between committed spending and mean shortfall across historical scenarios. The HSF is the frontier's most conservative special case: the committed spending level at which every historical scenario succeeds and mean shortfall is zero. Moving along the frontier toward higher committed spending implies accepting shortfalls in some scenarios, but these shortfalls are explicit, quantified, and visible — unlike the binary pass/fail framing of probability of success (PoS), which records only whether a plan fails, not by how much.

When historical data may not reflect prospective conditions, the same framework stress-tests the plan via a *Synthetic Spending Floor* (SSF) computed from Monte Carlo scenarios under explicitly stated return assumptions — complementing the HSF while clearly distinguishing what the future may hold from what the past has shown.

The remainder of this article is organized as follows. We first review the safe withdrawal rate (SWR) literature and the limitations of PoS, then describe Owl, the open-source optimizer used throughout. We introduce the efficient frontier framework, illustrate it through two case studies, and present historical and synthetic frontier results. We close with practical implications and extensions, including dynamic spending, annuities, and stochastic longevity.

BACKGROUND

The Safe Withdrawal Rate

The SWR has the appeal of simplicity and a worst-case guarantee: by construction, no historical sequence defeats it, and its single actionable number made it easy for practitioners to understand and adopt. The Trinity Study (Cooley et al., 1998) extended this analysis across multiple allocation mixes and time horizons. The rule has attracted substantial criticism over the years — for its reliance on a limited historical sample, its insensitivity to current valuations and yields, and its assumption of a rigid spending path (Scott et al., 2009; Pfau, 2012b; Webb, 2021) — and yet it remains widely used in practice as a conservative planning benchmark (Kitces, 2008). Its persistence in the face of these criticisms is itself telling: despite the standard caveat that past performance does not guarantee future results, anchoring to historical evidence is a natural human response to deep uncertainty. Faced with an unknowable future, practitioners and retirees instinctively turn to what has actually happened (Tversky and Kahneman, 1974; Greenwood and Shleifer, 2014). Bengen's historical framing made the rule not just empirically defensible but psychologically compelling. Its deeper limitation, for our purposes, is discussed next.

The SWR is more akin to a "retirement index" — a long-term market indicator — than a practical guide for individual retirees: it reflects aggregate historical market performance but carries no information about the household it is applied to. For example, Social Security alone accounts for at least half of retirement income for approximately three in five beneficiaries aged 65 and older (Dushi et al., 2017; Social Security Administration, 2025b) — an income source that varies widely across households and interacts with taxes, Medicare, and account structure in ways no fixed rule can capture. Yet by applying a universal rate to a simplified all-portfolio case, the SWR ignores this financial structure entirely. It also has no mechanism to express a bequest constraint: a retiree wishing to leave a specific estate cannot use a fixed withdrawal rule to find the highest spending level consistent with that goal.

The HSF proposed here addresses both limitations directly: it computes the case-specific safe spending level using the same historical data, with bequest targets embedded as MILP constraints, while the efficient frontier reveals what spending is achievable above that floor and how much shortfall risk each choice entails. The SSF extends this to forward-calibrated scenarios, grounding the result in the retiree's specific financial structure, historical performance, and explicit assumptions about the future.

Probability of Success and Shortfall Risk

Any method anchored to the historical record shares its implicit distributional assumption: the available historical sequences are treated as representative of the future. Monte Carlo simulation relaxes this by learning return statistics from historical data while generating synthetic scenarios under explicitly specified return assumptions, and reporting the fraction in which the portfolio survives the full retirement horizon — the probability of success (PoS). In most planning tools, PoS is computed by year-by-year forward simulation: a spending rule — fixed or with dynamic guardrails — is applied sequentially, and any scenario in which the balance reaches zero before the horizon ends is counted as a failure. PoS has become the dominant planning metric. Its limitation is that it is binary: each scenario is either a success or a failure, with no account taken of the severity of failure. A scenario in which the portfolio depletes in the final month of a 30-year retirement counts identically to one in which it runs out 15 years early.

Blanchett (2007) observed that PoS should be supplemented with a measure of shortfall magnitude, a theme further developed by Blanchett et al. (2012) and Gardner and Pittman (2013), who formalized expected shortfall as a mortality-adjusted metric. By using a MILP optimizer to maximize spending subject to all financial constraints for each scenario, the shortfall measure derived here is exact and applies to any set of return scenarios, whether historical or synthetic.

OWL: OPTIMAL WEALTH LAB

The case studies presented below use Owl (Lacasse, 2026), an open-source retirement optimizer available at `https://github.com/mdlacasse/Owl`. Owl builds on a line of LP-based retirement optimization research (Ragsdale et al., 1994; Coopersmith and Sumutka, 2011; Welch, 2015, 2016, 2017), extending it with a comprehensive federal tax model: income taxes, long-term capital gains brackets, Medicare IRMAA (Income-Related Monthly Adjustment Amount) surcharges with the two-year income lag, ACA (Affordable Care Act) premium credits, Social Security taxability, required minimum distributions (RMDs), and Roth conversion limits. This level of detail may appear to over-model an uncertain future, but it is essential where the framework delivers its most actionable value: the first several years of retirement, when Roth conversion windows, Medicare bracket management, and RMD planning decisions are immediately consequential and still reversible.

A parallel line of research has addressed decumulation through stochastic programming and optimal control (Konicz and Mulvey, 2013; Forsyth, 2020; Forsyth et al., 2021); the MILP approach adopted here is complementary, enabling exact tax and benefit modeling in place of stylized assumptions. For each scenario, Owl solves a MILP across the retiree's accounts — taxable, tax-deferred, Roth, and health savings — to find the maximum sustainable spending scale; a typical solve takes a few seconds on a laptop. Owl supports historical return sequences (Damodaran, 2026) and a variety of adjustable stochastic return models; in spending optimization mode it collects the per-scenario spending scales and solves a fast commitment linear program (LP) to trace the efficient frontier.

THE EFFICIENT FRONTIER FRAMEWORK

Definition

Let $g_s \geq 0$ denote the *spending scale* for scenario s: the largest scalar multiplier of a normalized spending profile that remains sustainable under that scenario's return and inflation sequence while satisfying all financial and tax constraints. For a single retiree with a flat profile, g_s equals the constant annual spending amount.

The spending scale g_s depends on several plan-level inputs beyond the return sequence. The *spending profile* — a separate planner-specified assumption — sets the normalized shape of real spending over time: it can be flat, declining in later years, geared to active- and slow-retirement phases, reduced at a spouse's death, or a dynamic guardrail rule that adjusts annually to portfolio performance (see the Dynamic Spending section below). *Longevity* — the assumed planning horizon — directly bounds how long the portfolio must sustain spending; a longer horizon generally implies a lower HSF and is itself an uncertain input the planner must choose. *Constraints* such as bequest targets or the proceeds from a planned home sale enter as hard bounds in the MILP, directly shaping the feasible spending range. *Asset allocation* — the equity/bond mix, any other asset classes held, and whether the allocation is fixed or follows a glide path over time — determines the return and risk profile of each scenario and has a direct effect on the HSF; a more conservative allocation generally lowers both the HSF and the upside potential of the frontier. *Future income streams* — Social Security, pensions, annuities, part-time earnings, or rental income, whether fixed or variable — reduce portfolio withdrawal requirements in every scenario, raising the HSF because each dollar of guaranteed income is a dollar the portfolio need not supply. Finally, the projected *future environment* — tax law, scheduled regulatory changes, and the full range of applicable retirement income rules — is embedded in each scenario solve. Different choices on any of these dimensions yield different values of g_s and hence a different HSF, revealing what universal rules like the 4% SWR cannot.

> *The Historical Spending Floor (HSF) is the maximum real spending level that, under a fixed set of plan parameters and constraints, survives every historical scenario.*

The HSF extends Bengen's idea to any retirement case: it is computed from the same historical data, but for the retiree's actual financial structure rather than a simplified portfolio. It is the most conservative point on a family of spending choices: it eliminates shortfall entirely, but at the cost of committed spending that may fall well below what most historical scenarios could sustain.

The Spending/Shortfall Efficient Frontier

Suppose the retiree must commit to a single first-year spending level g^* before knowing which scenario will occur. In any scenario where g^* exceeds what that scenario can sustain, the plan faces a shortfall — a gap between the commitment and the maximum the optimizer finds for that scenario. At $g^* = \text{HSF}$, every scenario succeeds and the mean shortfall is zero. As g^* rises above the HSF, some scenarios begin to fail and the mean shortfall grows. The efficient frontier is the set of Pareto-optimal pairs of committed spending and mean shortfall.

Owl traces the frontier by sweeping a risk-aversion parameter from zero — maximize spending regardless of shortfalls, the most aggressive choice — to arbitrarily large values, which drives committed spending down to the HSF where every scenario succeeds. Intermediate values yield the full range of defensible spending options between these two extremes. The mean shortfall also has a precise statistical interpretation: divided by the fraction of scenarios that fail, it equals the Conditional Value-at-Risk (CVaR) of spending losses — a coherent risk measure widely used in financial risk management. The complete formulation is given in Appendix B. Exhibit 1 summarizes the analogy with Modern Portfolio Theory (Markowitz, 1952).

Rather than asking the retiree to choose λ directly — a risk-aversion parameter with no intuitive scale — Owl translates the frontier into the familiar practitioner metric: a target probability of success ρ. Because PoS is already the dominant planning language, expressing the frontier in those terms makes the framework immediately accessible without requiring any change in how retirees and advisors frame the spending conversation. For any target ρ (say, 85%), Owl identifies the highest g^* achieving that PoS and reports the mean shortfall alongside the worst-case or 5th-

Concept	Modern Portfolio Theory	This Article
Decision variable	Portfolio weights	Committed spending g^*
Maximize	Expected return	Committed spending g^*
Risk measure	Variance	Mean shortfall $\bar{\sigma}$
Risk aversion	λ	λ
Conservative extreme	Minimum-variance portfolio	HSF (zero shortfall)
Aggressive extreme	Maximum-return portfolio	Best-case scenario spending

Exhibit 1: Analogy between Modern Portfolio Theory and the spending/shortfall efficient frontier. The HSF plays the role of the minimum-variance portfolio: it eliminates risk entirely, but at the cost of a spending level that may be far below the typical outcome.

percentile spending and the median. The latter two are properties of the scenario ensemble and do not change with ρ; what changes is how g^* relates to them — how large the shortfall is as a fraction of the commitment, and how far the median lies above it. This context is entirely invisible in a standard PoS analysis.

Pfau (2013) proposed a broader framework for retirement income in which spending power, portfolio sustainability, and expected bequests form an efficient surface. The present article differs: the decision variable is a committed spending level rather than a product mix, shortfall magnitude is the risk measure, and the result is a precise two-dimensional frontier with a formal LP solution. The same framework applies to both historical scenarios and Monte Carlo scenario sets calibrated to current conditions; when applied to the latter, its conservative anchor defines the Synthetic Spending Floor (SSF), introduced later.

CASE STUDIES

We illustrate the efficient frontier framework with two retirement cases of increasing complexity. The first is a deliberately simplified benchmark that isolates the HSF concept and connects it directly to the familiar SWR literature. The second introduces a realistic couple with Social Security, tax-deferred and Roth accounts, and Medicare costs, demonstrating how Owl's MILP handles household complexity while producing the same frontier output.

Bill: A Stylised Benchmark

Bill is a 65-year-old single retiree with $1,000,000 in a Roth IRA (Individual Retirement Account), invested 50% in US equities (S&P 500) and 50% in 10-year Treasury notes. He has no Social Security, no pension, no Medicare costs, no taxable or tax-deferred accounts, and a flat (constant real) spending profile over a 30-year horizon. Because his savings are entirely in a Roth account, there are no income taxes on withdrawals — the MILP degenerates to a simple cash-flow problem. This case is designed to mirror Bengen's setup as closely as possible, allowing a clean comparison.

Running Owl over all 66 historical windows from 1928 to 1993, the minimum per-scenario spending scale falls on the 1966 start year. Bill's HSF is $36,666/yr — equivalent to a 3.67%

withdrawal rate on his $1,000,000 portfolio. The 1966 window is the same worst case identified by Bengen, for the same reason: the severe real return sequence of 1966–1981, with simultaneous equity losses and rising bond yields, is uniquely damaging for a retiree drawing down assets.

The 0.48% difference between Bill's 3.67% HSF and Bengen's 4.15% is entirely methodological and fully accounted for by two factors: beginning-of-year withdrawals and the use of 10-year rather than 5-year Treasury notes. The details and reconciliation are given in Appendix A. The key point is that the HSF framework, applied to Bill's stylized case, reproduces Bengen's result exactly once the underlying assumptions are aligned.

Bill's case is instructive as a benchmark, but it is not representative of real retirees. It isolates sequence-of-returns risk by removing tax complications, Social Security, Medicare, any partner, or bequest targets. The HSF for a real case may differ substantially from 3.67%, and generally favorably, as Social Security provides income independent of market performance, reducing the portfolio's exposure to sequence-of-returns risk.

Chris and Pat

Chris and Pat illustrate the HSF framework applied to a realistic retired couple. Chris, an engineer, retires at age 64; Pat, a marketing professional, at 62. They file jointly, plan to age 89 and 92 respectively, and own their home outright. Their nest egg is concentrated in Chris's tax-deferred accounts, reflecting decades of 401(k) contributions. Account balances are chosen so that after-tax savings match Bill's $1,000,000: tax-deferred assets of $937,000 at a 20% effective tax rate yield $750,000 after tax, plus $100,000 in Roth and $150,000 in taxable, totaling approximately $1,000,000. This design isolates the effect of account structure, Social Security, and taxes from the effect of total wealth: any difference in HSF relative to Bill's reflects the complexity of the real case, not a difference in starting resources.

Pat claims Social Security at 62 for early income; Chris is inclined to delay to 70 to maximize the household survivor benefit — a choice that Owl's Social Security age optimizer confirms under their specific return and longevity assumptions. Both claiming ages are fixed for the frontier analysis. Their financial profile appears in Exhibit 2.

Owl solves the MILP for each of the 66 historical windows from 1928 to 1993. The range ends in 1993 because later start years do not provide a complete sequence sufficient to cover the couple's 33-year planning horizon — chosen to reflect a realistic longevity target for a two-person household, in contrast to Bill's stylized 30-year horizon. Each solve jointly optimizes Roth conversions and account withdrawals, subject to US tax rules, RMDs, and Medicare costs, producing the spending scale g_s for that scenario.

The resulting HSF for Chris and Pat — the minimum g_s across all 66 scenarios — is anchored by the 1966 start year, consistent with Bill's case. Their per-scenario spending scales are higher than Bill's primarily because Social Security provides income independent of portfolio performance, reducing the withdrawal burden in every scenario — and despite the slightly longer planning horizon and the $400,000 bequest constraint that Bill does not face. This illustrates a key advantage of

Parameter	Value
Ages at plan start (2026)	63 (Chris), 60 (Pat); retire end of 2027 and 2028
Planning horizon	Age 89 (Chris), 92 (Pat)
Filing status	Married filing jointly
Taxable accounts	\$100,000 (Chris), \$50,000 (Pat)
Tax-deferred (IRA/401(k))	\$700,000 (Chris), \$237,000 (Pat)
Roth IRA	\$70,000 (Chris), \$30,000 (Pat)
Primary residence	\$450,000 (held to death; left as bequest)
401(k) contributions	Employee max + 4% employer match (through retirement)
Social Security PIA	\$2,000/mo (Chris, claiming age 70)†, \$1,400/mo (Pat, claiming age 62)
Spending profile	Flat; 40% reduction at passing of first-dying spouse
Asset allocation	60% equity / 40% bonds (fixed)
Heirs' tax rate on IRA	30%
Bequest target	\$400,000 (excluding primary residence; also serves as a reserve against longevity and long-term care risk)

Exhibit 2: Chris and Pat's financial profile. †Chris's claiming age confirmed by Owl's SS age optimizer in a preliminary run and fixed for the frontier analysis. PIA = Primary Insurance Amount (monthly benefit at full retirement age). The fixed 60/40 allocation isolates sequence-of-returns risk from allocation drift. The case file is available in the Owl repository (Lacasse, 2026).

the HSF framework over the universal SWR: it incorporates the full income-replacement value of Social Security and satisfies the \$400,000 bequest constraint simultaneously — both features that a fixed fractional rule applied to portfolio assets alone cannot capture.

HISTORICAL RESULTS

Exhibit 3 shows the efficient frontier for Bill's stylized case at an 85% target success rate. Exhibits 4 and 5 show the three-panel output for Chris and Pat at 85% and 70% target success rates. Each exhibit displays the PoS curve (left), the efficient frontier (center), and a bar chart of committed spending and shortfall by historical start year (right).

The 1966 scenario anchors the frontier's left endpoint at the HSF for both cases.

For Bill at an 85% target, committed spending is \$42,660/yr (4.27% of portfolio), with a mean shortfall of \$423/yr (1.0% of committed spending), a CVaR of \$3,098/yr (7.3%), and a median scenario spending of \$57,065/yr. Dropping to a 70% target raises committed spending to \$47,611/yr but increases the mean shortfall to \$1,444/yr (3.0%) and the CVaR to \$5,296/yr (11.1%).

At an 85% target, Chris and Pat can commit to \$99,003/yr in today's dollars, with a mean shortfall of only \$786/yr (0.8%), a CVaR of \$5,762/yr (5.8%), and a median scenario spending of \$123,145/yr — well above the commitment. The 1966 scenario produces a worst-case spending of \$88,908/yr, a shortfall of only 10.2% (compared to 14.1% for Bill). Knowing that the most damaging sequence in a century of US market history implies a roughly 10% spending reduction — not a catastrophic failure — fundamentally reframes what "risk" means in this plan.

```
Committed spending (today's $):  $42,660/yr
Spending-to-savings ratio:       4.27%  (ETR ratio 20%)
Target success rate:             85%  (actual: 86%)
Median scenario spending:        $57,065/yr
Worst-case scenario spending:    $36,666/yr  (14.1% shortfall)
Mean shortfall:                  $423/yr  (1.0% of committed)
CVaR (avg loss | failure):       $3,098/yr  (7.3% of committed)
Scenarios:                       66
```

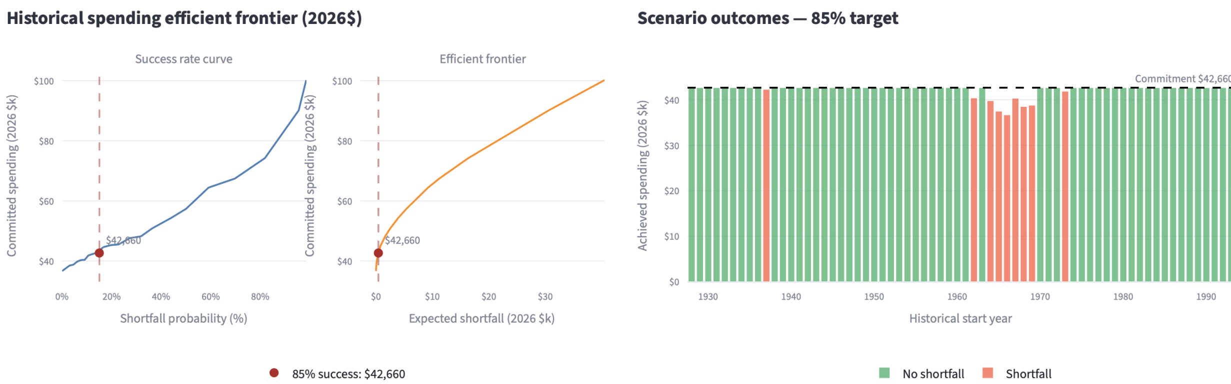

Exhibit 3: Bill's historical efficient frontier (66 scenarios, 1928–1993). *Left:* Committed spending versus shortfall probability. *Center:* Committed spending versus mean shortfall; the HSF ($36,666/yr) anchors the conservative endpoint where mean shortfall is zero. *Right:* Committed spending and shortfall by historical start year.

Accepting a 70% target raises committed spending to $106,948/yr (+8%), but the mean shortfall grows to $2,600/yr (2.4%), the CVaR to $9,535/yr (8.9%), and the worst-case shortfall to 16.6%. The median and worst-case scenario spending are unchanged between the two targets — they reflect properties of the scenario set, not of the chosen commitment level. What changes is how much the commitment exceeds the worst case.

Comparing Chris and Pat to Bill illuminates two structural advantages of the realistic case. First, Social Security income acts as a risk buffer: at 85% PoS, Bill's worst-case shortfall is 14.1% versus 10.2% for Chris and Pat, and Bill's CVaR of 7.3% exceeds theirs (5.8%) despite the same after-tax wealth. SS payments continue regardless of market performance, compressing the distribution of per-scenario spending scales: for Bill, the range from worst case to median is $20,399 (48%), while for Chris and Pat the absolute range is wider at $34,237 but represents only 35%. The signed percentages in Exhibit 6 make this compression visible at a glance: Bill's outcomes span −14.1% to +33.8% of committed spending, while Social Security narrows Chris and Pat's range to −10.2% to +24.4%. SS effectively places a floor under each scenario's g_s, reducing the variance across scenarios and with it the exposure to sequence-of-returns risk. Second, Chris and Pat's $400,000 bequest target is embedded directly as a constraint in Owl's MILP — the committed spending $99,003/yr is the highest achievable while satisfying that constraint across at least 85% of scenarios. A fixed fractional rule has no mechanism to express this trade-off: the SWR concept simply has no meaning when a bequest constraint is present.

The bar charts (right panels) make the cost of each choice concrete and scenario-specific: advi-

```
Committed spending (today's $):  $99,003/yr
Spending-to-savings ratio:       9.90%  (ETR ratio 20%)
Spending-to-savings note:        understated due to bequest of $400k
Target success rate:             85%  (actual: 86%)
Median scenario spending:        $123,145/yr
Worst-case scenario spending:    $88,908/yr  (10.2% shortfall)
Mean shortfall:                  $786/yr  (0.8% of committed)
CVaR (avg loss | failure):       $5,762/yr  (5.8% of committed)
Scenarios:                       66
```

Exhibit 4: Chris and Pat — historical scenarios (1928–1993), 85% target success rate. *Left:* Committed spending versus shortfall probability. *Center:* Committed spending versus mean shortfall; the HSF anchors the left endpoint at zero shortfall. *Right:* Committed spending and shortfall by historical start year; the 1966 sequence is the worst case.

sors and retirees can examine exactly which historical periods are responsible for shortfalls and by how much, rather than absorbing a single PoS figure.

Case	Target PoS	Committed	Mean shortfall	CVaR	Worst case	Median
Bill	85%	$42,660	$423 (1.0%)	$3,098 (7.3%)	$36,666 (−14.1%)	$57,065 (+33.8%)
Bill	70%	$47,611	$1,444 (3.0%)	$5,296 (11.1%)	$36,666 (−23.0%)	$57,065 (+19.9%)
Chris & Pat	85%	$99,003	$786 (0.8%)	$5,762 (5.8%)	$88,908 (−10.2%)	$123,145 (+24.4%)
Chris & Pat	70%	$106,948	$2,600 (2.4%)	$9,535 (8.9%)	$88,908 (−16.6%)	$123,145 (+15.1%)
Bill HSF: $36,666/yr (3.67%)	Chris & Pat HSF: $88,908/yr					

Exhibit 6: Summary of historical frontier results for Bill and Chris and Pat (66 scenarios, 1928–1993). Parenthetical percentages are fractions of committed spending; signs indicate deviation from committed (− below, + above). All dollar amounts are in today's dollars per year.

Relying on historical data carries an implicit assumption shared by all such approaches: that the 66 return sequences in the dataset are representative of the distribution of future returns. If prospective conditions differ systematically — lower expected returns, higher starting valuations, or a different inflation regime — the frontier will overstate sustainable spending. This limitation motivates the Synthetic Spending Floor introduced next, which replaces the historical record with a forward-calibrated Monte Carlo scenario set to stress-test the same framework under explicitly specified return assumptions.

Exhibit 5: Chris and Pat — historical scenarios (1928–1993), 70% target success rate. Compared to Exhibit 4, the bar chart reveals a substantially broader and deeper shortfall distribution, illustrating the concrete cost of targeting a lower success rate.

SYNTHETIC SPENDING FLOOR

Motivation and Definition

The decades that drove the historical record — including the post-war economic boom, the secular equity rally of the 1980s and 1990s, and the long decline in interest rates — may not be representative of the returns available to retirees who began their decumulation in 2020 or later. Low starting bond yields, elevated equity valuations, and the potential reversal of the secular bond trend suggest that prospective returns may differ from historical averages, a concern that motivated Finke et al. (2013) to adjust the safe withdrawal rate downward under prevailing yield conditions.

To address this, the same MILP framework can be run not on historical windows but on a large set of synthetic return sequences generated from a model calibrated to current or prospective conditions. The minimum spending that survives a target fraction of those scenarios plays the same role as the HSF, but under explicitly stated prospective assumptions. We define:

> *The Synthetic Spending Floor (SSF) is the maximum real spending level that, under a fixed set of plan parameters and constraints, survives at least 95% of a Monte Carlo scenario set calibrated to current return conditions.*

Unlike the HSF — where the finite historical ensemble makes the minimum g_s a stable quantity — the worst-case draw in N independent simulations decreases without bound as N grows, so the raw minimum is not a reproducible statistic. Instead, the 5th percentile serves as the conservative anchor of the frontier — playing the same role as the HSF — and converges to a stable estimate as N grows. It reflects today's expected return environment rather than the past century's average.[1] The SSF can be higher or lower than the HSF depending on whether prospective conditions are more or less favorable than historical ones.

Monte Carlo Setup

To demonstrate the SSF in practice, we use Owl's *histolognormal* model, in which log-returns for equities, bonds, and inflation are assumed jointly normally distributed, with the mean vector and covariance matrix estimated by maximum likelihood from the full 1928–2025 historical record. Each year's return vector is drawn independently from that distribution, and a scenario is the resulting sequence over the planning horizon. This parametric baseline follows Finke et al. (2013) and is recommended over the normal distribution by Collins et al. (2015), since portfolio values are products of gross-return factors and cannot turn negative. More sophisticated models — block bootstrap, stationary bootstrap (Politis and Romano, 1994), or GARCH (Generalized Autoregressive Conditional Heteroskedasticity) — that better capture mean reversion and autocorrelation are also available in Owl when behavior at extreme success-rate targets is of primary concern.

Owl ran 500 simulations for Chris and Pat, completing in approximately two minutes on a standard laptop.

Synthetic Frontier and Comparison with Historical

Exhibit 7 shows the Monte Carlo efficient frontier for Chris and Pat at an 85% target success rate, and Exhibit 8 summarizes both targets alongside the historical results.

Case	Target PoS	Committed	Mean shortfall	CVaR	5th-pct. spending	Median
Bill	85%	\$43,099	\$926 (2.1%)	\$6,345 (14.7%)	\$35,184 (−18.4%)	\$58,352 (+35.4%)
Bill	70%	\$48,950	\$2,146 (4.4%)	\$7,834 (16.0%)	\$35,184 (−28.1%)	\$58,352 (+19.2%)
Chris & Pat	85%	\$97,727	\$1,783 (1.8%)	\$12,215 (12.5%)	\$82,727 (−15.3%)	\$123,279 (+26.1%)
Chris & Pat	70%	\$108,780	\$4,140 (3.8%)	\$15,109 (13.9%)	\$82,727 (−23.9%)	\$123,279 (+13.3%)
Bill SSF: \$35,184/yr (3.52%)	Chris & Pat SSF: \$82,727/yr					

Exhibit 8: Monte Carlo frontier results for Bill and Chris and Pat (500 simulations, histolognormal model calibrated to 1928–1993). The 5th-percentile spending converges to a stable quantile as simulation count grows and is the appropriate tail indicator for Monte Carlo ensembles. Parenthetical percentages are fractions of committed spending; signs indicate deviation from committed (− below, + above). All dollar amounts are in today's dollars per year.

The Monte Carlo results are broadly consistent with the historical results — as expected, since the lognormal model is calibrated to the same historical data. For Chris and Pat at 85% PoS, com-

[1] The 5th percentile characterizes shortfall severity only when the target success rate $\rho < 95\%$, so that more than 5% of scenarios fall below committed spending. When $\rho \geq 95\%$, the 5th percentile lies in the success zone and does not reflect failure severity; in that case the mean shortfall divided by $(1-\rho)$ — the average shortfall per failing scenario — is the more informative tail statistic.

```
Committed spending (today's $):  $97,727/yr
Spending-to-savings ratio:       9.78%  (ETR ratio 20%)
Spending-to-savings note:        understated due to bequest of $400k
Target success rate:             85%  (actual: 85%)
Median scenario spending:        $123,279/yr
5th percentile spending:         $82,727/yr  (15.3% shortfall)
Mean shortfall:                  $1,783/yr  (1.8% of committed)
CVaR (avg loss | failure):       $12,215/yr  (12.5% of committed)
Rate method:                     histolognormal
Scenarios:                       500
```

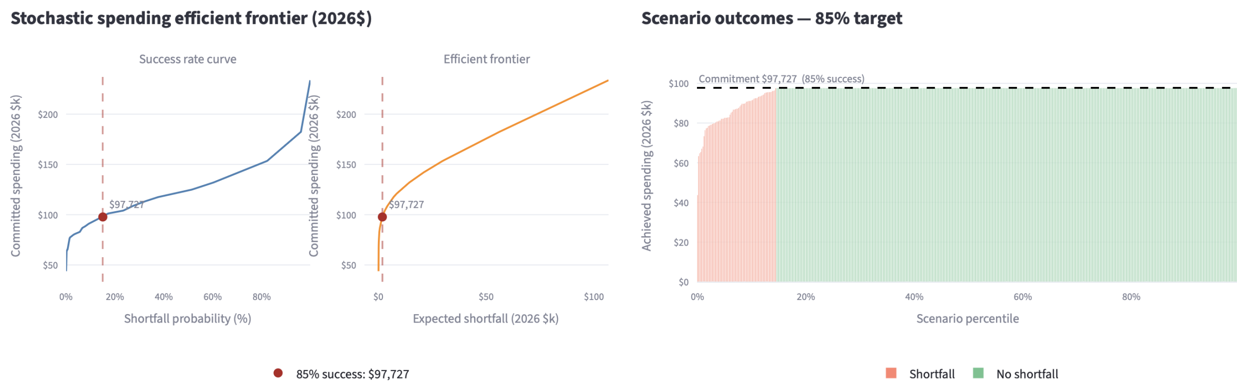

Exhibit 7: Chris and Pat — Monte Carlo scenarios (500 simulations, histolognormal model), 85% target success rate. *Left:* Committed spending versus shortfall probability. *Center:* Committed spending versus mean shortfall; the SSF anchors the conservative endpoint. *Right:* Achieved spending by scenario percentile, sorted from worst to best; red bars indicate shortfall scenarios.

mitted spending is $97,727/yr versus the historical $99,003, and the median is $123,279 versus $123,145 — close agreement that confirms the calibration is internally coherent. For Bill, the synthetic frontier yields $43,099/yr at 85% PoS versus the historical $42,660, again consistent. The SSF values — $35,184/yr for Bill (3.52%) and $82,727/yr for Chris and Pat — are close to but slightly below their historical counterparts ($36,666 and $88,908), reflecting that independent lognormal sampling occasionally generates sustained adverse sequences that the bounded historical record does not contain.

The differences between the two scenario sets are informative. The Monte Carlo mean shortfall at 85% PoS ($926/yr for Bill, $1,783/yr for Chris and Pat) exceeds the historical values ($423/yr and $786/yr), and the corresponding CVaRs ($6,345/yr and $12,215/yr) exceed the historical values ($3,098/yr and $5,762/yr), because independent sampling produces a wider tail than autocorrelated historical sequences. The 5th-percentile spending is likewise below the historical worst case for the same reason — independent random draws can generate more consecutive poor years than the autocorrelated historical record admits. This is a feature of the return model, not the retiree's actual risk landscape (Tharp, 2017; Campbell and Viceira, 2002), and it suggests using correlated return models when tail behavior at extreme success-rate targets matters most.

The historical PoS curve is approximately linear: each percentage-point reduction in the target buys a roughly proportional increase in committed spending. The Monte Carlo PoS curve shares this linearity in its middle range but shows nonlinearity near the extremes — a steep drop in committed spending as PoS approaches 100% (requiring defense against increasingly extreme tail draws) and rapid acceleration in mean shortfall as PoS approaches 0%. The historical curve's linearity arises because its 66 scenarios are bounded by the actual distributional structure of US market returns; the Monte Carlo curve's curvature is an artifact of the independent sampling assumption (Collins et al., 2015).

On simulation size. A natural instinct is to run more Monte Carlo simulations for greater accuracy. This holds for the mean shortfall and committed spending, whose estimation error scales as $1/\sqrt{N}$; a few hundred scenarios already yield tight estimates (Fitzpatrick and Tharp, 2022). The 5th-percentile SSF likewise stabilizes quickly, since it is estimated from $0.05N$ observations and the standard error of a sample quantile is of order $1/\sqrt{N}$. Shifting the conversation from worst-case spending to mean shortfall and a stable quantile is therefore not merely a presentational choice — it is the statistically appropriate summary of Monte Carlo tail risk.

PRACTICAL IMPLICATIONS

HSF and SSF as Complementary Anchors

The HSF and SSF serve different but complementary roles. The HSF is backward-looking: it is computed from the historical record and provides a guarantee of the same kind Bengen offered — no historical sequence would have defeated this spending level. It is therefore appropriate as a lower bound that retirees and their advisors can present with historical authority. The SSF is forward-looking: it is computed from a return model calibrated to current conditions and captures the possibility that the future differs systematically from the past. When the model suggests lower returns (higher cyclically adjusted price-to-earnings (CAPE) ratios, lower starting yields), the SSF falls below the HSF, serving as a warning. When both floors agree, it signals that prospective conditions are expected to mirror the historical record.

Together, the HSF and SSF bracket the planner's confidence interval on sustainable spending. A retiree whose HSF and SSF are close can commit near that level with confidence from both historical and prospective perspectives. One whose SSF is materially below the HSF faces a choice: trust the historical record, accept the model's forward-looking view, or target a committed spending level between the two floors.

The spending floor — HSF or SSF — marks the zero-shortfall extreme of the frontier, the spending analogue of holding all wealth in the minimum-variance portfolio. Just as Markowitz (1952) demonstrated that rational investors need not minimize variance to its theoretical extreme — the efficient frontier offers superior risk-adjusted outcomes for any given level of risk tolerance — the spending/shortfall frontier shows that rational retirees need not anchor to the floor: moving slightly away from it typically yields a substantially higher committed spending level for only a modest increase in mean shortfall.

Reframing the Spending Decision

The efficient frontier changes the retirement spending conversation in a meaningful way. Under the conventional PoS paradigm, the retiree negotiates a success rate threshold without knowing what happens in the scenarios below it. The frontier makes these scenarios visible: a mean shortfall of $X/yr, with tail spending of $Y/yr in the worst historical case or the 5th Monte Carlo percentile. For many retirees, this information will reveal that a lower PoS target is entirely reasonable: 70% sounds alarming in isolation, but the frontier shows that the mean shortfall across all scenarios amounts to only 2–4% of committed spending, with even the most extreme historical sequence falling short by 17%. Committing to the median scenario would correspond to a 50% success rate — and the median for Chris and Pat is $123,279/yr, well above any reasonable target.

The efficient frontier framework is also accessible to advisors using conventional balance-forward planning tools. Such tools test a user-specified spending level and report how many scenarios succeed — some go further and report shortfall magnitude in failing scenarios. The key difference is that balance-forward simulators cannot find the optimal spending level for each scenario: g_s is a decision variable, not a given. Because Owl's MILP maximizes spending subject to all constraints for each scenario, the resulting g_s is the true maximum sustainable spending and the shortfall measure is exact. Advisors using conventional simulators can in principle approximate the frontier by extracting per-scenario spending outcomes and applying the commitment LP as a post-processing step.

Dynamic Spending

Scott et al. (2009) demonstrated that financing a constant spending plan from a volatile investment strategy is fundamentally inefficient: it generates unnecessary shortfalls when markets underperform and wasteful surpluses when they outperform. A family of dynamic spending strategies — Guyton-Klinger guardrails (Guyton and Klinger, 2006), the RMD method (Larson, 2022), spending-flexibility rules (Pfau, 2012a), and others — address this by adjusting spending annually in response to portfolio performance.

The efficient frontier framework accommodates all of them. Because spending in each scenario is computed by a full MILP optimizer that accepts any normalized spending profile — including dynamic ones — the framework is not limited to flat real amounts. In practice, a dynamic rule simply defines a different spending profile, and the HSF and frontier are computed from it in exactly the same way.

The most natural implementation is annual re-evaluation: each year, as new information arrives — updated portfolio values, revised return forecasts, tax-law changes, or changed personal circumstances — Owl re-solves the MILP for the remaining horizon and recomputes the frontier. The new HSF reflects the portfolio's current state, and the retiree selects a new committed spending level for the coming year. Guyton-Klinger guardrails and the RMD method are both special cases: each corresponds to a specific path through the frontier over time, selecting a new point each year rather than holding a static commitment.

Annual re-evaluation also addresses an important practical concern: the HSF computed at retirement will differ from the HSF computed 5 or 10 years later, as the retiree's remaining horizon has shortened and the historical dataset has grown. Recomputing annually keeps the HSF grounded in the data actually relevant to the retiree's remaining plan.

The computational structure of the framework supports this cycle efficiently. Constructing the frontier requires one MILP solve per historical scenario — a few minutes for 66 historical windows on a standard laptop. Once those solves are complete, exploring the full range of target success rates is instantaneous: the commitment LP recombines the existing g_s values without additional MILP computation.

Extensions

The observation that Social Security compresses the distribution of per-scenario spending scales generalizes to any guaranteed income stream. Fixed annuities, defined-benefit pensions, and deferred income annuities (DIAs) all reduce portfolio withdrawal dependence by the same mechanism: payments continue regardless of market performance, placing a floor under each scenario's g_s. The HSF framework accommodates them directly — guaranteed income flows enter the cash-flow constraints exactly as Social Security does. A retiree weighing the purchase of an annuity against retaining full portfolio flexibility can compare HSFs and efficient frontiers under both scenarios, making the spending/risk trade-off of the annuity decision explicit and quantifiable.

The planning horizons in this paper are fixed deterministic inputs. The framework extends naturally to stochastic longevity: each scenario is assigned an independently drawn lifespan τ_s from a Social Security Administration (SSA) period life table (Social Security Administration, 2025a), coupling market risk and longevity risk within the same optimization. For a couple, the last-survivor horizon $\tau_s = \max(\tau_{1,s}, \tau_{2,s})$ is used. The MILP optimizer is solved with horizon τ_s for each scenario, producing a g_s that encodes both return and longevity risk. The commitment LP is entirely unchanged — it operates on whatever $\{g_s\}$ values the scenario solves produce. The resulting frontier spans the joint distribution of market and longevity risk, making longevity uncertainty explicit within the same framework rather than treating it as a fixed assumption.

CONCLUSION

Bengen's insight — that the historical record of market returns could ground retirement spending decisions — remains as attractive today as it was in 1994, not least because of its simplicity and psychological appeal. This article extends that insight in two directions.

The Historical Spending Floor (HSF) carries Bengen's idea forward: it is the highest real spending level that survives every historical return sequence, computed via MILP for a given retirement case. For a stylized single retiree with a pure Roth account and no tax complications, the HSF reproduces Bengen's number exactly once the underlying methodological assumptions are aligned (see Appendix A). For a real couple with Social Security, tax-deferred accounts, Medicare costs, and Roth conversion opportunities, the HSF is higher — primarily because Social Security provides income independent of market performance, reducing the portfolio withdrawal burden in every scenario. The MILP handles the associated tax complexity optimally, but the key drivers are features that a fixed fractional rule applied to portfolio assets cannot express: guaranteed income streams that reduce portfolio withdrawal requirements in every scenario, and bequest constraints that shape the feasible spending range from the outset.

The efficient frontier places the HSF in a richer context. Rather than identifying a single safe spending level, it maps the full trade-off between committed spending and mean shortfall across historical scenarios — a formally Pareto-optimal curve analogous to the Markowitz mean-variance frontier. Retirees can choose any point on this frontier based on their risk tolerance, and they can see exactly how much spending they gain and how much shortfall risk they accept at each choice. The HSF is the conservative anchor; everything to the right of it involves some risk, but that risk is explicit, quantified, and expressed in the same dollar units as the spending commitment itself.

The Synthetic Spending Floor (SSF) extends the same framework to Monte Carlo scenarios calibrated to current conditions. When HSF and SSF agree, prospective conditions are expected to mirror the historical record; when they diverge, the gap is itself informative. Despite the complexity of the underlying optimization — one MILP solve per scenario, across hundreds of scenarios — the full computation completes in minutes on a standard laptop.

The framework asks the same question Bengen asked in 1994 — what does the historical record tell us about safe spending? — but answers it for a specific retiree's situation, not for a stylized average case. The efficient frontier goes further: rather than stopping at the worst-case floor, it provides a rigorous framework for exploring the full range of spending and risk trade-offs, letting retirees make informed choices beyond the conservative extreme. And where historical data alone is insufficient, the Synthetic Spending Floor integrates forward-looking information — prevailing yields, current valuations, and today's return environment — into the same framework, grounding retirement decisions in both the past and the present.

DECLARATION OF SUBMISSION

The authors confirm that this manuscript is original work that has not been published elsewhere. It is not currently under review at any other publication in the Portfolio Management Research family or any other journal. The authors have read the PMR publication agreement and agree to its terms. The authors' institutions do not require that this paper be posted on any public-facing website, including SSRN, ResearchGate, or SciHub, prior to publication.

References

Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath (1999). Coherent measures of risk. *Mathematical Finance 9*(3), 203–228.

Bengen, W. P. (1994, October). Determining withdrawal rates using historical data. *Journal of Financial Planning*, 171–180.

Blanchett, D. M. (2007). Dynamic allocation strategies for distribution portfolios: Determining the optimal distribution glide path. *Journal of Financial Planning 20*(12), 68–81.

Blanchett, D. M., M. Kowara, and P. Chen (2012). Optimal withdrawal strategy for retirement portfolios. *Retirement Management Journal 2*(3), 7–20.

Campbell, J. Y. and L. M. Viceira (2002). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford: Oxford University Press.

Collins, P. J., H. D. Lam, and J. Stampfli (2015). How risky is your retirement income risk model? *Financial Services Review 24*, 193–216.

Cooley, P. L., C. M. Hubbard, and D. T. Walz (1998, February). Retirement savings: Choosing a sustainable withdrawal rate. *Journal of the American Association of Individual Investors*, 16–21. Also known as the Trinity Study.

Coopersmith, L. W. and A. R. Sumutka (2011, September). Tax-efficient retirement withdrawal planning using a linear programming model. *Journal of Financial Planning*.

Damodaran, A. (2026). Annual returns on stock, t.bills, t.bonds and inflation—united states (1928–present). NYU Stern School of Business, available at `https://pages.stern.nyu.edu/~adamodar/`. Historical annual returns dataset, updated annually.

Dushi, I., H. M. Iams, and B. Trenkamp (2017). The importance of social security benefits to the income of the aged population. *Social Security Bulletin 77*(2), 1–12. Available at `https://www.ssa.gov/policy/docs/ssb/v77n2/v77n2p1.html`.

Finke, M. S., W. D. Pfau, and D. M. Blanchett (2013, June). The 4 percent rule is not safe in a low-yield world. *Journal of Financial Planning 26*(6), 46–55.

Fitzpatrick, J. and D. Tharp (2022, May). Evaluating retirement spending risk: Monte carlo vs historical simulations. Kitces.com. Available at `https://www.kitces.com/blog/monte-carlo-simulation-historical-returns-sequence-risk-calculate-sustainable-spending-levels/`.

Forsyth, P. A. (2020). A stochastic control approach to defined contribution plan decumulation: "the nastiest, hardest problem in finance". *ASTIN Bulletin: The Journal of the IAA*. arXiv:2008.06598.

Forsyth, P. A., K. R. Vetzal, and G. Westmacott (2021). Optimal control of the decumulation of a retirement portfolio with variable spending and dynamic asset allocation. *ASTIN Bulletin: The Journal of the IAA 51*(3), 905–938.

Gardner, G. and S. Pittman (2013). Measuring the risk of running out of money in retirement. *Journal of Financial Planning 26*(12), 38–44.

Greenwood, R. and A. Shleifer (2014). Expectations of returns and expected returns. *Review of Financial Studies 27*(3), 714–746.

Guyton, J. T. and W. J. Klinger (2006, March). Decision rules and maximum initial withdrawal rates. *Journal of Financial Planning*.

Kitces, M. E. (2008, May). Resolving the paradox – is the safe withdrawal rate sometimes too safe? *The Kitces Report*.

Konicz, A. K. and J. M. Mulvey (2013). Applying a stochastic financial planning system for an individual: Immediate or deferred life annuities? *Journal of Retirement 1*(1), 90–103.

Lacasse, M.-D. (2026). Owl: Optimal wealth lab – a mixed-integer linear programming retirement financial planner. `https://github.com/mdlacasse/Owl`. Version 2026.04.21. Additional contributors are listed in the `AUTHORS` file at `https://github.com/mdlacasse/Owl/blob/main/AUTHORS`. The case file `Case_chris+pat.toml` is included in the repository for reproducibility.

Larson, S. J. (2022, January). Required minimum distributions as a retirement strategy: The trade-off between RMD volatility and the expected number of dollars paid out. *Journal of Financial Planning 35*(1), 60–67.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance 7*(1), 77–91.

Pfau, W. D. (2012a, March). Spending flexibility and safe withdrawal rates. *Journal of Financial Planning*.

Pfau, W. D. (2012b, January). Withdrawal rates, savings rates, and valuation-based asset allocation. *Journal of Financial Planning 25*(1), 36–48.

Pfau, W. D. (2013, February). A broader framework for determining an efficient frontier for retirement income. *Journal of Financial Planning*. SSRN 2151259.

Politis, D. N. and J. P. Romano (1994). The stationary bootstrap. *Journal of the American Statistical Association 89*(428), 1303–1313.

Ragsdale, C. T., A. F. Seila, and P. L. Little (1994). An optimization model for scheduling withdrawals from tax-deferred retirement accounts. *Financial Services Review 3*(2), 93–109.

Rockafellar, R. T. and S. Uryasev (2000). Optimization of conditional value-at-risk. *Journal of Risk 2*(3), 21–41.

Rockafellar, R. T. and S. Uryasev (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking and Finance 26*(7), 1443–1471.

Scott, J. S., W. F. Sharpe, and J. G. Watson (2009). The 4% rule—at what price? *Journal of Investment Management 7*(3), 31–48.

Social Security Administration (2025a). Actuarial life table, period life table 2021. `https://www.ssa.gov/oact/STATS/table4c6.html`. Accessed April 2026.

Social Security Administration (2025b). Fast facts & figures about social security, 2025. Technical report, Social Security Administration. Available at `https://www.ssa.gov/policy/docs/chartbooks/fast_facts/2025/fast_facts25.html`.

Tharp, D. (2017, July). Fat tails in Monte Carlo analysis vs. safe withdrawal rates. Kitces.com. Available at `https://www.kitces.com/blog/monte-carlo-analysis-risk-fat-tails-vs-safe-withdrawal-rates-rolling-historical-returns/`.

Tversky, A. and D. Kahneman (1974). Judgment under uncertainty: Heuristics and biases. *Science 185*(4157), 1124–1131.

Webb, A. (2021, April). What can scholarly research tell us about the merits of annuitization vs. drawing down unannuitized wealth? Technical Report Literature Review #004-2021, Retirement Income Institute.

Welch, J. S. (2015). Mitigating the impact of personal income taxes on retirement savings distributions. *Journal of Personal Finance*.

Welch, J. S. (2016). Measuring the financial consequences of IRA to Roth IRA conversions. *Journal of Personal Finance*.

Welch, J. S. (2017, August). A 3-step procedure for computing sustainable retirement savings withdrawals. *Journal of Financial Planning*.

APPENDIX

A. Reconciliation of Bill's HSF with Bengen's SWR

Bill's HSF of 3.67% differs from Bengen's 4.15% for two independent methodological reasons, each arising from the stylized assumptions of the respective analyses. Beginning-of-year withdrawals reduce the effective rate because funds are drawn before returns are applied; Bengen assumed end-of-year withdrawals (+0.32%). Owl adopts beginning-of-year withdrawals as the conservative and practically realistic choice: retirees cover living expenses at the start of each year, not after waiting for a year of investment returns. Owl uses 10-year Treasury notes, which suffered larger price losses than Bengen's 5-year notes during the rising-rate environment of 1966–1981 (+0.16%). Both effects are independent and additive. The 0.48% total gap also illustrates the sensitivity of the SWR to seemingly minor methodological choices (Pfau, 2012b; Cooley et al., 1998): two assumptions — withdrawal timing and bond maturity — that a practitioner might consider inconsequential together shift the rate by nearly half a percentage point, a material difference when applied to a seven-figure portfolio.

B. Commitment LP and CVaR Connection

Given N per-scenario spending scales $g_s \geq 0$ and a risk-aversion parameter $\lambda \geq 0$, the commitment LP is:

Factor	Bengen	Bill (Owl)
Withdrawal timing	End of year	Beginning of year
Bond type	5-year T-Notes	10-year T-Notes
Tax treatment	Simplified (not modeled)	None (pure Roth)
Planning horizon	30 years	30 years
Historical range	1926–1992	1928–1993
Safe withdrawal rate	4.15%	3.67%
+ beginning-of-year adjustment		+0.32%
+ 5-year vs. 10-year bond adjustment		+0.16%
Reconciled rate	4.15%	4.15% ✓

Appendix Exhibit A: Reconciliation of Bill's HSF with Bengen's SWR.

$$\begin{aligned}
\text{maximize} \quad & g^* - \frac{\lambda}{N}\sum_{s=1}^{N}\sigma_s \\
\text{subject to} \quad & \sigma_s \geq g^* - g_s, \qquad s = 1,\ldots,N \qquad (1)\\
& \sigma_s \geq 0, \qquad s = 1,\ldots,N \qquad (2)\\
& 0 \leq g^* \leq \max_s g_s. \qquad (3)
\end{aligned}$$

The $N+1$ decision variables are the scalar g^* and the N shortfall variables σ_s. Constraints (1)–(2) force $\sigma_s = \max(0, g^* - g_s)$ at optimality — the per-scenario spending deficit. The *mean shortfall* is then

$$\bar{\sigma}(g^*) \;=\; \frac{1}{N}\sum_{s=1}^{N}\sigma_s \;=\; \frac{1}{N}\sum_{s=1}^{N}\max\big(0,\, g^* - g_s\big), \qquad (4)$$

the average per-scenario spending deficit at commitment level g^*. Constraint (3) prevents g^* from exceeding the best achievable outcome. At $\lambda = 0$ the LP simply maximizes g^* (best-case, aggressive extreme). As $\lambda \to \infty$, g^* is driven to $\min_s g_s = \text{HSF}$ (zero shortfall, conservative extreme). Solving the LP for a grid of λ values traces the full Pareto curve; in practice, the grid is sampled on a logarithmic scale to obtain uniform visual density across the full curve.

If a scenario is infeasible — no plan satisfying all constraints exists under that return sequence — its spending scale is set to $g_s = 0$, contributing a full shortfall $\sigma_s = g^*$ for any positive commitment. The normalization constant N always equals the total number of scenarios requested, ensuring that infeasible scenarios are not silently excluded from the risk calculation.

The mean shortfall $\bar{\sigma}$ scaled by the failure rate, $\bar{\sigma}/(1-\rho)$ where ρ is the target success rate, equals the Conditional Value-at-Risk (CVaR) of spending losses at confidence level ρ (Rockafellar and Uryasev, 2000, 2002) — a coherent risk measure in the sense of Artzner et al. (1999). The commitment LP is therefore structurally identical to their CVaR minimization LP, with g^* playing the role of the Value-at-Risk threshold.

If spending surpluses — amounts by which scenario outcomes exceed the commitment — are treated as equally costly as shortfalls, the penalized term becomes the mean absolute deviation $\frac{1}{N}\sum_s |g_s - g^*|$, and the commitment LP generalizes to

$$\text{maximize} \quad g^* - \frac{\lambda}{N}\sum_{s=1}^{N} |g_s - g^*|. \tag{5}$$

The first-order condition yields $g^* = F^{-1}\left(\frac{1}{2} + \frac{1}{2\lambda}\right)$, where F is the empirical distribution of scenario outcomes. As $\lambda \to \infty$, g^* converges to the *median* of $\{g_s\}$ — the most conservative choice under symmetric loss. This gives the median column in the results tables a precise interpretation: it is the symmetric-loss solution at $\lambda \to \infty$, the committed spending level that minimizes expected absolute deviation from actual outcomes.