The Retirement Efficient Frontier, Part II: The Retirement Efficiency Score and Optimal Probability of Success

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Abstract

Part I of this series introduced the retirement efficient frontier — a Pareto curve trading committed spending against shortfall risk, anchored by a spending floor and characterized by the Conditional Value-at-Risk (CVaR) of spending losses, computed from either the historical record or a forward-calibrated Monte Carlo scenario set. This paper extends that framework with the *Retirement Efficiency Score* (RES): the ratio of upside spending above the floor to tail risk at a given target success rate, analogous to the Sharpe ratio on the mean–variance frontier. Its maximum, RES^*, and the associated optimal target rate ρ^* — the tangency point on the spending frontier — provide a principled alternative to choosing a success-rate target by convention. Worked examples illustrate how ρ^* and RES^* shift with asset allocation, Social Security claiming age, and the purchase of guaranteed income products, giving practitioners a single scalar for comparing retirement plan designs.

Highlights

- The Retirement Efficiency Score (RES) extends the retirement efficient frontier with a Sharpe-ratio analog that balances upside spending against tail risk.
- The optimal probability of success ρ^* is the tangency point on the spending frontier, replacing arbitrary threshold choices such as 85%.
- Worked examples show how RES^* and ρ^* respond to asset allocation, claiming-age, and annuity decisions.

Keywords: retirement income, probability of success, efficient frontier, Sharpe ratio, CVaR, spending floor, optimal stopping

JEL codes: G11, G51, D14

Part I of this series (Lacasse and Leonard, 2026) introduced the retirement efficient frontier, a Pareto curve in spending–risk space anchored at the Hard Spending Floor (HSF) and parameterized by the target probability of success ρ. A retiree who commits to spending $g(\rho)$ will fall short in at most $1-\rho$ fraction of historical scenarios, with tail losses summarized by the Conditional Value-at-Risk $\mathrm{CVaR}(\rho)$. The framework answers *how much* a retiree can spend at a given risk level, but leaves open the question of *which* risk level to target. This paper answers that question.

THE RETIREMENT EFFICIENCY SCORE

Definition and Sharpe Ratio Analogy

The spending frontier traces pairs $(g(\rho), \mathrm{CVaR}(\rho))$ as the target success rate ρ varies, analogous to the mean–variance efficient frontier in portfolio theory. This suggests a plan-quality scalar analogous to the Sharpe ratio. For the historical frontier, anchored at the HSF, define the *Retirement Efficiency Score* as

$$\mathrm{RES}(\rho) = \frac{g(\rho) - \mathrm{HSF}}{\mathrm{CVaR}(\rho)}, \quad \rho \in (0,1), \tag{1}$$

where $g(\rho)$ is the committed spending level at target success rate ρ and $\mathrm{CVaR}(\rho)$ is the CVaR of per-scenario spending losses at that same ρ. For the Monte Carlo frontier, anchored at the Synthetic Spending Floor (SSF) defined at the 95th percentile success rate, the floor and domain are adjusted:

$$\mathrm{RES}(\rho) = \frac{g(\rho) - \mathrm{SSF}}{\mathrm{CVaR}(\rho)}, \quad \rho \in (0, 0.95). \tag{2}$$

The domain is restricted to $\rho < 0.95$ because the SSF is ill-defined beyond that threshold — pushing the success rate above 95% has no meaning when the floor itself is defined there. Both numerator and denominator are in dollars per year, so $\mathrm{RES}(\rho)$ is dimensionless in both cases. Just as the Sharpe ratio is defined per portfolio and maximized to identify the tangency portfolio, $\mathrm{RES}(\rho)$ is defined per target rate and its maximum identifies the most efficient point on the spending frontier:

$$\mathrm{RES}^* = \max_{\rho} \mathrm{RES}(\rho), \qquad \rho^* = \operatorname*{argmax}_{\rho} \mathrm{RES}(\rho). \tag{3}$$

RES^* is the plan's summary efficiency score and ρ^* is its recommended target success rate — a principled alternative to choosing 85% or any other threshold by convention.

RES is most informative for *within-plan* comparisons, where plan structure is held fixed and a single dimension varies — asset allocation, claiming age, or the decision to purchase a guaranteed income product. For example, a retiree weighing the purchase of a deferred income annuity (DIA) or single premium immediate annuity (SPIA) can compute RES^* before and after the purchase. The annuity raises the floor and compresses the scenario distribution; the change in RES^*, read alongside the change in the floor, quantifies the efficiency cost of buying that safety — a trade-off the floor or PoS alone cannot reveal.

References

Lacasse, M.-D. and J. D. Leonard (2026). The retirement efficient frontier, Part I: Personalized spending floors and shortfall risk. *Journal of Retirement*. Forthcoming.