Supplementary Formulations for “Strengthened and Faster Linear Approximation to Joint Chance Constraints with Wasserstein Ambiguity”

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This document provides the detailed formulations of the chance-constrained UC and bilevel problems examined in the numerical case studies of the paper “Strengthened and Faster Linear Approximation to Joint Chance Constraints with Wasserstein Ambiguity”. To aid in understanding these formulations, we include the following nomenclature:

Nomenclature

Parameters

$\overline{P}_g^{\text{ramp}}$ Ramp up rate limit of generator g.

$\overline{P}_g$ Upper bound of power generation of generator g at time t.

$\underline{P}_g^{\text{ramp}}$ Ramp down rate limit of generator g.

$\underline{P}_g$ Lower bound of power generation of generator g at time t.

T_g^{dn} Minimal downtime of generator g.

T_g^{up} Minimal uptime of generator g.

$A_{g,t}$ Quadratic coefficient of the fuel cost function of generator g at time t.

$B_{g,t}$ Linear coefficient of the fuel cost function of generator g at time t.

$C_{g,t}^{\text{rs}}$ Unit cost of reserve of generator g at time t.

$C_{g,t}^{\text{sd}}$ Cost efficient of shutdown of generator g at time t.

$C_{g,t}^{\text{su}}$ Cost efficient of startup of generator g at time t.

$C_{g,t}$ Intercept coefficient of the fuel cost function of generator g at time t.

$C_{j,t}^{\text{cur}}$ Penalization coefficient of wind curtailment of wind farm j at time t.

$\overline{P}_l$ Power transmission capacity of transmission line l.

$S_{l,g}$ Power transfer distribution factor that measures the increase of flow on line l given a power injection at bus g.

$S_{l,j}$ Power transfer distribution factor that measures the increase of flow on line l given a power injection at bus j.

$S_{l,k}$ Power transfer distribution factor that measures the increase of flow on line l given a power injection at bus k.

$\overline{E}_s$ Energy capacity of storage system s.

$\overline{P}_s^{\text{ch}}$ Maximum charging power of storage system s.

$\overline{P}_s^{\text{dis}}$ Maximum discharging power of storage system s.

$\overline{R}_s^{\text{ch,dn}}$ Maximum charging downward reserve of storage system s.

$\overline{R}_s^{\text{ch,up}}$ Maximum charging upward reverse of storage system s.

$\overline{R}_s^{\text{dis,dn}}$ Maximum discharging downward reserve of storage system s.

$\overline{R}_s^{\text{dis,up}}$ Maximum discharging upward reserve of storage system s.

C_s^{ch} Marginal charging cost of storage system s.

C_s^{dis} Marginal discharging cost of storage system s.

E_s^{ini} Initial stored energy of storage system s.

$D_{k,t}$ Power demand of load k at time t.

$R_t^{\text{dn,ex}}$ Extra required downward reserve at time t.

$R_t^{\text{up,ex}}$ Extra required upward reserve at time t.

$w_{j,t}^{\text{fore}}$ The deterministic wind power forecast of wind farm j at time t.

ϵ The risk level of the RHS-WDRJCC.

κ_i The tunable hyperparameter in the proposed SFLA.

θ The Wasserstein radius.

M The big-M value.

N The number of samples (historical data) for random variables.

$Q_t^{\text{wm,dn}}$ The corresponding q_p of the proposed SFLA in the bilevel problem.

$Q_t^{\text{wm,up}}$ The corresponding q_p of the proposed SFLA in the bilevel problem.

w_t^{dn} The tunable hyperparameters of W-CVaR in the bilevel problem.

w_t^{up} The tunable hyperparameters of W-CVaR in the bilevel problem.

Random Variables

$\tilde{e}_{j,t}$ Wind forecast error of wind farm j at time t.

$\tilde{w}_{j,t}$ Actual power output of wind farm j at time t.

Indices

g Index of generators.

h, t Indices of time periods.

i Index of samples of a random variable.

j Index of wind turbines.

k Index of demands.

l Index of transmission lines.

s Index of storage systems.

Variables

$\hat{b}_{s,t}^{\mathrm{ch,dn}}$ Bid price for charging downward reserve of storage unit s in the day-ahead market at time t.

$\hat{b}_{s,t}^{\mathrm{ch,up}}$ Bid price for charging upward reserve of storage unit s in the day-ahead market at time t.

$\hat{b}_{s,t}^{\mathrm{ch}}$ Offer price for charging power of storage unit s in the day-ahead market at time t.

$\hat{b}_{s,t}^{\mathrm{dis,dn}}$ Bid price for discharging downward reserve of storage unit s in the day-ahead market at time t.

$\hat{b}_{s,t}^{\mathrm{dis,up}}$ Bid price for discharging upward reserve of storage unit s in the day-ahead market at time t.

$\hat{b}_{s,t}^{\mathrm{dis}}$ Bid price for discharging power of storage unit s in the day-ahead market at time t.

$\hat{p}_{s,t}^{\mathrm{ch}}$ Offer quantity for charging power of storage unit s in the day-ahead market at time t.

$\hat{p}_{s,t}^{\mathrm{dis}}$ Bid quantity for discharging power of storage unit s in the day-ahead market at time t.

$\hat{r}_{s,t}^{\mathrm{ch,dn}}$ Bid quantity for charging downward reserve of storage unit s in the day-ahead market at time t.

$\hat{r}_{s,t}^{\mathrm{ch,up}}$ Bid quantity for charging upward reserve of storage unit s in the day-ahead market at time t.

$\hat{r}_{s,t}^{\mathrm{dis,dn}}$ Bid quantity for discharging downward reserve of storage unit s in the day-ahead market at time t.

$\hat{r}_{s,t}^{\mathrm{dis,up}}$ Bid quantity for discharging upward reserve of storage unit s in the day-ahead market at time t.

μ Dual variables corresponding to inequality day-ahead lower-level constraints.

$c_{j,t}^{\mathrm{cr}}$ Scheduled wind curtailment cost of wind farm j at time t.

$c_{g,t}^{\mathrm{fc}}$ Scheduled fuel cost of generator g at time t.

$c_{g,t}^{\mathrm{rs}}$ Scheduled reserve cost of generator g at time t.

$c_{g,t}^{\mathrm{sd}}$ Scheduled shutdown cost of generator g at time t.

$c_{g,t}^{\mathrm{su}}$ Scheduled startup cost of generator g at time t.

$p_{g,t}$ Scheduled power of generator g in day-ahead market at time t.

$p_{s,t}^{\mathrm{ch}}$ Scheduled quantity for charging power of storage unit s in the day-ahead market at time t.

$p_{s,t}^{\mathrm{dis}}$ Scheduled quantity for discharging power of storage unit s in the day-ahead market at time t.

$r_{g,t}^{\mathrm{dn}}$ Scheduled downward reserve of generator g in day-ahead market at time t.

$r_{g,t}^{\mathrm{up}}$ Scheduled upward reserve of generator g in day-ahead market at time t.

$r_{s,t}^{\mathrm{ch,dn}}$ Scheduled quantity for charging downward reserve of storage unit s in the day-ahead market at time t.

$r_{s,t}^{\mathrm{ch,up}}$ Scheduled quantity for charging upward reserve of storage unit s in the day-ahead market at time t.

$r_{s,t}^{\mathrm{dis,dn}}$ Scheduled quantity for discharging downward reserve of storage unit s in the day-ahead market at time t.

$r_{s,t}^{\mathrm{dis,up}}$ Scheduled quantity for discharging upward reserve of storage unit s in the day-ahead market at time t.

$r_{t}^{\mathrm{wm,dn}}$ Scheduled quantity for downward reserve to tackle wind forecasting error at time t.

$r_{t}^{\mathrm{wm,up}}$ Scheduled quantity for upward reserve to tackle wind forecasting error at time t.

$u_{s,t}^{\mathrm{ch}}$ Binary decision variable indicating the charging mode of storage system s at time t.

$u_{s,t}^{\mathrm{dis}}$ Binary decision variable indicating the discharging mode of storage system s at time t.

$v_{g,t}$ Commitment state of generator g at time t.

$w_{j,t}^{\mathrm{cur}}$ Scheduled wind curtailment of wind farm j at time t.

$w_{j,t}^{\mathrm{sch}}$ Scheduled wind power of wind farm j at time t.

$m^{\mathrm{up}}, m^{\mathrm{dn}}$ Auxiliary variables for Bonferroni approximation in the bilevel problem.

$\alpha^{\mathrm{up}}, \alpha^{\mathrm{dn}}$ Auxiliary variables for Bonferroni approximation in the bilevel problem.

α_i, β, τ Auxiliary variables for W-CVaR in the bilevel problem.

$\beta^{\mathrm{up}}, \beta^{\mathrm{dn}}$ Auxiliary variables for Bonferroni approximation in the bilevel problem.

$\eta_t^{\mathrm{up}}, \eta_t^{\mathrm{dn}}$ Auxiliary variables for Bonferroni approximation in the bilevel problem.

u, v_i Auxiliary variables of the proposed SFLA and the existing LA method in the bilevel problem.

λ_t^{dn} Day-ahead market-clearing downward reserve price (as a dual variable) at time t.

λ_t^{en} Day-ahead market-clearing energy price (as a dual variable) at time t.

λ_t^{up} Day-ahead market-clearing upward reserve price (as a dual variable) at time t.

Appendix A: Formulation of Chance-constrained Unit Commitment

Following Yang et al. (2020), we formulate the UC problem as below:

$$
\begin{aligned}
\min \quad & \sum_{t\in[T]} \left(\sum_{g\in[G]} (c_{g,t}^{\mathrm{su}} + c_{g,t}^{\mathrm{sd}} + c_{g,t}^{\mathrm{fc}} + c_{g,t}^{\mathrm{rs}}) + \sum_{j\in[J]} c_{j,t}^{\mathrm{cr}} \right) && && \text{(1a)}\\
\text{s.t.} \quad & c_{g,t}^{\mathrm{su}} \ge C_{g,t}^{\mathrm{su}}(v_{g,t} - v_{g,t-1}),\ c_{g,t}^{\mathrm{su}} \ge 0, && \forall t\in[T], \forall g\in[G], && \text{(1b)}\\
& c_{g,t}^{\mathrm{sd}} \ge C_{g,t}^{\mathrm{sd}}(v_{g,t-1} - v_{g,t}),\ c_{g,t}^{\mathrm{sd}} \ge 0, && \forall t\in[T], \forall g\in[G], && \text{(1c)}\\
& c_{g,t}^{\mathrm{fc}} \ge A_{g,t}(p_{g,t})^2 + B_{g,t}p_{g,t} + C_{g,t}v_{g,t}, && \forall t\in[T], \forall g\in[G], && \text{(1d)}\\
& c_{g,t}^{\mathrm{rs}} = C_{g,t}^{\mathrm{rs}}(r_{g,t}^{\mathrm{up}} + r_{g,t}^{\mathrm{dn}}), && \forall t\in[T], \forall g\in[G], && \text{(1e)}\\
& c_{j,t}^{\mathrm{cr}} = C_{j,t}^{\mathrm{cur}} w_{j,t}^{\mathrm{cur}}, && \forall t\in[T], \forall j\in[J], && \text{(1f)}\\
& \sum_{g\in[G]} p_{g,t} + \sum_{j\in[J]} w_{j,t}^{\mathrm{sch}} = \sum_{k\in[K]} D_{k,t}, && \forall t\in[T], && \text{(1g)}\\
& p_{g,t} + r_{g,t}^{\mathrm{up}} \le v_g^t \overline{P}_g, && \forall t\in[T], \forall g\in[G], && \text{(1h)}\\
& v_{g,t}\underline{P}_g \le p_{g,t} - r_{g,t}^{\mathrm{dn}}, && \forall t\in[T], \forall g\in[G], && \text{(1i)}\\
& p_{g,t} - p_{g,t-1} \le \overline{P}_g^{\mathrm{ramp}} + (2 - v_{g,t-1} - v_{g,t})M, && \forall t\in[T], \forall g\in[G], && \text{(1j)}\\
& p_{g,t-1} - p_{g,t} \le \underline{P}_g^{\mathrm{ramp}} + (2 - v_{g,t-1} - v_{g,t})M, && \forall t\in[T], \forall g\in[G], && \text{(1k)}\\
& 0 \le r_{g,t}^{\mathrm{up}} \le \overline{R}_g^{\mathrm{up}}, && \forall t\in[T], \forall g\in[G], && \text{(1l)}\\
& 0 \le r_{g,t}^{\mathrm{dn}} \le \overline{R}_g^{\mathrm{dn}}, && \forall t\in[T], \forall g\in[G], && \text{(1m)}\\
& \sum_{\tau=t}^{t+T_g^{\mathrm{up}}-1} v_g^{\tau} \ge T_g^{\mathrm{up}}(v_{g,t} - v_{g,t-1}), && \forall t\in[1, T - T_g^{\mathrm{up}} + 1]_{\mathbb{Z}}, \forall g\in[G], && \text{(1n)}\\
& \sum_{\tau=t}^{T} [v_g^{\tau} - (v_g^t - v_{g,t-1})] \ge 0, && \forall t\in[T - T_g^{\mathrm{up}} + 2, T]_{\mathbb{Z}}, \forall g\in[G], && \text{(1o)}\\
& \sum_{\tau=t}^{t+T_g^{\mathrm{dn}}-1} (1 - v_g^{\tau}) \ge T_g^{\mathrm{dn}}(v_{g,t-1} - v_{g,t}), && \forall t\in[1, T - T_g^{\mathrm{dn}} + 1]_{\mathbb{Z}}, \forall g\in[G], && \text{(1p)}
\end{aligned}
$$

$$\sum_{\tau=t}^{T}[1-v_g^{\tau}-(v_g^{t-1}-v_{g,t})]\geq 0, \qquad \forall t\in[T-T_g^{\text{dn}}+2,T]_{\mathbb{Z}},\forall g\in[G], \tag{1q}$$

$$w_{j,t}^{\text{sch}}=w_{j,t}^{\text{fore}}-w_{j,t}^{\text{cur}}, \qquad \forall j\in[J],t\in[T], \tag{1r}$$

$$0\leq w_{j,t}^{\text{cur}}\leq w_{j,t}^{\text{fore}}, \qquad \forall j\in[J],t\in[T], \tag{1s}$$

$$\sup_{\mathbb{P}\in\mathcal{F}_N(\theta)}\ \mathbb{P}\left\{\begin{array}{ll} \quad\sum_{g\in[G]}r_{g,t}^{\text{up}}\geq\sum_{j\in[J]}w_{j,t}^{\text{sch}}-\sum_{j\in[J]}(\tilde{w}_{j,t}-w_{j,t}^{\text{cur}})+R_t^{\text{up,ex}} & \\ \Leftrightarrow\sum_{g\in[G]}r_{g,t}^{\text{up}}\geq-\sum_{j\in[J]}\tilde{e}_{j,t}+R_t^{\text{up,ex}}, & \forall t\in[T], \\ \quad\sum_{g\in[G]}r_{g,t}^{\text{dn}}\geq-\sum_{j\in[J]}w_{j,t}^{\text{sch}}+\sum_{j\in[J]}(\tilde{w}_{j,t}-w_{j,t}^{\text{cur}})+R_t^{\text{dn,ex}} & \\ \Leftrightarrow\sum_{g\in[G]}r_{g,t}^{\text{dn}}\geq\sum_{j\in[J]}\tilde{e}_{j,t}+R_t^{\text{dn,ex}}, & \forall t\in[T], \\ \sum_{g\in[G]}S_{l,g}p_{g,t}+\sum_{j\in[J]}S_{l,j}(w_{j,t}^{\text{sch}}+\tilde{e}_{j,t})-\sum_{k\in[K]}S_{l,k}D_{k,t}\leq\overline{P}_l, & \forall t\in[T],\forall l\in[L], \\ \sum_{g\in[G]}S_{l,g}p_{g,t}+\sum_{j\in[J]}S_{l,j}(w_{j,t}^{\text{sch}}+\tilde{e}_{j,t})-\sum_{k\in[K]}S_{l,k}D_{k,t}\geq-\overline{P}_l, & \forall t\in[T],\forall l\in[L] \end{array}\right\}\geq 1-\epsilon. \tag{1t}$$

The objective (1a) minimizes the sum of the start-up cost $c_{g,t}^{\text{su}}$, shut-down cost $c_{g,t}^{\text{sd}}$, quadratic fuel cost $c_{g,t}^{\text{fc}}$, reserve cost $c_{g,t}^{\text{rc}}$, and wind curtailment cost $c_{j,t}^{\text{cr}}$, which are modelled in constraints (1b)–(1f). The power balance constraints are modelled in constraint (1g). Constraints (1h) and (1i) limit the power output of generators considering upward and downward reserve capacity. The ramping constraints for generators are given in constraints (1j) and (1k) with big-M (can be set to $\overline{P}_g$). The reserve capacity of generators is restricted in constraints (1l) and (1m). Constraints (1n)–(1q) model the minimum up and down times of generators, where $[a,b]_{\mathbb{Z}}$ denotes the set of integers in interval $[a,b]$. Following Yang et al. (2020), we consider a random wind generation as a deterministic forecast plus a random forecasting error: $\tilde{w}_{j,t}=w_{j,t}^{\text{fore}}+\tilde{e}_w^t$ following Yang et al. (2020). Then, when specifying a certain amount of wind curtailment $w_{j,t}^{\text{cur}}$ subject to (1s), the scheduled wind generation $w_{j,t}^{\text{sch}}$ calculated by constraint (1r) can deviate from the actual wind generation after curtailment. A joint chance constraint (1t) is therefore established to 1) secure sufficient reserve to hedge the uncertain wind generation, and 2) secure the line thermal limits due to the uncertain power flow caused by the wind generation for all lines and all time steps.

Appendix B: Formulation of Bilevel Strategic Bidding

This section is organized as follows: Appendix B.1 and B.2 present the mathematical formulations for the upper-level and lower-level problems, respectively. Appendix B.3 reformulates the bilevel problem into a solvable single-level problem using KKT conditions when the proposed SFLA is applied, and Appendix B.4 presents the bilevel reformulation using other three benchmark methods that approximate RHS-WDRJCC. The formulation of the bilevel problem and the KKT-based solution scheme follow Nasrolahpour et al. (2018). Modifications have been made to integrate the uncertainty of wind power generation as an RHS-WDRJCC in the lower-level problem. Additionally, the second lower-level balancing market in Nasrolahpour et al. (2018) has been omitted, leaving it for future work.

B.1. Upper-level Problem: Storage System Profit Maximization

The upper-level problem is formulated to maximize the profit of the storage system from participating in day-ahead energy and reserve markets, as detailed in Problem (2). Readers are referred to Nasrolahpour et al. (2018) for a comprehensive introduction to the energy market dynamics. The primal variables of the upper-level problem are:

$$\Xi_{UL}=\{\hat{p}_{s,t}^{\text{ch}},\hat{p}_{s,t}^{\text{dis}},\hat{r}_{s,t}^{\text{dis,up}},\hat{r}_{s,t}^{\text{ch,up}},\hat{r}_{s,t}^{\text{ch,dn}},\hat{r}_{s,t}^{\text{dis,dn}},u_{s,t}^{\text{ch}},u_{s,t}^{\text{dis}},\hat{b}_{s,t}^{\text{ch}},\hat{b}_{s,t}^{\text{dis}},\hat{b}_{s,t}^{\text{ch,up}},\hat{b}_{s,t}^{\text{ch,dn}},\hat{b}_{s,t}^{\text{dis,up}},\hat{b}_{s,t}^{\text{dis,dn}}\}.$$

$$
\begin{aligned}
\max_{\Xi_{UL}} \quad & \sum_{t\in[T]}\sum_{s\in[S]}\left[-\left(\lambda_t^{\text{en}}+C_s^{\text{ch}}\right)p_{s,t}^{\text{ch}}+\left(\lambda_t^{\text{en}}-C_s^{\text{dis}}\right)p_{s,t}^{\text{dis}}+\left[\lambda_t^{\text{up}}\left(r_{s,t}^{\text{ch,up}}+r_{s,t}^{\text{dis,up}}\right)+\lambda_t^{\text{dn}}\left(r_{s,t}^{\text{ch,dn}}+r_{s,t}^{\text{dis,dn}}\right)\right]\right] && && \text{(2a)}\\
\text{s.t.} \quad & u_{s,t}^{\text{dis}}+u_{s,t}^{\text{ch}}\le 1, && \forall s\in[S],\forall t\in[T], && \text{(2b)}\\
& u_{s,t}^{\text{ch}},u_{s,t}^{\text{dis}}\in\{0,1\}, && \forall s\in[S],\forall t\in[T], && \text{(2c)}\\
& 0\le \hat{p}_{s,t}^{\text{dis}}\le u_{s,t}^{\text{dis}}\overline{P}_s^{\text{dis}}, && \forall s\in[S],\forall t\in[T], && \text{(2d)}\\
& 0\le \hat{r}_{s,t}^{\text{dis,up}}\le u_{s,t}^{\text{dis}}\overline{R}_s^{\text{dis,up}}, && \forall s\in[S],\forall t\in[T], && \text{(2e)}\\
& 0\le \hat{r}_{s,t}^{\text{dis,dn}}\le u_{s,t}^{\text{dis}}\overline{R}_s^{\text{dis,dn}}, && \forall s\in[S],\forall t\in[T], && \text{(2f)}\\
& \hat{p}_{s,t}^{\text{dis}}+\hat{r}_{s,t}^{\text{dis,up}}\le u_{s,t}^{\text{dis}}\overline{P}_s^{\text{dis}}, && \forall s\in[S],\forall t\in[T], && \text{(2g)}\\
& \hat{r}_{s,t}^{\text{dis,dn}}-\hat{p}_{s,t}^{\text{dis}}\le 0, && \forall s\in[S],\forall t\in[T], && \text{(2h)}\\
& 0\le \hat{p}_{s,t}^{\text{ch}}\le \overline{P}_s^{\text{ch}}, && \forall s\in[S],\forall t\in[T], && \text{(2i)}\\
& 0\le \hat{r}_{s,t}^{\text{ch,up}}\le u_{s,t}^{\text{ch}}\overline{R}_s^{\text{ch,up}}, && \forall s\in[S],\forall t\in[T], && \text{(2j)}\\
& 0\le \hat{r}_{s,t}^{\text{ch,dn}}\le u_{s,t}^{\text{ch}}\overline{R}_s^{\text{ch,dn}}, && \forall s\in[S],\forall t\in[T], && \text{(2k)}\\
& \hat{p}_{s,t}^{\text{ch}}+\hat{r}_{s,t}^{\text{ch,dn}}\le u_{s,t}^{\text{ch}}\overline{P}_s^{\text{ch}}, && \forall s\in[S],\forall t\in[T], && \text{(2l)}\\
& \hat{r}_{s,t}^{\text{ch,up}}-\hat{p}_{s,t}^{\text{ch}}\le 0, && \forall s\in[S],\forall t\in[T], && \text{(2m)}\\
& \hat{b}_{s,t}^{\text{ch}},\hat{b}_{s,t}^{\text{ch,up}},\hat{b}_{s,t}^{\text{ch,dn}}\ge 0, && \forall s\in[S],\forall t\in[T], && \text{(2n)}\\
& \hat{b}_{s,t}^{\text{dis}},\hat{b}_{s,t}^{\text{dis,up}},\hat{b}_{s,t}^{\text{dis,dn}}\ge 0, && \forall s\in[S],\forall t\in[T], && \text{(2o)}\\
& E_s^{\text{ini}}+\eta^{\text{ch}}\sum_{h=1}^{t}(\hat{p}_{s,h}^{\text{ch}}+\hat{r}_{s,h}^{\text{ch,dn}})-\frac{1}{\eta^{\text{dis}}}\sum_{h=1}^{t}(\hat{p}_{s,h}^{\text{dis}}-\hat{r}_{s,h}^{\text{dis,dn}})\le \overline{E}_s, && \forall s\in[S],\forall t\in[T], && \text{(2p)}\\
& E_s^{\text{ini}}+\eta^{\text{ch}}\sum_{h=1}^{t}(\hat{p}_{s,h}^{\text{ch}}-\hat{r}_{s,h}^{\text{ch,up}})-\frac{1}{\eta^{\text{dis}}}\sum_{h=1}^{t}(\hat{p}_{s,h}^{\text{dis}}+\hat{r}_{s,h}^{\text{dis,up}})\ge 0, && \forall s\in[S],\forall t\in[T]. && \text{(2q)}
\end{aligned}
$$

The objective (2a) of the upper-level problem is to maximize storage profit derived from the day-ahead energy and reserve markets by strategically bidding in both charging and discharging modes. Profit calculations are based on the prices and quantities that are determined within the lower-level problem, which simulates the day-ahead market clearing process. Specifically, profits are derived from the energy prices λ_t^{en}, and the upward and downward reserves prices $\lambda_t^{\text{up}}, \lambda_t^{\text{dn}}$, combined with the quantities dispatched for energy $p_{s,t}^{\text{ch}}, p_{s,t}^{\text{dis}}$, and reserves $r_{s,t}^{\text{dis,up}}, r_{s,t}^{\text{ch,up}}, r_{s,t}^{\text{ch,dn}}, r_{s,t}^{\text{dis,dn}}$. The first two terms of the objective function reflect the costs associated with charging and the revenues from discharging, while the last two terms account for the revenues generated from providing upward and downward reserves.

Constraints (2b) and (2c) specify that the decisions to discharge or charge are binary and mutually exclusive. The subsequent constraints, from (2d) to (2f), limit the discharge quantity of energy and reserve bids to ensure they do not exceed the storage capacity. Constraints (2g) and (2h) further enforce the total discharge quantity of energy and reserves does not exceed the discharging limit of the storage system. Similar restrictions apply to charging mode in constraints (2i)– (2m). The nonnegative nature of bids/offers for energy and reserve of both operating modes are ensured in (2n) and (2o). The constraints for the upper and lower energy limits of storage systems based on bidding quantities, reflecting the state of charge of storage systems while considering both energy and reserve capacities, are modeled in equations (2p) and (2q), respectively. Note that we adopt a conservative setting that ensure the energy limits under the worst-case (100%) reserve activation.

B.2. Lower-level Problem: Clearing of Day-ahead Market (Joint Energy and Reserve Settlement)

The lower-level problem which represents the day-ahead joint energy and reserve market clearing is formulated in Problem (3). All dual variables are given in constraints after a colon. The upper-level variables are treated as parameters in the lower-level problem. Variable set

$$\Xi_{LL}^{\text{Primal}} = \{p_{s,t}^{\text{ch}}, p_{s,t}^{\text{dis}}, p_{g,t}, r_{s,t}^{\text{ch,up}}, r_{s,t}^{\text{ch,dn}}, r_{s,t}^{\text{dis,up}}, r_{s,t}^{\text{dis,dn}}, r_{g,t}^{\text{up}}, r_{g,t}^{\text{dn}}, w_{j,t}^{\text{sch}}, w_{j,t}^{\text{cur}}, r_t^{\text{wm,up}}, r_t^{\text{wm,dn}}, u, v_i\}$$

contains the primal variables of the lower-level problem (3), whereas the dual variables of Problem (3) are included in the variable set Ξ_{LL}^{Dual}:

$$\Xi_{LL}^{\text{Dual}} = \{\lambda_t^{\text{en}}, \lambda_t^{\text{up}}, \lambda_t^{\text{dn}}, \underline{\mu}_{g,t}, \overline{\mu}_{g,t}, \underline{\mu}_{s,t}^{\text{ch}}, \overline{\mu}_{s,t}^{\text{ch}}, \underline{\mu}_{s,t}^{\text{dis}}, \overline{\mu}_{s,t}^{\text{dis}}, \underline{\mu}_{s,t}^{\text{ch,up}}, \overline{\mu}_{s,t}^{\text{ch,up}}, \underline{\mu}_{s,t}^{\text{ch,dn}}, \overline{\mu}_{s,t}^{\text{ch,dn}}, \underline{\mu}_{s,t}^{\text{dis,up}}, \overline{\mu}_{s,t}^{\text{dis,up}},$$
$$\underline{\mu}_{s,t}^{\text{dis,dn}}, \overline{\mu}_{s,t}^{\text{dis,dn}}, \mu_{s,t}^{\text{ch,up}}, \mu_{s,t}^{\text{dis,dn}}, \underline{\mu}_{g,t}^{\text{up}}, \underline{\mu}_{g,t}^{\text{dn}}, \overline{\mu}_{g,t}^{\text{dn}}, \mu_{g,t}^{\text{up}}, \mu_{g,t}^{\text{dn}}, \mu_{j,t}^{\text{w,sch}}, \underline{\mu}_{j,t}^{\text{w,cur}}, \overline{\mu}_{j,t}^{\text{w,cur}}, \underline{\mu}^{\text{u}}, \underline{\mu}_i^{\text{v}}, \mu^1, \mu_{i,t}^2, \mu_{i,t}^3, \mu_t^4, \mu_t^5\}$$

$$\max_{\Xi_{LL}} \quad \sum_{t\in[T]} \Bigg[\sum_{s\in[S]} \left(\hat{b}_{s,t}^{\text{ch}} p_{s,t}^{\text{ch}} - \hat{b}_{s,t}^{\text{dis}} p_{s,t}^{\text{dis}}\right) - \sum_{g\in[G]} C_{g,t} p_{g,t} - \sum_{g\in[G]} C_{g,t}^{\text{rs}} \left(r_{g,t}^{\text{up}} + r_{g,t}^{\text{dn}}\right) - \sum_{j\in[J]} C_{j,t}^{\text{cur}} w_{j,t}^{\text{cur}}$$
$$- \sum_{s\in[S]} \left(\hat{b}_{s,t}^{\text{ch,up}} r_{s,t}^{\text{ch,up}} + \hat{b}_{s,t}^{\text{ch,dn}} r_{s,t}^{\text{ch,dn}} + \hat{b}_{s,t}^{\text{dis,up}} r_{s,t}^{\text{dis,up}} + \hat{b}_{s,t}^{\text{dis,dn}} r_{s,t}^{\text{dis,dn}}\right) \Bigg] \tag{3a}$$

$$\text{s.t.} \quad \sum_{g\in[G]} p_{g,t} + \sum_{s\in[S]} \left(p_{s,t}^{\text{dis}} - p_{s,t}^{\text{ch}}\right) + \sum_{j\in[J]} w_{j,t}^{\text{sch}} = \sum_{k\in[K]} D_{k,t} : \lambda_t^{\text{en}}, \qquad \forall t \in [T], \tag{3b}$$

$$\underline{P}_g \le p_{g,t} \le \overline{P}_g : \underline{\mu}_{g,t}, \overline{\mu}_{g,t}, \qquad \forall g \in [G], \forall t \in [T], \tag{3c}$$

$$0 \le p_{s,t}^{\text{ch}} \le \hat{p}_{s,t}^{\text{ch}} : \underline{\mu}_{s,t}^{\text{ch}}, \overline{\mu}_{s,t}^{\text{ch}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3d}$$

$$0 \le p_{s,t}^{\text{dis}} \le \hat{p}_{s,t}^{\text{dis}} : \underline{\mu}_{s,t}^{\text{dis}}, \overline{\mu}_{s,t}^{\text{dis}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3e}$$

$$0 \le r_{s,t}^{\text{ch,up}} \le \hat{r}_{s,t}^{\text{ch,up}} : \underline{\mu}_{s,t}^{\text{ch,up}}, \overline{\mu}_{s,t}^{\text{ch,up}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3f}$$

$$0 \le r_{s,t}^{\text{ch,dn}} \le \hat{r}_{s,t}^{\text{ch,dn}} : \underline{\mu}_{s,t}^{\text{ch,dn}}, \overline{\mu}_{s,t}^{\text{ch,dn}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3g}$$

$$0 \le r_{s,t}^{\text{dis,up}} \le \hat{r}_{s,t}^{\text{dis,up}} : \underline{\mu}_{s,t}^{\text{dis,up}}, \overline{\mu}_{s,t}^{\text{dis,up}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3h}$$

$$0 \le r_{s,t}^{\text{dis,dn}} \le \hat{r}_{s,t}^{\text{dis,dn}} : \underline{\mu}_{s,t}^{\text{dis,dn}}, \overline{\mu}_{s,t}^{\text{dis,dn}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3i}$$

$$r_{s,t}^{\text{ch,up}} - p_{s,t}^{\text{ch}} \le 0 : \mu_{s,t}^{\text{ch,up}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3j}$$

$$r_{s,t}^{\text{dis,dn}} - p_{s,t}^{\text{dis}} \le 0 : \mu_{s,t}^{\text{dis,dn}}, \qquad \forall s \in [S], \forall t \in [T], \tag{3k}$$

$$0 \le r_{g,t}^{\text{up}} \le \overline{R}_g^{\text{up}} : \underline{\mu}_{g,t}^{\text{up}}, \overline{\mu}_{g,t}^{\text{up}}, \qquad \forall g \in [G], \forall t \in [T], \tag{3l}$$

$$0 \le r_{g,t}^{\text{dn}} \le \overline{R}_g^{\text{dn}} : \underline{\mu}_{g,t}^{\text{dn}}, \overline{\mu}_{g,t}^{\text{dn}}, \qquad \forall g \in [G], \forall t \in [T], \tag{3m}$$

$$p_{g,t} + r_{g,t}^{\text{up}} \le \overline{P}_g : \mu_{g,t}^{\text{up}}, \qquad \forall g \in [G], \forall t \in [T], \tag{3n}$$

$$r_{g,t}^{\text{dn}} - p_{g,t} \le -\underline{P}_g : \mu_{g,t}^{\text{dn}}, \qquad \forall g \in [G], \forall t \in [T], \tag{3o}$$

$$w_{j,t}^{\text{sch}} = w_{j,t}^{\text{fore}} - w_{j,t}^{\text{cur}} : \mu_{j,t}^{\text{w,sch}}, \qquad \forall j \in [J], t \in [T], \tag{3p}$$

$$0 \le w_{j,t}^{\text{cur}} \le w_{j,t}^{\text{fore}} : \underline{\mu}_{j,t}^{\text{w,cur}}, \overline{\mu}_{j,t}^{\text{w,cur}}, \qquad \forall j \in [J], t \in [T], \tag{3q}$$

$$\sup_{\mathbb{P}\in\mathcal{F}_N^e(\theta)} \mathbb{P} \left\{ \begin{array}{l} r_t^{\text{wm,up}} \ge -\sum_{j\in[J]} \tilde{e}_{j,t}, \forall t \in [T], \\ r_t^{\text{wm,dn}} \ge \sum_{j\in[J]} \tilde{e}_{j,t}, \forall t \in [T], \end{array} \right\} \ge 1 - \epsilon \tag{3r}$$

$$\sum_{s\in[S]}\left(r_{s,t}^{\text{ch,up}}+r_{s,t}^{\text{dis,up}}\right)+\sum_{g\in[G]}r_{g,t}^{\text{up}}\geq r_t^{\text{wm,up}}+R_t^{\text{up,ex}}:\lambda_t^{\text{up}}, \qquad \forall t\in[T], \tag{3s}$$

$$\sum_{s\in[S]}\left(r_{s,t}^{\text{ch,dn}}+r_{s,t}^{\text{dis,dn}}\right)+\sum_{g\in[G]}r_{g,t}^{\text{dn}}\geq r_t^{\text{wm,dn}}+R_t^{\text{dn,ex}}:\lambda_t^{\text{dn}}, \qquad \forall t\in[T]. \tag{3t}$$

The lower-level problem is a day-ahead joint energy and reserve market clearing problem, aiming to maximize the overall social welfare. In addition to storage systems, upward and downward reserve requirements of the market can also be provided by generators. The first line of the objective function (3a) calculates the net operational benefits and costs associated with storage charging and discharging, alongside the costs incurred from generation. The subsequent lines detail the costs incurred from procuring reserve capacities from both storage units and generators and from wind curtailment.

Constraint (3b) ensures the balance of power by incorporating contributions from storage and the scheduled power generation from wind farms, denoted as $w_{j,t}^{\text{sch}}$. The associated energy price is reflected by the dual variable λ_t^{en}. Constraint (3c) specifies that the generation levels for generators are bounded within specified limits. Constraints (3d) and (3e) regulate the charging and discharging operations of storage systems, ensuring they remain within the submitted bid quantities. Similarly, constraints (3f)–(3k) specify the limits for storage reserve capacities based on the bid quantities. Constraints (3l)–(3o) define the upper and lower limits for upward and downward reserves provided by generators.

Constraint (3p) specifies the relationship between scheduled wind power and its expected forecast value, consistent with the UC formulation in constraint (1r). Similarly, the bounds for wind curtailment stated in (3q) align with those in (1s). Due to the uncertain wind forecasting error, sufficiency of upward and downward reserve capacity (i.e., $r_t^{\text{wm,up}}, r_t^{\text{wm,dn}}$) must be jointly guaranteed with high probability and it is formulated as an RHS-WDRJCC (3r). Constraint (3s) further states that the upward reserve capacity provided by storage and generators should be enough to cover the upward reverse due to wind forecasting error $r_t^{\text{wm,up}}$ and extra required upward reserve margin $R_t^{\text{up,ex}}$. Downward reserve capacities follow similarly, detailed in constraint (3t). The dual variables of constraints (3s) and (3t) are prices for upward and downward reserves, respectively.

Note that, because the random variables $\tilde{e}_{j,t}$ only appear as the summation form of $\sum_{j\in[J]}\tilde{e}_{j,t}$ in Problem (3), it suffices to consider $\sum_{j\in[J]}\tilde{e}_{j,t}$ as a single random variable to simplify the subsequent derivation.

B.3. KKT Conditions

Under the proposed SFLA, the RHS-WDRJCC (3r) is reformulated as:

$$-u\leq 0, -v_i\leq 0:\underline{\mu}^{\text{u}},\underline{\mu}_i^{\text{v}}, \qquad \forall i\in[N], \tag{4a}$$

$$-(\epsilon N u-\sum_{i\in[N]}v_i)\leq -\theta N:\mu^1, \tag{4b}$$

$$u-v_i-\kappa_i r_t^{\text{wm,up}}\leq \kappa_i\sum_{j\in[J]}\tilde{e}_{j,t,i}:\mu_{i,t}^2, \qquad \forall i\in[N]_t^{\text{up}}, t\in[T], \tag{4c}$$

$$u-v_i-\kappa_i r_t^{\text{wm,dn}}\leq -\kappa_i\sum_{j\in[J]}\tilde{e}_{j,t,i}:\mu_{i,t}^3, \qquad \forall i\in[N]_t^{\text{dn}}, t\in[T], \tag{4d}$$

$$u-r_t^{\text{wm,up}}\leq Q_t^{\text{wm,up}}:\mu_t^4, \qquad \forall t\in[T], \tag{4e}$$

$$u-r_t^{\text{wm,dn}}\leq Q_t^{\text{wm,dn}}:\mu_t^5, \qquad \forall t\in[T]. \tag{4f}$$

Note that the Appendix renames the auxiliary variables s and r_i used in the main content to u and v_i, respectively. This change prevents confusion with the storage index s and the reserve variable in the bilevel problem, which aligns with existing literature in storage strategic bidding to improve readability.

The proposed SFLA leads to convex reformulation of the RHS-WDRJCC. Consequently, by substituting the lower-level problem with its KKT conditions, the bilevel strategic bidding problem can be effectively transformed into a single-level Mathematical Program with Equilibrium Constraints (MPEC). KKT conditions of the lower-level problem (3) under the proposed SFLA (4) of the RHS-WDRJCC (3r) is presented in (5) and (6):

$$\text{Constraints (3b), (3p),} \tag{5a}$$
$$\hat{b}^{\text{dis}}_{s,t} - \lambda^{\text{en}}_t + \overline{\mu}^{\text{dis}}_{s,t} - \underline{\mu}^{\text{dis}}_{s,t} - \mu^{\text{dis,dn}}_{s,t} = 0 : p^{\text{dis}}_{s,t}, \quad \forall s \in [S], \forall t \in [T], \tag{5b}$$
$$-\hat{b}^{\text{ch}}_{s,t} + \lambda^{\text{en}}_t + \overline{\mu}^{\text{ch}}_{s,t} - \underline{\mu}^{\text{ch}}_{s,t} - \mu^{\text{ch,up}}_{s,t} = 0 : p^{\text{ch}}_{s,t}, \quad \forall s \in [S], \forall t \in [T], \tag{5c}$$
$$\hat{b}^{\text{dis,up}}_{s,t} - \lambda^{\text{up}}_t + \overline{\mu}^{\text{dis,up}}_{s,t} - \underline{\mu}^{\text{dis,up}}_{s,t} = 0 : r^{\text{dis,up}}_{s,t}, \quad \forall s \in [S], \forall t \in [T], \tag{5d}$$
$$\hat{b}^{\text{dis,dn}}_{s,t} - \lambda^{\text{dn}}_t + \overline{\mu}^{\text{dis,dn}}_{s,t} - \underline{\mu}^{\text{dis,dn}}_{s,t} + \mu^{\text{dis,dn}}_{s,t} = 0 : r^{\text{dis,dn}}_{s,t}, \quad \forall s \in [S], \forall t \in [T], \tag{5e}$$
$$\hat{b}^{\text{ch,up}}_{s,t} - \lambda^{\text{up}}_t + \overline{\mu}^{\text{ch,up}}_{s,t} - \underline{\mu}^{\text{ch,up}}_{s,t} + \mu^{\text{ch,up}}_{s,t} = 0 : r^{\text{ch,up}}_{s,t}, \quad \forall s \in [S], \forall t \in [T], \tag{5f}$$
$$\hat{b}^{\text{ch,dn}}_{s,t} - \lambda^{\text{dn}}_t + \overline{\mu}^{\text{ch,dn}}_{s,t} - \underline{\mu}^{\text{ch,dn}}_{s,t} = 0 : r^{\text{ch,dn}}_{s,t}, \quad \forall s \in [S], \forall t \in [T], \tag{5g}$$
$$C_{g,t} - \lambda^{\text{en}}_t + \overline{\mu}_{g,t} - \underline{\mu}_{g,t} + \mu^{\text{up}}_{g,t} - \mu^{\text{dn}}_{g,t} = 0 : p_{g,t}, \quad \forall g \in [G], \forall t \in [T], \tag{5h}$$
$$C^{\text{rs}}_{g,t} - \lambda^{\text{up}}_t + \overline{\mu}^{\text{up}}_{g,t} - \underline{\mu}^{\text{up}}_{g,t} + \mu^{\text{up}}_{g,t} = 0 : r^{\text{up}}_{g,t}, \quad \forall g \in [G], \forall t \in [T], \tag{5i}$$
$$C^{\text{rs}}_{g,t} - \lambda^{\text{dn}}_t + \overline{\mu}^{\text{dn}}_{g,t} - \underline{\mu}^{\text{dn}}_{g,t} + \mu^{\text{dn}}_{g,t} = 0 : r^{\text{dn}}_{g,t}, \quad \forall g \in [G], \forall t \in [T], \tag{5j}$$
$$\mu^{\text{w,sch}}_{j,t} - \lambda^{\text{en}}_t = 0 : w^{\text{sch}}_{j,t}, \quad \forall j \in [J], \forall t \in [T], \tag{5k}$$
$$C^{\text{cur}}_{j,t} + \mu^{\text{w,sch}}_{j,t} + \overline{\mu}^{\text{w,cur}}_{j,t} - \underline{\mu}^{\text{w,cur}}_{j,t} = 0 : w^{\text{cur}}_{j,t}, \quad \forall j \in [J], \forall t \in [T], \tag{5l}$$
$$\lambda^{\text{up}}_t - \sum_{i \in [N]^{\text{up}}_t} \kappa_i \mu^2_{i,t} - \mu^4_t = 0 : r^{\text{wm,up}}_t, \quad \forall t \in [T], \tag{5m}$$
$$\lambda^{\text{dn}}_t - \sum_{i \in [N]^{\text{dn}}_t} \kappa_i \mu^3_{i,t} - \mu^5_t = 0 : r^{\text{wm,dn}}_t, \quad \forall t \in [T], \tag{5n}$$
$$-\underline{\mu}^{v}_i + \mu^1 - \sum_{t \in [T]: i \in [N]^{\text{up}}_t} \mu^2_{i,t} - \sum_{t \in [T]: i \in [N]^{\text{dn}}_t} \mu^3_{i,t} = 0 : v_i, \quad \forall i \in [N], \tag{5o}$$
$$-\underline{\mu}^{u} - \epsilon N \mu^1 + \sum_{t \in [T]} \left(\sum_{i \in [N]^{\text{up}}_t} \mu^2_{i,t} + \sum_{i \in [N]^{\text{dn}}_t} \mu^3_{i,t} + \mu^4_t + \mu^5_t \right) = 0 : u, \tag{5p}$$

$$0 \le (p_{g,t} - \underline{P}_g) \perp \underline{\mu}_{g,t} \ge 0, \quad \forall g \in [G], \forall t \in [T], \tag{6a}$$
$$0 \le (\overline{P}_g - p_{g,t}) \perp \bar{\mu}_{g,t} \ge 0, \quad \forall g \in [G], \forall t \in [T], \tag{6b}$$
$$0 \le p^{\text{ch}}_{s,t} \perp \underline{\mu}^{\text{ch}}_{s,t} \ge 0, \quad \forall s \in [S], \forall t \in [T], \tag{6c}$$
$$0 \le (\hat{p}^{\text{ch}}_s - p^{\text{ch}}_{s,t}) \perp \bar{\mu}^{\text{ch}}_{s,t} \ge 0, \quad \forall s \in [S], \forall t \in [T], \tag{6d}$$
$$0 \le p^{\text{dis}}_{s,t} \perp \underline{\mu}^{\text{dis}}_{s,t} \ge 0, \quad \forall s \in [S], \forall t \in [T], \tag{6e}$$
$$0 \le (\hat{p}^{\text{dis}}_s - p^{\text{dis}}_{s,t}) \perp \bar{\mu}^{\text{dis}}_{s,t} \ge 0, \quad \forall s \in [S], \forall t \in [T], \tag{6f}$$

$$0 \leq r_{s,t}^{\text{ch,up}} \perp \underline{\mu}_{s,t}^{\text{ch,up}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6g}$$

$$0 \leq (\hat{r}_s^{\text{ch,up}} - r_{s,t}^{\text{ch,up}}) \perp \bar{\mu}_{s,t}^{\text{ch,up}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6h}$$

$$0 \leq r_{s,t}^{\text{ch,dn}} \perp \underline{\mu}_{s,t}^{\text{ch,dn}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6i}$$

$$0 \leq (\hat{r}_s^{\text{ch,dn}} - r_{s,t}^{\text{ch,dn}}) \perp \bar{\mu}_{s,t}^{\text{ch,dn}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6j}$$

$$0 \leq r_{s,t}^{\text{dis,up}} \perp \underline{\mu}_{s,t}^{\text{dis,up}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6k}$$

$$0 \leq (\hat{r}_s^{\text{dis,up}} - r_{s,t}^{\text{dis,up}}) \perp \bar{\mu}_{s,t}^{\text{dis,up}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6l}$$

$$0 \leq r_{s,t}^{\text{dis,dn}} \perp \underline{\mu}_{s,t}^{\text{dis,dn}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6m}$$

$$0 \leq (\hat{r}_s^{\text{dis,dn}} - r_{s,t}^{\text{dis,dn}}) \perp \bar{\mu}_{s,t}^{\text{dis,dn}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6n}$$

$$0 \leq (p_{s,t}^{\text{ch}} - r_{s,t}^{\text{ch,up}}) \perp \mu_{s,t}^{\text{ch,up}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6o}$$

$$0 \leq (p_{s,t}^{\text{dis}} - r_{s,t}^{\text{dis,dn}}) \perp \mu_{s,t}^{\text{dis,dn}} \geq 0, \qquad \forall s \in [S], \forall t \in [T], \tag{6p}$$

$$0 \leq r_{g,t}^{\text{up}} \perp \underline{\mu}_{g,t}^{\text{up}} \geq 0, \qquad \forall g \in [G], \forall t \in [T], \tag{6q}$$

$$0 \leq (\overline{R}_g^{\text{up}} - r_{g,t}^{\text{up}}) \perp \bar{\mu}_{g,t}^{\text{up}} \geq 0, \qquad \forall g \in [G], \forall t \in [T], \tag{6r}$$

$$0 \leq r_{g,t}^{\text{dn}} \perp \underline{\mu}_{g,t}^{\text{dn}} \geq 0, \qquad \forall g \in [G], \forall t \in [T], \tag{6s}$$

$$0 \leq (\overline{R}_g^{\text{dn}} - r_{g,t}^{\text{dn}}) \perp \bar{\mu}_{g,t}^{\text{dn}} \geq 0, \qquad \forall g \in [G], \forall t \in [T], \tag{6t}$$

$$0 \leq (\overline{P}_g - p_{g,t} - r_{g,t}^{\text{up}}) \perp \mu_{g,t}^{\text{up}} \geq 0, \qquad \forall g \in [G], \forall t \in [T], \tag{6u}$$

$$0 \leq (p_{g,t} - r_{g,t}^{\text{dn}} - \underline{P}_g) \perp \mu_{g,t}^{\text{dn}} \geq 0, \qquad \forall g \in [G], \forall t \in [T], \tag{6v}$$

$$0 \leq w_{j,t}^{\text{cur}} \perp \underline{\mu}_{j,t}^{\text{w,cur}} \geq 0, \qquad \forall j \in [J], \forall t \in [T], \tag{6w}$$

$$0 \leq (w_{j,t}^{\text{fore}} - w_{j,t}^{\text{cur}}) \perp \overline{\mu}_{j,t}^{\text{w,cur}} \geq 0, \qquad \forall j \in [J], \forall t \in [T], \tag{6x}$$

$$0 \leq \left(\sum_{s \in [S]} \left(r_{s,t}^{\text{ch,up}} + r_{s,t}^{\text{dis,up}} \right) + \sum_{g \in [G]} r_{g,t}^{\text{up}} - r_t^{\text{wm,up}} - R_t^{\text{up,ex}} \right) \perp \lambda_t^{\text{up}} \geq 0, \qquad \forall t \in [T], \tag{6y}$$

$$0 \leq \left(\sum_{s \in [S]} \left(r_{s,t}^{\text{ch,dn}} + r_{s,t}^{\text{dis,dn}} \right) + \sum_{g \in [G]} r_{g,t}^{\text{dn}} - r_t^{\text{wm,dn}} - R_t^{\text{dn,ex}} \right) \perp \lambda_t^{\text{dn}} \geq 0, \qquad \forall t \in [T], \tag{6z}$$

$$0 \leq u \perp \underline{\mu}^{\text{u}} \geq 0, \tag{6aa}$$

$$0 \leq v_i \perp \underline{\mu}_i^{\text{v}} \geq 0, \qquad \forall i \in [N], \tag{6ab}$$

$$0 \leq (-\theta N + \epsilon N u - \sum_{i \in [N]} v_i) \perp \mu^1 \geq 0, \tag{6ac}$$

$$0 \leq (\kappa_i \sum_{j \in [J]} \tilde{e}_{j,t,i} - (u - v_i - \kappa_i r_t^{\text{wm,up}})) \perp \mu_{i,t}^2 \geq 0, \qquad \forall i \in [N]_t^{\text{up}}, \forall t \in [T], \tag{6ad}$$

$$0 \leq (-\kappa_i \sum_{j \in [J]} \tilde{e}_{j,t,i} - (u - v_i - \kappa_i r_t^{\text{wm,dn}})) \perp \mu_{i,t}^3 \geq 0, \qquad \forall i \in [N]_t^{\text{dn}}, \forall t \in [T], \tag{6ae}$$

$$0 \leq (Q_t^{\text{wm,up}} - (u - r_t^{\text{wm,up}})) \perp \mu_t^4 \geq 0, \qquad \forall t \in [T], \tag{6af}$$

$$0 \leq (Q_t^{\text{wm,dn}} - (u - r_t^{\text{wm,dn}})) \perp \mu_t^5 \geq 0, \qquad \forall t \in [T]. \tag{6ag}$$

The final formulation results in a single-level problem where all KKT conditions are integrated into upper-level problem (2) as constraints, ensuring that the solution adheres to the optimality conditions of the lower-level problem. These KKT constraints (product terms) are further converted to a set of mixed-integer linear constraints with big-M. It is important to note that the upper-level objective function (2a) contains a non-convex bilinear term due to the product of price and quantity. This problem can be handled by the mixed-integer nonlinear programming feature in `Gurobi`.

B.4. Benchmark Formulation

This section presents the reformulation of the bilevel problem with three different linear approximation schemes (i.e., LA, W-CVaR, Bonferroni) for the RHS-WDRJCC (3r).

B.4.1. First Benchmark: LA Under the LA approach detailed in the main manuscript, the RHS-WDRJCC (3r) can be reformulated as:

$$-u \le 0, -v_i \le 0 : \underline{\mu}^{\mathrm{u}}, \underline{\mu}_i^{\mathrm{v}}, \qquad \forall i \in [N], \tag{7a}$$

$$-(\epsilon N u - \sum_{i\in[N]} v_i) \le -\theta N : \mu^1, \tag{7b}$$

$$u - v_i - \kappa_i r_t^{\mathrm{wm,up}} \le \kappa_i \sum_{j\in[J]} \tilde{e}_{j,t,i} : \mu_{i,t}^2, \qquad \forall i \in [N], \forall t \in [T], \tag{7c}$$

$$u - v_i - \kappa_i r_t^{\mathrm{wm,dn}} \le -\kappa_i \sum_{j\in[J]} \tilde{e}_{j,t,i} : \mu_{i,t}^3, \qquad \forall i \in [N], \forall t \in [T]. \tag{7d}$$

Note that the Appendix renames the auxiliary variables s and r_i used in the main content to u and v_i, respectively. This change prevents confusion with the storage index s and the reserve variable in the bilevel problem, which aligns with existing literature in storage strategic bidding to improve readability.

Accordingly, we replace constraints (5m), (5n), (5o), (5p), (6ad), (6ae), (6af), (6ag) with the following KKT conditions:

$$\lambda_t^{\mathrm{up}} - \sum_{i\in[N]} \kappa_i \mu_{i,t}^2 = 0 : r_t^{\mathrm{wm,up}}, \qquad \forall t \in [T], \tag{8a}$$

$$\lambda_t^{\mathrm{dn}} - \sum_{i\in[N]} \kappa_i \mu_{i,t}^3 = 0 : r_t^{\mathrm{wm,dn}}, \qquad \forall t \in [T], \tag{8b}$$

$$-\underline{\mu}_i^v + \mu^1 - \sum_{t\in[T]} \mu_{i,t}^2 - \sum_{t\in[T]} \mu_{i,t}^3 = 0 : v_i, \qquad \forall i \in [N], \tag{8c}$$

$$-\underline{\mu}^u - \epsilon N \mu^1 + \sum_{t\in[T]} \left(\sum_{i\in[N]} \mu_{i,t}^2 + \sum_{i\in[N]} \mu_{i,t}^3 \right) = 0 : u, \tag{8d}$$

$$0 \le (\kappa_i \sum_{j\in[J]} \tilde{e}_{j,t,i} - (u - v_i - \kappa_i r_t^{\mathrm{wm,up}})) \perp \mu_{i,t}^2 \ge 0, \qquad \forall i \in [N], \forall t \in [T], \tag{8e}$$

$$0 \le (-\kappa_i \sum_{j\in[J]} \tilde{e}_{j,t,i} - (u - v_i - \kappa_i r_t^{\mathrm{wm,dn}})) \perp \mu_{i,t}^3 \ge 0, \qquad \forall i \in [N], \forall t \in [T]. \tag{8f}$$

which then leads to the corresponding single-level reformulation as in Appendix B.3.

B.4.2. Second Benchmark: W-CVaR Under the W-CVaR approximation approach detailed in the main manuscript, the RHS-WDRJCC (3r) can be reformulated as:

$$-\alpha_i \le 0, \beta \in \mathbb{R}, \tau \in \mathbb{R} : \underline{\mu}_i^{\alpha}, \qquad \forall i \in [N], \tag{9a}$$

$$\tau + \frac{1}{\epsilon}(\theta\beta + \frac{1}{N}\sum_{i\in[N]} \alpha_i) \le 0 : \mu^1, \tag{9b}$$

$$-\tau - \alpha_i - w_t^{\mathrm{up}} r_t^{\mathrm{wm,up}} \le w_t^{\mathrm{up}} \sum_{j\in[J]} \tilde{e}_{j,t,i} : \mu_{i,t}^2, \qquad \forall i \in [N], \forall t \in [T], \tag{9c}$$

$$-\tau-\alpha_i-w_t^{\mathrm{dn}}r_t^{\mathrm{wm,dn}}\le -w_t^{\mathrm{dn}}\sum_{j\in[J]}\tilde{e}_{j,t,i}:\mu_{i,t}^3,\qquad \forall i\in[N],\forall t\in[T], \tag{9d}$$

$$w_t^{\mathrm{up}}-\beta\le 0:\mu_t^4,\qquad \forall t\in[T], \tag{9e}$$

$$w_t^{\mathrm{dn}}-\beta\le 0:\mu_t^5,\qquad \forall t\in[T]. \tag{9f}$$

Accordingly, we replace constraints (5m), (5n), (5o), (5p), (6aa), (6ab), (6ac), (6ad), (6ae), (6af), (6ag) with the following KKT conditions:

$$\lambda_t^{\mathrm{up}}-\sum_{i\in[N]}w_t^{\mathrm{up}}\mu_{i,t}^2=0:r_t^{\mathrm{wm,up}},\qquad \forall t\in[T], \tag{10a}$$

$$\lambda_t^{\mathrm{dn}}-\sum_{i\in[N]}w_t^{\mathrm{dn}}\mu_{i,t}^3=0:r_t^{\mathrm{wm,dn}},\qquad \forall t\in[T], \tag{10b}$$

$$-\underline{\mu}_i^{\alpha}+\frac{1}{\epsilon N}\mu^1-\sum_{t\in[T]}\mu_{i,t}^2-\sum_{t\in[T]}\mu_{i,t}^3=0:\alpha_i,\qquad \forall i\in[N], \tag{10c}$$

$$\frac{\theta}{\epsilon}\mu^1-\sum_{t\in[T]}\left(\mu_t^4+\mu_t^5\right)=0:\beta, \tag{10d}$$

$$\mu^1-\sum_{t\in[T]}\sum_{i\in[N]}\left(\mu_{i,t}^2+\mu_{i,t}^3\right)=0:\tau, \tag{10e}$$

$$0\le\alpha_i\perp\underline{\mu}_i^{\alpha}\ge 0,\qquad \forall i\in[N], \tag{10f}$$

$$0\le-(\tau+\frac{1}{\epsilon}(\theta\beta+\frac{1}{N}\sum_{i\in[N]}\alpha_i))\perp\mu^1\ge 0, \tag{10g}$$

$$0\le(w_t^{\mathrm{up}}\sum_{j\in[J]}\tilde{e}_{j,t,i}+\tau+\alpha_i+w_t^{\mathrm{up}}r_t^{\mathrm{wm,up}})\perp\mu_{i,t}^2\ge 0,\qquad \forall i\in[N],\forall t\in[T], \tag{10h}$$

$$0\le(-w_t^{\mathrm{dn}}\sum_{j\in[J]}\tilde{e}_{j,t,i}+\tau+\alpha_i+w_t^{\mathrm{dn}}r_t^{\mathrm{wm,dn}})\perp\mu_{i,t}^3\ge 0,\qquad \forall i\in[N],\forall t\in[T], \tag{10i}$$

$$0\le(\beta-w_t^{\mathrm{up}})\perp\mu_t^4\ge 0,\qquad \forall t\in[T], \tag{10j}$$

$$0\le(\beta-w_t^{\mathrm{dn}})\perp\mu_t^5\ge 0,\qquad \forall t\in[T]. \tag{10k}$$

which then leads to the corresponding single-level reformulation as in Appendix B.3.

B.4.3. Third Benchmark: Bonferroni Approximation Under the Bonferroni approximation approach detailed in the main manuscript, it involves first computing the worst-case VaR for each individual chance constraint, namely η_t^{up} and η_t^{dn}, as follows:

$$\min_{\alpha^{\mathrm{up}},\beta^{\mathrm{up}},m^{\mathrm{up}},\eta^{\mathrm{up}}}\quad \eta_t^{\mathrm{up}} \tag{11a}$$

$$\text{s.t.}\quad \theta\beta^{\mathrm{up}}+\frac{1}{N}\sum_{i\in[N]}\alpha_i^{\mathrm{up}}\le\frac{\epsilon}{2T}, \tag{11b}$$

$$\alpha_i^{\mathrm{up}}\ge 1-m_i^{\mathrm{up}}\left(\eta_t^{\mathrm{up}}+\sum_{j\in[J]}\tilde{e}_{j,t,i}\right),\qquad \forall i\in[N], \tag{11c}$$

$$\beta^{\mathrm{up}}\ge m_i^{\mathrm{up}},\qquad \forall i\in[N], \tag{11d}$$

$$\alpha_i^{\mathrm{up}}\ge 0,\quad m_i^{\mathrm{up}}\ge 0,\qquad \forall i\in[N], \tag{11e}$$

$$\min_{\alpha^{\mathrm{dn}},\beta^{\mathrm{dn}},m^{\mathrm{dn}},\eta^{\mathrm{dn}}}\quad \eta_t^{\mathrm{dn}} \tag{12a}$$

$$\text{s.t.} \quad \theta\beta^{\text{dn}} + \frac{1}{N}\sum_{i\in[N]} \alpha_i^{\text{dn}} \le \frac{\epsilon}{2T}, \tag{12b}$$

$$\alpha_i^{\text{dn}} \ge 1 - m_i^{\text{dn}}\left(\eta_t^{\text{dn}} - \sum_{j\in[J]} \tilde{e}_{j,t,i}\right), \quad \forall i \in [N], \tag{12c}$$

$$\beta^{\text{dn}} \ge m_i^{\text{dn}}, \quad \forall i \in [N], \tag{12d}$$

$$\alpha_i^{\text{dn}} \ge 0, \quad m_i^{\text{dn}} \ge 0, \quad \forall i \in [N], \tag{12e}$$

where we use superscript η^{up} and η^{dn} to distinguish chance constraints for the upward reserve $r_t^{\text{wm,up}}$ and downward reserve $r_t^{\text{wm,dn}}$ to facilitate readability of the following formulation.

We denote the optimal solutions of Problems (11) and (12) are $(\eta_t^{\text{up}*}, \eta_t^{\text{dn}*})$, respectively. Then the RHS-WDRJCC (3r) under the Bonferroni approximation approach becomes

$$-r_t^{\text{wm,up}} \le -\eta_t^{\text{up}*} : \mu_t^1, \quad \forall t \in [T],$$

$$-r_t^{\text{wm,dn}} \le -\eta_t^{\text{dn}*} : \mu_t^2, \quad \forall t \in [T].$$

Accordingly, we replace constraints (5m), (5n), (5o), (5p), (6aa), (6ab), (6ac), (6ad), (6ae), (6af), (6ag) with the following KKT conditions:

$$\lambda_t^{\text{up}} - \mu_t^1 = 0 : r_t^{\text{wm,up}}, \quad \forall t \in [T], \tag{13a}$$

$$\lambda_t^{\text{dn}} - \mu_t^2 = 0 : r_t^{\text{wm,dn}}, \quad \forall t \in [T], \tag{13b}$$

$$0 \le (r_t^{\text{wm,up}} - \eta_t^{\text{up}*}) \perp \mu_t^1 \ge 0, \quad \forall t \in [T], \tag{13c}$$

$$0 \le (r_t^{\text{wm,up}} - \eta_t^{\text{dn}*}) \perp \mu_t^2 \ge 0, \quad \forall t \in [T]. \tag{13d}$$

which then leads to the corresponding single-level reformulation as discussed in Appendix B.3.

References

Nasrolahpour E, Kazempour J, Zareipour H, Rosehart WD (2018) A bilevel model for participation of a storage system in energy and reserve markets. *IEEE Transactions on Sustainable Energy* 9(2):582–598.

Yang Y, Wu W, Wang B, Li M (2020) Analytical reformulation for stochastic unit commitment considering wind power uncertainty with gaussian mixture model. *IEEE Transactions on Power Systems* 35(4):2769–2782.