

EUROCODE 2

WORKED EXAMPLES

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Rev A 31-03-2017: The form of this document has been revised to offer an easy navigation through the publication to the reader:

- added active bookmarks offer a full navigation pane in ADOBE ACROBAT.
- active hyperlinks in the table of contents have been added
- Better resolution for the figures
- Minor form revisions like unresolved symbols, numbering, spelling, alignment, tabs, borders, footer...

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This publication is based on the publication: "Guida all'uso dell'eurocodice 2" prepared by AICAP; the Italian Association for Reinforced and Prestressed Concrete, on behalf of the Italian Cement Organization AITEC, and on background documents prepared by the Eurocode 2 Project Teams Members, during the preparation of the EN version of Eurocode 2 (prof A.W. Beeby, prof H. Corres Peiretti, prof J. Walraven, prof B. Westerberg, prof R.V. Whitman).

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FOREWORD

The introduction of Eurocodes is a challenge and opportunity for the European cement and concrete industry. These design codes, considered to be the most advanced in the world, will lead to a common understanding of the design principles for concrete structures for owners, operators and users, design engineers, contractors and the manufacturers of concrete products. The advantages of unified codes include the preparation of common design aids and software and the establishment of a common understanding of research and development needs in Europe.

As with any new design code, it is important to have an understanding of the principles and background, as well as design aids to assist in the design process. The European cement and concrete industry represented by CEMBUREAU, BIBM and ERMCO recognised this need and set up a task group to prepare two documents, Commentary to EN 1992 and Worked Examples to EN 1992. The Commentary to EN 1992 captures the background to the code and Worked Examples to EN 1992 demonstrates the practical application of the code. Both the documents were prepared by a team led by Professor Giuseppe Mancini, Chairman of CEN TC 250/SC2 Concrete Structures, and peer reviewed by three eminent engineers who played a leading role in the development of the concrete Eurocode: Professor Narayanan, Professor Spehl and Professor Walraven.

This is an excellent example of pan-European collaboration and CEMBUREAU, BIBM and ERMCO are delighted to make these authoritative documents available to design engineers, software developers and all others with an interest in promoting excellence in concrete design throughout Europe. As chairman of the Task Group, I would like to thank the authors, peer reviewers and members of the joint Task Force for working efficiently and effectively in producing these documents.

Dr Pal Chana
Chairman, CEMBUREAU/BIBM/ERMCO TF 5.5 "Eurocodes"

Attributable Foreword to the Commentary and Worked Examples to EC2

Eurocodes are one of the most advanced suite of structural codes in the world. They embody the collective experience and knowledge of whole of Europe. They are born out of an ambitious programme initiated by the European Union. With a wealth of code writing experience in Europe, it was possible to approach the task in a rational and logical manner. Eurocodes reflect the results of research in material technology and structural behaviour in the last fifty years and they incorporate all modern trends in structural design.

Like many current national codes in Europe, Eurocode 2 (EC 2) for concrete structures draws heavily on the CEB Model Code. And yet the presentation and terminology, conditioned by the agreed format for Eurocodes, might obscure the similarities to many national codes. Also EC 2 in common with other Eurocodes, tends to be general in character and this might present difficulty to some designers at least initially. The problems of coming to terms with a new set of codes by busy practising engineers cannot be underestimated. This is the backdrop to the publication of ‘Commentary and Worked Examples to EC 2’ by Professor Mancini and his colleagues. Commissioned by CEMBUREAU, BIBM, EFCA and ERMCO this publication should prove immensely valuable to designers in discovering the background to many of the code requirements. This publication will assist in building confidence in the new code, which offers tools for the design of economic and innovative concrete structures. The publication brings together many of the documents produced by the Project Team during the development of the code. The document is rich in theoretical explanations and draws on much recent research. Comparisons with the ENV stage of EC2 are also provided in a number of cases. The chapter on EN 1990 (Basis of structural design) is an added bonus and will be appreciated by practioners. Worked examples further illustrate the application of the code and should promote understanding.

The commentary will prove an authentic companion to EC 2 and deserves every success.

Professor R S Narayanan
Chairman CEN/TC 250/SC2 (2002 – 2005)

Foreword to Commentary to Eurocode 2 and Worked Examples

When a new code is made, or an existing code is updated, a number of principles should be regarded:

1. Codes should be based on clear and scientifically well founded theories, consistent and coherent, corresponding to a good representation of the structural behaviour and of the material physics.
2. Codes should be transparent. That means that the writers should be aware, that the code is not prepared for those who make it, but for those who will use it.
3. New developments should be recognized as much as possible, but not at the cost of too complex theoretical formulations.
4. A code should be open-minded, which means that it cannot be based on one certain theory, excluding others. Models with different degrees of complexity may be offered.
5. A code should be simple enough to be handled by practicing engineers without considerable problems. On the other hand simplicity should not lead to significant lack of accuracy. Here the word “accuracy” should be well understood. Often so-called “accurate” formulations, derived by scientists, cannot lead to very accurate results, because the input values cannot be estimated with accuracy.
6. A code may have different levels of sophistication. For instance simple, practical rules can be given, leading to conservative and robust designs. As an alternative more detailed design rules may be offered, consuming more calculation time, but resulting in more accurate and economic results.

For writing a Eurocode, like EC-2, another important condition applies. International consensus had to be reached, but not on the cost of significant concessions with regard to quality. A lot of effort was invested to achieve all those goals.

It is a rule for every project, that it should not be considered as finalized if implementation has not been taken care of. This book may, further to courses and trainings on a national and international level, serve as an essential and valuable contribution to this implementation. It contains extensive background information on the recommendations and rules found in EC2. It is important that this background information is well documented and practically available, as such increasing the transparency. I would like to thank my colleagues of the Project Team, especially Robin Whittle, Bo Westerberg, Hugo Corres and Konrad Zilch, for helping in getting together all background information. Also my colleague Giuseppe Mancini and his Italian team are gratefully acknowledged for providing a set of very illustrative and practical working examples. Finally I would like to thank CEMBURAU, BIBM, EFCA and ERMCO for their initiative, support and advice to bring out this publication.

Joost Walraven
Convenor of Project Team for EC2 (1998 -2002)

NAVIGATION KEY

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SECTION 2. BASIS OF DESIGN - WORKED EXAMPLES

EXAMPLE 2.1. ULS combinations of actions for a continuous beam [EC2 – clause 2.4]

A continuous beam on four bearings is subjected to the following loads:

Self-weight	G_{k1}
Permanent imposed load	G_{k2}
Service imposed load	Q_{k1}

Note. In this example and in the following ones, a single characteristic value is taken for self-weight and permanent imposed load, respectively G_{k1} and G_{k2} , because of their small variability.

EQU – Static equilibrium (Set A)

Factors of Set A should be used in the verification of holding down devices for the uplift of bearings at end span, as indicated in Fig. 2.1.

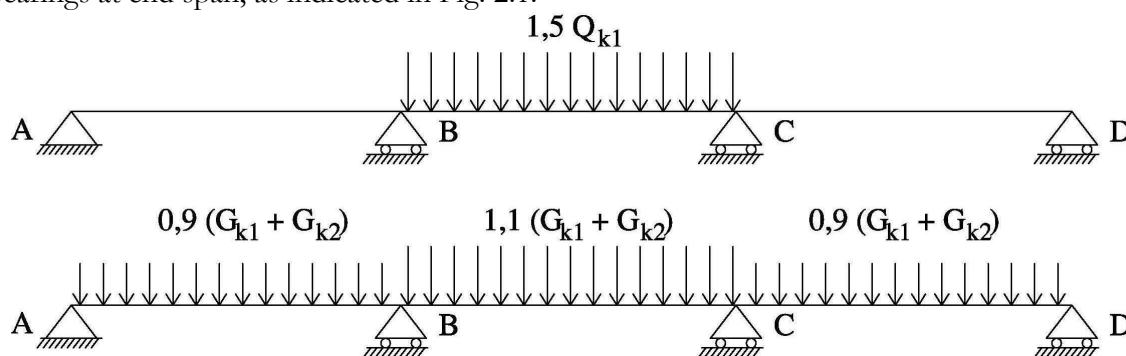


Fig. 2.1. Load combination for verification of holding down devices at the end bearings.

STR – Bending moment verification at mid span (Set B)

Unlike in the verification of static equilibrium, the partial safety factor for permanent loads in the verification of bending moment in the middle of the central span, is the same for all spans: $\gamma_G = 1.35$ (Fig. 2.2).

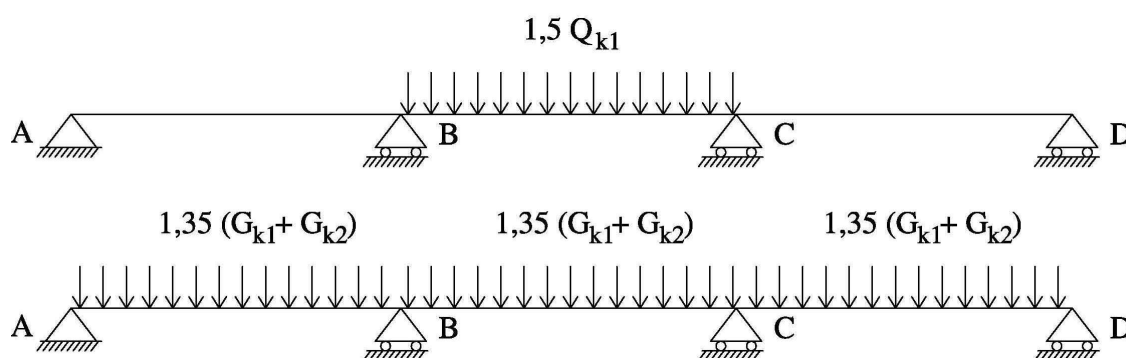


Fig. 2.2. Load combination for verification of bending moment in the BC span.

EXAMPLE 2.2 ULS combinations of actions for a canopy [EC2 – clause 2.4]

The canopy is subjected to the following loads:

Self-weight	G_{k1}
Permanent imposed load	G_{k2}
Snow imposed load	Q_{k1}

EQU – Static equilibrium (Set A)

Factors to be taken for the verification of overturning are those of Set A, as in Fig. 2.3.

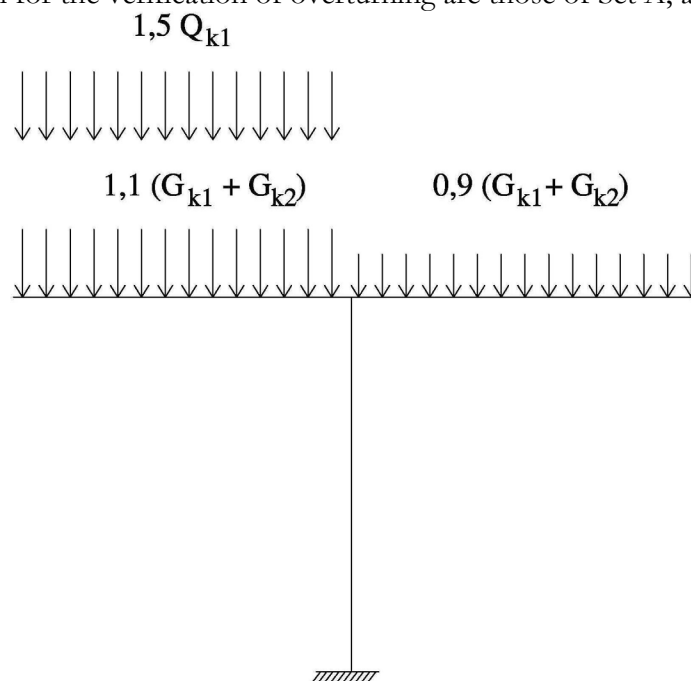


Fig. 2.3. Load combination for verification of static equilibrium.

STR – Verification of resistance of a column (Set B)

The partial factor to be taken for permanent loads in the verification of maximum compression stresses and of bending with axial force in the column is the same ($\gamma_G = 1.35$) for all spans.

The variable imposed load is distributed over the full length of the canopy in the first case, and only on half of it for the verification of bending with axial force.

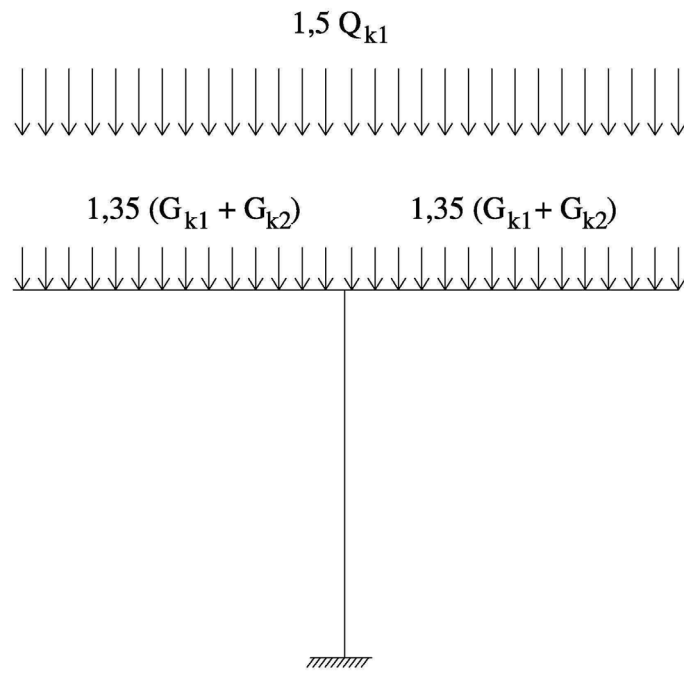


Fig. 2.4. Load combination for the compression stresses verification of the column.

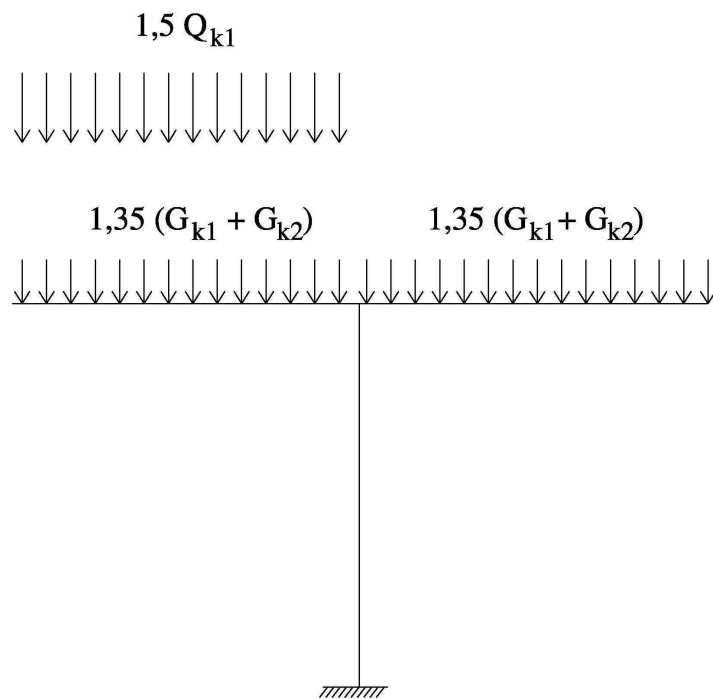


Fig. 2.5. Load combination for the verification of bending with axial force of the column.

EXAMPLE 2.3 ULS combination of action - residential concrete framed building [EC2 – clause 2.4]

The permanent imposed load is indicated as G_k . Variable actions are listed in table 2.1.

Table 2.1. Variable actions on a residential concrete building.

	Variable actions		
	serviceability imposed load	snow on roofing (for sites under 1000 m a.s.l.)	wind
Characteristic value Q_k	$Q_{k,es}$	$Q_{k,n}$	$F_{k,w}$
Combination value $\psi_0 Q_k$	$0.7 Q_{k,es}$	$0.5 Q_{k,n}$	$0.6 F_{k,w}$

N.B. The values of partial factors are those recommended by EN1990, but they may be defined in the National Annex.

Basic combinations for the verification of the superstructure - STR (Set B) (eq. 6.10-EN1990)

Predominant action: wind

favourable vertical loads (fig. 2.6, a)

$$1.0 \cdot G_k + 1.5 \cdot F_{k,w}$$

unfavourable vertical loads (fig. 2.6, b)

$$1.35 \cdot G_k + 1.5 \cdot (F_{k,w} + 0.5 \cdot Q_{k,n} + 0.7 \cdot Q_{k,es}) = 1.35 \cdot G_k + 1.5 \cdot F_{k,w} + 0.75 \cdot Q_{k,n} + 1.05 \cdot Q_{k,es}$$

Predominant action: snow (fig. 2.6, c)

$$1.35 \cdot G_k + 1.5 \cdot (Q_{k,n} + 0.7 \cdot Q_{k,es} + 0.6 \cdot F_{k,w}) = 1.35 \cdot G_k + 1.5 \cdot Q_{k,n} + 1.05 \cdot Q_{k,es} + 0.9 \cdot F_{k,w}$$

Predominant action: service load (fig. 2.6, d)

$$1.35 \cdot G_k + 1.5 \cdot (Q_{k,es} + 0.5 \cdot Q_{k,n} + 0.6 \cdot F_{k,w}) = 1.35 \cdot G_k + 1.5 \cdot Q_{k,es} + 0.75 \cdot Q_{k,n} + 0.9 \cdot F_{k,w}$$

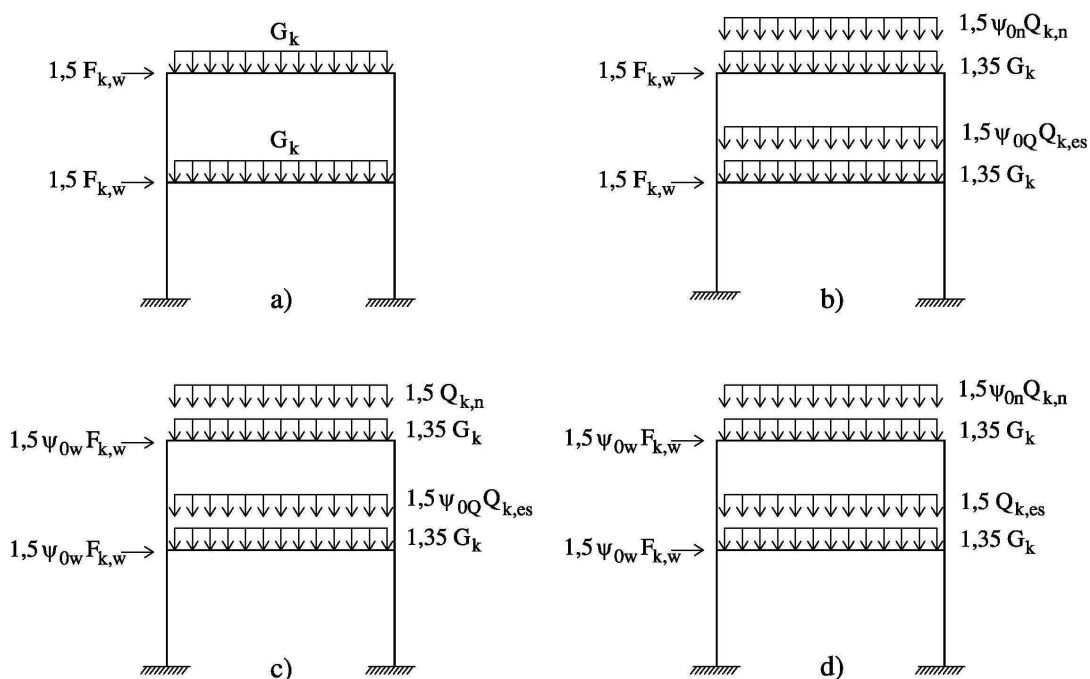


Fig. 2.6. Basic combinations for the verification of the superstructure (Set B): a) Wind predominant, favourable vertical loads; b) Wind predominant, unfavourable vertical loads; c) Snow load predominant; d) service load predominant.

Basic combinations for the verification of foundations and ground resistance – STR/GEO
[eq. 6.10-EN1990]

EN1990 allows for three different approaches; the approach to be used is chosen in the National Annex. For completeness and in order to clarify what is indicated in Tables 2.15 and 2.16, the basic combinations of actions for all the three approaches provided by EN1990 are given below.

Approach 1

The design values of Set C and Set B of geotechnical actions and of all other actions from the structure, or on the structure, are applied in separate calculations. Heavier values are usually given by Set C for the geotechnical verifications (ground resistance verification), and by Set B for the verification of the concrete structural elements of the foundation.

Set C (geotechnical verifications)

Predominant action: wind (favourable vertical loads) (fig. 2.7, a)

$$1.0 \cdot G_k + 1.3 \cdot F_{k,w}$$

Predominant action: wind (unfavourable vertical loads) (fig. 2.7, b)

$$1.0 \cdot G_k + 1.3 \cdot F_{k,w} + 1.3 \cdot 0.5 \cdot Q_{k,n} + 1.3 \cdot 0.7 \cdot Q_{k,es} = 1.0 \cdot G_k + 1.3 \cdot F_{k,w} + 0.65 \cdot Q_{k,n} + 0.91 \cdot Q_{k,es}$$

Predominant action: snow (fig. 2.7, c)

$$1.0 \cdot G_k + 1.3 \cdot Q_{k,n} + 1.3 \cdot 0.7 \cdot Q_{k,es} + 1.3 \cdot 0.6 \cdot F_{k,w} = 1.0 \cdot G_k + 1.3 \cdot Q_{k,n} + 0.91 \cdot Q_{k,es} + 0.78 \cdot F_{k,w}$$

Predominant action: service load (fig. 2.7, d)

$$1.0 \cdot G_k + 1.3 \cdot Q_{k,es} + 1.3 \cdot 0.5 \cdot Q_{k,n} + 1.3 \cdot 0.6 \cdot F_{k,w} = 1.0 \cdot G_k + 1.3 \cdot Q_{k,n} + 0.65 \cdot Q_{k,es} + 0.78 \cdot F_{k,w}$$

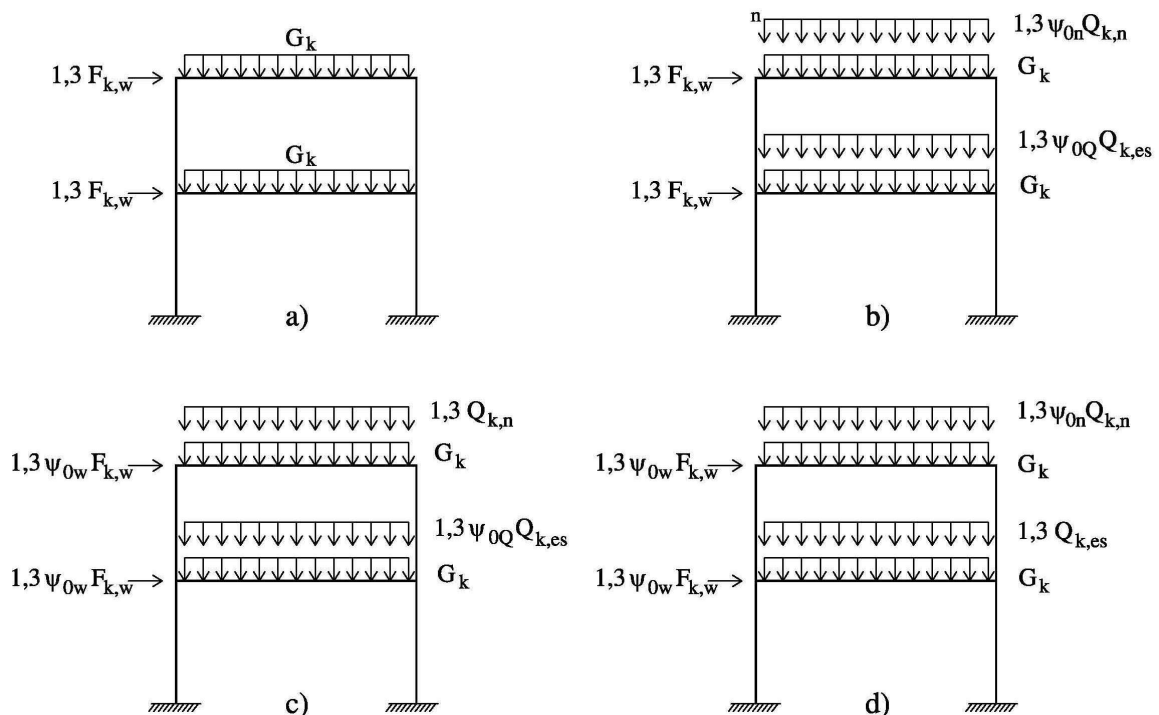


Fig. 2.7. Basic combinations for the verification of the foundations (Set C): a) Wind predominant, favourable vertical loads; b) Wind predominant, unfavourable vertical loads; c) Snow load predominant; d) service load predominant.

Set B (verification of concrete structural elements of foundations)

$$1.0 \cdot G_k + 1.5 \cdot Q_{k,w}$$

$$1.35 \cdot G_k + 1.5 \cdot F_{k,w} + 0.75 \cdot Q_{k,n} + 1.05 \cdot Q_{k,es}$$

$$1.35 \cdot G_k + 1.5 \cdot Q_{k,n} + 1.05 \cdot Q_{k,es} + 0.9 \cdot F_{k,w}$$

$$1.35 \cdot G_k + 1.5 \cdot Q_{k,es} + 0.75 \cdot Q_{k,n} + 0.9 \cdot F_{k,w}$$

Approach 2

The same combinations used for the superstructure (i.e. Set B) are used.

Approach 3

Factors from Set C for geotechnical actions and from Set B for other actions are used in one calculation. This case, as geotechnical actions are not present, can be referred to Set B, i.e. to approach 2.

EXAMPLE 2.4 ULS combinations of actions on a reinforced concrete retaining wall [EC2 – clause 2.4]

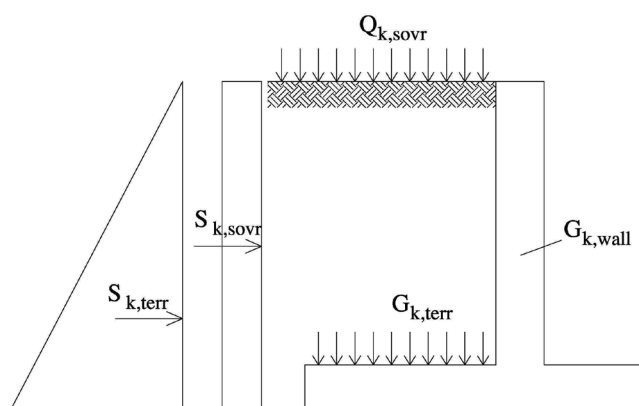


Fig. 2.8. Actions on a retaining wall in reinforced concrete

EQU - (static equilibrium of rigid body: verification of global stability to heave and sliding) (Set A)

Only that part of the embankment beyond the foundation footing is considered for the verification of global stability to heave and sliding (Fig. 2.9).

$$1.1 \cdot S_{k,terr} + 0.9 \cdot (G_{k,wall} + G_{k,terr}) + 1.5 \cdot S_{k,sovr}$$

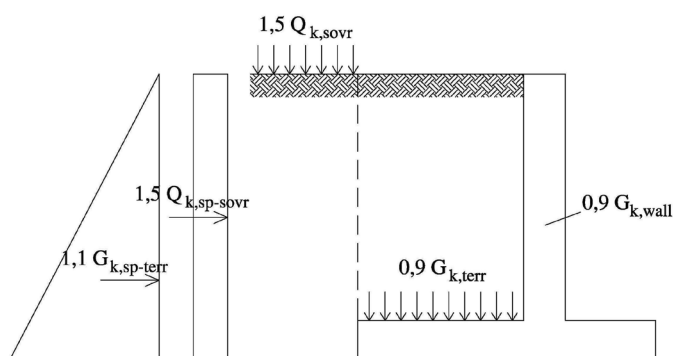


Fig. 2.9. Actions for EQU ULS verification of a retaining wall in reinforced concrete

STR/GEO - (ground pressure and verification of resistance of wall and footing)

Approach 1

Design values from Set C and from Set B are applied in separate calculations to the geotechnical actions and to all other actions from the structure or on the structure.

Set C

$$1.0 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.3 \cdot S_{k,sovr}$$

Set B

$$1.35 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

$$1.35 \cdot S_{k,terr} + 1.35 \cdot G_{k,wall} + 1.35 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

$$1.35 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.35 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

$$1.35 \cdot S_{k,terr} + 1.35 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

Note: For all the above-listed combinations, two possibilities must be considered: either that the surcharge concerns only the part of embankment beyond the foundation footing (Fig. 2.10a), or that it acts on the whole surface of the embankment (Fig. 2.10b).

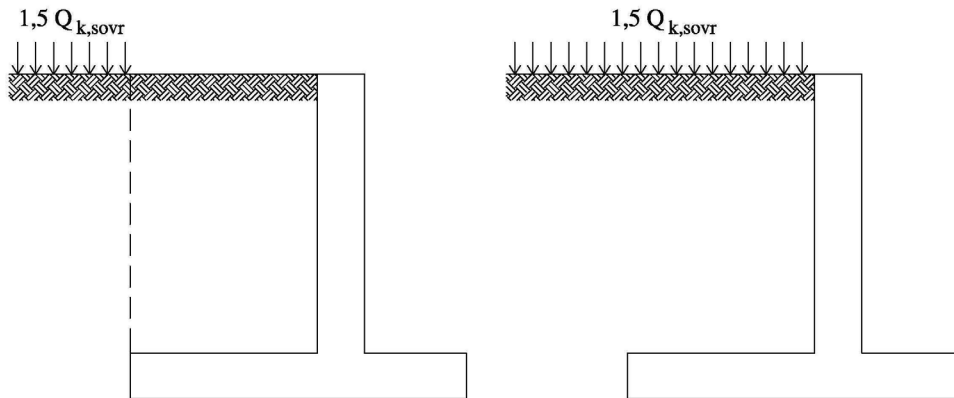


Fig. 2.10. Possible load cases of surcharge on the embankment.

For brevity, only cases in relation with case b), i.e. with surcharge acting on the whole surface of embankment, are given below.

The following figures show loads in relation to the combinations obtained with Set B partial safety factors.

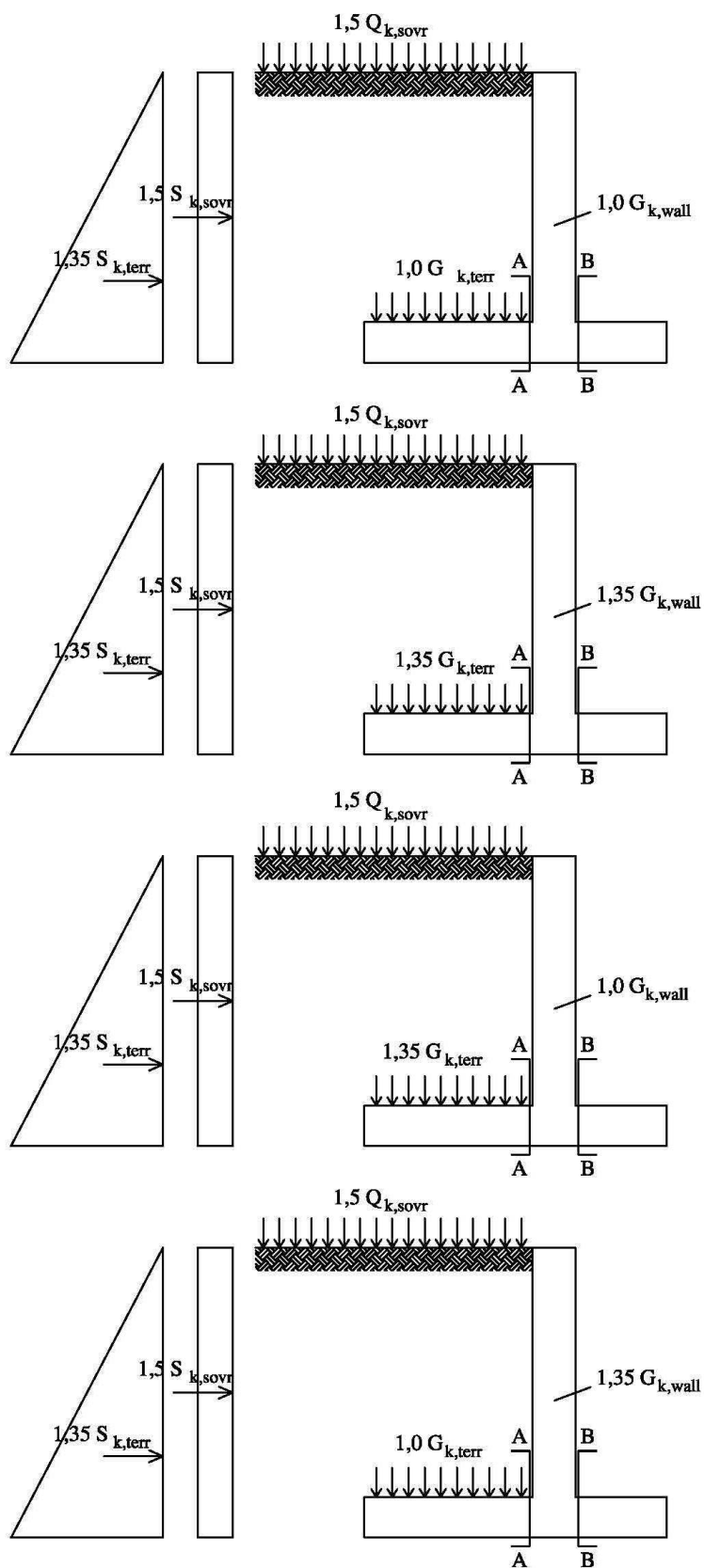


Fig. 2.11. Actions for GEO/STR ULS verification of a retaining wall in reinforced concrete.

Approach 2

Set B is used.

Approach 3

Factors from Set C for geotechnical actions and from Set B for other actions are used in one calculation.

$$1.0 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.3 \cdot Q_{k,sovr} + 1.3 \cdot S_{k,sovr}$$

$$1.0 \cdot S_{k,terr} + 1.35 \cdot G_{k,wall} + 1.35 \cdot G_{k,terr} + 1.3 \cdot Q_{k,sovr} + 1.3 \cdot S_{k,sovr}$$

$$1.0 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.35 \cdot G_{k,terr} + 1.3 \cdot Q_{k,sovr} + 1.3 \cdot S_{k,sovr}$$

$$1.0 \cdot S_{k,terr} + 1.35 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.3 \cdot Q_{k,sovr} + 1.3 \cdot S_{k,sovr}$$

A numeric example is given below.

EXAMPLE 2.5 Concrete retaining wall: global stability and ground resistance verifications [EC2 – clause 2.4]

The assumption is initially made that the surcharge acts only on the part of embankment beyond the foundation footing.

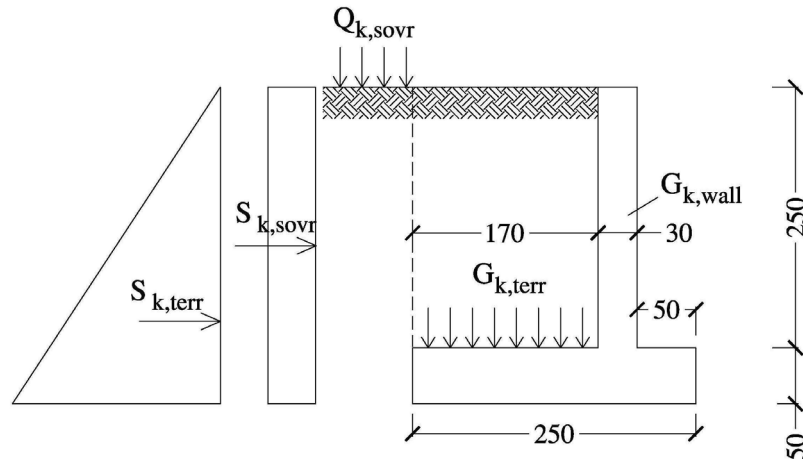


Fig. 2.12 Wall dimensions and actions on the wall (surcharge outside the foundation footing).

weight density:	$\gamma = 18 \text{ kN/m}^3$
angle of shearing resistance:	$\varphi = 30^\circ$
factor of horiz. active earth pressure:	$K_a = 0.33$
wall-ground interface friction angle:	$\delta = 0^\circ$
self-weight of wall:	$P_{k,wall} = 0.30 \cdot 2.50 \cdot 25 = 18.75 \text{ kN/m}$
self-weight of footing:	$P_{k,foot} = 0.50 \cdot 2.50 \cdot 25 = 31.25 \text{ kN/m}$
	$G_{k,wall} = P_{k,wall} + P_{k,foot} = 18.75 + 31.25 = 50 \text{ kN/m}$
self weight of ground above footing:	$G_{k,ground} = 18 \cdot 2.50 \cdot 1.70 = 76.5 \text{ kN/m}$
surcharge on embankment:	$Q_{k,surch} = 10 \text{ kN/m}^2$
ground horizontal force:	$S_{k,ground} = 26.73 \text{ kN/m}$
surcharge horizontal force:	$S_{k,surch} = 9.9 \text{ kN/m}$

Verification to failure by sliding

Slide force

Ground horizontal force ($\gamma_G = 1.1$): $S_{ground} = 1.1 \cdot 26.73 = 29.40 \text{ kN/m}$

Surcharge horizontal ($\gamma_Q = 1.5$): $S_{sur} = 1.5 \cdot 9.90 = 14.85 \text{ kN/m}$

Sliding force: $F_{slide} = 29.40 + 14.85 = 44.25 \text{ kN/m}$

Resistant force

(in the assumption of ground-flooring friction factor = 0.57)

wall self-weight ($\gamma_G = 0.9$): $F_{stab,wall} = 0.9 \cdot (0.57 \cdot 18.75) = 9.62 \text{ kN/m}$

footing self-weight ($\gamma_G = 0.9$): $F_{stab,foot} = 0.9 \cdot (0.57 \cdot 31.25) = 16.03 \text{ kN/m}$

ground self-weight ($\gamma_G = 0.9$): $F_{stab,ground} = 0.9 \cdot (0.57 \cdot 76.5) = 39.24 \text{ kN/m}$

resistant force: $F_{stab} = 9.62 + 16.03 + 39.24 = 64.89 \text{ kN/m}$

The safety factor for sliding is:

$$FS = F_{stab} / F_{rib} = 64.89 / 44.25 = 1.466$$

Verification to Overturningoverturning moment

moment from ground lateral force ($\gamma_G=1.1$): $M_{S,ground} = 1.1 \cdot (26.73 \cdot 3.00 / 3) = 29.40 \text{ kNm/m}$

moment from surcharge lateral force ($\gamma_Q=1.5$): $M_{S,surch} = 1.5 \cdot (9.90 \cdot 1.50) = 22.28 \text{ kNm/m}$

overturning moment: $M_{rib} = 29.40 + 22.28 = 51.68 \text{ kNm/m}$

stabilizing moment

moment wall self-weight ($\gamma_G=0.9$): $M_{stab,wall} = 0.9 \cdot (18.75 \cdot 0.65) = 10.97 \text{ kNm/m}$

moment footing self-weight ($\gamma_G=0.9$): $M_{stab,foot} = 0.9 \cdot (31.25 \cdot 1.25) = 35.16 \text{ kNm/m}$

moment ground self-weight ($\gamma_G=0.9$): $M_{stab,ground} = 0.9 \cdot (76.5 \cdot 1.65) = 113.60 \text{ kNm/m}$

stabilizing moment: $M_{stab} = 10.97 + 35.16 + 113.60 = 159.73 \text{ kNm/m}$

safety factor to global stability

$$FS = M_{stab} / M_{rib} = 159.73 / 51.68 = 3.09$$

Contact pressure on ground

Approach 2, i.e. Set B if partial factors, is used.

By taking 1.0 and 1.35 as the partial factors for the self-weight of the wall and of the ground above the foundation footing respectively, we obtain four different combinations as seen above:

first combination

$$1.35 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

second combination

$$1.35 \cdot S_{k,terr} + 1.35 \cdot G_{k,wall} + 1.35 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

third combination

$$1.35 \cdot S_{k,terr} + 1.0 \cdot G_{k,wall} + 1.35 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

fourth combination

$$1.35 \cdot S_{k,terr} + 1.35 \cdot G_{k,wall} + 1.0 \cdot G_{k,terr} + 1.5 \cdot Q_{k,sovr} + 1.5 \cdot S_{k,sovr}$$

the contact pressure on ground is calculated, for the first of the fourth above-mentioned combinations, as follows:

moment vs. centre of mass of the footing

moment from ground lateral force ($\gamma_G=1.35$): $M_{S,terr} = 1.35 \cdot (26.73 \cdot 3.00 / 3) = 36.08 \text{ kNm/m}$

moment from surcharge lateral force ($\gamma_Q=1.5$): $M_{S,sovr} = 1.5 \cdot (9.90 \cdot 1.50) = 22.28 \text{ kNm/m}$

moment from wall self-weight ($\gamma_G=1.0$): $M_{wall} = 1.0 \cdot (18.75 \cdot 0.60) = 11.25 \text{ kNm/m}$

moment from footing self-weight ($\gamma_G=1.0$): $M_{foot} = 0 \text{ kNm/m}$

moment from ground self-weight ($\gamma_G=1.0$): $M_{ground} = -1.0 \cdot (76.5 \cdot 0.40) = -30.6 \text{ kNm/m}$

$$\text{Total moment } M_{tot} = 36.08 + 22.28 + 11.25 - 30.6 = 39.01 \text{ kNm/m}$$

Vertical load

Wall self-weight ($\gamma_G=1.0$): $P_{\text{wall}} = 1.0 \cdot (18.75) = 18.75 \text{ kNm/m}$

Footing self-weight ($\gamma_G=1.0$): $P_{\text{foot}} = 1.0 \cdot (31.25) = 31.25 \text{ kNm/m}$

Ground self-weight ($\gamma_G=1.0$): $P_{\text{ground}} = 1.0 \cdot (76.5) = 76.5 \text{ kNm/m}$

Total load $P_{\text{tot}} = 18.75 + 31.25 + 76.5 = 126.5 \text{ kNm/m}$

Eccentricity $e = M_{\text{tot}} / P_{\text{tot}} = 39.01 / 126.5 = 0.31 \text{ m} \leq B/6 = 2.50/6 = 41.67 \text{ cm}$

Max pressure on ground $\sigma = P_{\text{tot}} / 2.50 + M_{\text{tot}} \cdot 6 / 2.50^2 = 88.05 \text{ kN/m}^2 = 0.088 \text{ MPa}$

The results given at Table 2.2 are obtained by repeating the calculation for the three remaining combinations of partial factors.

The maximal pressure on ground is achieved with the second combination, i.e. for the one in which the wall self-weight and the self-weight of the ground above the footing are both multiplied by 1.35.

For the verification of the contact pressure, the possibility that the surcharge acts on the whole embankment surface must be also considered. (Fig. 2.13); the values given at Table 2.3 are obtained by repeating the calculation for this situation.

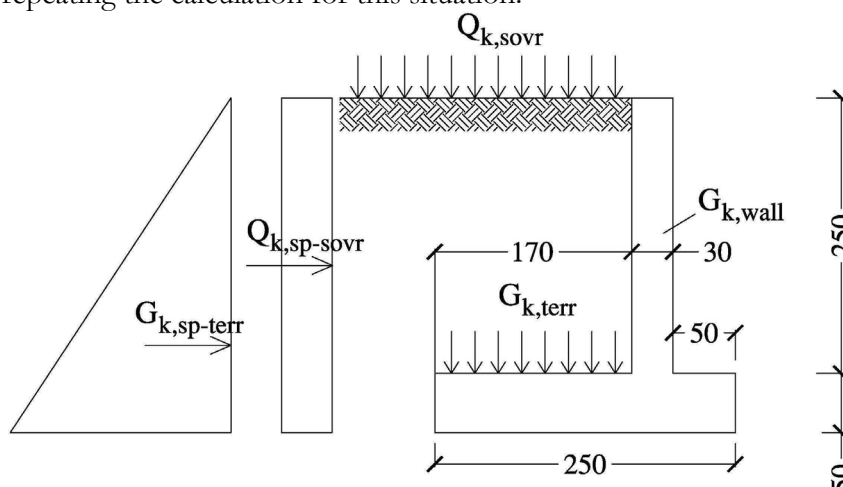


Fig. 2.13. Dimensions of the retaining wall of the numeric example with surcharge on the whole embankment.

Table 2.2. Max pressure for four different combinations of partial factors of permanent loads (surcharge outside the foundation footing).

Combination	first	second	third	fourth
$M_{S,ground}$ (kNm/m)	36.08 ($\gamma_Q=1.35$)	36.08 ($\gamma_Q=1.35$)	36.08 ($\gamma_Q=1.35$)	36.08 ($\gamma_Q=1.35$)
$M_{S,surch}$ (kNm/m)	22.28 ($\gamma_Q=1.5$)	22.28 ($\gamma_Q=1.5$)	22.28 ($\gamma_Q=1.5$)	22.28 ($\gamma_Q=1.5$)
M_{wall} (kNm/m)	11.25 ($\gamma_G=1.0$)	15.19 ($\gamma_G=1.35$)	11.25 ($\gamma_G=1.0$)	15.19 ($\gamma_G=1.35$)
M_{ground} (kNm/m)	-30.60 ($\gamma_G=1.0$)	-41.31 ($\gamma_G=1.35$)	-41.31 ($\gamma_G=1.35$)	-30.60 ($\gamma_G=1.0$)
M_{tot} (kNm/m)	39.01	32.24	28.30	42.95
P_{wall} (kN/m)	18.75 ($\gamma_G=1.0$)	25.31 ($\gamma_G=1.35$)	18.75 ($\gamma_G=1.0$)	25.31 ($\gamma_G=1.35$)
P_{foot} (kN/m)	31.25 ($\gamma_G=1.0$)	42.19 ($\gamma_G=1.35$)	31.25 ($\gamma_G=1.0$)	42.19 ($\gamma_G=1.35$)
P_{ground}	76.50	103.28	103.28	76.50

(kN/m)	$(\gamma_G=1.0)$	$(\gamma_G=1.35)$	$(\gamma_G=1.35)$	$(\gamma_G=1.0)$
P_{tot} (kN/m)	126.50	170.78	153.28	144
<i>eccentricity (m)</i>	0.31	0.19	0.18	0.30
<i>pressure on ground (kN/m^2)</i>	88.05	99.26	88.48	98.83

Table 2.3. Max pressure on ground for four different combinations of partial factors of permanent loads (surcharge on the whole foundation footing).

Combination	first	second	third	fourth
$M_{S_{ground}}$ (kNm/m)	36.08 $(\gamma_Q=1.35)$	36.08 $(\gamma_Q=1.35)$	36.08 $(\gamma_Q=1.35)$	36.08 $(\gamma_Q=1.35)$
$M_{S_{surch}}$ (kNm/m)	22.28 $(\gamma_Q=1.5)$	22.28 $(\gamma_Q=1.5)$	22.28 $(\gamma_Q=1.5)$	22.28 $(\gamma_Q=1.5)$
M_{wall} (kNm/m)	11.25 $(\gamma_G=1.0)$	15.19 $(\gamma_G=1.35)$	11.25 $(\gamma_G=1.0)$	15.19 $(\gamma_G=1.35)$
M_{ground} (kNm/m)	-30.60 $(\gamma_G=1.0)$	-41.31 $(\gamma_G=1.35)$	-41.31 $(\gamma_G=1.35)$	-30.60 $(\gamma_G=1.0)$
M_{surch} (kNm/m)	-10.20 $(\gamma_Q=1.5)$	-10.20 $(\gamma_Q=1.5)$	-10.20 $(\gamma_Q=1.5)$	-10.20 $(\gamma_Q=1.5)$
M_{tot} (kNm/m)	28.81	22.04	18.10	32.75
P_{wall} (kN/m)	18.75 $(\gamma_G=1.0)$	25.31 $(\gamma_G=1.35)$	18.75 $(\gamma_G=1.0)$	25.31 $(\gamma_G=1.35)$
P_{foot} (kN/m)	31.25 $(\gamma_G=1.0)$	42.19 $(\gamma_G=1.35)$	31.25 $(\gamma_G=1.0)$	42.19 $(\gamma_G=1.35)$
P_{terr} (kN/m)	76.50 $(\gamma_G=1.0)$	103.28 $(\gamma_G=1.35)$	103.28 $(\gamma_G=1.35)$	76.50 $(\gamma_G=1.0)$
P_{surch} (kN/m)	25.50 $(\gamma_Q=1.5)$	25.50 $(\gamma_Q=1.5)$	25.50 $(\gamma_Q=1.5)$	25.50 $(\gamma_Q=1.5)$
P_{tot} (kN/m)	152.0	196.28	178.78	169.50
<i>eccentricity (m)</i>	0.19	0.11	0.10	0.19
<i>pressure on ground (kN/m^2)</i>	88.46	99.67	88.89	99.24

The two additional lines, not present in Table 2.2 and here highlighted in bold, correspond to the moment and to the vertical load resulting from the surcharge above the footing.

The max pressure on ground is achieved once again for the second combination and its value is here higher than the one calculated in the previous scheme.

SECTION 4. DURABILITY - WORKED EXAMPLES

EXAMPLE 4.1 [EC2 clause 4.4]

Design the concrete cover of a reinforced concrete beam with exposure class XC1.

The concrete in use has resistance class C25/30.

Bottom longitudinal bars are $5 \phi 20$; the stirrups are $\phi 8$ at 100 mm.

The max aggregate size is: $d_g = 20 \text{ mm}$ ($< 32 \text{ mm}$).

The design working life of the structure is 50 years.

Normal quality control is put in place.

Refer to Fig. 4.1.

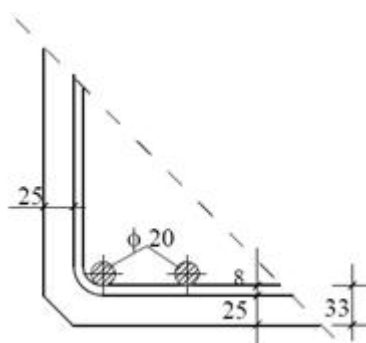


Fig. 4.1.

From table E.1N - EC2 we see that, in order to obtain an adequate concrete durability, the reference (min.) concrete strength class for exposure class XC1 is C20/25; the resistance class adopted (C25/30) is suitable as it is higher than the reference strength class.

The structural class is S4.

First, the concrete cover for the stirrups is calculated.

With:

$$c_{\min,b} = 8 \text{ mm}$$

We obtain from table 4.4N - EC2:

$$c_{\min,dur} = 15 \text{ mm}$$

Moreover:

$$\Delta c_{dur,\gamma} = 0 ;$$

$$\Delta c_{dur,st} = 0 ;$$

$$\Delta c_{dur,add} = 0 .$$

From relation (3.2):

$$c_{\min} = \max (c_{\min,b}; c_{\min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add}; 10 \text{ mm}) = \max (8; 15 + 0 - 0 - 0; 10 \text{ mm}) = 15 \text{ mm}$$

Moreover:

$$\Delta c_{\text{dev}} = 10 \text{ mm.}$$

We obtain from relation (3.1):

$$c_{\text{nom}} = c_{\text{min}} + \Delta c_{\text{dev}} = 15 + 10 = 25 \text{ mm .}$$

If we now calculate now the concrete cover for longitudinal reinforcement bars,

we have:

$$c_{\text{min,b}} = 20 \text{ mm.}$$

We obtain from table 4.4N - EC2:

$$c_{\text{min,dur}} = 15 \text{ mm .}$$

Moreover:

$$\Delta c_{\text{dur},\gamma} = 0 ;$$

$$\Delta c_{\text{dur,st}} = 0 ;$$

$$\Delta c_{\text{dur,add}} = 0 .$$

From relation (3.2):

$$c_{\text{min}} = \max (20; 15 + 0 - 0 - 0; 10 \text{ mm}) = 20 \text{ mm .}$$

Moreover: $\Delta c_{\text{dev}} = 10 \text{ mm.}$

We obtain from relation (3.1):

$$c_{\text{nom}} = 20 + 10 = 30 \text{ mm .}$$

The concrete cover for the stirrups is “dominant”. In this case, the concrete cover for longitudinal bars is increased to: $25 + 8 = 33 \text{ mm .}$

EXAMPLE 4.2 [EC2 clause 4.4]

Design the concrete cover for a reinforced concrete beam placed outside a residential building situated close to the coast.

The exposure class is XS1.

We originally assume concrete with strength class C25/30.

The longitudinal reinforcement bars are $5 \phi 20$; the stirrups are $\phi 8$ at 100 mm .

The maximal aggregate size is: $d_g = 20 \text{ mm}$ ($< 32 \text{ mm}$).

The design working life of the structure is 50 years.

A normal quality control is put in place.

Refer to Fig. 4.2.

From table E.1N - EC2 we find that, in order to obtain an adequate concrete durability, the reference (min.) concrete strength class for exposure class XS1 is C30/37; the concrete strength class must therefore be increased from the originally assumed C25/30 to C30/37, even if the actions on concrete were compatible with strength class C25/30.

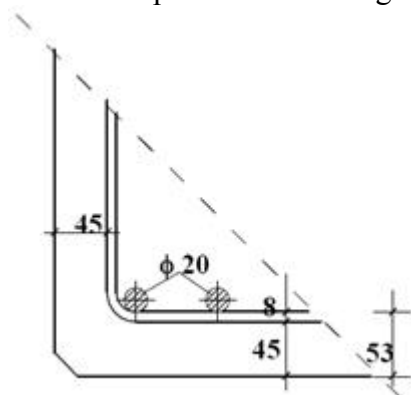


Fig. 4.2.

In accordance with what has been stated in example 4.1, we design the minimum concrete cover with reference to both the stirrups and the longitudinal bars.

The structural class is S4

We obtain ($c_{\min, \text{dur}} = 35 \text{ mm}$; $\Delta c_{\text{dev}} = 10 \text{ mm}$):

- for the stirrups: $c_{\text{nom}} = 45 \text{ mm}$;
- for the longitudinal bars: $c_{\text{nom}} = 45 \text{ mm}$.

The concrete cover for the stirrups is “dominant”. In this case, the concrete cover for longitudinal bars is increased to: $45 + 8 = 53 \text{ mm}$.

EXAMPLE 4.3 [EC2 clause 4.4]

Calculate the concrete cover of a TT precast element, made of prestressed reinforced concrete, placed outside an industrial building situated close to the coast.

The exposure class is XS1.

We use concrete with strength class C45/55.

At the lower side of the two ribbings of the TT element we have:

- longitudinal ϕ 12 reinforcement bars;
- ϕ 8 stirrups at 100 mm ;
- strands ϕ 0.5” .

The maximal aggregate size is: $d_g = 16$ mm.

The design working life of the structure is 50 years.

An accurate quality control of concrete production is put in place.

Refer to Fig. 4.3.

We find out from table E.1N - EC2 that for exposure class XS1, the minimum concrete strength class is C30/37; strength class C45/55 is therefore adequate.

The original structural class is S4.

In accordance with table 4.3N:

- the structural class is reduced by 1 as the concrete used (C45/55) is of strength class higher than C40/50;
- the structural class is reduced by 1 as special quality control of the concrete production is ensured

We then refer to structural class S2.

Calculating first the concrete cover for stirrups.

We have:

$$c_{\min,b} = 8 \text{ mm} .$$

We obtain from table 4.4N - EC2:

$$c_{\min,dur} = 25 \text{ mm} .$$

Moreover:

$$\Delta c_{dur,\gamma} = 0 ;$$

$$\Delta c_{dur,st} = 0 ;$$

$$\Delta c_{dur,add} = 0 .$$

From relation (3.2):

$$\begin{aligned} c_{\min} &= \max (c_{\min,b} ; c_{\min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add} ; 10 \text{ mm}) = \\ &= \max (8 ; 25 + 0 - 0 - 0 ; 10 \text{ mm}) = 25 \text{ mm} . \end{aligned}$$

Considering that the TT element is cast under procedures subjected to a highly efficient quality control, in which the concrete cover length is also assessed, the value of Δc_{dev} can be taken as 5 mm.

We obtain from relation (3.1):

$$c_{nom} = c_{min} + \Delta c_{dev} = 25 + 5 = 30 \text{ mm} .$$

Calculating now the concrete cover for longitudinal bars.

We have:

$$c_{min,b} = 12 \text{ mm} .$$

We obtain from table 4.4N - EC2:

$$c_{min,dur} = 25 \text{ mm} .$$

Moreover:

$$\Delta c_{dur,\gamma} = 0 ;$$

$$\Delta c_{dur,st} = 0 ;$$

$$\Delta c_{dur,add} = 0 .$$

From relation (3.2):

$$\begin{aligned} c_{min} &= \max (c_{min,b} ; c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add} ; 10 \text{ mm}) = \\ &= \max (12 ; 25 + 0 - 0 - 0 ; 10 \text{ mm}) = 25 \text{ mm} . \end{aligned}$$

We obtain from relation (3.1):

$$c_{nom} = c_{min} + \Delta c_{dev} = 25 + 5 = 30 \text{ mm} .$$

Note that for the ordinary reinforcement bars, the concrete cover for stirrups is “dominant”. In this case, the concrete cover for longitudinal bars is increased to: $30 + 8 = 38 \text{ mm}$.

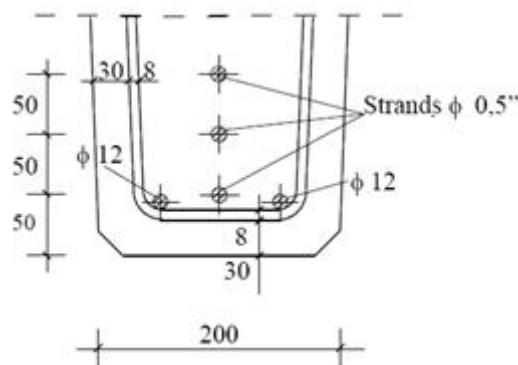


Fig. 4.3.

Calculating now the concrete cover for strands.

We have:

$$c_{\min,b} = 1.5 \cdot 12.5 = 18.8 \text{ mm} .$$

We obtain from table 4.5N - EC2:

$$c_{\min,dur} = 35 \text{ mm} .$$

Moreover:

$$\Delta c_{dur,\gamma} = 0 ;$$

$$\Delta c_{dur,st} = 0 ;$$

$$\Delta c_{dur,add} = 0 .$$

From relation (3.2):

$$c_{\min} = \max (18.8; 35 + 0 - 0 - 0; 10 \text{ mm}) = 35 \text{ mm} .$$

Moreover:

$$\Delta c_{dev} = 5 \text{ mm} .$$

From relation (3.1):

$$c_{nom} = 35 + 5 = 40 \text{ mm} .$$

The first strand's axis is placed at 50mm from the lower end of the ribbing of the TT element. The concrete cover for the lower strands of the TT element (one for each ribbing) is therefore equal to 43mm.

SECTION 6. ULTIMATE LIMIT STATES - WORKED EXAMPLES

GENERAL NOTE: Eurocode 2 permits to use a various steel yielding grades ranging from 400 MPa to 600 MPa. In particular the examples are developed using S450 steel with ductility grade C, which is used in southern Europe and generally in seismic areas. Some example is developed using S500 too.

EXAMPLE 6.1 (Concrete C30/37) [EC2 clause 6.1]

Geometrical data: $b = 500$ mm; $h = 1000$ mm; $d' = 50$ mm; $d = 950$ mm.

Steel and concrete resistance, β_1 and β_2 factors and x_1, x_2 values are shown in Table 6.1.

Basis: β_1 means the ratio between the area of the parabola – rectangle diagram at certain deformation ϵ_c and the area of rectangle at the same deformation.

β_2 is the “position factor”, the ratio between the distance of the resultant of parabola – rectangle diagram at certain deformation ϵ_c from ϵ_c and the deformation ϵ_c itself.

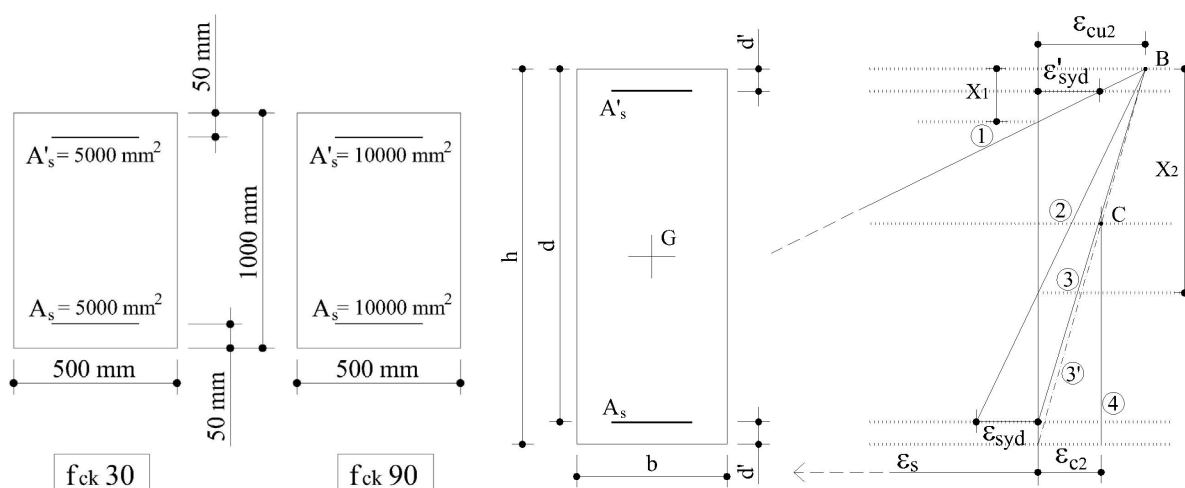


Fig. 6.1. Geometrical data and Possible strain distributions at the ultimate limit states

Table 6.1 Material data, β_1 and β_2 factors and neutral axis depth.

Example	f_{yk} (MPa)	f_{yd} (MPa)	f_{ck} (MPa)	f_{cd} (MPa)	β_1	β_2	x_1 (mm)	x_2 (mm)
6.1	450	391	30	17	0.80	0.40	113.5	608.0
6.2	450	391	90	51	0.56	0.35	203.0	541.5

First the N_{Rd} values corresponding to the 4 configurations of the plane section are calculated.

$$N_{Rd1} = 0.8 \cdot 500 \cdot 113.5 \cdot 17 \cdot 10^{-3} = 772 \text{ kN}$$

$$N_{Rd2} = 0.8 \cdot 500 \cdot 608.0 \cdot 17 \cdot 10^{-3} = 4134 \text{ kN.}$$

The maximum moment resistance $M_{Rd,max} = 2821.2$ kNm goes alongside it.

$$N_{Rd3} = 0.8 \cdot 500 \cdot 950 \cdot 17 \cdot 10^{-3} + 5000 \cdot 391 \cdot 10^{-3} = 6460 + 1955 = 8415 \text{ kN}$$

$$N_{Rd4} = 0.8 \cdot 500 \cdot 1000 \cdot 17 \cdot 10^{-3} + 5000 \cdot 391 \cdot 10^{-3} = 8500 + 3910 = 12410 \text{ kN}$$

M_{Rd3} must also be known. This results: $M_{Rd3} = 6460 \cdot (500 - 0.4 \cdot 950) \cdot 10^{-3} = 1655 \text{ kNm}$

Subsequently, for a chosen value of N_{Ed} in each interval between two following values of N_{Rd} written above and one smaller than N_{Rd1} , the neutral axis x , M_{Rd} , and the eccentricity

$e = \frac{M_{Rd}}{N_{Ed}}$ are calculated. Their values are shown in Table 6.2.

Table 6.2. Values of axial force, depth of neutral axis, moment resistance, eccentricity.

N_{Ed} (kN)	X (m)	M_{Rd} (kNm)	e (m)
600	0.105	2031	3.38
2000	0.294	2524	1.26
5000	0.666	2606	0.52
10000	virtual neutral axis	1000	0.10

As an example the calculation related to $N_{Ed} = 5000 \text{ kN}$ is shown.

The equation of equilibrium to shifting for determination of x is written:

$$x^2 - \left(\frac{5000000 - 5000 \cdot 391 - 5000 \cdot 0.0035 \cdot 200000 \cdot 5000}{0.80 \cdot 500 \cdot 17} \right) x - \left(\frac{0.0035 \cdot 200000 \cdot 5000 \cdot 950}{0.80 \cdot 500 \cdot 17} \right) = 0$$

Developing, it results:

$$x^2 + 66.91x - 488970 = 0$$

which is satisfied for $x = 666 \text{ mm}$

The stress in the lower reinforcement is: $\sigma_s = 0.0035 \cdot 200000 \cdot \left(\frac{950}{666} - 1 \right) = 297 \text{ N/mm}^2$

The moment resistance is:

$$M_{Rd} = 5000 \cdot 391 \cdot (500 - 50) + 5000 \cdot 297 \cdot (500 - 50) + 0.80 \cdot 666 \cdot 500 \cdot 17 \cdot (500 - 0.40 \cdot 666) = 2606 \cdot 10^6 \text{ Nmm} = 2606 \text{ kNm}$$

and the eccentricity $e = \frac{2606}{5000} = 0.52 \text{ m}$

EXAMPLE 6.2 (Concrete C90/105) [EC2 clause 6.1]

For geometrical and mechanical data refer to example 6.1.

Values of N_{Rd} corresponding to the 4 configurations of the plane section and of M_{Rd3} :

$$N_{Rd1} = 2899 \text{ kN}$$

$$N_{Rd2} = 7732 \text{ kN.} \quad M_{Rd,max} = 6948.7 \text{ kNm is associated to it.}$$

$$N_{Rd3} = 13566 + 3910 = 17476 \text{ kN}$$

$$N_{Rd4} = 14280 + 7820 = 22100 \text{ kN}$$

$$M_{Rd3} = 13566 (0.5 - 0.35 \cdot 0.619) + 3910 \cdot (0.50 - 0.05) = 4031 \text{ kNm}$$

Applying the explained procedure x , M_{Rd} and the eccentricity e were calculated for the chosen values of N_{Ed} .

The results are shown in Table 6.3

Table 6.3. Values of axial load, depth of neutral axis, moment resistance, eccentricity

N_{Ed} (kN)	x (m)	M_{Rd} (kNm)	e (m)
1500	0.142	4194	2.80
5000	0.350	5403	1.08
10000	0.619	5514	0.55
19000	virtual neutral axis	2702	0.14

EXAMPLE 6.3 Calculation of $V_{Rd,c}$ for a prestressed beam [EC2 clause 6.2]

Rectangular section $b_w = 100$ mm, $h = 200$ mm, $d = 175$ mm. No longitudinal or transverse reinforcement bars are present. Class C40 concrete. Average prestressing $\sigma_{cp} = 5.0$ MPa.

Design tensile resistance in accordance with:

$$f_{ctd} = \alpha_{ct} f_{ctk, 0.05} / \gamma_C = 1 \cdot 2.5 / 1.5 = 1.66 \text{ MPa}$$

Cracked sections subjected to bending moment.

$$V_{Rd,c} = (v_{min} + k_1 \sigma_{cp}) b_w d$$

where $v_{min} = 0.626$ and $k_1 = 0.15$. It results:

$$V_{Rd,c} = (0.626 + 0.15 \cdot 5.0) \cdot 100 \cdot 175 = 24.08 \text{ kN}$$

Non-cracked sections subjected to bending moment. With $\alpha_1 = 1$ it results

$$I = 100 \cdot \frac{200^3}{12} = 66.66 \cdot 10^6 \text{ mm}^4$$

$$S = 100 \cdot 100 \cdot 50 = 500 \cdot 10^3 \text{ mm}^3$$

$$V_{Rd,c} = \frac{100 \cdot 66.66 \cdot 10^6}{500 \cdot 10^3} \sqrt{(1.66)^2 + 1.66 \cdot 5.0} = 44.33 \text{ kN}$$

EXAMPLE 6.4 Determination of shear resistance given the section geometry and mechanics [EC2 clause 6.2]

Rectangular or T-shaped beam, with

$$b_w = 150 \text{ mm},$$

$$h = 600 \text{ mm},$$

$$d = 550 \text{ mm},$$

$$z = 500 \text{ mm};$$

vertical stirrups diameter 12 mm, 2 legs ($A_{sw} = 226 \text{ mm}^2$), $s = 150 \text{ mm}$, $f_{yd} = 391 \text{ MPa}$.

The example is developed for three classes of concrete.

a) $f_{ck} = 30 \text{ MPa}$; $f_{cd} = 17 \text{ MPa}$; $v = 0.616$

$$\frac{A_{sw} f_{ywd}}{b_w s v f_{cd}} = \sin^2 \theta \quad \text{obtained from } V_{Rd,s} = V_{Rd,max}$$

$$\text{it results: } \sin^2 \theta = \frac{226 \cdot 391}{150 \cdot 150 \cdot 0.616 \cdot 17} = 0.375 \quad \text{hence } \cot \theta = 1.29$$

$$\text{Then } V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot \theta = \frac{226}{150} \cdot 500 \cdot 391 \cdot 1.29 \cdot 10^{-3} = 380 \text{ kN}$$

b) For the same section and reinforcement, with $f_{ck} = 60 \text{ MPa}$, $f_{cd} = 34 \text{ MPa}$; $v = 0.532$, proceeding as above it results:

$$\sin^2 \theta = \frac{226 \cdot 391}{150 \cdot 150 \cdot 0.532 \cdot 34} = 0.2171 \quad \text{hence } \cot \theta = 1.90$$

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot \theta = \frac{226}{150} \cdot 500 \cdot 391 \cdot 1.90 \cdot 10^{-3} = 560 \text{ kN}$$

c) For the same section and reinforcement, with $f_{ck} = 90 \text{ MPa}$, $f_{cd} = 51 \text{ MPa}$; $v = 0.512$, proceeding as above it results:

$$\sin^2 \theta = \frac{226 \cdot 391}{150 \cdot 150 \cdot 0.512 \cdot 51} = 0.1504 \quad \text{hence } \cot \theta = 2.38$$

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot \theta = \frac{226}{150} \cdot 500 \cdot 391 \cdot 2.38 \cdot 10^{-3} = 701 \text{ kN}$$

Determination of reinforcement (vertical stirrups) given the beam and shear action V_{Ed}

Rectangular beam $b_w = 200$ mm, $h = 800$ mm, $d = 750$ mm, $z = 675$ mm; vertical stirrups $f_{ywd} = 391$ MPa. Three cases are shown, with varying values of V_{Ed} and of f_{ck} .

- $V_{Ed} = 600$ kN; $f_{ck} = 30$ MPa ; $f_{cd} = 17$ MPa ; $v = 0.616$

$$\text{Then } \theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{(\alpha_{cw} v f_{cd}) b_w z} = \frac{1}{2} \arcsin \frac{2 \cdot 600000}{(1 \cdot 0.616 \cdot 17) \cdot 200 \cdot 675} = 29.0^\circ$$

$$\text{hence } \cot \theta = 1.80$$

$$\text{It results: } \frac{A_{sw}}{s} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot \theta} = \frac{600000}{675 \cdot 391 \cdot 1.80} = 1.263 \text{ mm}^2 / \text{mm}$$

which is satisfied with 2-leg stirrups $\phi 12/170$ mm.

The tensile force in the tensioned longitudinal reinforcement necessary for bending must be increased by $\Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \cdot 600000 \cdot 1.80 = 540$ kN

- $V_{Ed} = 900$ kN; $f_{ck} = 60$ MPa ; $f_{cd} = 34$ MPa ; $v = 0.532$

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{(\alpha_{cw} v f_{cd}) b_w z} = \frac{1}{2} \arcsin \frac{2 \cdot 900000}{(1 \cdot 0.532 \cdot 34) \cdot 200 \cdot 675} = 23.74^\circ$$

$$\text{hence } \cot \theta = 2.27$$

$$\text{Then with it results } \frac{A_{sw}}{s} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot \theta} = \frac{900000}{675 \cdot 391 \cdot 2.27} = 1.50 \text{ mm}^2 / \text{mm}$$

which is satisfied with 2-leg stirrups $\phi 12/150$ mm.

The tensile force in the tensioned longitudinal reinforcement necessary for bending must be increased by $\Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \cdot 900000 \cdot 2.27 = 1021$ kN

- $V_{Ed} = 1200$ kN; $f_{ck} = 90$ MPa ; $f_{cd} = 51$ MPa ; $v = 0.512$

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{(\alpha_{cw} v f_{cd}) b_w z} = \frac{1}{2} \arcsin \frac{2 \cdot 1200000}{0.512 \cdot 51 \cdot 200 \cdot 675} = 21.45^\circ$$

As θ is smaller than 21.8° , $\cot \theta = 2.50$

$$\text{Hence } \frac{A_{sw}}{s} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot \theta} = \frac{1200000}{675 \cdot 391 \cdot 2.50} = 1.82 \text{ mm}^2 / \text{mm}$$

which is satisfied with 2-leg stirrups $\phi 12/120$ mm.

The tensile force in the tensioned longitudinal reinforcement necessary for bending must be increased by $\Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \cdot 1200000 \cdot 2.50 = 1500$ kN

EXAMPLE 6.4b – the same above, with steel S500C $f_{yd} = 435$ MPa. [EC2 clause 6.2]

The example is developed for three classes of concrete.

a) $f_{ck} = 30$ MPa ; $f_{cd} = 17$ MPa ; $v = 0.616$

$$\frac{A_{sw} f_{ywd}}{b_w s v f_{cd}} = \sin^2 \theta \quad \text{obtained for } V_{Rd,s} = V_{Rd,max}$$

it results: $\sin^2 \theta = \frac{226 \cdot 435}{150 \cdot 150 \cdot 0.616 \cdot 17} = 0.417$ hence $\cot \theta = 1.18$

Then $V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot \theta = \frac{226}{150} \cdot 500 \cdot 435 \cdot 1.18 \cdot 10^{-3} = 387$ kN

b) For the same section and reinforcement, with $f_{ck} = 60$ MPa , $f_{cd} = 34$ MPa; $v = 0.532$, proceeding as above it results:

$\sin^2 \theta = \frac{226 \cdot 435}{150 \cdot 150 \cdot 0.532 \cdot 34} = 0.242$ hence $\cot \theta = 1.77$

$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot \theta = \frac{226}{150} \cdot 500 \cdot 435 \cdot 1.77 \cdot 10^{-3} = 580$ kN

c) For the same section and reinforcement, with $f_{ck} = 90$ MPa, $f_{cd} = 51$ MPa; $v = 0.512$, proceeding as above it results:

$\sin^2 \theta = \frac{226 \cdot 435}{150 \cdot 150 \cdot 0.512 \cdot 51} = 0.167$ hence $\cot \theta = 2.23$

$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot \theta = \frac{226}{150} \cdot 500 \cdot 435 \cdot 2.23 \cdot 10^{-3} = 731$ kN

Determination of reinforcement (vertical stirrups) given the beam and shear action V_{Ed}

Rectangular beam $b_w = 200$ mm, $h = 800$ mm, $d = 750$ mm, $z = 675$ mm; vertical stirrups $f_{ywd} = 435$ MPa. Three cases are shown, with varying values of V_{Ed} and of f_{ck} .

• $V_{Ed} = 600$ kN; $f_{ck} = 30$ MPa ; $f_{cd} = 17$ MPa ; $v = 0.616$ then

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{(\alpha_{cw} v f_{cd}) b_w z} = \frac{1}{2} \arcsin \frac{2 \cdot 600000}{(1 \cdot 0.616 \cdot 17) \cdot 200 \cdot 675} = 29.0^\circ \text{ hence } \cot \theta = 1.80$$

It results: $\frac{A_{sw}}{s} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot \theta} = \frac{600000}{675 \cdot 435 \cdot 1.80} = 1.135$ mm² / mm

which is satisfied with 2-leg stirrups $\phi 12/190$ mm.

The tensile force in the tensioned longitudinal reinforcement necessary for bending must be increased by $\Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \cdot 600000 \cdot 1.80 = 540$ kN

- $V_{Ed} = 900 \text{ kN}$; $f_{ck} = 60 \text{ MPa}$; $f_{cd} = 34 \text{ MPa}$; $v = 0.532$

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{(\alpha_{cw} v f_{cd}) b_w z} = \frac{1}{2} \arcsin \frac{2 \cdot 900000}{(1 \cdot 0.532 \cdot 34) \cdot 200 \cdot 675} = 23.74^\circ \text{ hence } \cot \theta = 2.27$$

$$\text{Then with it results } \frac{A_{sw}}{s} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot \theta} = \frac{900000}{675 \cdot 435 \cdot 2.27} = 1.35 \text{ mm}^2 / \text{mm}$$

which is satisfied with 2-leg stirrups $\phi 12/160 \text{ mm}$.

The tensile force in the tensioned longitudinal reinforcement necessary for bending must be increased by $\Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \cdot 900000 \cdot 2.27 = 1021 \text{ kN}$

- $V_{Ed} = 1200 \text{ kN}$; $f_{ck} = 90 \text{ MPa}$; $f_{cd} = 51 \text{ MPa}$; $v = 0.512$

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{(\alpha_{cw} v f_{cd}) b_w z} = \frac{1}{2} \arcsin \frac{2 \cdot 1200000}{0.512 \cdot 51 \cdot 200 \cdot 675} = 21.45^\circ$$

As θ is smaller than 21.8° , $\cot \theta = 2.50$

$$\text{Hence } \frac{A_{sw}}{s} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot \theta} = \frac{1200000}{675 \cdot 435 \cdot 2.50} = 1.63 \text{ mm}^2 / \text{mm}$$

which is satisfied with 2-leg stirrups $\phi 12/130 \text{ mm}$.

The tensile force in the tensioned longitudinal reinforcement necessary for bending must be increased by $\Delta F_{td} = 0.5 V_{Ed} \cot \theta = 0.5 \cdot 1200000 \cdot 2.50 = 1500 \text{ kN}$

EXAMPLE 6.5 [EC2 clause 6.2]

Rectangular or T-shaped beam, with

$$b_w = 150 \text{ mm}$$

$$h = 800 \text{ mm}$$

$$d = 750 \text{ mm}$$

$$z = 675 \text{ mm};$$

$$f_{ck} = 30 \text{ MPa}; f_{cd} = 17 \text{ MPa}; v = 0.616$$

Reinforcement:

inclined stirrups 45° ($\cot \alpha = 1.0$), diameter 10 mm, 2 legs ($A_{sw} = 157 \text{ mm}^2$), $s = 150 \text{ mm}$, $f_{yd} = 391 \text{ MPa}$.

Calculation of shear resistance

•Ductility is first verified by $\frac{A_{sw,max} f_{ywd}}{b_w s} \leq 0.5 \cdot \frac{\alpha_{cw} v_1 f_{cd}}{\sin \alpha}$

And replacing $\frac{157 \cdot 391}{150 \cdot 150} \leq 0.5 \cdot \frac{1 \cdot 0.616 \cdot 17}{0.707} = 2.72 < 7.40$

•The angle θ of simultaneous concrete – reinforcement steel collapse

It results $\cot \theta = \sqrt{\frac{b s v f_{cd}}{A_{sw} f_{ywd} \sin \alpha} - 1}$

and, replacing $\cot \theta = \sqrt{\frac{150 \cdot 150 \cdot 0.616 \cdot 17}{157 \cdot 391 \cdot 0.707} - 1} = 2.10$

c) Calculation of V_{Rd}

It results: $V_{Rd,s} = \frac{157}{150} \cdot 675 \cdot 391 \cdot (2.10 + 1.0) \cdot 0.707 \cdot 10^{-3} = 605.4 \text{ kN}$

•Increase of tensile force the longitudinal bar ($V_{Ed} = V_{Rd,s}$)

$$\Delta F_{td} = 0.5 V_{Rd,s} (\cot \theta - \cot \alpha) = 0.5 \cdot 605.4 \cdot (2.10 - 1.0) = 333 \text{ kN}$$

EXAMPLE 6.6 [EC2 clause 6.3]

Ring rectangular section, Fig. 6.2, with depth 1500 mm, width 1000 mm, $d = 1450$ mm, with 200 mm wide vertical members and 150 mm wide horizontal members.

Materials:

$$f_{ck} = 30 \text{ MPa}$$

$$f_{yk} = 500 \text{ MPa}$$

Results of actions:

$$V_{Ed} = 1300 \text{ kN (force parallel to the larger side)}$$

$$T_{Ed} = 700 \text{ kNm}$$

Design resistances:

$$f_{cd} = 0.85 \cdot (30/1.5) = 17.0 \text{ MPa}$$

$$v = 0.7[1 - 30/250] = 0.616$$

$$v f_{cd} = 10.5 \text{ MPa}$$

$$f_{yd} = 500/1.15 = 435 \text{ MPa}$$

Geometric elements:

$$u_k = 2(1500 - 150) + 2(1000 - 200) = 4300 \text{ mm}$$

$$A_k = 1350 \cdot 800 = 1080000 \text{ mm}^2$$

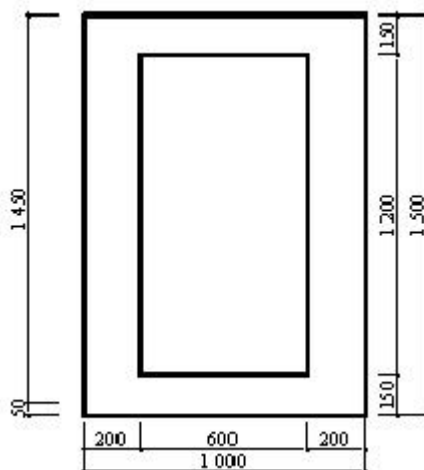


Fig. 6.2. Ring section subjected to torsion and shear

The maximum equivalent shear in each of the vertical members is (z refers to the length of the vertical member):

$$V_{Ed}^* = V_{Ed} / 2 + (T_{Ed} \cdot z) / (2 \cdot A_k) = [1300 \cdot 10^3 / 2 + (700 \cdot 10^6 \cdot 1350) / (2 \cdot 1.08 \cdot 10^6)] \cdot 10^{-3} = 1087 \text{ kN}$$

Verification of compressed concrete with $\cot \theta = 1$. It results:

$$V_{Rd,max} = t z v f_{cd} \sin \theta \cos \theta = 200 \cdot 1350 \cdot 10.5 \cdot 0.707 \cdot 0.707 = 1417 \text{ kN} > V_{Ed}^*$$

Determination of angle θ :

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}^*}{v f_{ctd} t z} = \frac{1}{2} \arcsin \frac{2 \cdot 1087000}{10.5 \cdot 200 \cdot 1350} = 25.03^\circ \quad \text{hence } \cot \theta = 2.14$$

Reinforcement of vertical members:

$$(A_{sw}/s) = V_{Ed}^* / (z f_{yd} \cot \theta) = (1087 \cdot 10^3) / (1350 \cdot 435 \cdot 2.14) = 0.865 \text{ mm}^2/\text{mm}$$

which can be carried out with 2-legs 12 mm bars, pitch 200 mm; pitch is in accordance with [9.2.3(3)-EC2].

Reinforcement of horizontal members, subjected to torsion only:

$$(A_{sw}/s) = T_{Ed} / (2 \cdot A_k \cdot f_{yd} \cdot \cot \theta) = 700 \cdot 10^6 / (2 \cdot 1.08 \cdot 10^6 \cdot 435 \cdot 2.14) = 0.348 \text{ mm}^2/\text{mm}$$

which can be carried out with 8 mm wide, 2 legs stirrups, pitch 200 mm.

Longitudinal reinforcement for torsion:

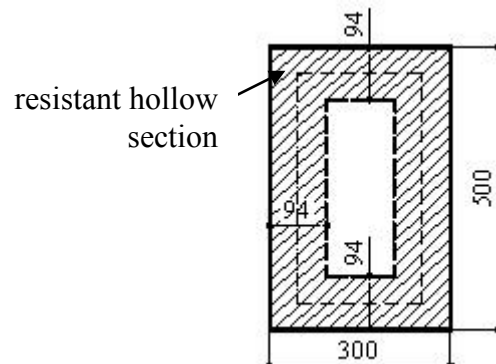
$$A_{sl} = T_{Ed} \cdot u_k \cdot \cot \theta / (2 \cdot A_k \cdot f_{yd}) = 700 \cdot 10^6 \cdot 4300 \cdot 2.14 / (2 \cdot 1080000 \cdot 435) = 6855 \text{ mm}^2$$

to be distributed on the section, with particular attention to the corner bars.

Longitudinal reinforcement for shear:

$$A_{sl} = V_{Ed} \cdot \cot \theta / (2 \cdot f_{yd}) = 1300000 \cdot 2.14 / (2 \cdot 435) = 3198 \text{ mm}^2$$

To be placed at the lower end.

EXAMPLE 6.7 Shear – Torsion interaction diagrams [EC2 clause 6.3]*Fig. 6.3. Rectangular section subjected to shear and torsion*

Example: full rectangular section $b = 300 \text{ mm}$, $h = 500 \text{ mm}$, $z = 400 \text{ mm}$ (Fig. 6.3)

Materials:

$$f_{ck} = 30 \text{ MPa}$$

$$f_{cd} = 0.85 \cdot (30/1.5) = 17.0 \text{ MPa}$$

$$v = 0.7 \cdot \left(1 - \frac{30}{250}\right) = 0.616; v f_{cd} = 10.5 \text{ MPa}$$

$$f_{yk} = 450 \text{ MPa}; f_{yd} = 391 \text{ MPa}$$

$$\alpha_{cw} = 1$$

Geometric elements

$$A = 150000 \text{ mm}^2$$

$$u = 1600 \text{ mm}$$

$$t = A/u = 94 \text{ mm}$$

$$A_k = (500 - 94) \cdot (300 - 94) = 83636 \text{ mm}^2$$

$$\text{Assumption: } \theta = 26.56^\circ (\cot \theta = 2.0)$$

$$\text{It results: } V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v \cdot f_{cd} / (\cot \theta + \tan \theta) = 10.5 \cdot 300 \cdot 400 / (2 + 0.5) = 504 \text{ kN}$$

and for the taken $z = 400 \text{ mm}$

$$T_{Rd,max} = 2 \cdot 10.5 \cdot 83636 \cdot 94 \cdot 0.4471 \cdot 0.8945 = 66 \text{ kNm}$$

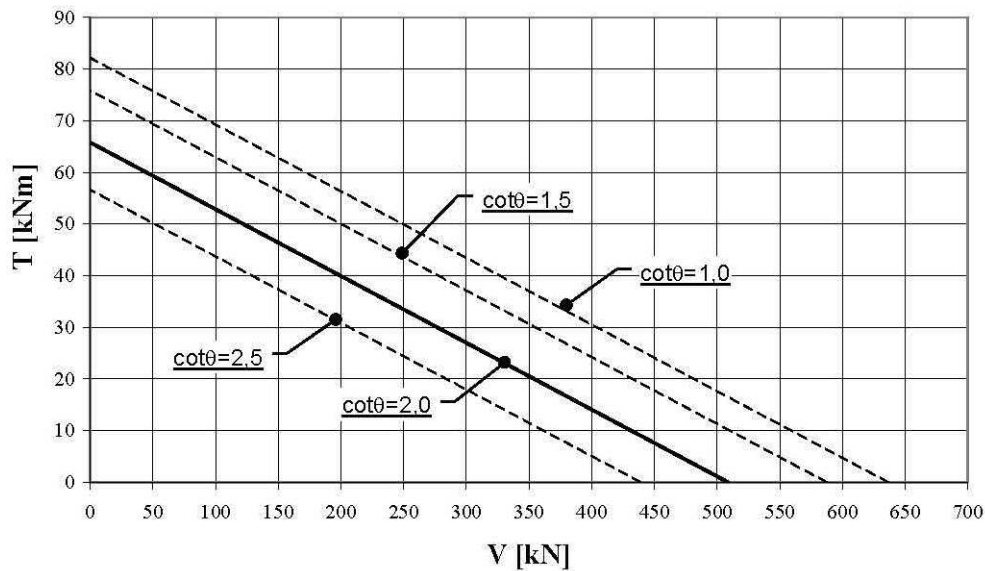


Fig. 6.4. *V-T interaction diagram for highly stressed section*

The diagram is shown in Fig. 6.4. Points below the straight line that connects the resistance values on the two axis represent safety situations. For instance, if $V_{Ed} = 350$ kN is taken, it results that the maximum compatible torsion moment is 20 kNm.

On the figure other diagrams in relation with different θ values are shown as dotted lines.

Second case: light action effects

Same section and materials as in the previous case. The safety condition (absence of cracking) is expressed by:

$$T_{Ed}/T_{Rd,c} + V_{Ed}/V_{Rd,c} \leq 1 \quad [(6.31)\text{-EC2}]$$

where $T_{Rd,c}$ is the value of the torsion cracking moment:

$\tau = f_{ctd} = f_{ctk} / \gamma_c = 2.0/1.5 = 1.3$ MPa (f_{ctk} deducted from Table [3.1-EC2]). It results therefore:

$$T_{Rd,c} = f_{ctd} \cdot t \cdot 2A_k = 1.3 \cdot 94 \cdot 2 \cdot 83636 = 20.4 \text{ kNm}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_l f_{ck})^{1/3} \right] \cdot b_w d$$

In this expression, $\rho = 0.01$; moreover, it results:

$$C_{Rd,c} = 0.18/1.5 = 0.12$$

$$k = 1 + \sqrt{\frac{200}{500}} = 1.63$$

$$(100\rho_l f_{ck})^{1/3} = (100 \cdot 0.01 \cdot 30)^{1/3} = (30)^{1/3}$$

Taking $d = 450$ mm it results:

$$V_{Rd,c} = 0.12 \cdot 1.63 \cdot (30)^{1/3} \cdot 300 \cdot 450 = 82.0 \text{ kN}$$

The diagram is shown in Fig.6.5

The section, in the range of action effects defined by the interaction diagram, should have a minimal reinforcement in accordance with [9.2.2 (5)-EC2] and [9.2.2 (6)-EC2]. Namely, the minimal quantity of stirrups must be in accordance with [9.5N-EC2], which prescribes for shear:

$$(A_{sw} / s \cdot b_w)_{\min} = \frac{0.08 \sqrt{f_{ck}}}{f_{yk}} = \frac{0.08 \sqrt{30}}{450} = 0.010 = 0.010$$

with s not larger than $0.75d = 0.75 \cdot 450 = 337$ mm.

Because of the torsion, stirrups must be closed and their pitch must not be larger than $u/8$, i.e. 200 mm. For instance, stirrups of 6 mm diameter with 180 mm pitch can be placed. It results : $A_{sw}/s \cdot b_w = 2 \cdot 28 / (180 \cdot 300) = 0.0010$

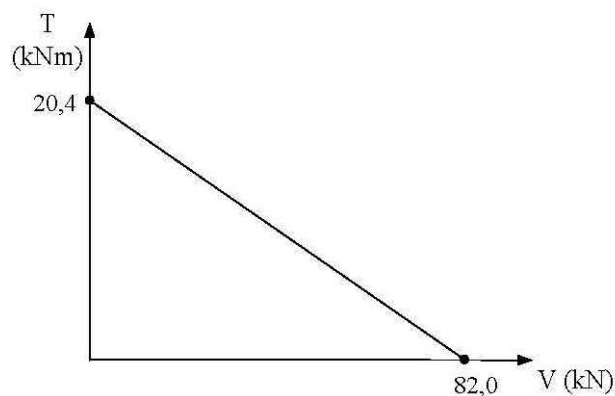


Fig. 6.5. *V-T interaction diagram for lightly stressed section*

EXAMPLE 6.8 Wall beam [EC2 clause 6.5]

Geometry: 5400 x 3000 mm beam (depth $b = 250$ mm), 400 x 250 mm columns, columns reinforcement $6\phi 20$

We state that the strut location C_2 is 200 cm from the bottom reinforcement, so that the inner drive arm is equal to the elastic solution in the case of a wall beam with ratio $1/h=2$, that is $0.67 h$; it suggests to use the range $(0.6 \div 0.7) \cdot l$ as values for the lever arm, lower than the case of a slender beam with the same span.

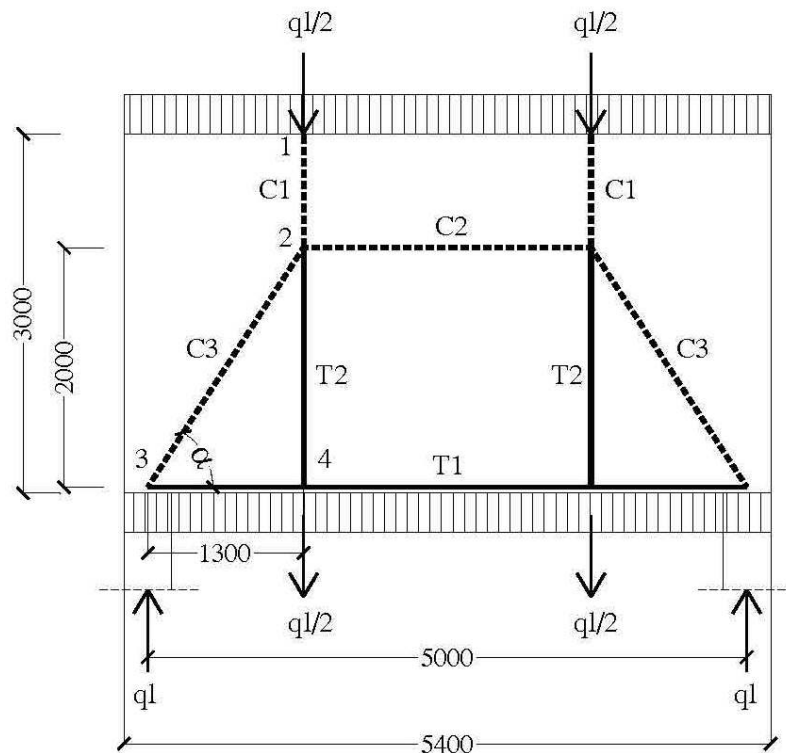


Fig. 6.6. 5400 x 3000 mm wall beam.

Materials: concrete C25/30 $f_{ck} = 25$ MPa, steel B450C $f_{yk} = 450$ MPa

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 25}{1.5} = 14.17 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2$$

nodes compressive strength:

compressed nodes

$$\sigma_{1Rd,max} = k_1 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = 1.18 \left(1 - \frac{25}{250}\right) 14.17 = 15 \text{ N/mm}^2$$

nodes tensioned – compressed by anchor logs in a fixed direction

$$\sigma_{2Rd,max} = k_2 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = \left(1 - \frac{25}{250}\right) 14.17 = 12.75 \text{ N/mm}^2$$

nodes tensioned – compressed by anchor logs in different directions

$$\sigma_{3Rd,max} = k_3 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = 0.88 \left(1 - \frac{25}{250}\right) 14.17 = 11.22 \text{ N/mm}^2$$

Actions

Distributed load: 150 kN/m upper surface and 150 kN/m lower surface

Columns reaction

$$R = (150 + 150) \cdot 5.40 / 2 = 810 \text{ kN}$$

Evaluation of stresses in lattice bars

Equilibrium node 1 $C_1 = \frac{q l}{2} = 405 \text{ kN}$

Equilibrium node 3 $C_3 = \frac{R}{\sin \alpha} = 966 \text{ kN}$ (where $\alpha = \arctg \frac{2000}{1300} = 56.98^\circ$)

$$T_1 = C_3 \cos \alpha = 526 \text{ kN}$$

Equilibrium node 2 $C_2 = C_3 \cos \alpha = T_1 = 526 \text{ kN}$

Equilibrium node 4 $T_2 = \frac{q l}{2} = 405 \text{ kN}$

Tension rods

The tension rod T_1 requires a steel area not lower than:

$$A_{s1} \geq \frac{526000}{391.3} = 1344 \text{ mm}^2 \quad \text{we use } 6\phi 18 = 1524 \text{ mm}^2,$$

the reinforcement of the lower tension rod are located at the height of $0.12 h = 360 \text{ mm}$

The tension rod T_2 requires a steel area not lower than:

$$A_{s1} \geq \frac{405000}{391.3} = 1035 \text{ mm}^2 \quad \text{We use } 4\phi 20 = 1257 \text{ mm}^2$$

Nodes verification

Node 3

The node geometry is unambiguously defined by the column width, the wall depth (250 mm), the height of the side on which the lower bars are distributed and by the strut C_3 fall (Fig. 6.7)

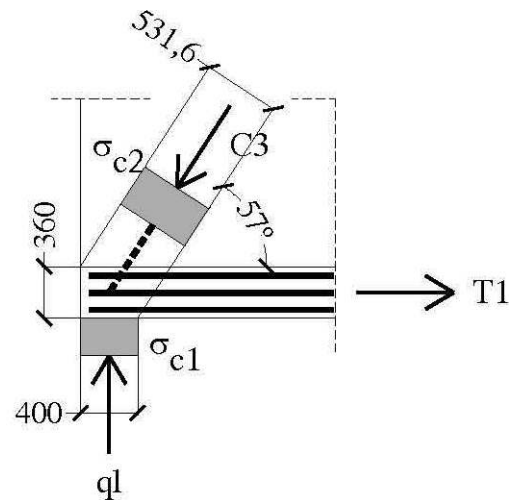


Fig. 6.7. Node 3, left support.

The node 3 is a compressed-stressed node by a single direction reinforcement anchor, then it is mandatory to verify that the maximal concrete compression is not higher than the value:

$$\sigma_{2Rd,max} = 12.75 \text{ N / mm}^2$$

$$\sigma_{c1} = \frac{810000}{400 \cdot 250} = 8.1 \text{ N / mm}^2 \leq \sigma_{2Rd,max}$$

Remark as the verification of the column contact pressure is satisfied even without taking into account the longitudinal reinforcement (6 $\phi 20$) present in the column.

$$\sigma_{c2} = \frac{966000}{531.6 \cdot 250} = 7.27 \text{ N / mm}^2 \leq \sigma_{2Rd,max}$$

EXAMPLE 6.9 Thick short corbel, $a_c < h_c/2$ [EC2 clause 6.5]

Geometry: 250 x 400 mm cantilever (width $b = 400$ mm), 150 x 300 load plate, beam
 $b \times h = 400 \times 400$ mm

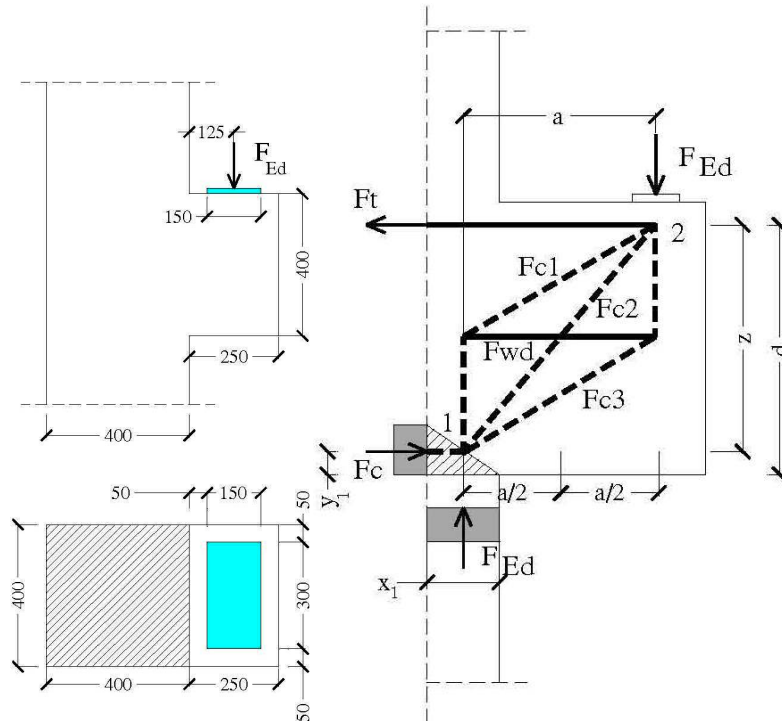


Fig. 6.8. 250 x 400 mm thick cantilever beam.

Fig. 6.9. Cantilever beam S&T model.

Materials: concrete C35/45 $f_{ck} = 35$ MPa, steel B450C $f_{yk} = 450$ MPa

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 35}{1.5} = 19.83 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2$$

nodes compressive strength:

compressed nodes

$$\sigma_{1Rd,max} = k_1 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = 1.18 \left(1 - \frac{35}{250}\right) 19.83 = 20.12 \text{ N/mm}^2$$

nodes tensioned – compressed by anchor logs in a fixed direction

$$\sigma_{2Rd,max} = k_2 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = \left(1 - \frac{35}{250}\right) 19.83 = 17.05 \text{ N/mm}^2$$

nodes tensioned – compressed by anchor logs in different directions

$$\sigma_{3Rd,max} = k_3 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = 0.88 \left(1 - \frac{35}{250}\right) 19.83 = 15 \text{ N/mm}^2$$

Actions

$$F_{Ed} = 700 \text{ kN}$$

Load eccentricity with respect to the column side: $e = 125 \text{ mm}$ (Fig. 6.8)

The beam vertical strut width is evaluated by setting the compressive stress equal to $\sigma_{1Rd,max}$:

$$x_1 = \frac{F_{Ed}}{\sigma_{1Rd,max} b} = \frac{700000}{20.12 \cdot 400} \cong 87 \text{ mm}$$

the node 1 is located $x_1/2 \cong 44 \text{ mm}$ from the outer column side (Fig. 6.9)

We state that the upper reinforcement is located 40 mm from the upper cantilever side; the distance y_1 of the node 1 from the lower border is evaluated setting the internal drive arm z equal to $0.8 \cdot d$ ($z = 0.8 \cdot 360 = 288 \text{ mm}$):

$$y_1 = 0.2d = 0.2 \cdot 360 = 72 \text{ mm}$$

$$\text{rotational equilibrium: } F_{Ed} a = F_c z \quad 700000 \cdot (125 + 44) = F_c \cdot 288$$

$$F_c = F_t = \frac{700000 \cdot (125 + 44)}{288} = 410763 \text{ N} \cong 411 \text{ kN}$$

node 1 verification:

$$\sigma = \frac{F_c}{b (2y_1)} = \frac{411000}{400 (2 \cdot 72)} = 7.14 \text{ N/mm}^2 \leq \sigma_{1Rd,max} = 20.12 \text{ N/mm}^2$$

Main upper reinforcement design:

$$A_s = \frac{F_t}{f_{yd}} = \frac{411000}{391.3} = 1050 \text{ mm}^2 \text{ we use } 8\phi 14 (A_s = 1232 \text{ mm}^2)$$

Secondary upper reinforcement design:

The beam proposed in EC2 is indeterminate, then it is not possible to evaluate the stresses for each single bar by equilibrium equations only, but we need to know the stiffness of the two elementary beams shown in Fig. 6.10 in order to make the partition of the diagonal stress

$$\left(F_{diag} = \frac{F_c}{\cos\theta} = \frac{F_{Ed}}{\sin\theta} \right) \text{ between them;}$$

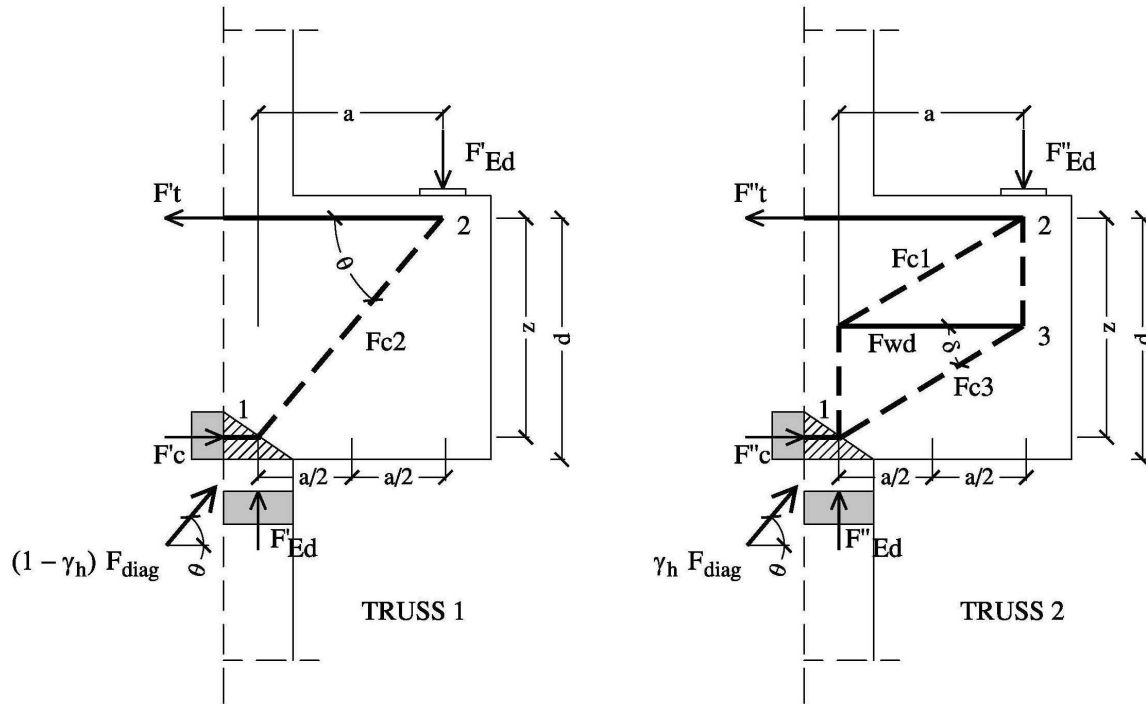


Fig. 6.10. S&T model resolution in two elementary beams and partition of the diagonal stress F_{diag} .

based on the trend of main compressive stresses resulting from linear elastic analysis at finite elements, some researcher of Stuttgart have determined the two rates in which F_{diag} is divided, and they have provided the following expression of stress in the secondary reinforcement (MC90 par. 6.8.2.2.1):

$$F_{wd} = \frac{2 \frac{z}{a} - 1}{3 + F_{Ed} / F_c} F_c = \frac{2 \cdot \frac{288}{125 + 44} - 1}{3 + 700 / 411} 411 \approx 211 \text{ kN}$$

$$A_{sw} = \frac{F_{wd}}{f_{yd}} = \frac{211000}{391.3} \approx 539 \text{ mm}^2 \geq k_1 \cdot A_s = 0.25 \cdot 1232 = 308 \text{ mm}^2$$

we use 5 stirrups $\phi 10$, double armed ($A_{sw} = 785 \text{ mm}^2$)

node 2 verification, below the load plate:

The node 2 is a tied-compressed node, where the main reinforcement is anchored; the compressive stress below the load plate is:

$$\sigma = \frac{F_{Ed}}{150 \cdot 300} = \frac{700000}{45000} = 15.56 \text{ N/mm}^2 \leq \sigma_{2Rd,max} = 17.05 \text{ N/mm}^2$$

EXAMPLE 6.10 Thick cantilever beam, $a_c > h_c/2$ [EC2 clause 6.5]

Geometry: 325 x 300 mm cantilever beam (width $b = 400$ mm), 150 x 220 mm load plate, 400 x 400 mm column

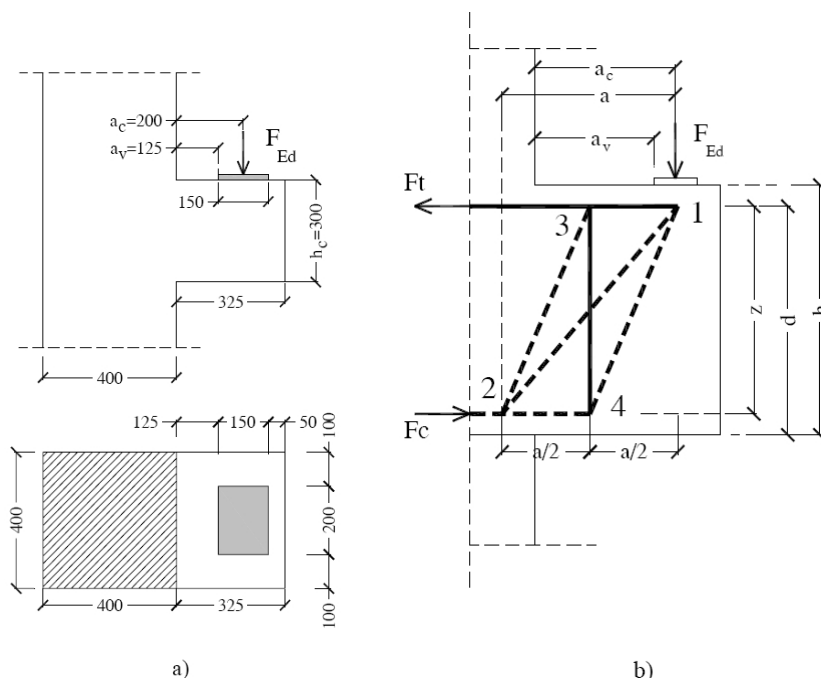


Fig. 6.11. 325 x 300 mm cantilever.

Fig. 6.12. *Cantilever S&T model.*

The model proposed in EC2 (Fig. 6.12) is indeterminate, then as in the previous example one more boundary condition is needed to evaluate the stresses values in the rods;

The stress F_{wd} in the vertical tension rod is evaluated assuming a linear relation between F_{wd} and the a value, in the range $F_{wd} = 0$ when $a = z/2$ and $F_{wd} = F_{Ed}$ when $a = 2 \cdot z$. This assumption corresponds to the statement that when $a \leq z/2$ (a very thick cantilever), the resistant beam is the beam 1 only (Fig. 6.13a) and when $a \geq 2 \cdot z$ the beam 2 only (Fig. 6.13b).

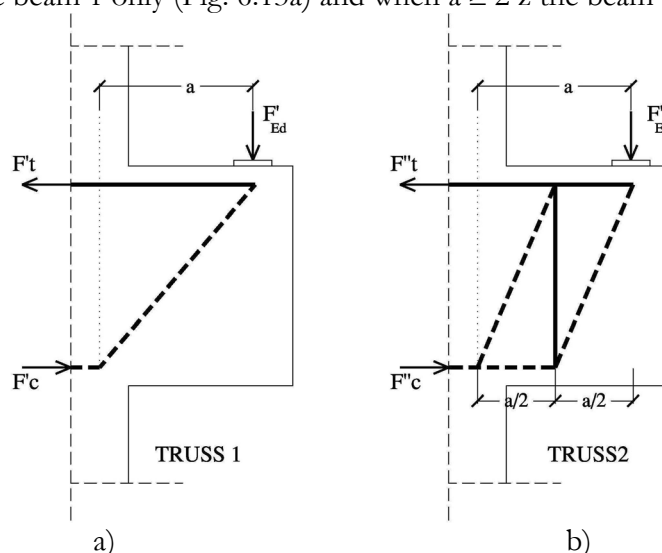


Fig. 6.13. Elementary beams of the S&T model.

Assumed this statement, the expression for F_{wd} is:

$$F_{wd} = F_{w1} a + F_{w2}$$

when the two conditions $F_{wd}(a = \frac{z}{2}) = 0$ and $F_{wd}(a = 2z) = F_{Ed}$ are imposed, some trivial algebra leads to:

$$F_{w1} = \frac{2}{3} \frac{F_{Ed}}{z} \quad \text{and} \quad F_{w2} = -\frac{F_{Ed}}{3};$$

in conclusion, the expression for F_{wd} as a function of a is the following:

$$F_{wd} = \frac{2}{3} \frac{F_{Ed}}{z} a - \frac{F_{Ed}}{3} = F_{Ed} \frac{2a/z - 1}{3}.$$

Materials: concrete C35/45 $f_{ck} = 35$ MPa, steel B450C $f_{yk} = 450$ MPa

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 35}{1.5} = 19.83 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2$$

Nodes compression resistance (same values of the previous example):

Compressed nodes

$$\sigma_{1Rd,max} = 20.12 \text{ N/mm}^2$$

tied-compressed nodes with tension rods in one direction

$$\sigma_{2Rd,max} = 17.05 \text{ N/mm}^2$$

tied-compressed nodes with tension rods in different directions

$$\sigma_{3Rd,max} = 15 \text{ N/mm}^2$$

Actions:

$$F_{Ed} = 500 \text{ kN}$$

Load eccentricity with respect to the column outer side: $e = 200$ mm

The column vertical strut width is evaluated setting the compressive stress equal to $\sigma_{1Rd,max}$:

$$x_1 = \frac{F_{Ed}}{\sigma_{1Rd,max} b} = \frac{500000}{20.12 \cdot 400} \cong 62 \text{ mm}$$

node 1 is located $x_1/2 = 31$ mm from the outer side of the column;

the upper reinforcement is stated to be 40 mm from the cantilever outer side; the distance y_1 of the node 1 from the lower border is calculated setting the internal drive arm z to be $0.8 \cdot d$ ($z = 0.8 \cdot 260 = 208$ mm):

$$y_1 = 0.2d = 0.2 \cdot 260 = 52 \text{ mm}$$

rotational equilibrium:

$$F_{Ed} \left(a_c + \frac{x_l}{2} \right) = F_c z \quad 500000 (200+31) = F_c \cdot 208$$

$$F_c = F_t = \frac{500000 \cdot (200+31)}{208} = 555288 \text{ N} \cong 556 \text{ kN}$$

node 1 verification

$$\sigma = \frac{F_c}{b(2y_1)} = \frac{556000}{400 (2 \cdot 52)} = 13.37 \text{ N/mm}^2 \leq \sigma_{1Rd,max} = 20.12 \text{ N/mm}^2$$

Main upper reinforcement design:

$$A_s = \frac{F_t}{f_{yd}} = \frac{556000}{391.3} = 1421 \text{ mm}^2 \quad \text{we use } 8\phi 16 (A_s = 1608 \text{ mm}^2)$$

Secondary reinforcement design:

(the expression deduced at the beginning of this example is used)

$$F_w = \frac{2 \frac{a}{z} - 1}{3} F_{Ed} \cong 204 \text{ kN}$$

$$A_w = \frac{F_w}{f_{yd}} = \frac{204000}{391.3} = 521 \text{ mm}^2$$

EC2 suggests a minimum secondary reinforcement of:

$$A_w \geq k_2 \frac{F_{Ed}}{f_{yd}} = 0.5 \frac{500000}{391.3} = 639 \text{ mm}^2 \quad \text{we use } 3 \text{ stirrups } \phi 12 (A_s = 678 \text{ mm}^2)$$

node 2 verification, below the load plate:

The node 2 is a compressed-stressed node, in which the main reinforcement is anchored; the compressive stress below the load plate is:

$$\sigma = \frac{F_{Ed}}{150 \cdot 220} = \frac{500000}{33000} = 15.15 \text{ N/mm}^2 \leq \sigma_{2Rd,max} = 17.05 \text{ N/mm}^2$$

EXAMPLE 6.11 Gerber beam [EC2 clause 6.5]

Two different strut-tie trusses can be considered for the design of a Gerber beam, eventually in a combined configuration [EC2 (10.9.4.6)], (Fig. 6.14). Even if the EC2 allows the possibility to use only one strut and then only one reinforcement arrangement, we remark as the scheme b) results to be poor under load, because of the complete lack of reinforcement for the bottom border of the beam. It seems to be opportune to combine the type b) reinforcement with the type a) one, and the latter will carry at least half of the beam reaction.

On the other hand, if only the scheme a) is used, it is necessary to consider a longitudinal top reinforcement to anchor both the vertical stirrups and the confining reinforcement of the tilted strut C_1 .

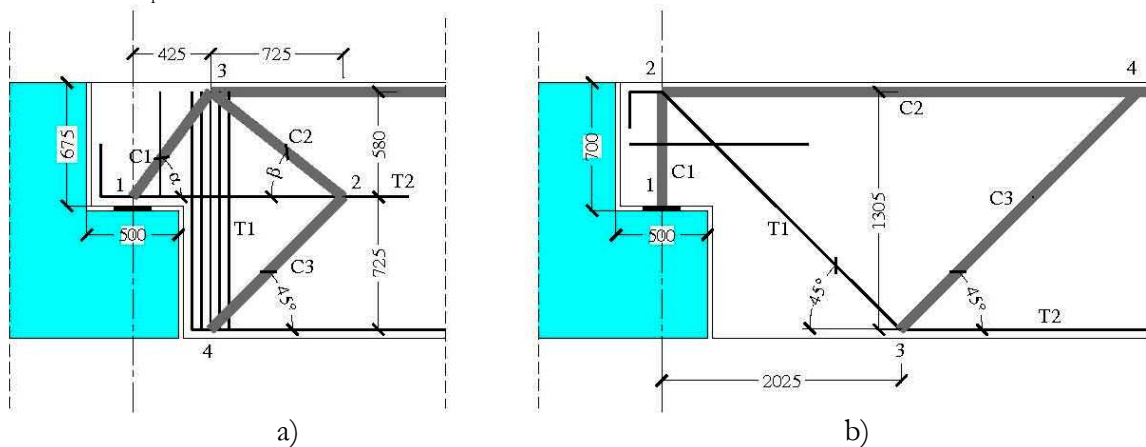


Fig. 6.14. Possible strut and tie models for a Gerber beam.

Hereafter we report the partition of the support reaction between the two trusses.

Materials:

concrete	C25/30	$f_{ck} = 25 \text{ MPa},$
steel	B450C	$f_{yk} = 450 \text{ MPa}$
		$E_s = 200000 \text{ MPa} \quad [(3.2.7 (4)\text{-EC2})]$

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 25}{1.5} = 14.17 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2$$

Actions:

Distributed load: 250 kN/m

Beam span: 8000 mm

$R_{Sdu} = 1000 \text{ kN}$

Bending moment in the beam mid-span: $M_{Sdu} = 2000 \text{ kNm}$

Beam section: $b \times h = 800 \times 1400 \text{ mm}$

Bottom longitudinal reinforcement (A_s): $10\phi 24 = 4524 \text{ mm}^2$

Top longitudinal reinforcement (A'_s): $8\phi 20 = 2513 \text{ mm}^2$

Truss a) $R = R_{Sdu} / 2 = 500 \text{ kN}$

Definition of the truss rods position

The compressed longitudinal bar has a width equal to the depth x of the section neutral axis and then it is $x/2$ from the top border; the depth of the neutral axis is evaluated from the section translation equilibrium:

$$0.8 b x f_{cd} + E_s \varepsilon'_s A'_s = f_{yd} A_s$$

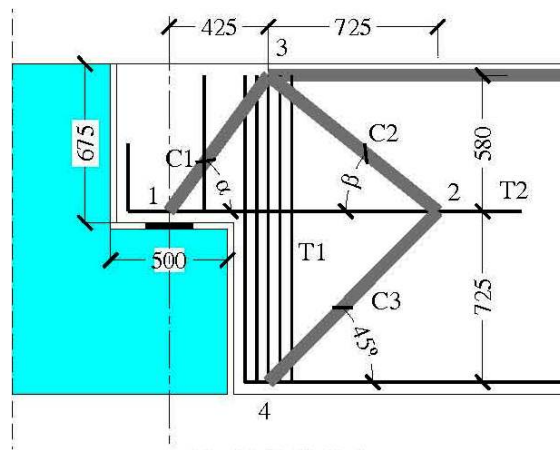


Fig. 6.15. Truss a.

$$\varepsilon'_s = \frac{0.0035}{x} \cdot (x - d') \quad \text{where } d' = 50 \text{ mm is the distance of the upper surface reinforcement}$$

$$0.8 b x f_{cd} + E_s 0.0035 \frac{x - 50}{x} A'_s = f_{yd} A_s$$

and then:

$$x = 99 \text{ mm}$$

$$\varepsilon'_s = \frac{0.0035}{99} \cdot (99 - 50) = 0.00173 \leq \frac{f_{yd}}{E_s} = \frac{391.3}{200000} = 0.00196$$

then the compressed steel strain results lower than the strain in the elastic limit, as stated in the calculation;

the compressive stress in the concrete is

$$C = 0.8 b x f_{cd} = 0.8 \cdot 800 \cdot 99 \cdot 14.17$$

(applied at $0.4 \cdot x \cong 40 \text{ mm}$ from the upper surface)

while the top reinforcement stress is:

$$C' = E_s \varepsilon'_s A'_s = 200000 \cdot 0.00173 \cdot 2513$$

(applied at 50 mm from the upper surface)

the compression net force ($C + C'$) results to be applied at 45 mm from the beam upper surface, then the horizontal strut has the axis at $675 - 50 - 45 = 580$ mm from the tension rod T2.

Calculation of the truss rods stresses

Node 1 equilibrium:

$$\alpha = \arctg \frac{580}{425} = 53,77^\circ \quad C_1 = \frac{R}{\sin \alpha} = 620 \text{ kN} \quad T_2 = C_1 \cdot \cos \alpha = 366 \text{ kN}$$

$$\text{Node 2 equilibrium: } \beta = \arctg \frac{580}{725} = 38,66^\circ$$

$$C_2 \cos \beta + C_3 \cos 45^\circ = T_2 \quad C_2 = \frac{T_2}{\sin \beta + \cos \beta} = 260 \text{ kN}$$

$\Rightarrow$

$$C_2 \sin \beta = C_3 \sin 45^\circ \quad C_3 = \frac{\sin \beta}{\sin 45^\circ} \cdot C_2 = 230 \text{ kN}$$

Node 3 equilibrium:

$$T_1 = C_1 \sin \alpha + C_2 \sin \beta = 663 \text{ kN}$$

Tension rods design

$$\text{the tension rod } T_1 \text{ needs a steel area not lower than: } A_{s1} \geq \frac{663000}{391.3} = 1694 \text{ mm}^2$$

we use 5 stirrups $\phi 16$ double arm ($A_{s1} = 2000 \text{ mm}^2$)

$$\text{the tension rod } T_2 \text{ needs a steel area not lower than: } A_{s1} \geq \frac{366000}{391.3} = 935 \text{ mm}^2$$

we use 5 $\phi 16$ ($A_{s1} = 1000 \text{ mm}^2$).

Truss b) $R = R_{Sdu} / 2 = 500 \text{ kN}$

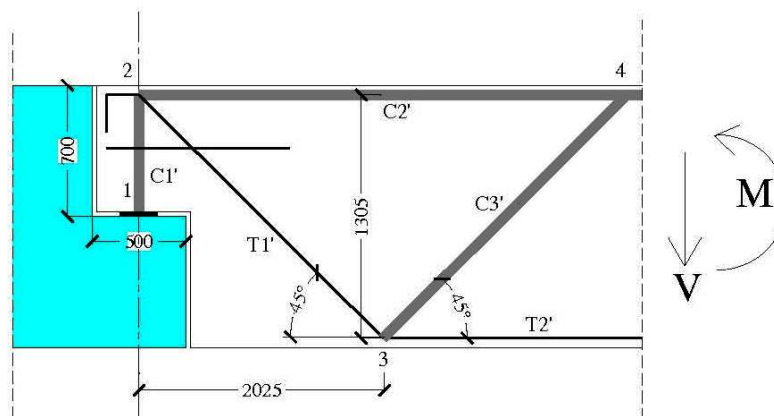


Fig. 6.16. Calculation scheme for the truss b bars stresses.

Calculation of the truss rods stresses

node 1 equilibrium $C'_1 = 500 \text{ kN}$

node 2 equilibrium

$$C'_2 = C'_1 = 500 \text{ kN}$$

$$T'_1 = \sqrt{2} \cdot C'_1 = 707 \text{ kN}$$

node 3 equilibrium

$$C'_3 = T'_1 = 707 \text{ kN}$$

$$T'_2 = (T'_1 + C'_3) \cdot \cos 45^\circ = 1000 \text{ kN}$$

Tension rods

for tension rod T'_2 it is necessary to adopt a steel area not lower than:

$$A_{s1} \geq \frac{1000000}{391.3} = 2556 \text{ mm}^2$$

$6\phi 24 = 2712 \text{ mm}^2$ are adopted,

a lower reinforcement area would be sufficient for tension rod T'_1 but for question of bar anchoring the same reinforcement as in T'_2 is adopted.

EXAMPLE 6.12 Pile cap [EC2 clause 6.5]

Geometry: 4500 x 4500 mm plinth (thickness $b=1500$ mm), 2000 x 700 mm columns, diameter 800 mm piles

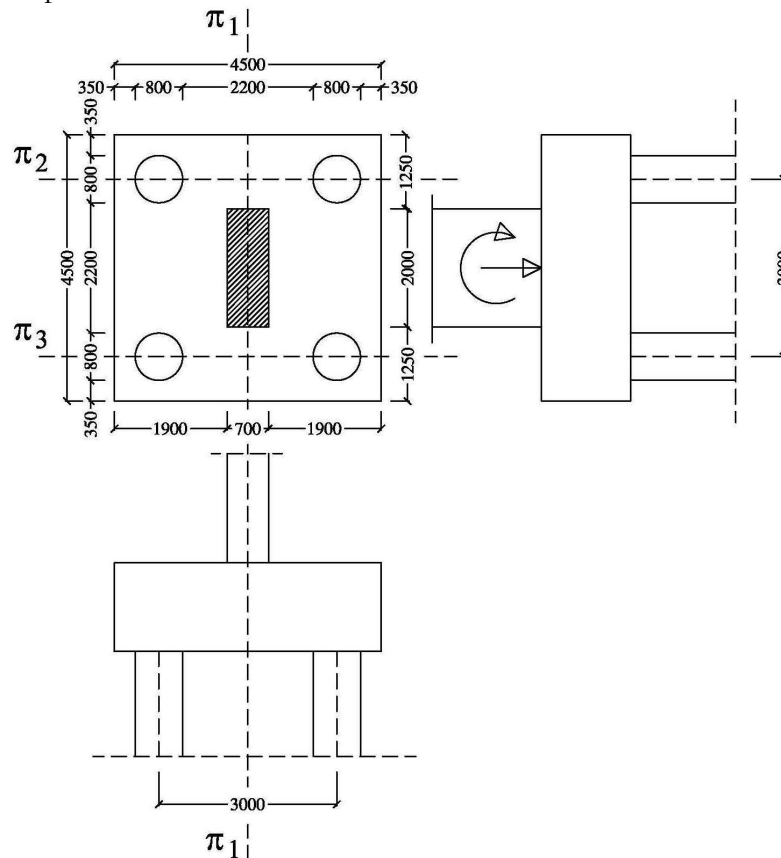


Fig. 6.17. Log plinth on pilings.

Materials: concrete C25/30 $f_{ck} = 25$ MPa, steel B450C $f_{yk} = 450$ MPa

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 25}{1.5} = 14.17 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2$$

Nodes compression resistance (same values as in the example 6.8)

Compressed nodes $\sigma_{1Rd,max} = 15 \text{ N/mm}^2$

tied-compressed nodes with tension rods in one direction $\sigma_{2Rd,max} = 12.75 \text{ N/mm}^2$

tied-compressed nodes with tension rods in different directions $\sigma_{3Rd,max} = 11.22 \text{ N/mm}^2$

Pedestal pile

$$N_{sd} = 2000 \text{ kN}$$

$$M_{sd} = 4000 \text{ kNm}$$

Tied reinforcement in the pile: $8 \phi 26$ ($A_s = 4248 \text{ mm}^2$)

The compressive stress F_c in the concrete and the steel tension F_s on the pedestal pile are evaluated from the ULS verification for normal stresses of the section itself:

$$F_s = f_{yd} A_s = 391.3 \cdot 4248 = 1662242 \text{ N} = 1662 \text{ kN}$$

$$N_{sd} = 0.8 b x f_{cd} - F_s \Rightarrow 2000000 = 0.8 \cdot 700 \cdot x \cdot 14.17 - 1662242 \Rightarrow x = 462 \text{ mm}$$

The compressive stress in the concrete is:

$$C = 0.8 b x f_{cd} = 0.8 \cdot 700 \cdot 462 \cdot 14.17 = 3666062 \text{ N} = 3666 \text{ kN}$$

(applied at $0.4 \cdot x \cong 185 \text{ mm}$ from the upper surface)

piles stress

pile stresses are evaluated considering the column actions transfer in two steps:

in the first step, the transfer of the forces F_c e F_s happens in the plane π_1 (Fig. 6.17) till to the orthogonal planes π_2 and π_3 , then in the second step the transfer is inside the planes π_2 and π_3 till to the piles;

the truss-tie beam in Fig. 6.18 is relative to the transfer in the plane π_1 :

compression: $A' = (M_{sd}/3.00 + N_{sd}/2) = (4000/3.00 + 2000/2) = 2333 \text{ kN}$

tension: $B' = (M_{sd}/3.00 - N_{sd}/2) = (4000/3.00 - 2000/2) = 333 \text{ kN}$

for each compressed pile: $A = A'/2 = 1167 \text{ kN}$

for each tied pile: $B = B'/2 = 167 \text{ kN}$

In the evaluation of stresses on piles, the plinth own weight is considered negligible.

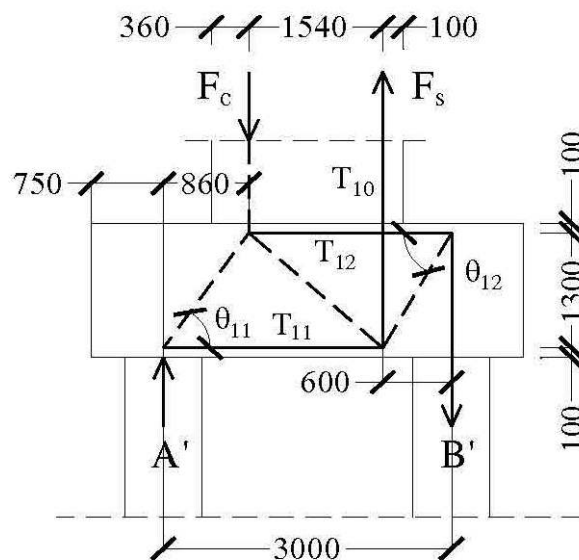


Fig. 6.18. S&T model in the plane π_1 .

$$\theta_{11} = \arctg (1300 / 860) = 56.5^\circ$$

$$\theta_{12} = \arctg (1300 / 600) = 65.2^\circ$$

$$T_{10} = F_s = 1662 \text{ kN}$$

$$T_{11} = A' \cot \theta_{11} = 2333 \cot 26.5^\circ = 1544 \text{ kN}$$

$$T_{12} = B' \cot \theta_{12} = 333 \cot 65.2^\circ = 154 \text{ kN}$$

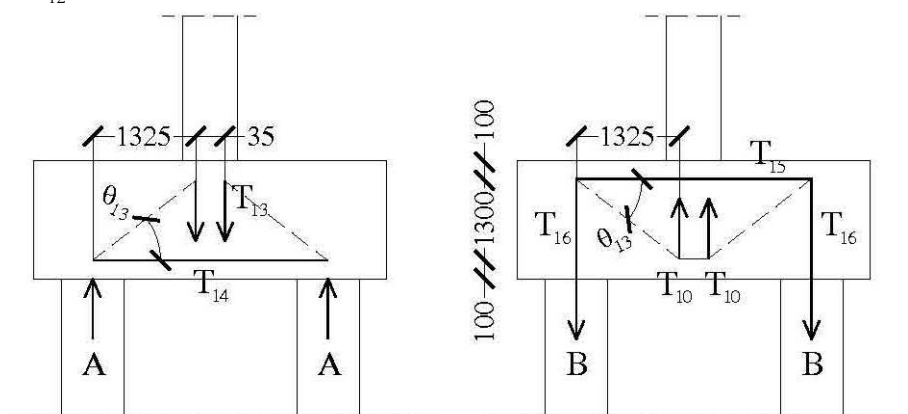


Fig. 6.19. Trusses in plan π_2 and in plan π_3 .

$$\theta_{13} = \arctg (1300 / 1325) = 44.5^\circ$$

$$T_{13} = A = 1167 \text{ kN}$$

$$T_{14} = A \cot \theta_{13} = 1167 \cot 44.5^\circ = 1188 \text{ kN}$$

$$T_{15} = B \cot \theta_{13} = 167 \cot 44.5^\circ = 170 \text{ kN}$$

$$T_{16} = B = 167 \text{ kN}$$

design of tension rods

Table 6.3.

Tension rod	Force (kN)	Required reinforcement (mm ²)	Bars
10 (plinth tied reinforcement)	1662	4248	8 ϕ 26
11	1544	3946	9 ϕ 24
12	154	394	1 ϕ 12/20 (6 ϕ 12)
13	1167	2982	stirrups 10 ϕ 20
14	1188	3036	7 ϕ 24
15	170	434	1 ϕ 12/20 (5 ϕ 12)
16	167	427	Pile reinforcement

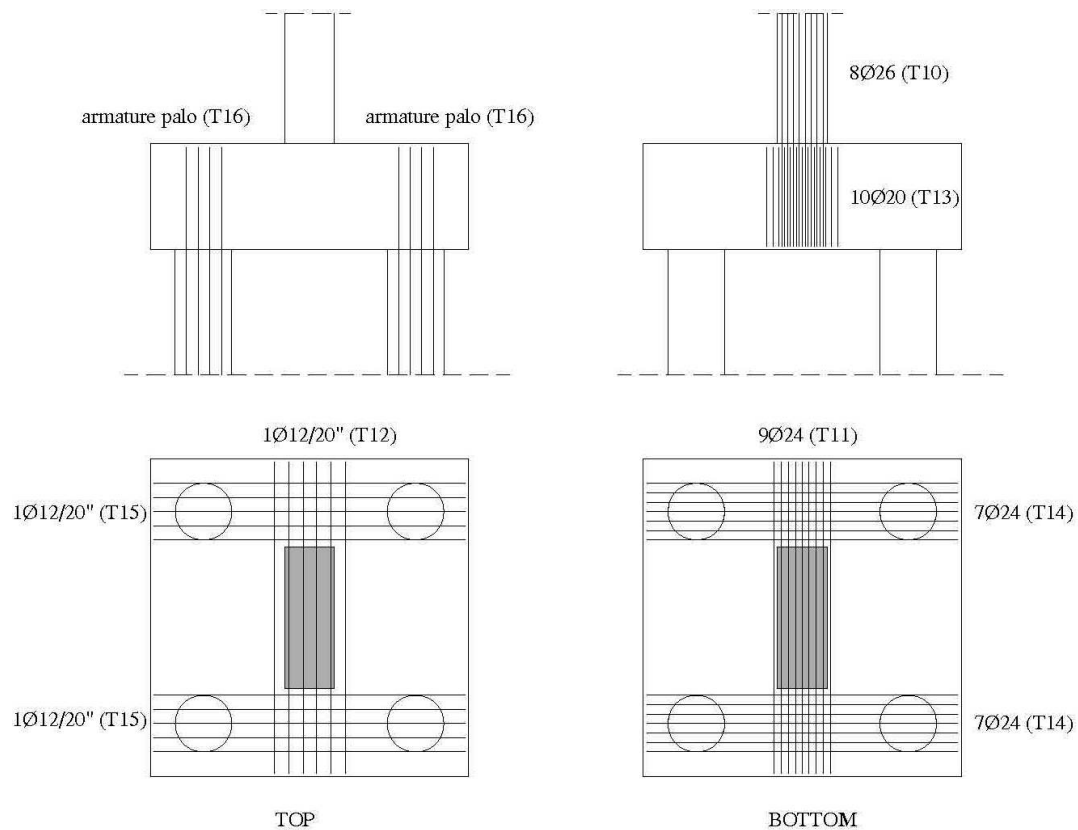


Fig. 6.20. Schematic placement of reinforcements.

Nodes verification

Concentrated nodes are only present at the pedestal pile and on the piles top. In these latter, the compressive stresses are very small as a consequence of the piles section large area.:

$$\sigma_c = \frac{A}{\pi \cdot r^2} = \frac{2333000}{\pi \cdot 400^2} = 4.64 \text{ N/mm}^2$$

EXAMPLE 6.13 Variable height beam [EC2 clause 6.5]

Geometry: length 22500 mm, rectangular section 300 x 3500 mm and 300 x 2000 mm

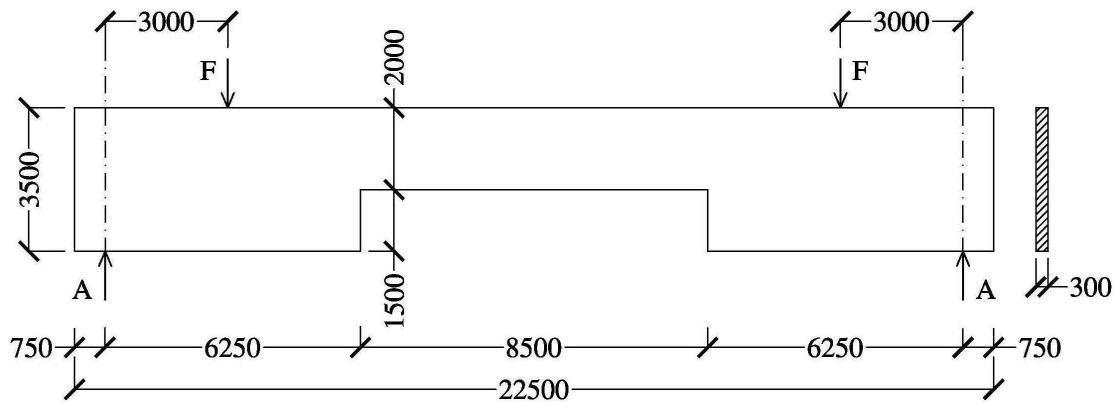


Fig. 6.21. Variable height beam

Materials: concrete C30/37 $f_{ck} = 30$ MPa, steel B450C $f_{yk} = 450$ MPa

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 30}{1.5} = 17 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2$$

Nodes compressive resistance:

compressed nodes (EC2 eq. 6.60)

$$\sigma_{1Rd,max} = k_1 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = 1.18 \left(1 - \frac{30}{250}\right) 17 = 17.65 \text{ N/mm}^2$$

tied-compressed nodes with tension rods in one direction

$$\sigma_{2Rd,max} = k_2 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = \left(1 - \frac{30}{250}\right) 17 = 14.96 \text{ N/mm}^2$$

tied-compressed nodes with tension rods in different directions

$$\sigma_{3Rd,max} = k_3 \frac{\left(1 - \frac{f_{ck}}{250}\right)}{0.85} f_{cd} = 0.88 \left(1 - \frac{30}{250}\right) 17 = 13.16 \text{ N/mm}^2$$

loads

$$F = 1200 \text{ kN}$$

(the own weight of the beam is negligible)

strut&tie model identification

Beam partitioning in two regions B and D

The region standing on the middle section is a continuity region (B), while the remaining part of the beam is composed of D type regions.

The boundary conditions for the stress in the region B.

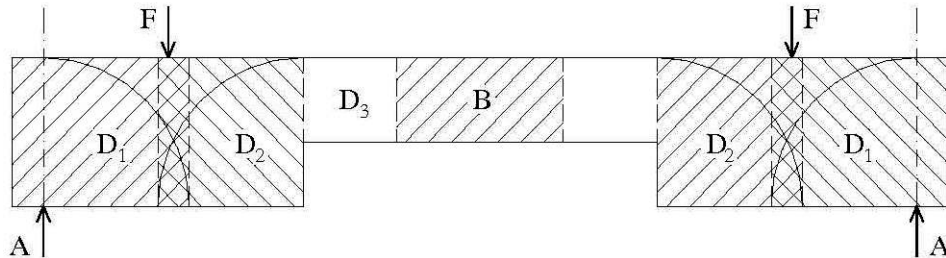


Fig. 6.22. Identification of B and D regions.

Stresses evaluation for the bars of the S&T model

$$T_{\max} = 1200 \text{ kN}$$

$$M_{\max} = 1200 \cdot 3.00 = 3600 \text{ kNm} = 3.6 \cdot 10^9 \text{ Nmm}$$

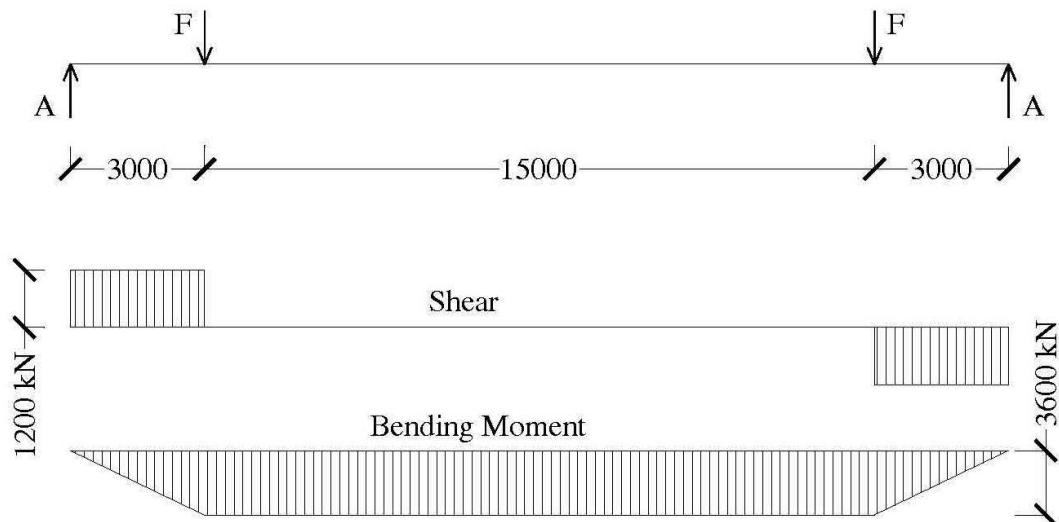


Fig. 6.23. Shear and bending moment diagrams.

Calculation of stresses in the region B

The stress-block diagram is used for the concrete compressive stresses distribution;
rotational equilibrium:

$$f_{cd} \cdot 0.8 \cdot x \cdot b \cdot (d - 0.4 x) = 3.6 \cdot 10^9$$

$$17 \cdot 0.8 \cdot x \cdot 300 \cdot (1900 - 0.4 x) = 3.6 \cdot 10^9$$

$$7752000 \cdot x - 1632 \cdot x^2 = 3.6 \cdot 10^9 \quad \Rightarrow \quad x = 522 \text{ mm}$$

$$C = f_{cd} \cdot 0.8 \cdot x \cdot b = 17 \cdot 0.8 \cdot 522 \cdot 300 = 2129760 \text{ N} = 2130 \text{ kN}$$

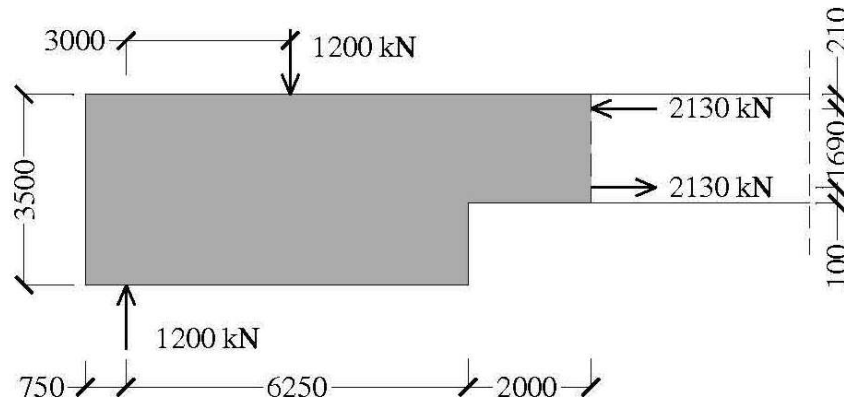
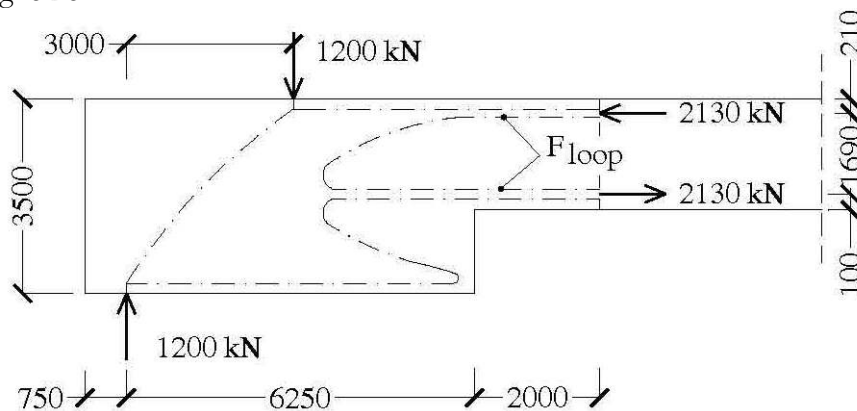
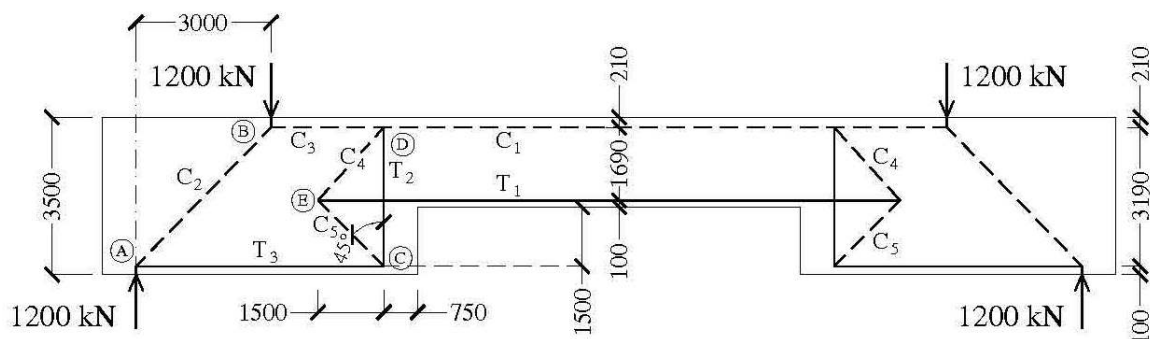
Identification of boundary stresses in the region D**Fig. 6.24.** Reactions and boundary stresses in the region D.strut&tie model

Fig. 6.25 shows the load paths characterized by Schlaich in the strut&tie model identification, shown in Fig. 6.26.

**Fig. 6.25.** Load paths.**Fig. 6.26.** Strut and tie model.

The strut C_2 tilting is

$$\theta = \arctg \frac{3190}{3000} = 46.76^\circ$$

while the strut C_4 tilting is

$$\theta_1 = \arctg \frac{1690}{1500} = 48.41^\circ.$$

The following table reports the value for the stresses in the different beam elements.

Table 6.4.

C_1	See stresses evaluation in the region B	2130 kN
T_1	$T_1 = C_1$	2130 kN
C_2	$C_2 = F/\sin \theta$ (Node A vertical equilibrium)	1647 kN
T_3	$T_3 = C_2 \cos \theta$ (Node A horizontal equilibrium.)	1128 kN
T_2	$T_2 = T_3$, because C_5 is 45° tilted (node C equil.)	1128 kN
C_3	$C_3 = C_2 \cos \theta = T_3$ (Node B horizontal equil.)	1128 kN
F_{loop}	$F_{\text{loop}} = C_1 - C_3$	1002 kN
C_4	$C_4 = F_{\text{loop}}/\cos \theta$	1509 kN
C_5	$C_5 = T_2 \cdot \sqrt{2}$ (Node C vertical equil.)	1595 kN

Steel tension rods design

EC2 point 9.7 suggests that the minimum reinforcement for the wall beams is the 0.10 % of the concrete area, and not less than $150 \text{ mm}^2/\text{m}$, and it has to be disposed on both sides of the structural member and in both directions. Bars $\phi 12 / 20''$ ($=565 \text{ mm}^2/\text{m} > 0.10 \% \cdot 300 \cdot 1000 = 300 \text{ mm}^2/\text{m}$ e di $150 \text{ mm}^2/\text{m}$) are used.

The following table reports the evaluation for the reinforcement area required for the three tension bars T_1 , T_2 and T_3 .

Table 6.5.

T_1	$A_s = 2.13 \cdot 10^6 / 391.3 = 5443 \text{ mm}^2$	$18 \phi 20 = 5655 \text{ mm}^2$
T_2	$A_s = 1.128 \cdot 10^6 / 391.3 = 2883 \text{ mm}^2$ on 1.50 m length	stirrups $\phi 12 / 10''$ 2 legs = $2260 \text{ mm}^2/\text{m}$ ($2260 \cdot 1.50 = 3390 \text{ mm}^2$)
T_3	$A_s = 1.128 \cdot 10^6 / 391.3 = 2883 \text{ mm}^2$ on two layers	$10 \phi 20 = 3142 \text{ mm}^2$

Verification of nodes

Node A (left support)

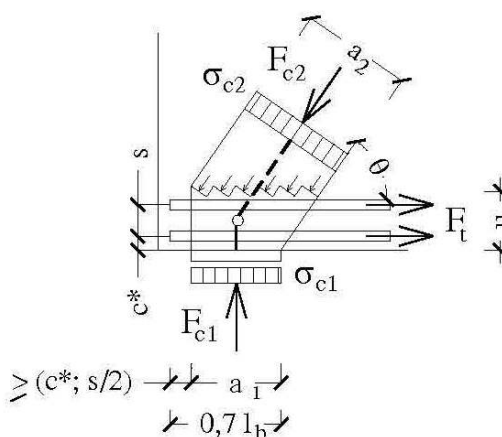


Fig. 6.27. Node A.

tied-compressed nodes with tension rods in one direction [(6.61)-EC2]

$$\sigma_{2Rd,max} = 14.96 \text{ N/mm}^2$$

Loading plate area:

$$A \geq \frac{F_{c1}}{\sigma_{2Rd,max}} = \frac{1.2 \cdot 10^6}{14.96} = 80214 \text{ mm}^2$$

a 300 x 300 mm plate ($A = 90000 \text{ mm}^2$) is used

the reinforcement for the tension rod T_3 is loaded on two layers (Fig. 6.27): $u = 150 \text{ mm}$

$$a_1 = 300 \text{ mm}$$

$$a_2 = 300 \sin 46.76^\circ + 150 \cos 46.76^\circ = 219 + 103 = 322 \text{ mm}$$

$$\sigma_{c2} = \frac{1.647 \cdot 10^6}{300 \cdot 322} = 17.05 \text{ N/mm}^2 > 14.96 \text{ N/mm}^2$$

u has to be higher (it is mandatory a reinforcement on more than two layers, or an increase of the plate length); this last choice is adopted, and the length is increased from 300 to 400 mm:

$$a_2 = 400 \sin 46.76^\circ + 150 \cos 46.76^\circ = 291 + 103 = 394 \text{ mm}$$

$$\sigma_{c2} = \frac{1.647 \cdot 10^6}{300 \cdot 394} = 13.93 \text{ N/mm}^2 \leq 14.96 \text{ N/mm}^2$$

Node B

Compressed nodes

$$\sigma_{1Rd,max} = 17.65 \text{ N/mm}^2$$

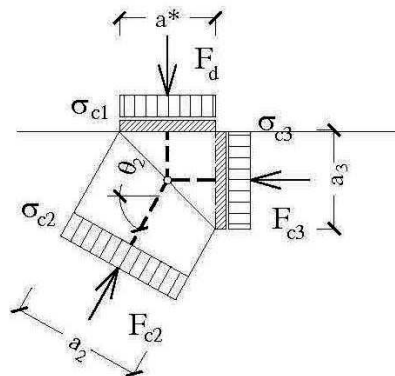


Fig. 6.28. Node B.

$a_3 = 522 \text{ mm}$ (coincident with the depth of the neutral axis in the region B)

$$\sigma_{c3} = \frac{C_3}{300 \cdot 522} = \frac{1.128 \cdot 10^6}{300 \cdot 522} = 7.2 \text{ N/mm}^2 \leq 17.65 \text{ N/mm}^2$$

load plate dimensions:

$$a^* \geq \frac{1.2 \cdot 10^6}{300 \cdot 17.65} = 227 \text{ mm}$$

a 300 x 300 mm plate is used

Strut verification

The compressive range for each strut (only exception, the strut C_1 , which stress has been verified before in the forces evaluation for the region B) can spread between the two ends, in this way the maximal stresses are in the nodes.

The transversal stress for the split of the most stressed strut (C_2) is:

$$T_s \leq 0.25 \cdot C_2 = 0.25 \cdot 1647 = 412 \text{ kN};$$

and then, for the reinforcement required to carry this stress:

$$A_s = \frac{412000}{391.3} = 1053 \text{ mm}^2,$$

then the minimum reinforcement (1 ϕ 12 / 20" on both sides and in both directions, that is $a_s = 1130 \text{ mm}^2/\text{m}$) is enough to carry the transversal stresses.

EXAMPLE 6.14. 3500 kN concentrated load [EC2 clause 6.5]

3500 kN load on a 800x500 rectangular column by a 300x250 mm cushion

Materials:

concrete	C30/37	$f_{ck} = 30 \text{ MPa}$,
steel	B450C	$f_{yk} = 450 \text{ MPa}$, $E_s = 200000 \text{ MPa}$

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} = \frac{0.85 \cdot 30}{1.5} = 17 \text{ N/mm}^2,$$

$$f_{yd} = \frac{f_{yk}}{1.15} = \frac{450}{1.15} = 391.3 \text{ N/mm}^2,$$

loading area

$$A_{c0} = 300 \cdot 250 = 75000 \text{ mm}^2$$

dimensions of the load distribution area

$$d_2 \leq 3 d_1 = 3 \cdot 300 = 900 \text{ mm}$$

$$b_2 \leq 3 b_1 = 3 \cdot 250 = 750 \text{ mm}$$

maximal load distribution area

$$A_{c1} = 900 \cdot 750 = 675000 \text{ mm}^2$$

load distribution height

$$h \geq (b_2 - b_1) = 750 - 250 = 500 \text{ mm}$$

$$\Rightarrow h = 600 \text{ mm}$$

$$h \geq (d_2 - d_1) = 900 - 300 = 600 \text{ mm}$$

Ultimate compressive stress

$$F_{Rdu} = A_{c0} f_{cd} \sqrt{A_{c1} / A_{c0}} = 75000 \cdot 17 \cdot \sqrt{675000 / 75000} = 3825 \text{ kN} \leq 3.0 f_{cd} A_{c0} = 3.0 \cdot 17 \cdot 75000 = 3.825 \cdot 10^6 \text{ N}$$

It is worth to observe that the F_{Rdu} upper limit corresponds to the the maximal value $A_{c1} = 3 A_{c0}$ for the load distribution area, just as in this example; the 3500kN load results to be lower than F_{Rdu} .

Reinforcement design

Point [6.7(4)-EC2] recommends the use of a suitable reinforcement capable to sustain the transversal **shrinkage** stresses and point [6.7(1)P-EC2] sends the reader to paragraph [(6.5)-EC2] to analyse this topic.

In this case there is a partial discontinuity, because the strut width (500 mm) is lower than the distribution height (600 mm), then:

$$a = 250 \text{ mm}$$

$$b = 500 \text{ mm}$$

$$T = \frac{F}{4} \frac{b-a}{b} = \frac{3500}{4} \frac{500-250}{500} = 437.5 \text{ kN}$$

the steel area required to carry 'T' is:

$$A_s \geq \frac{T}{f_{yd}} = \frac{437500}{391.3} = 1118 \text{ mm}^2$$

using 10 mm diameter bars, 15 bars are required for a total area of:

$$A_s = 15 \cdot 78.5 = 1178 \text{ mm}^2.$$

EXAMPLE 6.15 Slabs^{1,2} [EC2 clause 5.10 – 6.1 – 6.2 – 7.2 – 7.3 – 7.4]

As two dimensional member a prestressed concrete slab is analysed: the actual structure is described in the following point.

6.15.1 Description of the structure

The design example proposed in this section is related to a railway bridge deck made up by a continuous slab on three spans with two orders of prestressing tendons (longitudinal and transverse prestressing). The slab is designed in category A (see Eurocode 2, Part 2, table 4.118) for fatigue reasons. The deck rests on abutments and circular piers and has a overall breadth of 13.60 m with two side-walks of 1.40 m width, two ballast retaining walls and, in the middle, two track spacing of 5.0 m. The slab presents a constant thickness of 1.50 m for a central zone 7.0 m width, whilst is tapered towards the extremity with a final height of 0.6 m. Figs 6.29 and 6.30 represent the principal geometric dimension of the slab bridge and supports' scheme.

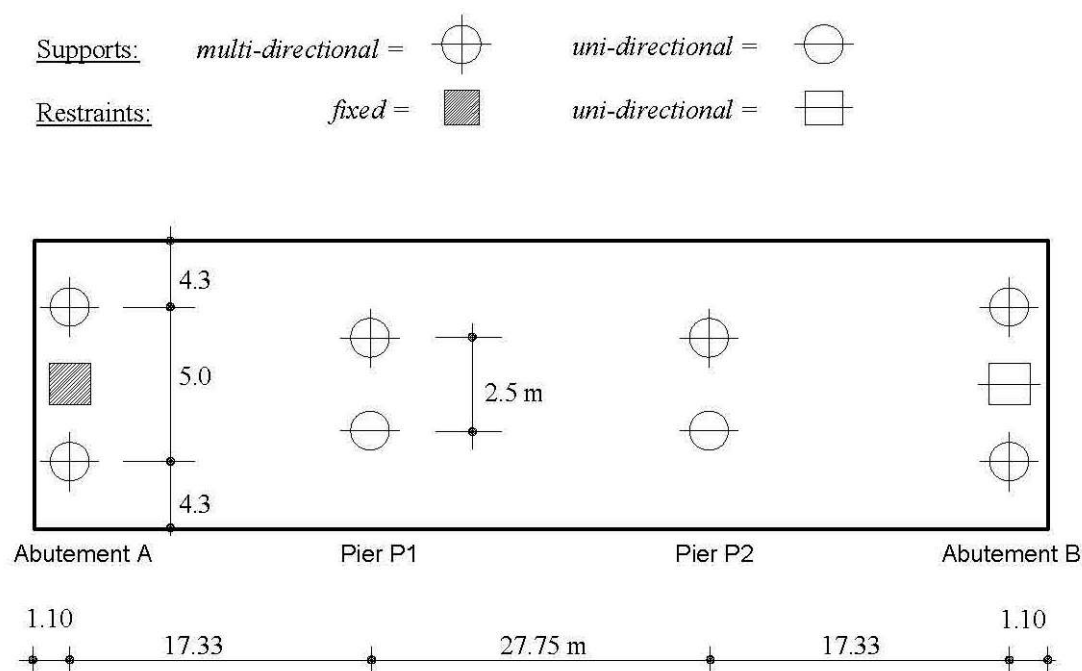


Fig. 6.29. Plan view of the structure and supports' scheme

¹ Example taken from example 7.2 “slabs” by prof. Mancini, FIB Bulletin n°3, “Structural Concrete Textbook on Behaviour, Design and Performance Vol. 3: Durability - Design for Fire Resistance - Member Design - Maintenance, Assessment and Repair - Practical aspects” Manual - textbook (292 pages, ISBN 978-2-88394-043-7, December 1999).

² See too EN 1992-2 Eurocode 2, bridge design.

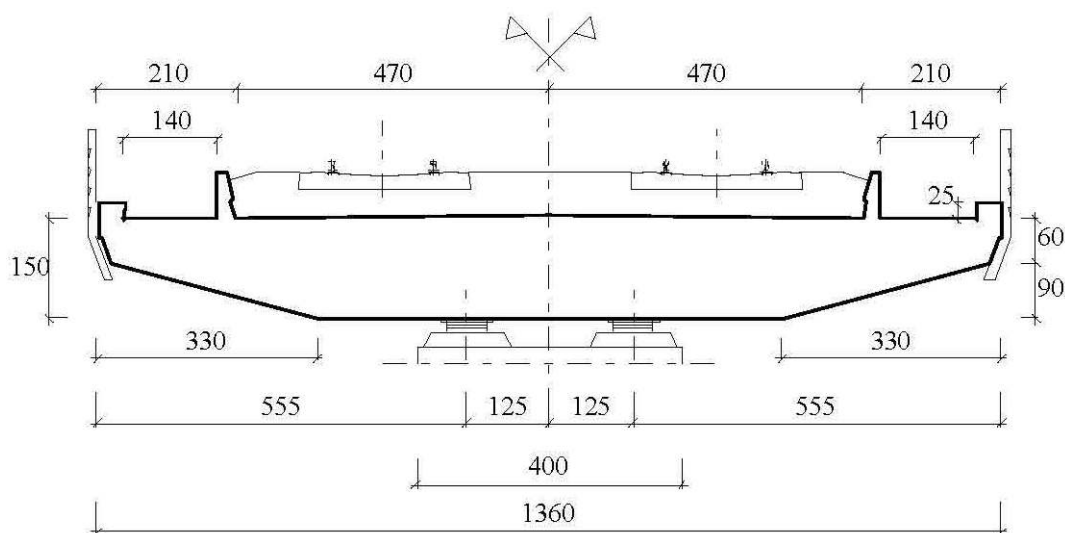


Fig. 6.30. Geometric dimensions of bridge cross section

Material properties

The following materials properties have been considered:

- Concrete Grade 35:
 - compressive design strength: $f_{cd} = 23.3 \text{ MPa}$;
 - compressive resistance for uncracked zones: $f_{cd1} = 17.1 \text{ MPa}$;
 - compressive resistance for cracked zones: $f_{cd2} = 12.0 \text{ MPa}$;
 - mean value of tensile strength: $f_{ctm} = 3.23 \text{ MPa}$;
 - modulus of elasticity: $E_c = 29.7 \cdot 10^3 \text{ MPa}$;
 - shear modulus: $G = 12.4 \cdot 10^3 \text{ MPa}$;
 - Poisson ratio: $\nu = 0.2$
- Prestressing steel, (strands $\phi 0.6''$):
 - 0.1% proof stress: $f_{p0.1k} = 1600 \text{ MPa}$
 - total elongation at maximum load: $\varepsilon_{pu} > 35\%$
 - modulus of elasticity: $E_p = 195.0 \cdot 10^3 \text{ MPa}$;
- Reinforcing steel, Grade 500:
 - design strength: $f_{yk} = 500.0 \text{ MPa}$;
 - modulus of elasticity: $E_s = 200.0 \cdot 10^3 \text{ MPa}$.

Concrete cover

As environmental condition an Exposure Class 2 may be considered (Humid environment with frost: exterior components exposed to frost).

The minimum concrete cover for Class 2 is equal to 25 mm, which should be added to the tolerance value of 10mm; as a consequence the nominal value for concrete cover results:

$$c_{nom} = c_{min} + 10 = 25 + 10 = 35 \text{ mm}$$

adopted in the calculations.

6.15.2 Structural model

To evaluate the internal actions on the structure a linear FEM analysis has been performed adopting shell elements to represent the reinforced slab; this kind of element takes account of all the slab and plate components as well as the out-of-plane shear forces. The thickness of shell elements has been assumed constant for the inner zone of the slab and stepped to fashion the tapered extremity. In Fig 6.31 and 6.32 the FEM model is sketched and the different thick of the element is reported too.

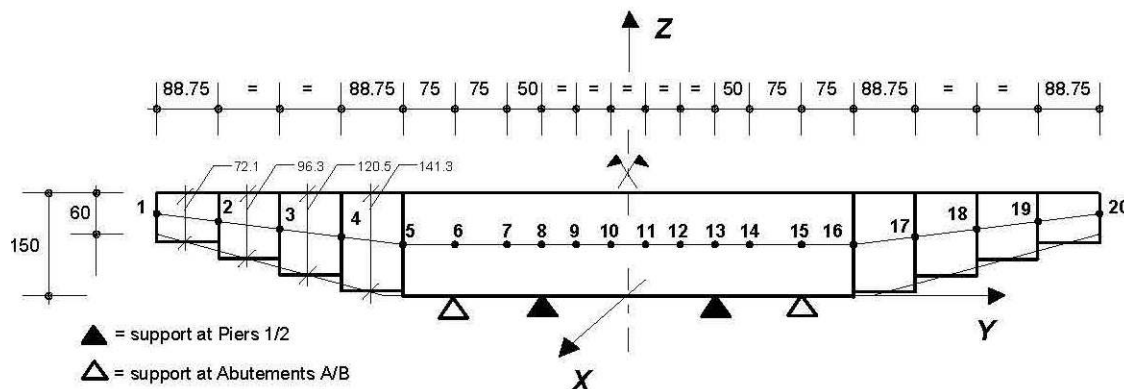


Fig. 6.31. Transverse view of FEM model

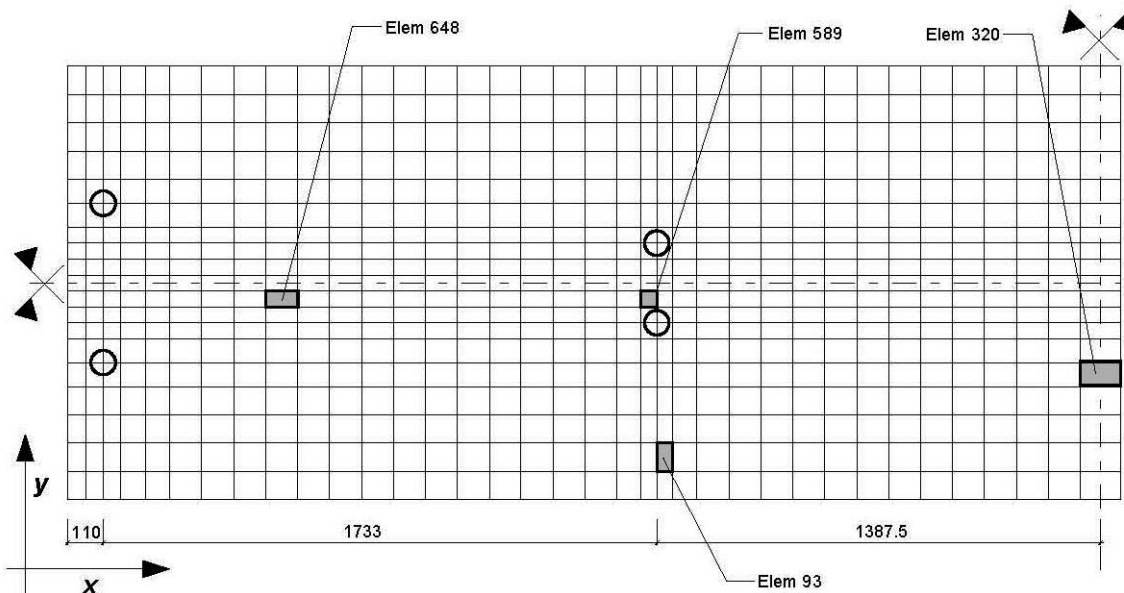


Fig. 6.32. Plan of FEM model and considered elements

The adopted shell elements are oriented with the following guidelines:

- local axis 2 is oriented as global axis Y of the deck;
- local axis 3 is oriented in the opposite direction of global axis X of the deck;
- local axis 1 is oriented as global axis Z of the deck.

Positive forces for FEM program output are reported in Fig. 6.33:

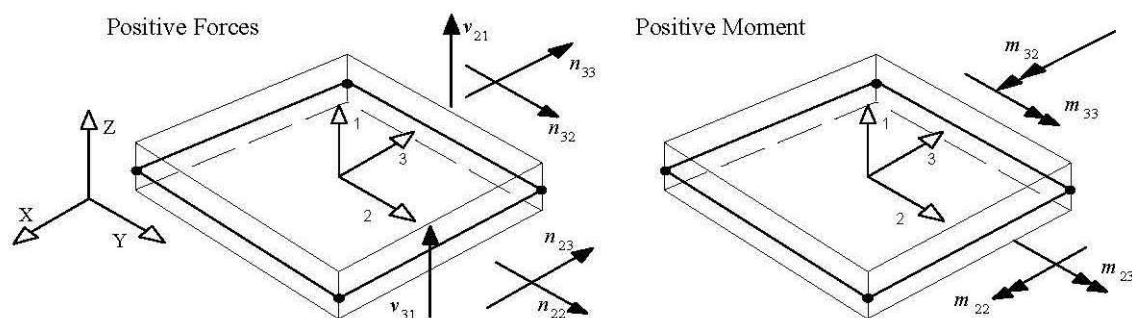


Fig. 6.33. Positive actions for FEM elements

Restraints

The external restraints have been introduced in the FEM model considering their real geometric dimensions; thus, few nodes have been restrained by means of spring elements in order to represent only an individual restraint or support. Fig. 6.34 shows a symbolic notation for the external restraints with the nodes involved.

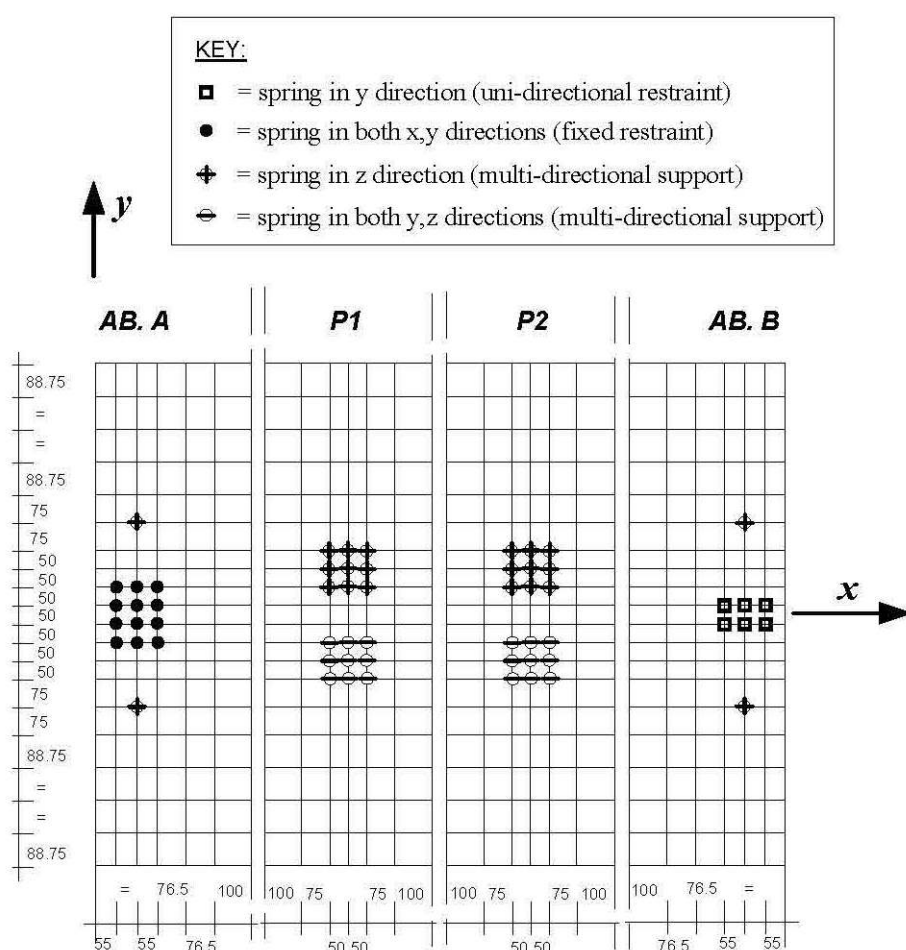


Fig. 6.34. External restraints on the FEM model

The elastic constant of the spring restraining elements is calculated to have the same stiffness of the substructure (abutments or piers) on which the slab is rested. For the x and y directions, it may be assumed that the pier, or the abutment front wall, behaves like a

single column fixed at the base and free at his top, so that the relevant $K_{x/y}$ stiffness is valuable as:

$$K_{x/y} = \frac{3EI}{H^3}$$

where E is the Young modulus, I the inertia and H the height of the column. For the vertical direction, the intrinsic stiffness of pot-bearing is assumed, considering the substructure vertical behaviour as rigid.

For the sake of simplicity the calculus of the relevant stiffness is omitted and the final values of the spring constants are reported in Table 6.6 .

Table 6.6. *Stiffness for restraining elements*

Location	$K_{x,tot}$ 10 ⁶ kN/m	$K_{y,tot}$ 10 ⁶ kN/m	$K_{z,tot}$ 10 ⁶ kN/m
Abutment A	9.55	178.80	10.02
Pier P1	-	4.74	11.61
Pier P2	-	2.66	11.61
Abutment B	-	2.78	10.02

It can be noticed that the previous values are referred to the overall stiffness of the restraint, thus the elastic constant of any individual spring element may be obtained dividing the K values of Table 6.6 by the number of element representing the restraint or the supports.

Prestressing forces

Two orders of prestressing tendons are arranged (in longitudinal and transverse directions) in order to avoid any tensile stress in concrete at service (required by railway code). The initial tensile stress of tendon is:

$$\sigma_{po,max} = 0.85 f_{p0.1k} = 0.85 \times 1600 = 1360 \text{ MPa.}$$

The number of tendons is 39 for the longitudinal direction and 64 for the transverse one. Each tendon is built up with 19 strands $\phi 0.6''$ having an area of 1.39 cm². Fig. 6.35 reports tendon's layout for half deck, being symmetrically disposed.

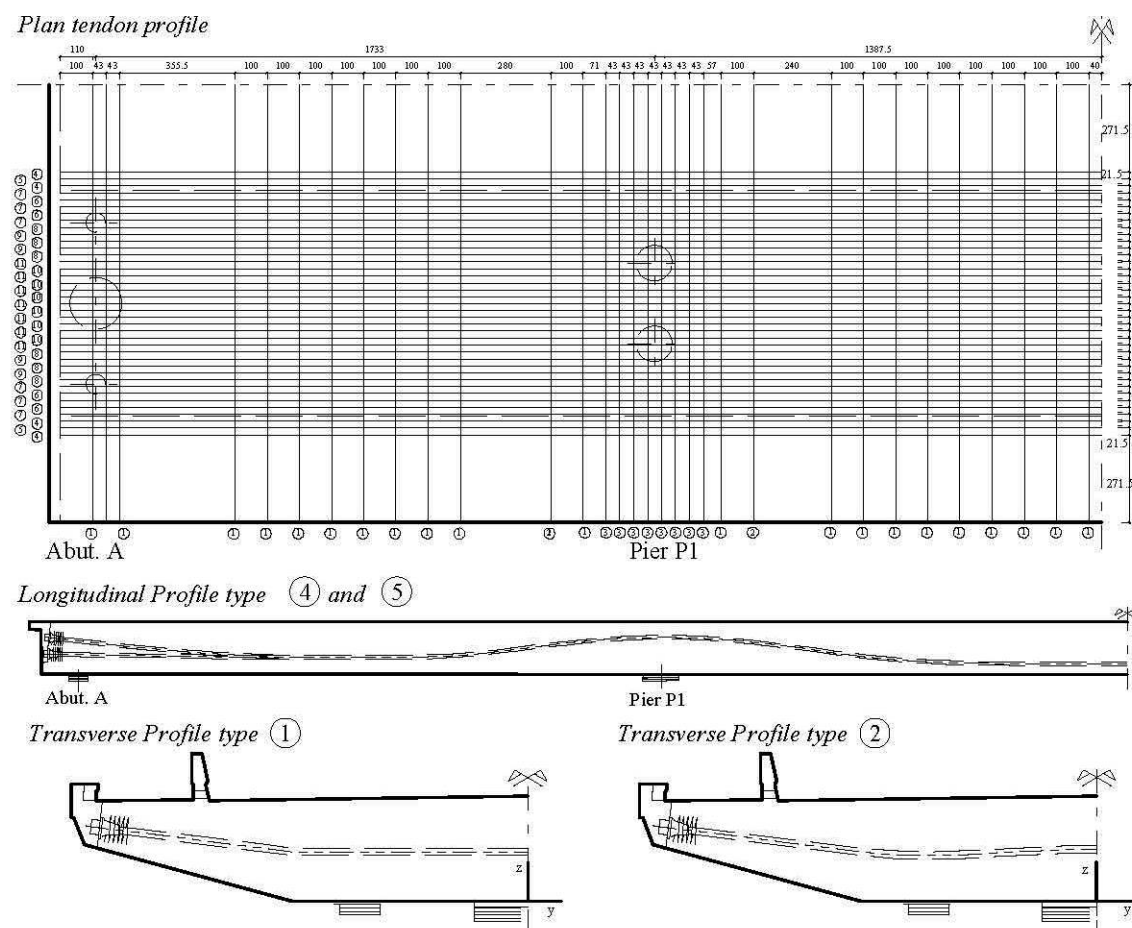


Fig. 6.35. Plan and principal section of tendon layout

Immediate losses of prestressing due to friction have been evaluated by means of the following expression:

$$\sigma_{po}(x) = \sigma_{po,max} \cdot e^{-\mu(\alpha + kx)}$$

with:

$\mu = 0.19$ coefficient of friction between the tendons and their sheathing;

$k = 0.01$ rad/m unintentional angular deviation.

Prestressing has to be introduced in the FEM model in order to calculate the hyperstatic actions that arise in the structural scheme. Considering prestressing as an external load, it is possible to introduce it by means of two inclined forces at anchorages (representing actions at the extremity) and of a system of equivalent loads along tendon's profile (representing tendon curvature and losses due to friction): these actions per tendon, should be applied consistently at the nodes of FEM model.

The equivalent loads may be calculated subdividing the tendon profile into elementary segments and evaluating the internal action able to equilibrate the external one due to end actions deriving by the prestressing.

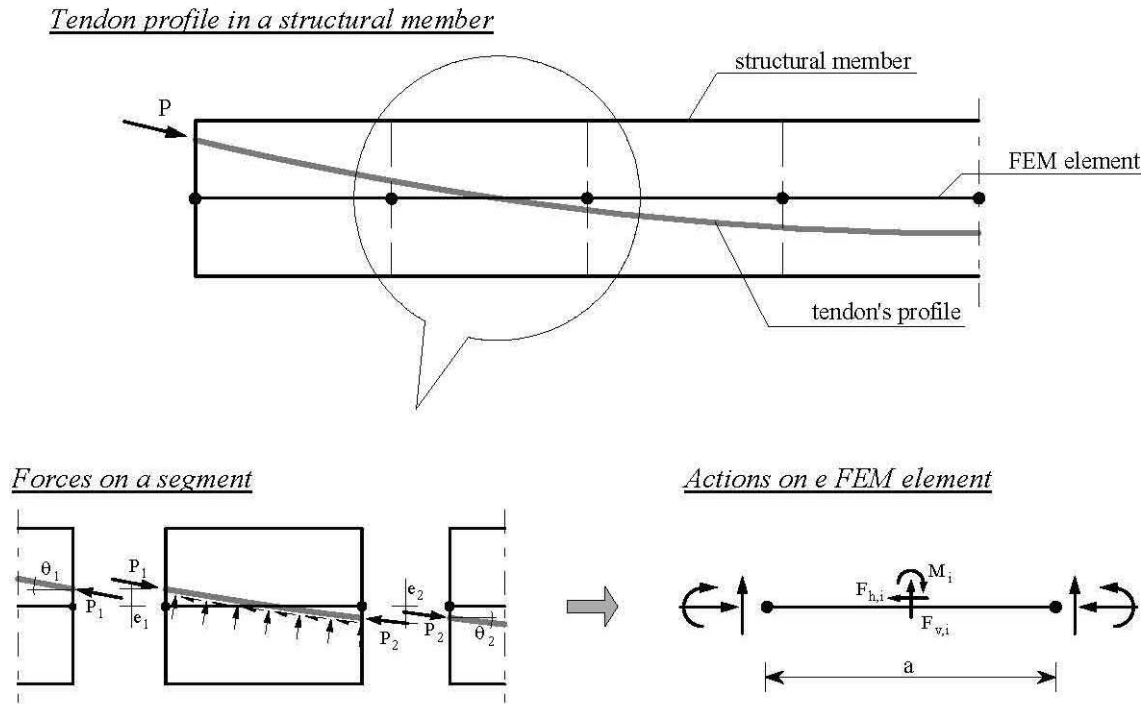


Fig. 6.36. Effect of prestressing on a segment and equivalent loads

Fig. 6.36 represents the forces acting on a segment of concrete due to a curved prestressing tendon; if the inclination of the cable changes from θ_1 to θ_2 while the prestress force changes from P_1 to P_2 due to friction, the equilibrating vertical and horizontal forces in the i -segment result:

$$F_{v,i} = P_2 \sin \theta_2 - P_1 \sin \theta_1 \quad ; \quad F_{h,i} = P_2 \cos \theta_2 - P_1 \cos \theta_1$$

while the balancing moment turns out:

$$M_i = (P_2 \cos \theta_2 e_2 - P_1 \cos \theta_1 e_1) - (P_2 \sin \theta_2 - P_1 \sin \theta_1) a/2$$

The above procedure should be repeated for all the segments. It can be notice that the forces at the end of each segment extremity are the same with opposite signs, depending on whether the right or the left segment is considered; these forces cancel out themselves with the exception at anchorages. Finally, for each tendon, the forces at the extremity of the cable plus the equilibrating system for each segment, shall be introduced in the FEM model.

The choice of the position of the elementary segments is relative to the kind of element adopted in the FEM model. If beam elements are used, it is possible to introduce a point load (or moment) whether along the element body or at nodes, consequently the segment extremities may be placed indifferently at nodes or at the middle of the element. With shell elements, only nodal forces can be considered so that it is necessary to place segment extremities within two sequential nodes; furthermore, due to the two-dimensional scheme, one has to consider the transverse position of the tendon that, in general, do not coincide with a nodal alignment. As a simple rule, the indications of Fig. 6.37 may be followed.

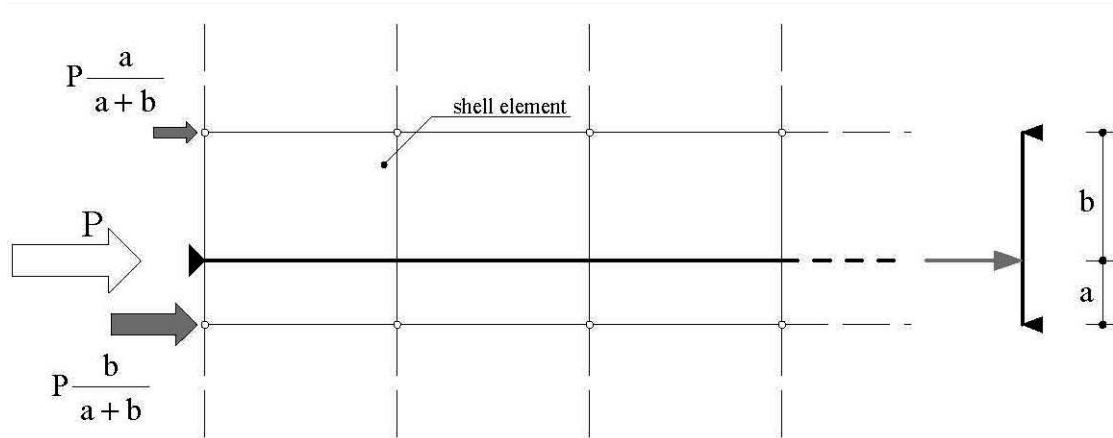


Fig. 6.37. Transverse distribution of prestressing

Time-dependent prestressing losses

Time-dependent losses of prestress may be evaluated by means of the following equation:

$$\Delta\sigma_{p,c+s+r} = \frac{\varepsilon_{cs}(t_{\infty}, t_0) E_s + \Delta\sigma_{pr} + \alpha \phi(t_{\infty}, t_0) (\sigma_{cg} + \sigma_{cp0})}{1 + \alpha \frac{A_p}{A_c} \left[\left(1 + \frac{A_c}{I_c} z_{cp}^2 \right) (1 + 0.8 \phi(t_{\infty}, t_0)) \right]}$$

where:

$\Delta\sigma_{p,c+s+r}$: loss of initial tendon stress due to creep and shrinkage of concrete and relaxation of steel, between time t_0 and time t_{∞} ;

$t_0 = 28$ days: age of concrete at prestressing time;

$t_{\infty} = 25550$ days.: corresponding to a life-time of 70 years;

$\varepsilon_{cs}(t_{\infty}, t_0)$: shrinkage strain at time t_{∞} calculated from:

$$\varepsilon_{cs}(t_{\infty}, t_0) = \varepsilon_{cs0} \times \beta_s(t_{\infty} - t_0) = -0.127 \times 10^{-3}$$

where: $\varepsilon_{cs0} = \varepsilon_s(f_{cm}) \times \beta_{RH}$ with:

$$\varepsilon_s(f_{cm}) = [160 + 10 \beta_{sc} (9 - f_{cm} / f_{cm0})] \times 10^{-6} = 0.000395$$

f_{cm} = mean compressive strength of concrete at 28 days = $f_{ck} + 8$ MPa;

$f_{cm0} = 10$ MPa;

$\beta_{sc} = 5$ for rapid hardening cements;

$$\beta_{RH} = -1.55 \left[1 - \left(\frac{RH}{100} \right)^3 \right] = -1.018;$$

$RH = 70$ % relative humidity of the ambient atmosphere;

$$\beta_s(t_{\infty} - t_0) = \sqrt{\frac{t_{\infty} - t_0}{0.035 \cdot h^2 + t_{\infty} - t_0}} = 0.574$$

$h = (2A_c / u) = 1217$ mm notional size of member;

$A_c = 17.43 \times 10^6 \text{ mm}^2$ gross section of the beam;

$u = 28640 \text{ mm}$ perimeter of the member in contact with the atmosphere;

$\phi(t_\infty, t_0)$: creep coefficient at time t_∞ calculated from:

$\phi(t_\infty, t_0) = \phi_0 \times \beta_c(t_\infty - t_0) = 1.5708$ where:

$\phi_0 = \phi_{RH} \times \beta(f_{cm}) \times \beta(t_0) = 1.598$ with

$$\phi_{RH} = 1 + \frac{1 - RH/100}{0.1 \sqrt[3]{h}} = 1.281;$$

$$\beta(f_{cm}) = \frac{5.3}{\sqrt{f_{cm}/f_{cm0}}} = 2.556;$$

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}} = 0.488$$

$$\beta_c(t_\infty - t_0) = \left(\frac{t_\infty - t_0}{\beta_H + t_\infty - t_0} \right)^{0.3} = 0.983 \quad \text{with}$$

$$\beta_H = 1.5 \left[1 + (0.012 RH)^{18} \right] h + 250 = 2155 > 1500 \rightarrow 1500$$

If the improved prediction model of chapter 3 is used, the following values for $\varepsilon_{cs}(t_\infty, t_0)$ and for $\phi(t_\infty, t_0)$ may be evaluated:

$$\varepsilon_{cs}(t_\infty, t_0) = 182.62 \times 10^{-6} \quad ; \quad \Phi(t_\infty, t_0) = 1.5754$$

in good agreement with the previous one, at least for creep value.

$\Delta\sigma_{pr}$: loss of prestressing due to relaxation of steel calculated for a reduced initial tensile stress of $\sigma_p = \sigma_{pgo} - 0.3 \Delta\sigma_{p,c+s+r}$ (where σ_{pgo} is the effective initial stress in tendons due to dead load and prestressing) and evaluated as percentage by the following formula:

$$\rho_t = \rho_{1000h} \left(\frac{t}{1000} \right)^{0.19} = \rho_{1000h} \times 3 \quad \text{where}$$

ρ_t = is the relaxation after t hours; for $t > 50$ years $\rho_t = \rho_{1000h} \times 3$;

ρ_{1000h} = is the relaxation after 1000 hours evaluated from Fig. 6.38;

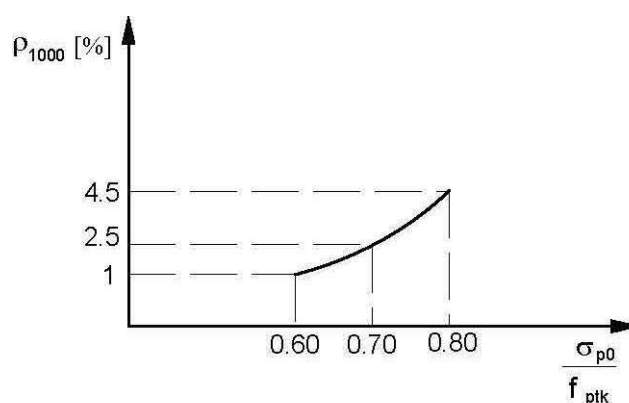


Fig. 6.38. Relaxation losses in % at 1000 hours for Class 2

σ_c : stress on concrete at level of pretensioned steel due to self-weight and permanent load;

σ_{cp0} : stress on concrete at level of pretensioned steel due to prestressing;

$\alpha = E_s/E_c$: modulus of elasticity ratio;

A_p : area of prestressing steel at the considered level;

A_c : area of concrete gross section;

I_c : inertia of concrete gross section;

z_{cp} : lever arm between centroid of concrete gross section and prestressing steel.

Time-dependent losses of prestressing should be calculated for each tendon along his profile so that a correct value may be used for each element. As a reference, the maximum value of prestressing losses, as percentage of initial steel tension, turn out:

longitudinal tendon: 19% at anchorage and 14% at pier axis;

transverse tendon: 18% at anchorage and 12% at midspan.

The effects of losses are taken into account with the same procedure used for the prestressing, but as actions of opposite sign.

6.15.3 Actions

The external loads applied on the structure should be evaluated according to the provisions of Eurocode 1.3 Traffic Load on Bridges. As vertical train load the load model LM71 plus the load models SW (SW/0 and SW/2 respectively) have been adopted with an α coefficient of 1.1. For the LM71, the 4 point loads have been reduced in an equivalent uniform load by smearing their characteristic value Q_{vk} along the influence length so that a $q_{vk,1}$ may be obtained:

$$Q_{vk} = 1.1 \times 250 \times \phi_{din} = 319.6 \text{ kN} \quad \rightarrow \quad q_{vk,1} = 319.6 / 1.6 = 199.75 \text{ kN/m}$$

where ϕ_{din} , being the dynamic factor equal to 1.162, is evaluated below.

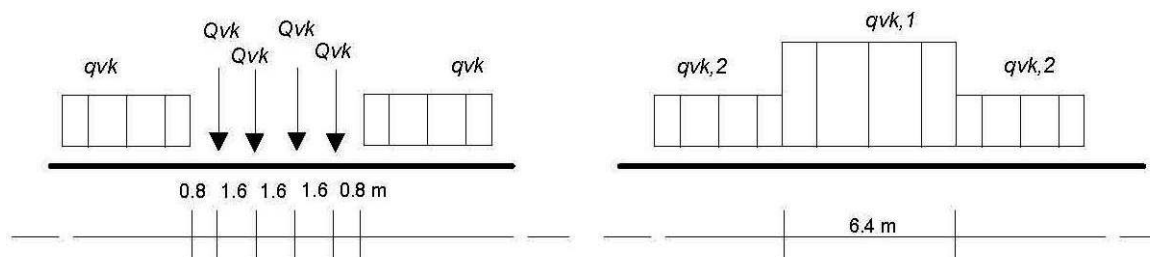


Fig. 6.39. Adopted load arrangement for LM71 load model

The uniformly distributed load q_{vk} according to Eurocode 1.3 is:

$$q_{vk} = 1.1 \times 80 \times \phi_{\text{din}} \quad \rightarrow \quad q_{vk,2} = 102.3 \text{ kN/m}$$

without any limitation in length. Fig. 6.39 shows the LM71 arrangement adopted in the calculations.

The load model SW/0 is represented in Fig. 6.40 and its characteristic value results:

$$q_{vk} = 1.1 \times 133 \times \phi_{\text{din}} = 170.0 \text{ kN/m}$$

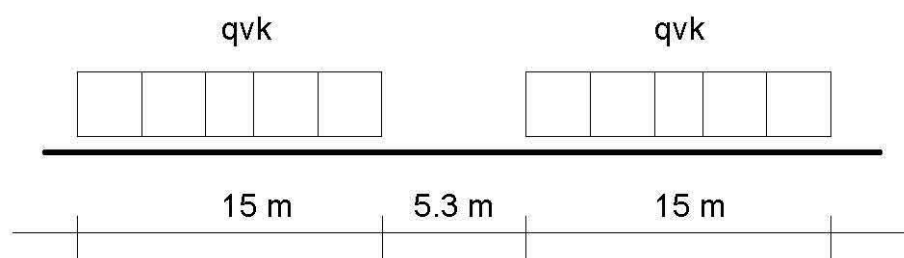


Fig. 6.40. Load model SW/0

The load model SW/2 is represented in Fig. 6.41 and its characteristic value results:

$$q_{vk} = 1.1 \times 150 \times \phi_{\text{din}} = 174.3 \text{ kN/m}$$

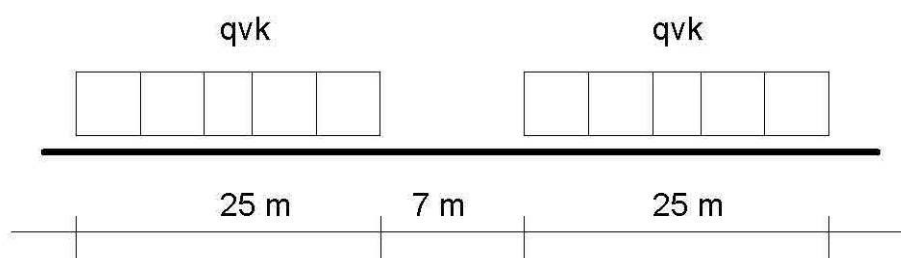


Fig. 6.41. Load model SW/2

The previous load model LM71, SW/0 and SW/2 have been introduced in the FEM analysis considering a spreading ratio of 4:1 in the ballast and of 1:1 in the concrete up to the middle plane of the slab. In the following as left track is denoted the track which has a positive value for the y co-ordinate, while right truck the other one. Fig. 6.42 shows which elements are involved by spreading effects, therefore subjected to variable load.

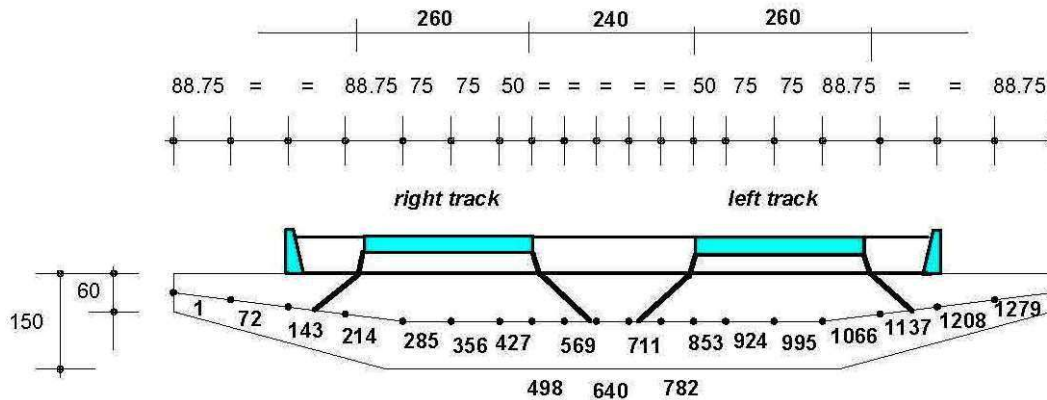


Fig. 6.42. Spreading effects on FEM model and loaded elements

The dynamic factor ϕ is calculated by means of the following expression (track with standard maintenance):

$$\phi_3 = \frac{2.16}{\sqrt{L_\phi} - 0.2} + 0.73 = 1.162$$

where L_ϕ is the *determinant* length defined in the Eurocode 1.3 as:

$$L_\phi = 1.3 \frac{L_1 + L_2 + L_3}{3} = 1.3 \frac{17.33 + 27.75 + 17.33}{3} = 27.04 \text{ m}$$

Several other actions, arising from variable loads, should be considered in the analysis (as traction and braking, centrifugal forces, derailment, wind pressure, differential temperature variation etc.) but, for the sake of simplicity, in these calculations only the following actions have been considered (introduced in the mathematical model in different steps):

- STEP 1: Self-weight of the structure: adopting a unit weight value of $\gamma = 25 \text{ kN/m}^3$;
- STEP 2: Prestressing forces at time of tensioning;
- STEP 3: Prestressing forces after time-dependent losses:

in the calculations, a limit value of tensile stress in tendon equal to $0.6 \times f_{ptk}$ after allowance for losses (t_∞), has been considered, according to the provisions of the applied Railway Code to avoid the risk of brittle failure due to stress corrosion.

- STEP 4: Track load comprehensive of;

rails, sleepers and ballast (waterproofing included) evaluated as a cover with a nominal height of 0.8 m and a unit weight of $\gamma = 18 \text{ kN/m}^3$, so that for a width of 9.5 m, an uniformly distributed load results:

$$g_{\text{ballast}} = 0.8 \times 1.8 \times 9.5 = 136.8 \text{ kN/m};$$

- STEP 5: Others permanent loads composed by;

transverse gradient for drain water, assumed as a load of 1.25 kN/m^2 it turns out:

$$g_{\text{drain}} = 1.25 \times 9.5 = 11.875 \text{ kN/m};$$

ballast retaining walls (with a cross section area of 0.25 m^2 and unit weight of 25 kN/m^3)

$$g_{\text{walls}} = 25 \times 0.25 = 6.25 \text{ kN/m} \quad \text{for each};$$

ducts:

$$g_{\text{ducts}} = 3 \text{ kN/m} \quad \text{for each};$$

border curbs (with a cross section area of 0.1 m^2 and unit weight of 25 kN/m^3):

$$g_{\text{reinf beam}} = 25 \times 0.25 = 6.25 \text{ kN/m} \quad \text{for each};$$

noise barriers:

$$g_{\text{barriers}} = 8.00 \text{ kN/m} \quad \text{for each};$$

- STEP 6: Variable loads for maximum bending moment on first span ($x = 6.18 \text{ m}$);
the applied load is a LM71 model on the left track with the following longitudinal arrangement:

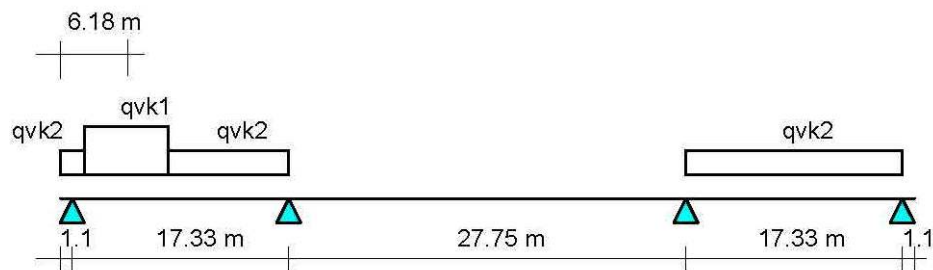


Fig. 6.43. LM71 arrangement for Load Step 5

plus a SW/2 train on the right track with the following longitudinal arrangement:

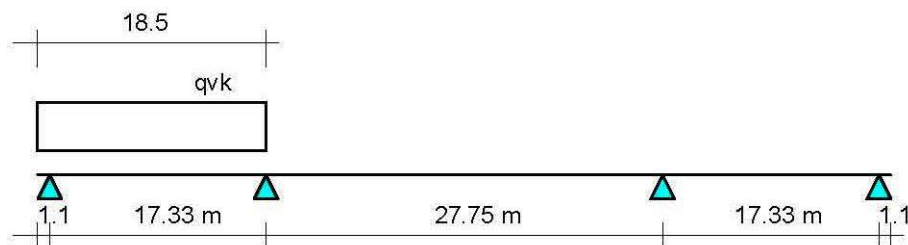


Fig. 6.44. SW/2 arrangement for Load Step 5

- STEP 7: Variable loads for minimum bending moment at pier P1 ($x = 18.43 \text{ m}$);
the applied load is a SW/0 model on the left track with the following longitudinal arrangement:

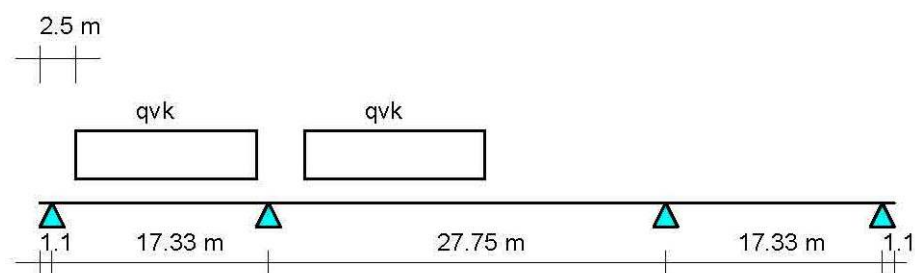


Fig. 6.45. SW/0 arrangement for Load Step 6

plus a SW/2 train on the right track with the following longitudinal arrangement:

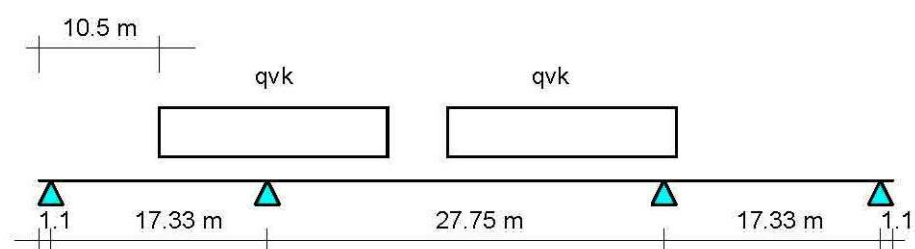


Fig. 6.46. SW/2 arrangement for Load Step 6

- STEP 8: Variable loads for max bending moment on second span ($x = 32.305$ m);
the applied load is a LM71 model on the left track with the following longitudinal arrangement:

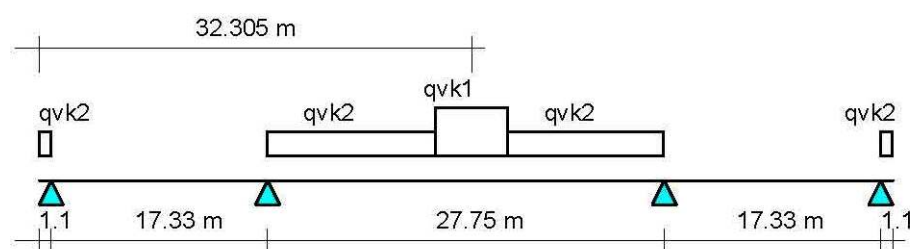


Fig. 6.47. LM71 arrangement for Load Step 7

plus a SW/2 train on the right track with the following longitudinal arrangement:

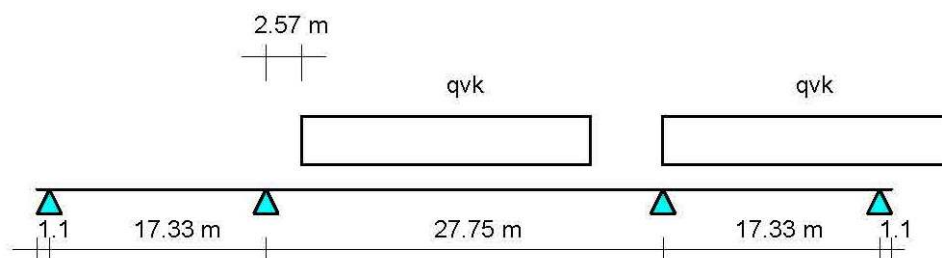


Fig. 6.48. SW/2 arrangement for Load Step 7

6.15.4 Combinations of Actions

The design values for the external actions have been calculated adopting the combinations of loads specified in the applied Code as follow indicated in the symbolic presentation:

- Ultimate Limit State

$$S_d = S \left\{ \gamma_{G1} G_{1k} + \gamma_{G2} G_{2k} + \gamma_p P_k + \gamma_Q \left(Q_{1k} + \sum_{i>1} \Psi_{oi} Q_{ik} \right) \right\}$$

- Serviceability Limit State: rare combination

$$S_d = S \left\{ G_{1k} + G_{2k} + P_k + Q_{1k} + \sum_{i>1} \Psi_{oi} Q_{ik} \right\}$$

- Serviceability Limit State: quasi-permanent combination

$$S_d = S \left\{ G_{1k} + G_{2k} + P_k + \sum_{i>1} \Psi_{2i} Q_{ik} \right\}$$

where:

G_{1k} = characteristic value of the action due to self-weight and permanent loads, ballast excluded;

G_{2k} = characteristic value of action due to ballast self-weight;

P_k = characteristic value of action due to prestress;

Q_{1k} = characteristic value of action due to the base variable action;

Q_{ik} = characteristic value action due to of the other independent variable loads;

γ_1 = partial factor of self-weight and permanent loads, ballast excluded, equal to 1.4 for unfavourable effect and 1.0 for favourable effect;

γ_2 = partial factor of ballast load equal to 1.8 for unfavourable effect and 1.0 for favourable effect;

γ_p = partial factor of prestress load equal to 1.2 for unfavourable effect and 0.9 for favourable effect;

γ_Q = partial factor of variable loads equal to 1.5 for unfavourable effect and 0.0 for favourable effect;

Ψ_{oi} = combination factor of variable loads equal to 0.8;

Ψ_{2i} = combination factor of variable loads for quasi-permanent combination at service, equal to 0.6.

6.15.5 Verification at Serviceability Limit State

The verification at serviceability limit state is relative to the following conditions:

- stress limitation at tensioning;
- stress limitation at service;
- crack widths;
- deformation.

Verification at tensioning

At time of tensioning, no tensile stress should be present in the extreme fibres of the slab and the maximum compressive stress should not exceed the limit value of $0.6 \times f_{ck} = 21$ MPa. For the sake of simplicity, one reports the verification related to the four elements showed in fig ii, as subjected to the higher stress level.

The external actions are calculated adopting the rare combination with only the load steps 1 and 2. From FEM analysis, the value of n_{22} , m_{22} , n_{33} , m_{33} , n_{23} , m_{23} are evaluated so that it results:

$$\begin{aligned} \sigma_{y,t} = \sigma_{22,t} &= \frac{n_{22}}{h} - \frac{6 m_{22}}{h^2} & ; & & \sigma_{y,b} = \sigma_{22,b} &= \frac{n_{22}}{h} + \frac{6 m_{22}}{h^2} \\ \sigma_{x,t} = \sigma_{33,t} &= \frac{n_{33}}{h} - \frac{6 m_{33}}{h^2} & ; & & \sigma_{x,b} = \sigma_{33,b} &= \frac{n_{33}}{h} + \frac{6 m_{33}}{h^2} \\ \sigma_{xy,t} = \sigma_{23,t} &= \frac{n_{23}}{h} - \frac{6 m_{23}}{h^2} & ; & & \sigma_{xy,b} = \sigma_{23,b} &= \frac{n_{23}}{h} + \frac{6 m_{23}}{h^2} \end{aligned}$$

where the subscripts t and b indicate respectively top and bottom fibre. The angles of principal directions (for which is $\sigma_{xy} = 0$) are:

$$\theta_1 = \frac{1}{2} \arctan \left(\frac{2 \sigma_{23}}{\sigma_{22} - \sigma_{33}} \right) \quad ; \quad \theta_2 = \theta_1 + 90^\circ$$

and the principal stresses result:

$$\sigma_{1,t/b} = \sigma_{22,t/b} \cos^2(\theta_1) + \sigma_{33,t/b} \sin^2(\theta_1) + \sigma_{23,t/b} \sin(2\theta_1)$$

$$\sigma_{2,t/b} = \sigma_{22,t/b} \cos^2(\theta_2) + \sigma_{33,t/b} \sin^2(\theta_2) + \sigma_{23,t/b} \sin(2\theta_2)$$

Referring to the elements marked in Fig.6.32 one obtains:

Table 6.7.

Elem.	h [m]	n_{22} [kN/m]	n_{33} [kN/m]	n_{23} [kN/m]	m_{22} [kNm/m]	m_{33} [kNm/m]	m_{23} [kNm/m]
648	1.5	-3091	-13159	6	-225	-2176	0
93	0.963	-7806	-8526	75	743	456	-51
320	1.5	-3516	-10418	1	-45	-812	0
589	1.5	-4280	-10007	-67	653	1945	20

Table 6.8.

$\sigma_{22,b}$ [Mpa]	$\sigma_{33,b}$ [Mpa]	$\sigma_{23,b}$ [Mpa]	$\sigma_{22,t}$ [Mpa]	$\sigma_{33,t}$ [Mpa]	$\sigma_{23,t}$ [Mpa]	$\theta_{1,b}$ [°]	$\theta_{2,b}$ [°]	$\theta_{1,t}$ [°]	$\theta_{2,t}$ [°]	$\sigma_{1,b}$ [Mpa]	$\sigma_{2,b}$ [Mpa]	$\sigma_{1,t}$ [Mpa]	$\sigma_{2,t}$ [Mpa]
-2.66	-14.58	0.00	-1.46	-2.97	0.00	0.02	89.98	0.15	89.85	-2.66	-14.58	-1.46	-2.97
-3.30	-5.90	-0.25	-12.91	-11.80	0.41	-5.48	95.48	-18.17	108.17	-3.27	-5.83	-13.05	-12.15
-2.46	-9.11	0.00	-2.22	-4.78	0.00	0.01	89.99	0.01	89.99	-2.46	-9.11	-2.22	-4.78
-1.11	-1.48	0.01	-4.59	-11.86	-0.10	1.29	88.71	-0.77	90.77	-1.11	-1.48	-4.59	-11.85

which not exceed the limit one.

Verification of limit state of stress limitation in concrete

The serviceability limit states checked in this section are relative only to stress limitation, ensuring that, under service load conditions, concrete extreme stresses do not exceed the corresponding limit, for the quasi-permanent and the rare combinations. The limit stresses for concrete are:

Quasi-permanent combination: Compressive stress = $0.4 \times f_{ck} = 14.00$ MPa

Rare combination: Compressive stress = $0.6 \times f_{ck} = 21.00$ MPa

Applying to the structural FEM model the variable loads and combining them according to the railway code provisions, one obtain the maxima stress values at top and bottom fibres that have to be lower than the corresponding limit. One reports the results relative to the four elements indicated in Fig. 6.32.

Quasi-Permanent Combination

Table 6.9.

Elem.	h [m]	n_{22} [kN/m]	n_{33} [kN/m]	n_{23} [kN/m]	m_{22} [kNm/m]	m_{33} [kNm/m]	m_{23} [kNm/m]	$\sigma_{1,b}$ [Mpa]	$\sigma_{2,b}$ [Mpa]	$\sigma_{1,t}$ [Mpa]	$\sigma_{2,t}$ [Mpa]
648	1.5	-2420	-10152	4	-236	-1576	4	-2.24	-10.97	-0.98	-2.57
93	0.963	-6233	-6347	50	589	108	-37	-2.65	-5.86	-10.31	-7.37
320	1.5	-3539	-7855	2	81	233	4	-2.14	-4.62	-2.58	-5.86
589	1.5	-2736	-7900	-3	-151	-396	0	-2.23	-6.32	-1.42	-4.21

Rare Combination

Table 6.10.

Elem.	h [m]	n_{22} [kN/m]	n_{33} [kN/m]	n_{23} [kN/m]	m_{22} [kNm/m]	m_{33} [kNm/m]	m_{23} [kNm/m]	$\sigma_{1,b}$ [Mpa]	$\sigma_{2,b}$ [Mpa]	$\sigma_{1,t}$ [Mpa]	$\sigma_{2,t}$ [Mpa]
648	1.5	-2238	-10270	3	-226	-615	4	-2.09	-8.49	-0.89	-5.21
93	0.963	-6284	-6033	4	577	-133	-62	-2.76	-7.02	-10.29	-5.51
320	1.5	-2604	-7479	7	7	1279	-9	-1.72	-1.58	-1.75	-8.40
589	1.5	-3791	-8243	-55	-689	-1275	-26	-4.36	-8.89	-0.69	-2.09

Verification of Serviceability Limit State of Cracking

The characteristic crack width should be calculated according to the provisions of Model

Code 90. It has to be noticed, however, that from stress calculation neither for the quasi-permanent combination nor in the rare one, the maximum stress results tensile. Therefore, no specific reinforcement is required and it is sufficient to arrange the minimum amount of reinforcing steel, able to ensure a ductile behaviour in case of corrosion of prestressing steel.

Deformation

Deformation limitation is carried out to control the maximum vertical deflection for passengers comfort. The limit values δ/L (deflection/span Length) are given by the Eurocode 1.3 as a function of the span length and the train speed. The limit value for maximum vertical deflection is calculated considering a span length of 27.75 m (central span) and a train speed over 280 km/h; according to the provisions of the Code, it results:

$$\frac{\delta}{L} = \frac{1}{1600}$$

that should be multiplied by a factor 1.1 for continuous structures; finally, the following limit may be achieved:

$$\frac{\delta_{\text{lim}}}{L} = \frac{1.1}{1600} = \frac{1}{1455}$$

As a consequence of the transient nature of this event, the elastic deflection, calculated by the FEM model, is relative to the only live load; the check shall be performed loading only one track, reading the maximum deflection in correspondence of the track axis. Having loaded the right track with a LM71 load model plus dynamic allowance, placed in the middle of the central span, the obtained δ/L value is:

$$\frac{\delta_{\text{effective}}}{L} = \frac{0.0055}{27.75} = \frac{1}{5045}$$

and it results lower than the corresponding limit.

It can be noticed that no further calculation is requested because, due to prestressing effect, the structure remains entirely compressed, so that the elastic value, calculated by the FEM analysis, has to be considered.

6.15.6 Verification of Ultimate Limit State

Verification at ULS should regard the structure as a whole and its component parts, analysing the resistance of the critical regions. In addition to the analysis of ULS of several shell element under the relevant combination of internal actions, in this example some case of *detailing* are investigated, i.e.:

- bursting force at anchorage of prestressing tendon;
- spalling force at anchorage of prestressing tendon;
- punching under support plate.

Slab ultimate limit state

Verification at ULS has been performed adopting the sandwich model for shell elements. The internal actions in a shell element at ULS are sketched in Fig. 6.49.

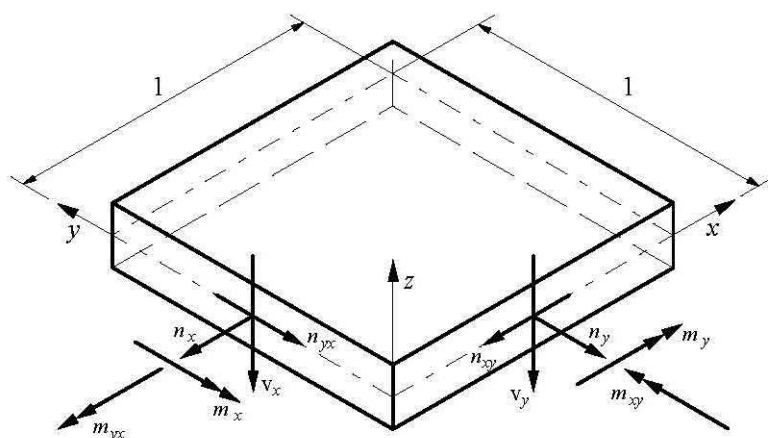


Fig. 6.49. Internal actions at ULS in a shell elements

Let us consider in this section only four elements on the whole (see Fig.6.32). The external actions are derived from FEM model using the load step for trains which leads to the maximum values and combining the results according to the relevant combination formula. For the investigated elements, turns out (on brackets the notation of Fig. 6.49):

Table 6.11.

Elem.	h	$n_{Sd,y}$ (n_{22})	$n_{Sd,x}$ (n_{33})	$n_{Sd,xy}$ (n_{23})	$m_{Sd,y}$ (m_{22})	$m_{Sd,x}$ (m_{33})	$m_{Sd,xy}$ (m_{23})	$v_{Sd,y}$ (v_{13})	$v_{Sd,x}$ (v_{12})
	[m]	[kN/m]	[kN/m]	[kN/m]	[kNm/m]	[kNm/m]	[kNm/m]	[kN/m]	[kN/m]
648	1.5	-1779	-9096	5	-239	470	-14	-6	-5
93	0.963	-5746	-4610	-63	499	-671	-75	89	-150
320	1.5	-2130	-5922	10	38	3241	-13	20	47
589	1.5	-3865	-7748	-54	-1950	-4274	-41	-1124	-1095

As first step, one may design the inner layer checking if specific shear reinforcement is required or not. In fact, it is possible to calculate the principal shear $v_o^2 = v_x^2 + v_y^2$, on the principal shear direction φ_o (such that $\tan \varphi_o = v_y/v_x$), and to check that it turn out:

$$v_o < v_{Rd1} = 0.12 \xi (100 \rho f_{ck})^{1/3} d$$

where v_{Rd1} is specified in chapter 6.4.2.3 of MC 90 and $\rho = \rho_x \cos^2 \varphi_o + \rho_y \sin^2 \varphi_o$. If the is not satisfied, specific shear reinforcement shall be arranged (vertical stirrups) and diagonal compressive forces in concrete shall be checked. According to CEB Bulletin 223, and having set a minimum amount of longitudinal and transverse reinforcement in the bottom and top layer of $A_{s,x} = A_{s,y} = 22.6 \text{ cm}^2/\text{m}$ placed at 0.07 m from the external face, the following table may be calculated for the elements considered.

Table 6.12.

Elem.	d [m]	ϕ_o [°]	ρ_o [-]	v_o [kN/m]	v_{Rd1} [kN/m]	θ [°]	F_{Scw} [kN/m]	F_{Rcw} [kN/m]	A_s/s^2 [cm ² /m ²]	n_{xc} [kN/m]	n_{yc} [kN/m]	n_{xyc} [kN/m]
648	1.43	51.18	0.00158	7	417	26.56	-	-	-	0.0	0.0	0.0
93	0.893	-30.77	0.00253	174	327	26.56	-	-	-	0.0	0.0	0.0
320	1.43	23.14	0.00158	51	417	26.56	-	-	-	0.0	0.0	0.0
589	1.43	45.76	0.00158	1569	417	26.56	3509	13860	14.0	763.9	805.6	784.5

with variation of slab components due to v_x and v_y (i.e. n_{xc} , n_{yc} and n_{xyc}) only for element number 589.

The outer layers should be designed supposing an initial thickness for both layers not lesser than twice the concrete cover evaluated at the centroid of reinforcement. One assumes:

$$t_s = t_i = 2 \times 0.07 = 0.14 \text{ m}$$

so that, internal lever arm z and in plane actions may be evaluated for the outer layers of each element referring to Fig. 6.50 and by means of the following equations:

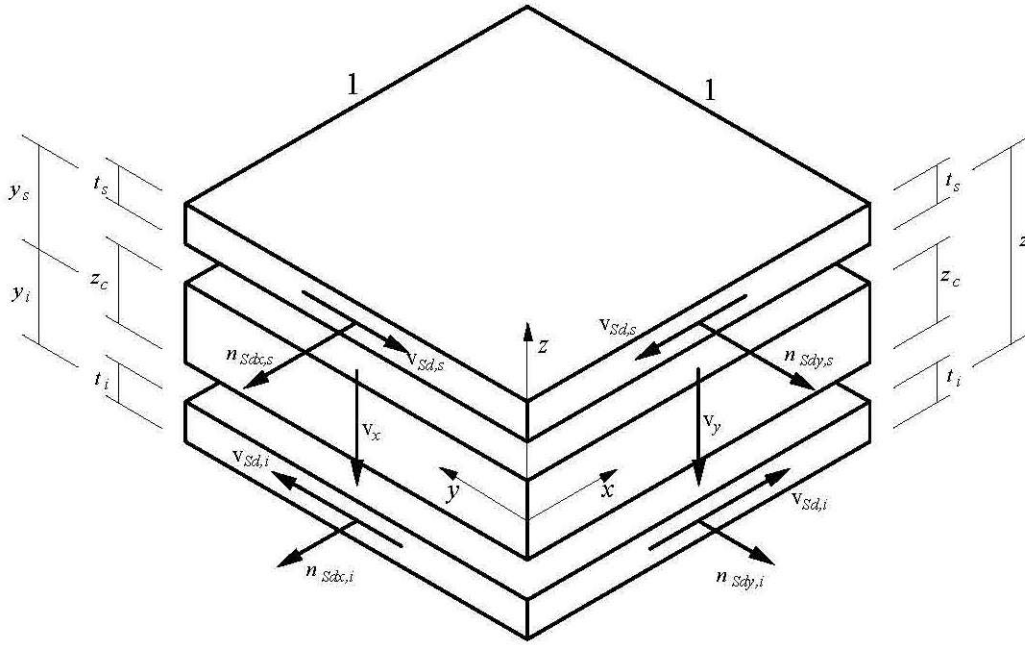


Fig. 6.50. Internal forces in the different layers

$$n_{Sdx,s} = n_x \frac{z - y_s}{z} + \frac{m_x}{z} + \left(\frac{1}{2} \frac{v_x^2}{v_0} \cot \theta \right)$$

$$n_{Sdx,i} = n_x \frac{z - y_i}{z} - \frac{m_x}{z} + \left(\frac{1}{2} \frac{v_x^2}{v_0} \cot \theta \right)$$

$$n_{Sdy,s} = n_y \frac{z - y_s}{z} + \frac{m_y}{z} + \left(\frac{1}{2} \frac{v_y^2}{v_0} \cot \theta \right)$$

$$n_{Sdy,i} = n_y \frac{z - y_i}{z} - \frac{m_y}{z} + \left(\frac{1}{2} \frac{v_y^2}{v_0} \cot \theta \right)$$

$$v_{Sd,s} = n_{xy} \frac{z - y_s}{z} - \frac{m_{xy}}{z} + \left(\frac{1}{2} \frac{v_x v_y}{v_0} \cot \theta \right)$$

$$v_{Sd,i} = n_{xy} \frac{z - y_i}{z} + \frac{m_{xy}}{z} + \left(\frac{1}{2} \frac{v_x v_y}{v_0} \cot \theta \right)$$

where terms on brackets have be summed if shear reinforcement is required. In the design

procedure is convenient to reach the minimum amount of reinforcement, so that a value of 45° for θ angle may be adopted.

For the chosen elements it turns out:

Table 6.13.

Elem.	h [m]	t_s [m]	t_i [m]	t_c [m]	y_s [m]	y_i [m]	z [m]	$n_{Sdy,s}$ [kN/m]	$n_{Sdx,s}$ [kN/m]	$V_{Sd,s}$ [kN/m]	$n_{Sdy,i}$ [kN/m]	$n_{Sdx,i}$ [kN/m]	$V_{Sd,i}$ [kN/m]
648	1.5	0.140	0.140	1.220	0.680	0.680	1.360	-713.5	-4893.3	13.0	-1065.1	-4202.2	-7.7
93	0.963	0.140	0.140	0.683	0.412	0.412	0.823	-3479.6	-1489.8	59.3	-2266.7	-3120.5	-122.3
320	1.5	0.140	0.140	1.220	0.680	0.680	1.360	-1093.1	-5344.2	14.8	-1037.3	-577.9	-4.4
589	1.5	0.140	0.140	1.220	0.680	0.680	1.360	307.0	32.7	787.8	-2560.7	-6252.6	726.9

At this stage each layer may be designed by applying the following equations ($\theta = 45^\circ$):

$$\sigma_c t = \frac{V_{Sd}}{\sin \theta \cos \theta} \leq f_{cd2} t \quad \text{safety verification on concrete side}$$

$$n_{Rdx} = n_{Sdx} + V_{Sd} \cot \theta \quad \text{required resistance along x direction}$$

$$n_{Rdy} = n_{Sdy} + \frac{V_{Sd}}{\cot \theta} \quad \text{required resistance along y direction}$$

from which, if result satisfied, the reinforcement areas may be calculated as:

$$A_{sx} = \frac{n_{Rdx}}{f_{yd}} \quad ; \quad A_{sy} = \frac{n_{Rdy}}{f_{yd}}$$

If concrete strength requirement is not satisfied, an increase layer thickness shall be provided until verification is met; in this case new values for the layer action having changed z value.

It can be notice that if n_{Rdx} or n_{Rdy} value are negative, a compression force is present along that direction and no reinforcement is required; if both the n_{Rdx} and n_{Rdy} are negative it is possible to omit the reinforcement in both the directions but, in this case the verification is performed along the principal compression direction in the concrete subjected to biaxial compression and the checking equation is:

$$\sigma_c t = \frac{n_{Sdx} + n_{Sdy}}{2} + \sqrt{\frac{(n_{Sdx} - n_{Sdy})^2}{4} + V_{Sd}^2} \leq f_{cd1} t$$

For the considered elements, one obtains:

Table 6.14.

Elem.	Top Layer Design				Bottom Layer Design			
	σ_c [MPa]	$f_{cd1/2}$ [kN/m]	A_{sy} [cm ² /m]	A_{sx} [cm ² /m]	σ_c [MPa]	$f_{cd1/2}$ [kN/m]	A_{sy} [cm ² /m]	A_{sx} [cm ² /m]
648	-35.0	-17.1	0.0	0.0	-30.0	-17.1	0.0	0.0
93	-24.9	-17.1	0.0	0.0	-22.4	-17.1	0.0	0.0
320	-38.2	-17.1	0.0	0.0	-7.4	-17.1	0.0	0.0
589	-11.3	-12.0	25.2	18.9	-45.6	-17.1	0.0	0.0

It can be notice that verification for concrete in compression is not satisfied for any layers except for element 589 top layer and element 320 bottom layer. Thus, an increasing of layer thickness is required and new values of plate actions are obtained:

Table 6.15.

Elem.	h [m]	t _s [m]	t _i [m]	t _c [m]	y _s [m]	y _i [m]	Z [m]	n _{Sdy,s} [kN/m]	n _{Sdx,s} [kN/m]	V _{Sd,s} [kN/m]	n _{Sdy,i} [kN/m]	n _{Sdx,i} [kN/m]	V _{Sd,i} [kN/m]
648	1.5	0.300	0.240	0.960	0.600	0.630	1.230	-716.6	-5040.8	14.1	-1062.0	-4054.7	-8.8
93	0.963	0.220	0.190	0.553	0.372	0.387	0.758	-3588.5	-1465.5	66.5	-2157.8	-3144.8	-129.4
320	1.5	0.340	0.140	1.020	0.580	0.680	1.260	-1179.8	-5768.3	16.0	-950.6	-153.8	-5.5
589	1.5	0.140	0.430	0.930	0.680	0.535	1.215	708.7	870.1	794.7	-2962.5	-7090.0	720.0

which lead to the following values:

Table 6.16.

Elem.	Top Layer Design				Bottom Layer Design			
	σ _c [MPa]	f _{cd1/2} [kN/m]	A _{sy} [cm ² /m]	A _{sx} [cm ² /m]	σ _c [MPa]	f _{cd1/2} [kN/m]	A _{sy} [cm ² /m]	A _{sx} [cm ² /m]
648	-16.8	-17.1	0.0	0.0	-16.9	-17.1	0.0	0.0
93	-16.3	-17.1	0.0	0.0	-16.6	-17.1	0.0	0.0
320	-17.0	-17.1	0.0	0.0	-6.8	-17.1	0.0	0.0
589	-11.4	-12.0	34.6	38.3	-16.8	-17.1	0.0	0.0

Of course, minima values should be adopted for A_{sx} and A_{sy} if no reinforcement areas are required. For element 589, the A_{sx} and A_{sy} value are required at the centroid of the layer, whereas they are arranged at 0.07 m from the external surface of the slab in an eccentric position with respect to middle plane of the layer; so, the amount of reinforcement provided has to be changed to restore equilibrium conditions. This variation may be assessed with the aid of the mechanism pictured in Fig. 6.51:

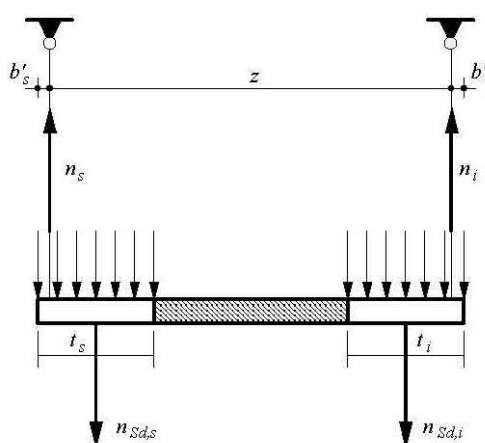


Fig. 6.51. Shell element equilibrium in one direction with two reinforcement layers only

The new forces acting on the reinforcements become:

$$n_s = \frac{n_{Sd,s} \left(h - \frac{t_s}{2} - b'_i \right) + n_{Sd,i} \left(\frac{t_i}{2} - b'_i \right)}{z}$$

$$n_i = n_{Sd,s} + n_{Sd,i} - n_s$$

For the investigated elements, the following areas have been detected.

Table 6.17.

Elem.	Forces referred to tension steel level				Top layer reinf		Bottom layer reinf	
	$n_{s,y}$ [kN/m]	$n_{i,y}$ [kN/m]	$n_{s,x}$ [kN/m]	$n_{i,x}$ [kN/m]	A_{sy} [cm ² /m]	A_{sx} [cm ² /m]	A_{sy} [cm ² /m]	A_{sx} [cm ² /m]
648	-702.5	-1070.8	-5026.6	-4063.6	0.0	0.0	0.0	0.0
93	-3522.0	-2287.2	-1399.0	-3274.3	0.0	0.0	0.0	0.0
320	-1163.8	-956.1	-5752.3	-159.3	0.0	0.0	0.0	0.0
589	1503.5	-2242.5	1664.8	-6370.0	34.6	38.3	0.0	0.0

The previous procedure should be repeated for all the elements of the structural model finding the amount of reinforcement to provide in the slab; it is useful, to control the structural behaviour and for a best fitted reinforcement layout, to summarise the results in a visual map.

Verification to Bursting Force

For the calculation of the bursting force the symmetric prism analogy is used, evaluating the height of the prism so that his centroid results coincident with the centroid of prestressing tendons. For the sake of simplicity, only the longitudinal direction of prestressing tendon is considered with respect to the vertical plane, but transverse force due to bursting effect should be also calculated in the horizontal plane and for transverse prestressing too.

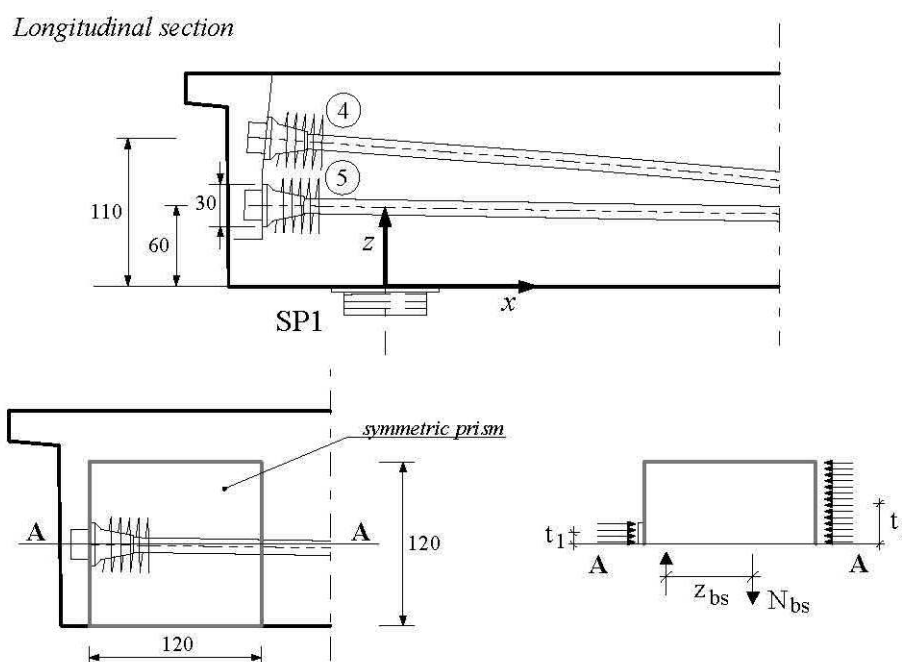


Fig. 6.52. Geometric dimension for bursting calculation

Checking situation is represented in Fig. 6.52, and the most unfavourable situation

occurs when a single tendon is tensioned; considering the lower level of tendon (first tensioned) the height of the prism results:

$$h_{bs} = 2 \times 0.6 = 1.2 \text{ m}$$

and his length, for end anchored tendon, is:

$$l_{bs} = h_{bs} = 1.2 \text{ m}$$

while the width follows from the possible enlargement of the anchor plate that may be assumed equal to 0.43 m, corresponding to the transverse spacing of longitudinal lower tendons.

The design force per tendon has been evaluated by means of the following expression:

$$F_{Sd} = \frac{f_{ptk}}{1.15} A_{sp} = \frac{1800}{1.15} (139 \times 19) \times 10^{-3} = 4134 \text{ kN}$$

The bursting force follows from the moment equilibrium along section A-A:

$$N_{bs} = \frac{0.5 (n_1 + n_2) t_2 - n_1 t_1}{z_{bs}} \gamma_1 F_{Sd} = 852.6 \text{ kN}$$

where:

- $t_1 = 0.075 \text{ m}$ distance between the centroid of tendons above section A-A to the centroid of the prism;
- $t_2 = 0.300 \text{ m}$ distance between the centroid of concrete stress block above section A-A to the centroid of the prism;
- n_1, n_2 numbers of tendons above and below section A-A, respectively: considering the anchor plate as rigid a value of 0.5 may be assumed;
- $\gamma_1 = 1.1$ supplementary partial safety factor against overstressing by overpumping.

Bursting force shall be resisted by an area of reinforcement steel of:

$$A_{s,bs} = N_{bs} / f_{yd} = 19.61 \text{ cm}^2$$

distributed within $l_{bs}/3$ to l_{bs} , (i.e. from 0.40 m to 1.20 m) from the anchor plate. Thus the effective area on a meter length, may be found by the following:

$$\frac{A_{s,bs}}{s \times s} = \frac{A_{s,bs}}{b \times \frac{2}{3} l_{bs}} = \frac{19.61}{0.43 (1.20 - 0.4)} = 57.0 \text{ cm}^2/\text{m}^2$$

that may be provided with ties having diameter of 22 mm and spacing both transversally and longitudinally of 250 mm (see Fig. 6.53); in fact $\phi 22/25 \times 25$ corresponds to $60.82 \text{ cm}^2/\text{m}^2$.

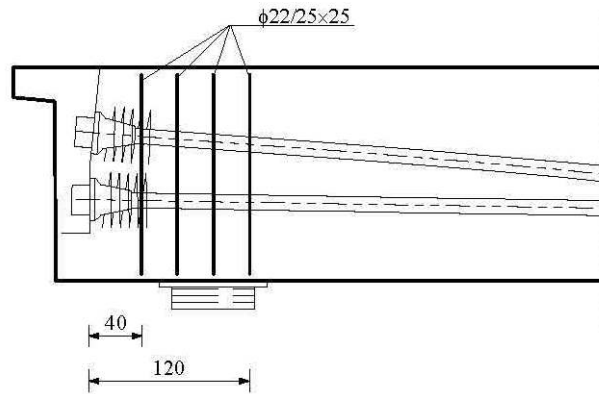


Fig. 6.53. Bursting reinforcement arrangement

Verification to spalling force

The spalling force may be calculated with the equivalent prism analogy. As for bursting verification, only the longitudinal direction is considered; furthermore, spalling effects arise if upper tendon are tensioned firstly (the eccentricity leads to tension stresses). Thus, a section with a breadth of 0.43 m and a height of 1.50 m has to be verified for one tendon tensioning.

The length of the prism for end anchored tendon, is equal to the overall height of the section, i.e. $l_{sl} = 1.50$ m. Considering an eccentricity for upper prestressing tendon of 0.35 m, the extreme stresses at the end of prism length are calculated by means of the beam theory; for a prestressing force $F_{Sd} = 4134.0$ kN they result (negative if compressive):

$$\left. \begin{array}{l} \sigma_{top} \\ \sigma_{bottom} \end{array} \right\} = F_{Sd} \left(-\frac{1}{0.43 \times 1.50} \mp \frac{0.35 \times 6}{0.43 \times 1.50^2} \right) = \left\{ \begin{array}{l} -15.38 \text{ MPa} \\ +2.56 \text{ MPa} \end{array} \right.$$

The section along which no shear force results, is placed at 0.428 m from slab bottom fibre (see Fig 6.54) and the moment for equilibrium turns out:

$$M_{sl} = \frac{2}{3} \sigma_{bottom} \times 0.214^2 \times 0.43 \times 10^3 = 33.61 \text{ kNm}$$

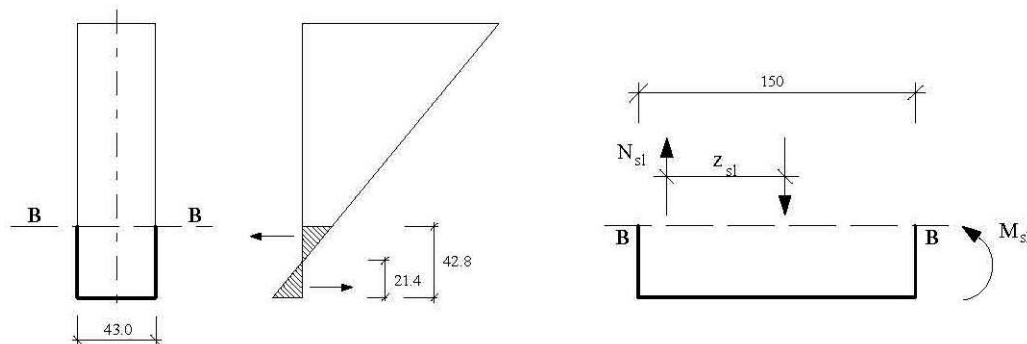


Fig. 6.54. Calculation scheme for spalling

Assuming $z_{sl} = 0.5 \times l_{sl}$ and $b_{sl} = 0.43$ m, the maximum spalling force turns out:

$$N_{sl} = M_{sl} / z_{sl} = 44.81 \text{ kN}$$

Disregarding any concrete tensile resistance, the amount of reinforcement is:

$$A_s = N_{sl} / f_{yd} = 1.031 \text{ cm}^2$$

placed parallel to the end face in its close vicinity.

SECTION 7. Serviceability limit states – worked examples

EXAMPLE 7.1 Evaluation of service stresses [EC2 clause 7.2]

Evaluate the normal compressive force and of the associated bending moment in the section of Fig. 7.1, with the boundary conditions

$$\begin{aligned} &\sigma_c(y=h) = 0; & \sigma_c(y=0) &= k_1 f_{ck} \\ \text{a) } &\sigma_c(y=0) = k_2 f_{ck}; & \text{b) } &\sigma_s(y=d) = k_3 f_{sk} \end{aligned}$$

Then, evaluate the materials strains from the stresses c)

$$N_0 = -800 \text{ kN}; M_0 = 400 \text{ kNm}.$$

Finally, calculate the couples M , N associated to the three paths d) $M/N = -0.5$; e) $N = N_0 = -800 \text{ kN}$; f) $M = M_0 = 400 \text{ kNm}$, that, linearly changing M , N , or M , N , respectively with constant normal force or constant bending, keep the section to the ultimate tension state under load.

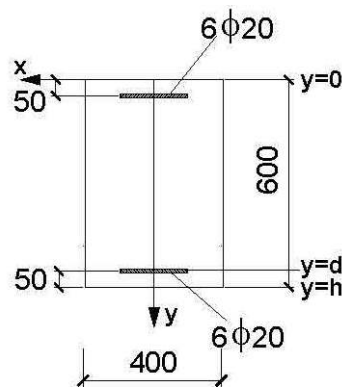


Fig. 7.1. Rectangular section, calculation of service stresses.

The following data are given:

$$f_{ck} = 30 \text{ MPa}, f_{yk} = 450 \text{ MPa}, \alpha_e = 15.$$

Considering Fig. 7.1, we have

$$d = 550 \text{ mm}; d' = 50 \text{ mm}; h = 600 \text{ mm} \quad A_s = 6 \cdot 314 = 1884 \text{ mm}^2; \beta = 1$$

The boundary conditions from the first exercise set the neutral axis on the border of the bottom section, that is $y_n = h$. For this value it results

$$e = \frac{400 \cdot 600^3 / 12 + 400 \cdot 600 \cdot (300)^2 + 15 \cdot 1884 [50^2 + 550^2]}{-400 \cdot 600^2 / 2 + 15 \cdot 1884 (-600)} + 300$$

$$\text{and then } e = -120.65 \text{ mm}, S_{yn}^* = -88.96 \cdot 10^6 \text{ mm}^3.$$

The second condition in the first exercise, assuming $k_2 = 0.45$, can be written as

$$\frac{N(-600)}{-88.96 \cdot 10^6} = -0.45 \cdot 30$$

and then

$$N = -2001.16 \text{ kN}, \quad M = N \cdot e = -2001.16 \cdot (-120.65) \cdot 10^{-3} = 241.49 \text{ kNm}.$$

The tension stress postulated by the second exercise gives the following expression for the neutral axis

$$y_n = \frac{d}{1 + \frac{k_3 f_{yk}}{\alpha_e k_2 f_{ck}}} = \frac{550}{1 + \frac{0.8 \cdot 450}{15 \cdot 0.6 \cdot 30}} = 235.71 \text{ mm}$$

and the compressed steel tension and the stress components are

$$\sigma'_s = \sigma_s \frac{d' - y_n}{d - y_n} = \sigma_s \frac{50 - 235.7}{550 - 235.7} = -0.59 \sigma_s$$

$$N = -0.6 \cdot 30 \cdot 400 \cdot 235.7 / 2 + 0.8 \cdot 450 \cdot 1884 \cdot (1 - 0.59) \cdot 10^{-3} = -570.48 \text{ kN}$$

$$M = (-0.6 \cdot 30 \cdot 400 \cdot 235.7 / 2 \cdot (235.7 / 3 - 300) + 0.8 \cdot 450 \cdot 1884 \cdot (1 - 0.59) \cdot 250) \cdot 10^{-6} = 457.5 \text{ kNm}$$

$$e = -457.5 \cdot 10^6 / 570.48 \cdot 10^3 = -801.95 \text{ mm}$$

Considering the third exercise

$$e = -\frac{400}{800} \cdot 10^{-3} = -500 \text{ mm and } \frac{\frac{400}{3} y_n^3 + 15 \cdot 1884 \left[(550 - y_n)^2 + (50 - y_n)^2 \right]}{-\frac{400}{2} y_n^2 + 15 \cdot 1884 [600 - 2 \cdot y_n]} + y_n = -200$$

this equation is iteratively solved:

$$y_n = 272.3 \text{ mm}, \quad S_{y_n}^* = -13263 \cdot 10^3 \text{ mm}^3$$

and then the tensional state is

$$\sigma_c = -\frac{800 \cdot 10^3 (-272.3)}{-13.263 \cdot 10^6} = -16.42 \text{ MPa}$$

$$\sigma_s = \frac{16.42}{272.3} \cdot (550 - 272.3) \cdot 15 = 251.18 \text{ MPa}$$

$$\sigma'_s = \frac{16.42}{272.3} \cdot (50 - 272.3) \cdot 15 = -201.07 \text{ MPa}$$

Because the condition $e = -500 \text{ mm}$ implies that the neutral axis position is lower than the one previously evaluated assuming the maximal stresses for both materials, the ultimate tension state corresponds to the maximal tension admitted for concrete. If we consider to change M , N keeping constant the eccentricity, the tensional state change proportionally and we can state

$$\frac{N}{N_0} = \frac{M}{M_0} = \frac{0.6 f_{ck}}{\sigma_c} = 1.096.$$

Once the concrete ultimate compressive limit state is reached, the stress is

$$N = 1.096 N_0 = -876.80 \text{ kN}; \quad M = 1.096 M_0 = 438.4 \text{ kNm}.$$

Working with constant normal force ($N = N_0$) the ultimate limit state for the concrete tension

leads to

$$\frac{N_0(-y_n)}{S_{yn}^*} = -0.6f_{ck}$$

and then

$$\frac{-y_n}{-\frac{400}{2}y_n^2 + 15 \cdot 1884[600 - 2 \cdot y_n]} = \frac{0.6 \cdot 30}{800 \cdot 10^3}$$

Solving with respect to y_n

$$y_n^2 + 60y_n - 84795 = 0$$

$$y_n = -30.25 + \sqrt{30.25^2 + 84795} = 262.51 \text{ mm}$$

$$\sigma'_s = 0.6 \cdot 30 \cdot \frac{(50 - 262.51)}{262.51} \cdot 15 = -218.57 \text{ MPa}$$

$$\sigma_s = 0.6 \cdot 30 \cdot \frac{(550 - 262.51)}{262.51} \cdot 15 = 295.69 \text{ MPa}$$

and then

$$M = \left[-0.6 \cdot 30 \cdot \frac{400 \cdot 262.51}{2} \cdot \left(\frac{262.51}{3} - 300 \right) + 1884 \cdot (295.69 + 218.57) \cdot 250 \right] \cdot 10^{-6} = 442.56 \text{ kNm}$$

$$e = -\frac{442.56 \cdot 10^6}{800 \cdot 10^3} = -553.2 \text{ mm}$$

Keeping constant the bending moment ($M = M_0$), the limit state condition for the concrete stress is

$$\frac{N(-y_n)}{S_{yn}^*} = -0.6f_{ck} \quad \text{and then} \quad e = \frac{M_0}{N} = \frac{M_0(y_n)}{0.6f_{ck} \cdot S_{yn}^*}$$

and

$$\frac{I_{yn}^*}{S_{yn}^*} - \frac{M_0(y_n)}{0.6 \cdot f_{ck} \cdot S_{yn}^*} + y_n = \frac{h}{2} \quad \text{As} \quad \frac{M_0}{0.6 \cdot f_{ck}} = \frac{400 \cdot 10^6}{0.6 \cdot 30} = 22.22 \cdot 10^6 \text{ mm}^3$$

the previous numeric form becomes

$$\frac{\frac{400}{3}y_n^3 + 15 \cdot 1884 \left[(550 - y_n)^2 + (50 - y_n)^2 \right] - 22.22 \cdot 10^6 \cdot y_n}{-\frac{400}{2}y_n^2 + 15 \cdot 1884[600 - 2 \cdot y_n]} + y_n = 300$$

and iteratively solving

$$y_n = 395 \text{ mm}$$

$$\sigma'_s = 0.6 \cdot 30 \cdot \frac{(50 - 395)}{395} \cdot 15 = -235.82 \text{ MPa}$$

$$\sigma_s = 0.6 \cdot 30 \cdot \frac{(550 - 395)}{395} \cdot 15 = 105.95 \text{ MPa}$$

$$N = \left[-0.6 \cdot 30 \cdot \frac{400 \cdot 395}{2} + 1884 \cdot (105.95 + 235.82) \right] = -1666.67 \text{ kN}$$

$$M = \left[-0.6 \cdot 30 \cdot \frac{400 \cdot 395}{2} \cdot \left(\frac{395}{3} - 300 \right) + 1884 \cdot (105.95 + 235.82) \cdot 250 \right] \cdot 10^{-6} = 400.34 \text{ kNm}$$

$$e = -240 \text{ mm}$$

Fig. 7.2 reports the results obtained in the evaluation in terms of forces and stresses.

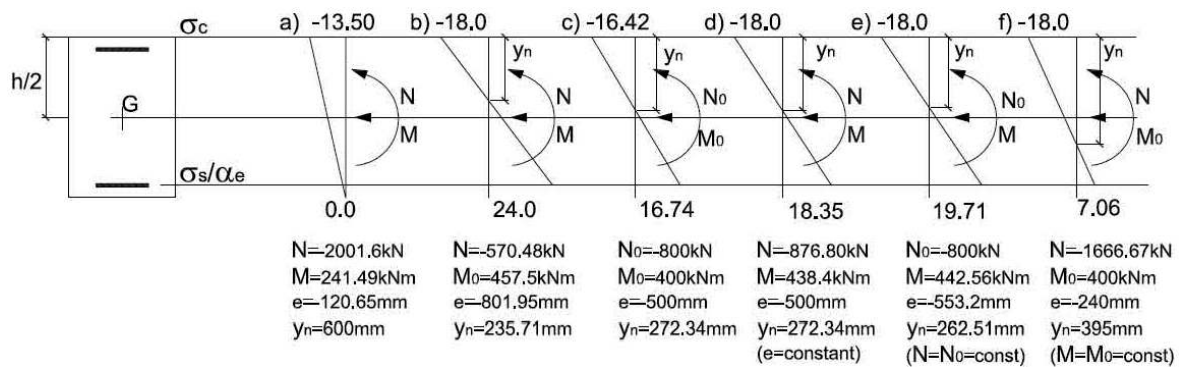


Fig. 7.2. Results for different limit distributions of stresses.

As a remark, just in the case c) the concrete tension limit state under load is not reached while in the case a) ($k_1=0.45$) and the other cases b) d) e) f) ($k_1=0.65$) respectively reach the tension ultimate states under load associated to non-linear viscosity phenomena and minimal tension in the presence of particular combinations. On the other hand, the tension ultimate state under load for tied steel is got just in the case b).

EXAMPLE 7.2 Design of minimum reinforcement [EC2 clause 7.3.2]

Let's consider the section in Fig. 7.3 with the following geometry:

$$A = 1.925 \cdot 10^6 \text{ mm}^2; v_G = 809 \text{ mm}; I = 71.82 \cdot 10^{10} \text{ mm}^4;$$

$$W_i = 7.25 \cdot 10^8 \text{ mm}^3; r^2 = I/A = 39.35 \cdot 10^4 \text{ mm}^2$$

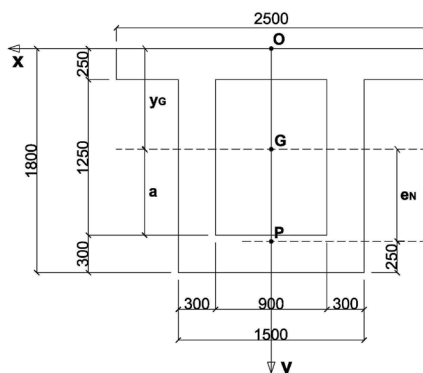


Fig. 7.3. Box - section, design of minimum reinforcement.

Evaluate the minimum reinforcement into the bottom slab in the following cases:

- Application of the first cracking moment M_{cr}
- Application of an axial compressive force $N = -6000$ kN, applied in the point P at 250 mm from the bottom border of the corresponding cracking moment.

Consider the following data:

$$f_{ck} = 45 \text{ MPa}; \quad f_{ct,eff} = 3.8 \text{ MPa}; \quad \sigma_s = 200 \text{ MPa}; \quad k = 0.65 \text{ (} h_w > 1 \text{ m)}$$

The given statements imply:

$$\alpha_s = 250 / 1800 = 0.1388$$

$$\alpha_f = 300 / 1800 = 0.1667$$

$$\beta = 1 - \alpha_s - \alpha_f = 0.6945$$

$$\rho_{s,\min}^0 = 0.65 \cdot 3.8 / 200 = 0.01235$$

Case a)

The application of cracking moment is associated to the neutral axis position $y_n = y_G$, and then $\xi = 809 / 1800 = 0.4494$

It results also

$$\frac{1 - \alpha_s - \alpha_f}{2} = 0.4860 > \xi$$

and for the web

$$\rho_{s,min} = 0.01235 \cdot 0.4 \left[1 - \frac{3}{4} \frac{0.6945 - 2(1 - 0.1667 - 0.4494)}{1 - 0.4494} \right] (1 - 0.1667 - 0.4494) = 0.00208$$

$$A_{s,min} = 0.00208 \cdot 300 \cdot 1800 = 1123 \text{ mm}^2$$

this reinforcement has to be put in the web tied area with height over the bottom slab $a = 1800 - 809 - 300 = 691 \text{ mm}$.

We use $(5+5)\phi 12 \text{ mm}$ equivalent to 1130 mm^2 .

Referring to the bottom slab we get

$$1 - \frac{9}{8} \alpha_f = 0.812 > \xi$$

and it follows:

$$\rho_{s,min} = 0.01235 \cdot 0.45 \frac{2(1 - 0.4494) - 0.1667}{1 - 0.4494} = 0.00943$$

$$A_{s,min} = 0.00943 \cdot 300 \cdot 1500 = 4243 \text{ mm}^2$$

We use $(14+14)\phi 14 \text{ mm}$ equivalent to 4312 mm^2 .

The reinforcement scheme is report in Fig. 7.4

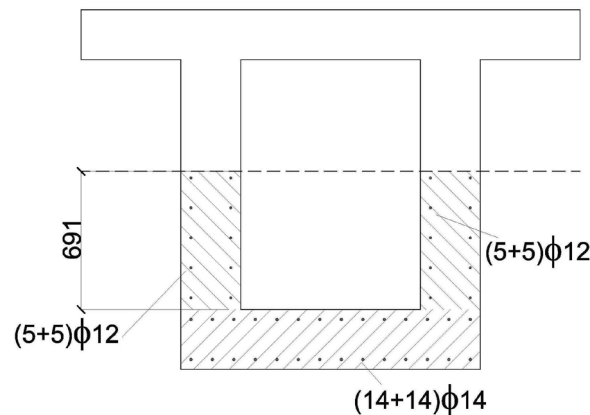


Fig. 7.4. Minimum reinforcement, case (a).

Case b)

The cracking moment associated to the axial force $N = -6000 \text{ kN}$, with eccentricity $e_N = 1800 - 809 - 250 = 741 \text{ mm}$ derives from the relation

$$M_{cr} = \left[-\frac{N}{A} \left(1 + e_N \frac{A}{W_i} \right) + f_{ct,eff} \right] W_i$$

and then:

$$M_{cr} = \left[\frac{6000 \cdot 10^3}{1.825 \cdot 10^6} \left(1 + \frac{741 \cdot 1.825 \cdot 10^6}{7.25 \cdot 10^8} \right) + 3.8 \right] 7.25 \cdot 10^8 \cdot 10^{-6} = 9585 \text{ kNm}$$

the eccentricity of the normal force in the presence of M_{cr} is then:

$$e = -9585 \cdot 10^3 / 6000 + 741 = -856 \text{ mm}$$

and the neutral axis position results from the relation

$$y_n = y_G - \frac{r^2}{e} = 809 + \frac{39.35 \cdot 10^4}{856} = 1269 \text{ mm}, \xi = 0.7050$$

Considering the web, with $\frac{h}{h^*} = 1.8$ we deduce:

$$\rho_{s,\min} = 0.01235 \cdot 0.4 \left[1 - \frac{0.6945 - 2(1 - 0.1667 - 0.7050)}{3 \cdot 1.8(1 - 0.7050)} \right] (1 - 0.1667 - 0.7050) = 0.00046$$

$$A_{s,\min} = 0.00046 \cdot 300 \cdot 1800 = 248 \text{ mm}^2$$

We use 4 $\phi 10$ equivalent to 314 mm^2 .

The bars have to be located in the tied part of the web for an extension

$$a = 1800 - 1269 - 300 = 231 \text{ mm}$$

over the bottom slab

In the bottom slab we have:

$$1 - \frac{9}{8} \alpha_f = 0.812 > \xi$$

and it results

$$\rho_{s,\min} = 0.01235 \cdot 0.45 \frac{2(1 - 0.705) - 0.1667}{1 - 0.705} = 0.00797$$

$$A_{s,\min} = 0.00797 \cdot 300 \cdot 1500 = 3586 \text{ mm}^2$$

We use $(12+12)\phi 14$ mm equivalent to 3692 mm^2 .

The reinforcement scheme is reported in Fig. 7.5

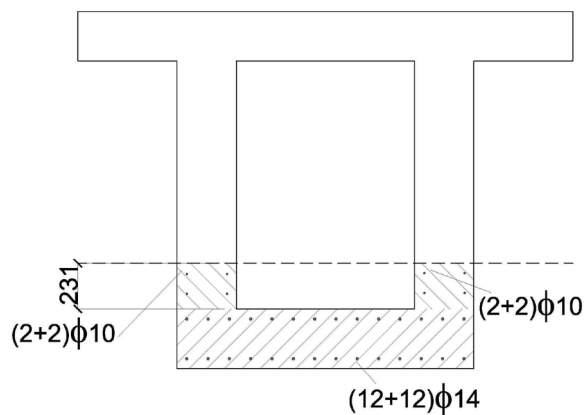


Fig. 7.5. Minimum reinforcement, case (b).

EXAMPLE 7.3 Evaluation of crack amplitude [EC2 clause 7.3.4]

The crack width can be written as:

$$w_k = \frac{\sigma_s}{E_s} \left[1 - \frac{\sigma_{s,cr}}{\sigma_s} \right] \cdot \left[k_3 \cdot c + k_1 k_2 k_4 \frac{\phi}{\rho_s} \lambda \right] \quad (7.1)$$

with

$$\sigma_{s,cr} = k_t \cdot f_{ct,eff} \frac{\lambda}{\rho_s} \left(1 + \alpha_e \frac{\rho_s}{\lambda} \right) \quad (7.2a)$$

$$\lambda = \min \left[2.5(1-\delta); \frac{1-\xi}{3}; \frac{1}{2} \right] \quad (7.3)$$

Assuming the prescribed values $k_3=3.4$, $k_4=0.425$ and considering the bending case ($k_2=0.5$) with improved bound reinforcement ($k_1=0.8$), the (7.2) we get

$$w_k = \frac{\sigma_s}{E_s} \left[1 - \frac{\sigma_{s,cr}}{\sigma_s} \right] \cdot \left[3.4 \cdot c + 0.17 \frac{\phi}{\rho_s} \lambda \right] \quad (7.4)$$

The (7.4) can be immediately used as verification formula.

As an **example** let's consider the section in Fig. 7.6

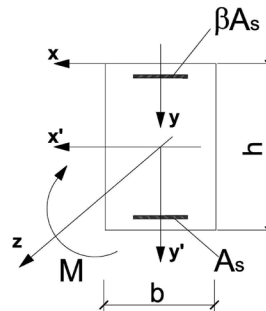


Fig. 7.6. Reinforced concrete section, cracks amplitude evaluation

assuming $\alpha_e = 15$, $d=548\text{mm}$, $d'=46.0\text{mm}$, $c=40\text{mm}$, $b=400\text{mm}$, $h=600\text{mm}$, $M=300\text{kNm}$, $A_s=2712\text{mm}^2$ ($6\phi 24$), $A_s'=452\text{mm}^2$ ($4\phi 12$), $f_{ck}=30\text{MPa}$, $f_{ct,eff}=f_{ctm}=2.9\text{MPa}$

Referring to a short time action ($k_t=0.6$).

It results then

$$\beta = 452/2712 = 0.167, \quad \delta = 548/600 = 0.913, \quad \delta' = 460/600 = 0.0767,$$

$$\rho_s = 2712 / (400 \cdot 600) = 0.0113$$

And the equation for the neutral axis y_n is

$$\frac{-400}{2} y_n^2 + 15 \cdot 2712 \left[548 - y_n + 0.167 (46 - y_n) \right] = 0$$

and then

$$y_n^2 + 237.4y_n - 113026 = 0$$

$$y_n = -118.7 + \sqrt{118.7^2 + 113026} = 237.8 \text{ mm} \quad , \quad \xi = \frac{y_n}{h} = \frac{237.8}{600} = 0.3963$$

The second order moment results

$$I_{y_n}^* = \frac{400}{3} 237.8^3 + 15 \cdot 2712 \left[(548 - 237.8)^2 + 0.167 (46 - 237.8)^2 \right] = 5.96 \cdot 10^9 \text{ mm}^4$$

and we deduce $\sigma_s = 15 \cdot 300 \cdot 10^6 \cdot (548 - 237.8) / 5.96 \cdot 10^9 = 234 \text{ MPa}$

the λ value to be adopted is the lowest between $2.5(1-0.913)=0.2175$; $(1-0.3963)/3=0.2012$; 0.5. Then $\lambda=0.2012$.

The adopted statements lead to

$$\sigma_{s,cr} = 0.6 \cdot 2.9 \frac{0.2012}{0.0113} \left(1 + 15 \frac{0.0113}{0.2012} \right) = 57.08 \text{ MPa}$$

$$w_k = \frac{234}{2 \cdot 10^5} \left[1 - \frac{57.08}{234} \right] \cdot \left[3.4 \cdot 40 + 0.17 \frac{24}{0.0113} 0.2012 \right] = 0.184 \text{ mm}$$

EXAMPLE 7.4 Design formulas derivation for the cracking limit state [EC2 clause 7.4]

7.4.1 Exact method

It is interesting to develop the (7.4) to use it as a design formula. In particular, stated b , h , d , d' , b , and fixed M , we want to deduce the metal reinforcement amount A_s and its design tension σ_s in order to have a crack amplitude w_k lower than the fixed value $\bar{w}_k$. The adimensional calculus leads to

$$-\frac{1}{2}\xi^2 - \alpha_e \cdot \rho_s (1 + \beta)\xi + \alpha_e \cdot \rho_s (\delta + \beta \cdot \delta') = 0 \quad (7.5)$$

$$\sigma_s = \frac{\alpha_e v (\delta - \xi) f_{ctm} k_t}{2 \left[3n \cdot \rho_s \left[(\delta - \xi)^2 + \beta (\delta' - \xi)^2 \right] + \xi^3 \right]} \quad (7.6)$$

setting

$$v = \frac{M}{M_{cr}^0} = \frac{M}{k_t f_{ctm} \frac{b \cdot h^2}{6}} \quad (7.7)$$

Deducing ρ_s from (7.5) and with its substitution in the (7.6) we get

$$\rho_s = \frac{\xi^2}{2\alpha_e \left[-(1 + \beta)\xi + \delta + \beta\delta' \right]} \quad (7.8)$$

$$\frac{\alpha_e v (\delta - \xi) - 2\xi^3 p}{(\delta - \xi)^2 + \beta (\delta' - \xi)^2} = \frac{3\xi^2 p}{\delta + \beta\delta' - (1 + \beta)\xi} \quad (7.9)$$

$$\text{with } p = \sigma_s / (k_t f_{ctm}) \quad (7.10)$$

From (7.4), where $w = \bar{w}_k$, after some calculations we deduce

$$p = \frac{\bar{w}_k^0}{3.4 \cdot c + 0.17 \frac{\phi \cdot \lambda}{\rho_s}} + \frac{\lambda}{\rho_s} + n \quad (7.11)$$

$$\text{setting } \bar{w}_{0k} = \frac{E_s \bar{w}_k}{k_t f_{ctm}} \quad (7.12)$$

Combining (7.8) and (7.11), the (7.9) is

$$\frac{\alpha_e v (\delta - \xi)}{[(\delta - \xi)^2 + \beta(\delta' - \xi)^2]} = \left[\frac{\bar{w}_k^0 \cdot \xi^2}{3.4 \cdot c \cdot \xi^2 + 0.34 \alpha_e \cdot \phi \cdot \lambda [\delta + \beta \delta' - (1 + \beta) \xi]} + \frac{2 \alpha_e \cdot \lambda}{\xi^2} [\delta + \beta \delta' - (1 + \beta) \xi] + \alpha_e \right] \times$$

$$\times \left[\frac{3 \xi^2}{[\delta + \beta \delta' - (1 + \beta) \xi]} + \frac{2 \xi^3}{[(\delta - \xi)^2 + \beta(\delta' - \xi)^2]} \right] \quad (7.13)$$

the (7.13), numerically solved, allows the determination of the neutral axis position and then, using the (7.11) (7.8), the evaluation of the reinforcement tension and its amount. . If it is not the case, it is necessary to set in the (7.13) $\lambda = 2.5(1 - \delta)$ and then re-evaluating ξ , being the value $\lambda = 0.5$ practically impossible for bending problems.

The procedure, aimed to the determination of the reinforcement amount and its tension corresponding to fixed crack amplitude values and stress level, requires to set before the value of the bars diameter ϕ .

Alternatively, it is possible to set the tensional level σ_s , for example coincident with the permissible one, and to evaluate the corresponding reinforcement amount ρ_s and the maximal bars diameter. In this case, as the parameter p is defined, the neutral axis is obtained from (7.9), ρ_s from (7.7) and the maximal diameter derives from (7.11) solved with respect to ϕ , which assumes the form:

$$\phi_{\max} = \frac{\rho_s}{\lambda} \left[\frac{5.88 \rho_s \bar{w}_{ok}}{(p - \alpha_e) \rho_s - \lambda} - 2c \right] \quad (7.14)$$

7.4.2 Approximated method

The application of the procedure discussed above is quite laborious as it requires to iteratively solve the (7.13). An alternative procedure, easier to be applied, consist in the statement that the lever arm h_0 is constant and independent from ξ and equivalent to $0.9d$. In this way, we have $\sigma_s A_s 0.9d = M$ and then

$$\rho_s = 0.185v / (p\delta) \quad (7.15)$$

the (7.4) written for $w = \bar{w}_k$ immediately gives

$$\bar{w}_k = \frac{\sigma_s}{E_s} \left[1 - \frac{\lambda}{\rho_s \cdot p} \left(1 + \frac{\alpha_e \cdot \rho_s}{\lambda} \right) \right] \left(3.4c + 0.17 \frac{\phi \cdot \lambda}{\rho_s} \right) \quad (7.16)$$

aiming to a further simplification of the problem, let's state $\delta = 0.9$, $\lambda = 0.243$

and assuming by definition

$$v^* = \frac{v}{1 - \frac{\delta \cdot \lambda}{0.185v}} = \frac{v}{1 - \frac{1.18}{v}} \quad u_1 = \frac{c}{\phi} \quad u_2 = \frac{\bar{w}_{ok}}{\phi} \quad (7.17)$$

the (7.16) after some algebra has the form

$$p^2 + 5 \cdot v^* \left[3.4 u_1 \frac{v}{v^*} - 0.20 \frac{\alpha_e}{v} \right] p - v^* [17 \alpha_e \cdot u_1 + 5 u_2] = 0 \quad (7.18)$$

the (7.18) is easy to solve, and together with the (7.15) and (7.11), leads to the desired values ρ_s and σ_s .

In this case too, if we set the value of σ_s , the solution for (7.18) with respect to ϕ leads to the relation

$$\phi_{\max} = \frac{[17c(vp - \alpha_e v^*) - 5v^* \bar{w}_{ok}]}{\alpha_e v^* \frac{v}{v^*} p - p^2} \quad (7.19)$$

that defines the maximal bars diameter, which, associated to the reinforcement amount given by the (7.15), allows to satisfy the cracking ultimate state corresponding to a fixed value of the steel tension.

EXAMPLE 7.5 Application of the approximated method [EC2 clause 7.4]

Let's use the described procedure to the section in Fig. 7.7.

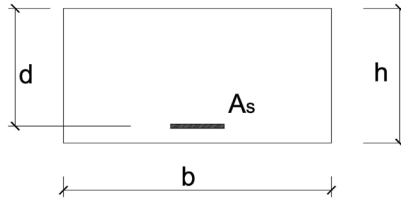


Fig. 7.7. Reinforced concrete Section, reinforcement design for the cracking ultimate state.

Assuming $b=1000\text{mm}$; $h=500\text{mm}$; $c=50\text{mm}$; $\phi=26\text{mm}$; $f_{ck}=33\text{MPa}$; $M=600\text{kNm}$, design the section to have a crack amplitude $\bar{w}_k = 0.30\text{mm}$, $\bar{w}_k = 0.20\text{mm}$, $\bar{w}_k = 0.10\text{mm}$.

It results

$$f_{ctm} = 0.3 \cdot 33^{2/3} = 3.086\text{MPa} \quad \delta = (500 - 63) / 500 = 0.874$$

$$M_{cr}^0 = 0.6 \cdot 3.086 \cdot (1000 \cdot 500^2 / 6) \cdot 10^{-6} = 77.15\text{kNm} \quad (\text{see ex. 7.1})$$

$$v = 600 / 77.15 = 7.77 \quad v^* = 7.77 / (1 - 1.18 / 7.77) = 9.16 \quad u_1 = 50 / 26 = 1.92$$

Defined $\bar{w}_k^{\max} = 0.30\text{mm}$ the maximal amplitude, in the three cases under examination we can set $\bar{w}_k = \bar{w}_k^{\max} \cdot k_w$ where $k_w = 1; 2/3; 1/3$. Then in a general form

$$\bar{w}_{ok} = \frac{0.3 \cdot 2 \cdot 10^5}{0.6 \cdot 3.086} \cdot k_w = 32404 \cdot k_w \quad u_2 = \frac{32404}{26} \cdot k_w = 1246 \cdot k_w$$

$$p^2 + 5 \cdot 9.16 \cdot \left[3.4 \cdot 1.92 \cdot \frac{7.77}{9.16} - \frac{0.20 \cdot 15}{7.77} \right] p - 9.16 \cdot [17 \cdot 15 \cdot 1.92 + 5 \cdot 1246 \cdot k_w] = 0$$

and then

$$p^2 + 235.93 p - 4485 - 57067 k_w = 0$$

and then

$$p(k_w) = -117.965 + \sqrt{18400.74 + 57067 k_w}$$

Using the previous relation, together with the (7.15) and (7.10), in the three cases here considered

$$k_w = 1, \quad \bar{w}_k = 0.3 \text{ mm}, \quad p(1) = 156.75$$

$$\rho_s(1) = \frac{0.185 \cdot 7.77}{156.75 \cdot 0.874} = 0.01049, \quad A_s(1) = 0.01049 \cdot 500 \cdot 1000 = 5245 \text{ mm}^2$$

$$\sigma_s(1) = 0.6 \cdot 3.086 \cdot 156.75 = 290 \text{ MPa}$$

$$k_w = 2/3, \quad \bar{w}_k = 0.2 \text{ mm}, \quad p(2/3) = 119.62$$

$$\rho_s(2/3) = \frac{0.185 \cdot 7.77}{119.62 \cdot 0.874} = 0.01375, \quad A_s(2/3) = 0.01375 \cdot 500 \cdot 1000 = 6875 \text{ mm}^2$$

$$\sigma_s (2/3) = 0.6 \cdot 3.086 \cdot 119.62 = 221 \text{ MPa}$$

$$k_w = 1/3, \quad \bar{w}_k = 0.1 \text{ mm}, \quad p (1/3) = 75.48$$

$$\rho_s (1/3) = \frac{0.185 \cdot 7.77}{75.48 \cdot 0.874} = 0.02179, \quad A_s (1/3) = 0.02179 \cdot 500 \cdot 1000 = 10895 \text{ mm}^2$$

$$\sigma_s (1/3) = 0.6 \cdot 3.086 \cdot 75.48 \cong 140 \text{ MPa}$$

The three sections are reported in Fig. 7.7; the metal areas are overestimated, and 26 mm diameter bars are used.

Let's verify the adopted design method in order to evaluate its precision. The following results are obtained:

$$k_w = 1, \quad \rho_s = 5310 / (500 \cdot 1000) = 0.01062$$

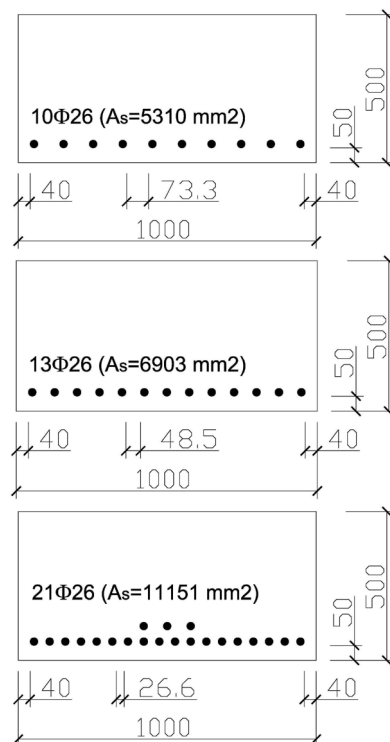
$$-\frac{1000}{2} y_n^2 + 15 \cdot 5310 \cdot (437 - y_n) = 0$$

$$y_n^2 + 159.3 y_n - 69614 = 0$$

$$y_n = -79.65 + \sqrt{79.65^2 + 69614} = 195.9 \text{ mm}, \quad \xi = 0.3918$$

$$I_{y_n}^* = \frac{1000 \cdot 195.9^3}{3} + 15 \cdot 5310 \cdot (437 - 195.9)^2 = 7.13 \cdot 10^9 \text{ mm}^4$$

$$\sigma_s = 15 \cdot 600 \cdot 10^6 \frac{437 - 195.9}{7.13 \cdot 10^9} = 304 \text{ MPa}$$



$$k_w = 1$$

$$\bar{w}_k = 0.3 \text{ mm}$$

$$\sigma_s = 290 \text{ MPa}$$

$$k_w = 2/3$$

$$\bar{w}_k = 0.2 \text{ mm}$$

$$\sigma_s = 221 \text{ MPa}$$

$$k_w = 1/3$$

$$\bar{w}_k = 0.1 \text{ mm}$$

$$\sigma_s = 140 \text{ MPa}$$

Fig. 7.8. Designed sections.

The lowest value for λ has to be chosen between

$$\lambda = 2.5 (1 - 0.874) = 0.315; \quad \lambda = (1 - 0.3918) / 3 = 0.2027$$

$$\text{Then } \sigma_{s,cr} = 0.6 \cdot 3.086 \cdot \frac{0.2027}{0.01062} \cdot \left(1 + 15 \cdot \frac{0.01062}{0.2027}\right) = 63.11 \text{ MPa}$$

$$w_k = \frac{304}{2 \cdot 10^5} \cdot \left(1 - \frac{63.11}{304}\right) \cdot \left(3.4 \cdot 50 + 0.17 \cdot 26 \cdot \frac{0.2027}{0.01062}\right) = 0.306 \text{ mm}$$

$$k_w = 2/3, \quad \rho_s = 69.03 / (50 \cdot 100) = 0.0138$$

$$-\frac{1000}{2} y_n^2 + 15 \cdot 6903 \cdot (437 - y_n) = 0$$

$$y_n^2 + 207.1 y_n - 90498 = 0$$

$$y_n = -103.5 + \sqrt{103.5^2 + 90498} = 214.6 \text{ mm}, \quad \xi = 0.4292$$

$$I_{y_n}^* = \frac{1000 \cdot 214.6^3}{3} + 15 \cdot 6903 \cdot (437 - 214.6)^2 = 8.41 \cdot 10^9 \text{ mm}^4$$

$$\sigma_s = 15 \cdot 600 \cdot 10^6 \frac{437 - 214.6}{8.41 \cdot 10^9} = 238 \text{ MPa}$$

$$\lambda = (1 - 0.4292) / 3 = 0.1903$$

$$\sigma_{s,cr} = 0.6 \cdot 3.086 \cdot \frac{0.1903}{0.0138} \cdot \left(1 + 15 \cdot \frac{0.0138}{0.1903}\right) = 53.31 \text{ MPa}$$

$$w_k = \frac{238}{2 \cdot 10^5} \cdot \left(1 - \frac{53.31}{238}\right) \cdot \left(3.4 \cdot 50 + 0.17 \cdot 26 \cdot \frac{0.1903}{0.0138}\right) = 0.213 \text{ mm}$$

$$k_w = 1/3, \quad \rho_s = 111.51 / (50 \cdot 100) = 0.0223$$

$$-\frac{1000}{2} y_n^2 + 15 \cdot [9558 \cdot (437 - y_n) + 1593 \cdot (385 - y_n)] = 0$$

$$y_n^2 + 334.5 y_n - 143704 = 0$$

$$y_n = -167.2 + \sqrt{167.2^2 + 143704} = 247.1 \text{ mm}, \quad \xi = 0.494$$

$$I_{y_n}^* = \frac{1000 \cdot 247.1^3}{3} + 15 \cdot 9558 \cdot (437 - 247.1)^2 + 15 \cdot 1593 \cdot (385 - 247.1)^2 = 1.06 \cdot 10^9 \text{ mm}^4$$

$$\sigma_s = 15 \cdot 600 \cdot 10^6 \frac{437 - 247.1}{1.06 \cdot 10^9} = 160 \text{ MPa}$$

$$\lambda = (1 - 0.494) / 3 = 0.1687$$

$$\sigma_{s,cr} = 0.6 \cdot 3.086 \cdot \frac{0.1687}{0.0223} \cdot \left(1 + 15 \cdot \frac{0.0223}{0.1687} \right) = 41.78 \text{ MPa}$$

$$w_k = \frac{160}{2 \cdot 10^5} \cdot \left(1 - \frac{41.78}{160} \right) \cdot \left(3.4 \cdot 50 + 0.17 \cdot 26 \cdot \frac{0.1687}{0.0223} \right) = 0.12 \text{ mm}$$

The obtained values are in good agreement with those evaluated within the design.

The values from the verification are slightly larger because of the fact that in the considered section the internal drive lever arm is lower than the approximated value $0.9d$ assumed in the approximated design procedure. In fact, being h_0/d the adimensional lever arm in units of effective height d , in the three cases we have

$$k_w = 1 \quad h_0/d = (43.70 - 19.59/3) / 43.70 = 0.85$$

$$k_w = 2/3 \quad h_0/d = (43.70 - 21.46/3) / 43.70 = 0.836$$

$$k_w = 1/3 \quad h_0/d = [(18 \cdot 160 \cdot 18.99 + 3 \cdot 160 \cdot 13.79^2 / 18.99) / (18 \cdot 160 + 3 \cdot 160 \cdot 13.79 / 18.99) + 2/3 \cdot 24.71] / 43.70 \cong 0.8$$

Let's remark that the presence of a compressed reinforcement is highly recommended to make ductile the section in the ultimate limit state. The reinforcement increase the lever arm of the section reducing the difference between the approximated values and those coming from the verification. The approximated method previously discussed can be successfully applied in the design of the ultimate crack state.

The obtained results are reported in the Tables 7.1 and 7.2 and they are shown in Fig. 7.9. Table 7.3 and Fig. 7.10 report numerical values and graphs for the maximal diameter and the required reinforcement expressed as a function of fixed values for σ_s . Stating a suitable precision for the approximated method, those values are evaluated using the (7.16) (7.14).

Table 7.1. *Approximated method.*

w_k (mm)	A_s (mm ²)	σ_s (MPa)	h_0/d
0.1	11151	140	0.9
0.2	6903	221	0.9
0.3	5310	190	0.9

Table 7.2. *Exact method.*

A_s (mm ²)	w_k (mm)	σ_s (MPa)	h_0/d
11151	0.120	160	0.811
6903	0.213	238	0.836
5310	0.306	304	0.85

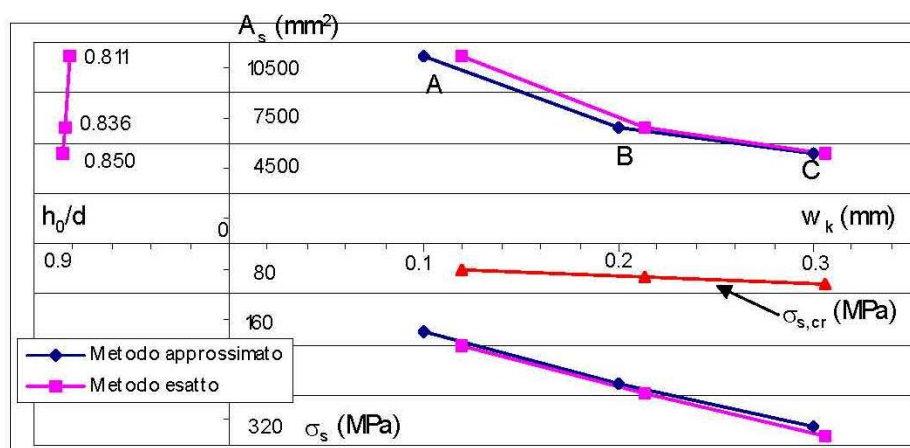
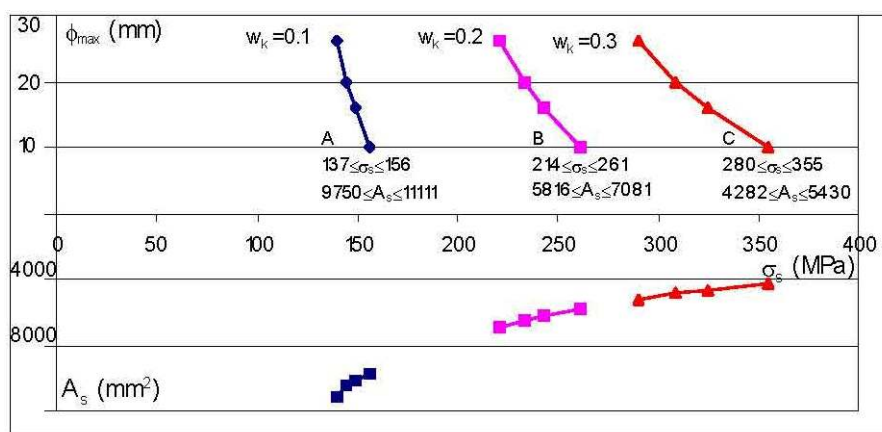


Fig. 7.9. *Comparison between the exact and approximated methods.*

Table 7.3. Approximated method – Determination of maximum diameter.

$w_k = 0.1 \text{ mm (A)}$			$w_k = 0.2 \text{ mm (B)}$			$w_k = 0.3 \text{ mm (C)}$		
σ_s (MPa)	$\phi_{\max}$ (mm)	A_s (mm ²)	σ_s (MPa)	$\phi_{\max}$ (mm)	A_s (mm ²)	σ_s (MPa)	$\phi_{\max}$ (mm)	A_s (mm ²)
137	30	11111	214	30	7001	280	30	5430
140	26	11151	221	26	6903	290	26	5310
145	20	10486	233	20	6508	309	20	4910
149	16	10205	243	16	6245	325	16	4672
156	10	9750	261	10	5816	355	10	4282

**Fig. 7.10.** Diagrams for Maximal diameter ($\phi_{\max}$) – Metal area (A_s) – Steel tension (σ_s).

EXAMPLE 7.6 Verification of limit state of deformation

Evaluate the vertical displacement in the mid-span of the beam in Fig. 7.11 with constant transversal section represented in Fig. 7.12

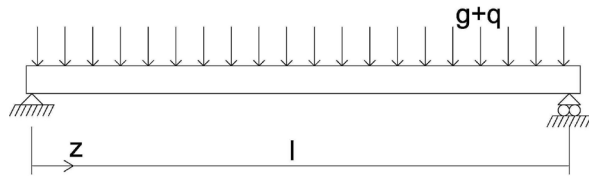
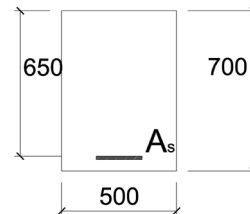


Fig. 7.11. deflected beam, deformation limit state.

Fig. 7.12. Transversal section.

Assume the following values for the main parameters

$f_{ck}=30\text{MPa}$; $g+q=40\text{kN/m}$; $g=2q$; $l=10\text{m}$; $A_s=3164\text{mm}^2$ ($7\phi 24$);

and solve the problem firstly in a cumulative way, stating $\alpha_e = E_s/E_c=15$.

Referring to the stage I, as indicated in Fig. 7.13,

$$A^* = 700 \cdot 500 + 15 \cdot 3164 = 397460\text{mm}^2$$

$$y_G^* = \frac{(700 \cdot 500 \cdot 350 + 15 \cdot 3164 \cdot 650)}{397460} = 385.8\text{mm}$$

$$I_I^* = \frac{500 \cdot 700^3}{12} + 500 \cdot 700 \cdot 35.8^2 + 15 \cdot 3164 \cdot (650 - 385.8)^2 = 18.05 \cdot 10^9\text{mm}^4$$

$$W_i^* = \frac{18.05 \cdot 10^9}{700 - 385.8} = 5.745 \cdot 10^7\text{mm}^3$$

From Table [3.2-EC2] we get $f_{ctm} = 0.30 \cdot 30^{2/3} = 2.9\text{MPa}$

and then the cracking moment results

$$M_{cr} = f_{ctm} W_i^* = 2.9 \cdot 5.745 \cdot 10^7 \cdot 10^{-6} = 166.6\text{kNm}$$

Considering the whole applied load then $M_{max} = 40 \cdot 10^2 / 8 = 500\text{kNm}$

$$\lambda = \frac{M_{max}}{M_{cr}} = \frac{500}{166.6} = 3$$

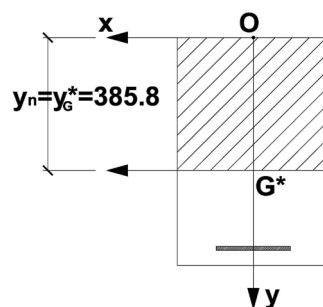


Fig. 7.13. Section at stage I.

In the stage II, as reported in Fig. 7.13,

$$-500 \cdot \frac{y_n^2}{2} + 15 \cdot 3164 \cdot (650 - y_n) = 0$$

$$y_n^2 + 189.84 y_n - 123396 = 0$$

$$y_n = -94.92 + \sqrt{94.92^2 + 123396} = 269 \text{ mm}$$

$$I_{II}^* = 500 \cdot \frac{269^3}{3} + 15 \cdot 3164 \cdot (650 - 269)^2 = 1.01 \cdot 10^{10} \text{ mm}^4$$

then

$$c = 18.05 / 10.13 = 1.78$$

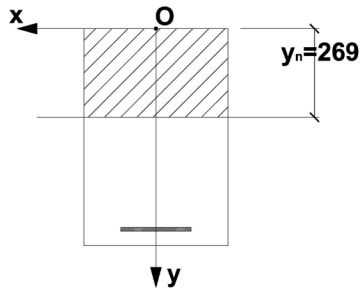


Fig. 7.14. Section at stage II.

The evaluation of the middle-spam displacement can be easily obtained using the relation (7.1) here expressed as

$$v(1/2) = v_I(1/2) \cdot \left(1 + \frac{\Delta v(1/2)}{v_I(1/2)} \right) \quad (7.20)$$

where v_I is the displacement calculated in the first step and $\Delta v(1/2)$ the increase of the displacement itself caused from the cracking, that can be expressed for symmetry reason

$$\Delta v\left(\frac{\ell}{2}\right) = 2(c-1) \frac{M_{\max} \ell^2}{E_c I_I^*} \left[\int_{\xi_1}^{\frac{1}{2}} f_M(\xi) g(\xi) d\xi - \int_{\xi_1}^{\frac{1}{2}} \beta \frac{M_{cr}^2}{M_{\max}^2} \frac{f_M(\xi)}{g(\xi)} d\xi \right], \quad \xi = \frac{z}{l} \quad (7.21)$$

where E_c is assumed to be $E_c = E_s/15$ in agreement with the introduced statement for the parameter α_c .

Defining the parameter $\lambda = M_{\max}/M_{cr}$ and considering that $f_M(\xi) = \xi/2$, $g(\xi) = 4(\xi - \xi^2)$, the equation (7.21) is written as

$$\Delta v\left(\frac{\ell}{2}\right) = (c-1) \frac{M_{\max} \ell^2}{E_c I_I^*} \left[\int_{\xi_1}^{\frac{1}{2}} 4(\xi^2 - \xi^3) d\xi - \frac{\beta}{4\lambda^2} \int_{\xi_1}^{\frac{1}{2}} \frac{d\xi}{1-\xi} \right] \quad (7.22)$$

Calculating the integrals on the right side of the equation we finally obtain

$$\Delta v\left(\frac{\ell}{2}\right) = (c-1) \frac{M_{\max} \ell^2}{E_c I_I^*} \left[\frac{5}{48} + \xi_1^4 - \frac{4}{3} \xi_1^3 - \frac{\beta}{4\lambda^2} \ln[2(1-\xi_1)] \right] \quad (7.23)$$

The abscissa ξ_1 , where the cracked part of the beam start, is given solving the equation

$$4(\xi_1 - \xi_1^2) = \frac{M_{cr}}{M_{max}} = \frac{1}{\lambda} \quad (7.24)$$

$$\text{and then } \xi_1 = \frac{1}{2} \left[1 - \sqrt{\frac{\lambda - 1}{\lambda}} \right] \quad (7.25)$$

$$\text{Finally, considering that } v_1 = \frac{5}{48} \frac{M_{max} \ell^2}{E_c I_1^*} \quad (7.26)$$

The (7.20) is expressed as

$$v\left(\frac{\ell}{2}\right) = \frac{5}{48} \frac{M_{max} \ell^2}{E_c I_1^*} \left\{ 1 + (c-1) \left[1 + \frac{48}{5} \left(\xi_1^4 - \frac{4}{3} \xi_1^3 \right) - \frac{12}{5} \frac{\beta}{\lambda^2} \ln[2(1-\xi_1)] \right] \right\} \quad (7.27)$$

If the value of c previously calculated is inserted in the (7.27) stating $\beta=1$ and letting λ changing in the range $1 \leq \lambda \leq \infty$, we obtain the curves reported in Fig. 7.15, that show as the increase of the ratio λ means a decrease for ξ_1 and the increase of $v(1/2)$ as a consequence of a larger cracked part of the beam.

In the same way, a concentrated load $Q=200\text{kN}$, producing the same maximal moment in the mid-span section, leads to the following expression for the section displacement

$$v\left(\frac{\ell}{2}\right) = \frac{M_{max} \ell^2}{12 E_c I_1^*} \left[1 + (c-1) \left[1 - 8 \xi_1^3 - \frac{3\beta}{\lambda^2} (1 - 2\xi_1) \right] \right] \quad (7.28)$$

$$\text{in this case } v_1 = M_{max} \frac{\ell^2}{12 E_c I_1^*} \quad (7.29)$$

$$\xi_1 = 1/(2\lambda). \quad (7.30)$$

The corresponding curves are reported in Fig. 7.15. We observe as the displacements in the two cases of distributed and concentrated load are respectively 0.93 and 0.88 of the displacement calculated in the stage II. Furthermore, for the same M_{max} , the displacement in case of concentrated load results to be lower because the linear trend of the relative bending moment is associated to a smaller region of the cracking beam with respect to the case of distributed load, that is characterized by a parabolic diagram of the bending moments.

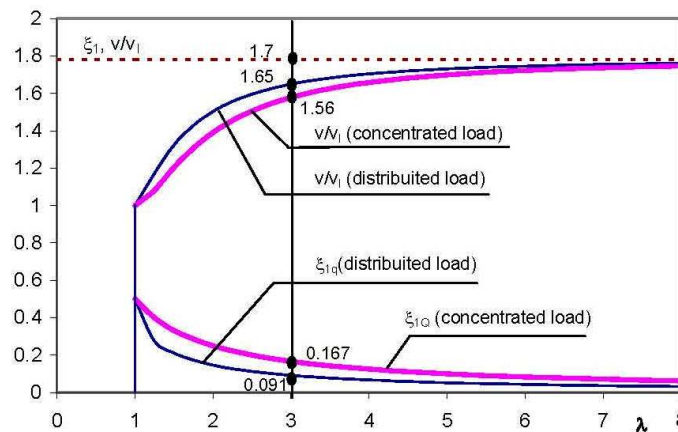


Fig. 7.15. Diagrams for v/v_1 , $\xi_1 - \lambda$.

The same problems can be solved in a generalized form evaluating numerically the displacement following the procedure expressed in (11.51). In this way, it is possible the evaluation the deformation of the whole beam, varying $\bar{z}$. The result, for a distributed load and for $\lambda=3$, is reported in Fig. 7.16, where graphs refer to a 20 folders division for the cracking part of the beam. Remark as the committed error in the evaluation of the mid-span deflection, as obtained comparing the values in Fig. 7.15 and Fig. 7.16, is about 4%. In particular, introducing the numerical values in the (7.26) (7.29) and using the results in Fig. 11.25, we have for the mid-span displacement:

•Distributed load

$$v_1\left(\frac{\ell}{2}\right) = \frac{5}{48} \cdot \frac{500 \cdot 10^6 \cdot 10^8 \cdot 15}{2 \cdot 10^5 \cdot 18.05 \cdot 10^9} = 21.64 \text{ mm}$$

$$v\left(\frac{\ell}{2}\right) = 1.65 \cdot 21.64 = 35.71 \text{ mm}$$

a) Concentrated load

$$v_1\left(\frac{\ell}{2}\right) = \frac{1}{12} \cdot \frac{500 \cdot 10^6 \cdot 10^8 \cdot 15}{2 \cdot 10^5 \cdot 18.05 \cdot 10^9} = 17.31 \text{ mm}$$

$$v\left(\frac{\ell}{2}\right) = 1.56 \cdot 17.31 = 27.00 \text{ mm}$$

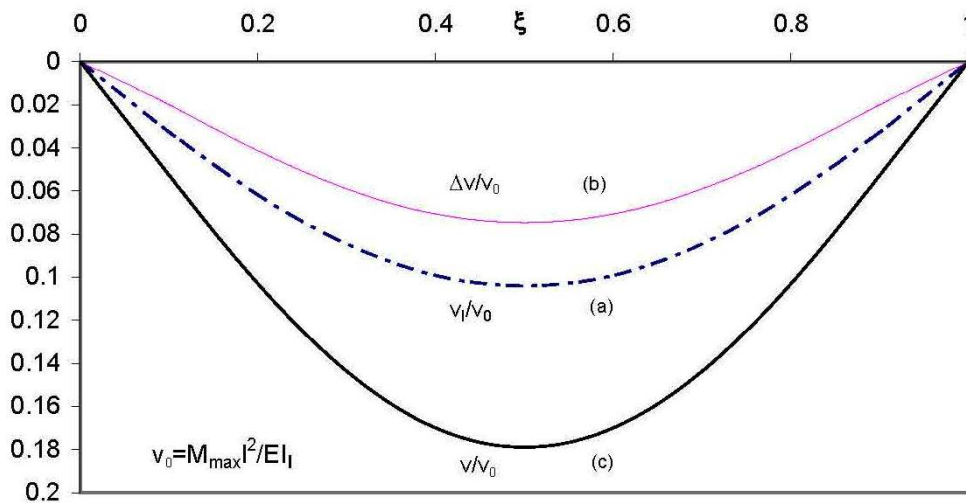


Fig. 7.16. Deformation in the stage I (a), displacement increase caused by the cracking (b) And total deformation (c).

SECTION 11. LIGHTWEIGHT CONCRETE – WORKED EXAMPLES

EXAMPLE 11.1 [EC2 Clause 11.3.1 – 11.3.2]

The criteria for design of the characteristic tensile strength (fractile 5% and 95%) and of the intersecting compressive elastic module for light concrete are shown below, in accordance with the instructions of paragraphs 11.3.1 and 11.3.2 of Eurocode 2.

Tensile strength

The average value of simple (axial) tensile strength, in lack of direct experimentation, can be taken equal to:

- for concrete of class $\leq$ LC 50/55 $f_{lctm} = 0.30 f_{lck}^{2/3} \eta_1$
- for concrete of class $>$ LC 50/55 $f_{lctm} = 2.12 \ln[1+(f_{lcm}/10)] \eta_1$

Where:

$$\eta_1 = 0.40 + 0.60 \rho / 2200$$

ρ = upper limit value of the concrete density, for the corresponding density class expressed in kg/m³;

f_{lck} = value of the characteristic cylinder compressive strength in MPa.

f_{lcm} = value of the average cylinder compressive strength in MPa.

The characteristic values of simple tensile strength, corresponding to fractiles 0.05 and 0.95, can be taken equal to:

$$\text{fractile 5\% : } f_{lctk,0.05} = 0.7 f_{lctm}$$

$$\text{fractile 95\% : } f_{lctk,0.95} = 1.3 f_{lctm}$$

Intersecting compressive elastic module

In lack of direct experimentation, the intersecting compressive elastic module at 28 days, which can be used as an indicative value for design of the deformability of structural members, can be estimated by the expression:

$$E_{lcm} = 22000 \left[\frac{f_{lcm}}{10} \right]^{0.3} \eta_E \quad [\text{MPa}]$$

where:

- f_{lcm} = value of the cylinder average compressive strength in MPa;

$$\bullet \eta_E = \left(\frac{\rho}{2200} \right)^2;$$

ρ = upper limit value of the concrete density, for corresponding density class in kg/m³. The results of calculation of the two above-mentioned mechanical features are shown and compared in the following table, for two different types of light concretes and for the corresponding ordinary concretes belonging to the same strength classes.

Table 11.1.

	Concrete type 1		Concrete type 2	
	Light	Ordinary	Light	Ordinary
f_{ck} [MPa]	35		60	
ρ [kg/m³]	1650	2400	2050	2400
f_{cm} [MPa]	43		68	
η_1	0.850	--	0.959	--
η_E	0.563	--	0.868	--
f_{ctm} [MPa]	2.7	3.2	4.2	4.4
$f_{\text{ctk};0.05}$ [MPa]	1.9	2.2	2.9	3.1
$f_{\text{ctk};0.95}$ [MPa]	3.5	4.2	5.4	5.7
E_{cm} [MPa]	19168	34077	33950	39100

EXAMPLE 11.2 [EC2 Clause 11.3.1 – 11.3.5 – 11.3.6 – 11.4 – 11.6]

The maximum moment that the reinforced concrete section of given dimensions, made of type 1 lightweight concrete, described in the previous example, is able to withstand when the reinforcement steel achieves the design elastic limit. The dimensions of the section are: $b=30$ cm, $h=50$ cm and $d=47$ cm.

The section in question is shown in Fig. 11.1 together with the strain diagram related to the failure mode recalled, which implies the simultaneous achievement of maximum contraction side concrete and of the strain corresponding to the design yield stress of the tensioned reinforcement steel.

In case one chooses, like in the previous example, to use the bilinear diagram to calculate the compressive strength on concrete, the limits of strain by compression have values $\varepsilon_{lc3} = 1.75\text{‰}$ and $\varepsilon_{lcu3} = 3.5\eta_1 = 2.98\text{‰}$.

The design strain corresponding to steel yielding, for $f_{yk}=450$ MPa, is $\varepsilon_{yd} = f_{yd}/(1.15 \times E_s) = 450/(1.15 \times 200000) = 1.96\text{‰}$. The distance of the neutral axis from the compressed upper edge is therefore $x = 28.3$ cm.

Two areas can be distinguished in the compressed zone: the first one is comprised between the upper edge and the chord placed at the level where the contraction is $\varepsilon_{lc3} = 1.75\text{‰}$. The compressive stress in it is constant and it is equal to $f_{lcd} = 0.85 f_{lck}/\gamma_c = 19.8$ MPa; the second remaining area is the one where compression on concrete linearly decreases from the value f_{lcd} to zero in correspondence of the neutral axis.

The resultant of compression forces is placed at a distance of around 10.5 cm from the compressed end of the section and is equal to $C = 1185$ kN. For the condition of equilibrium the resultant of compressions C is equal to the resultant of tractions T , to which corresponds a steel section A_s equal to $A_s = T/f_{yd} = 3030$ mm². The arm of internal forces is $h' = d - 10.5$ cm = 36.5 cm, from which the value of the moment resistance of the section can eventually be calculated as $M_{Rd} = 1185 \times 0.365 = 432.5$ kNm.

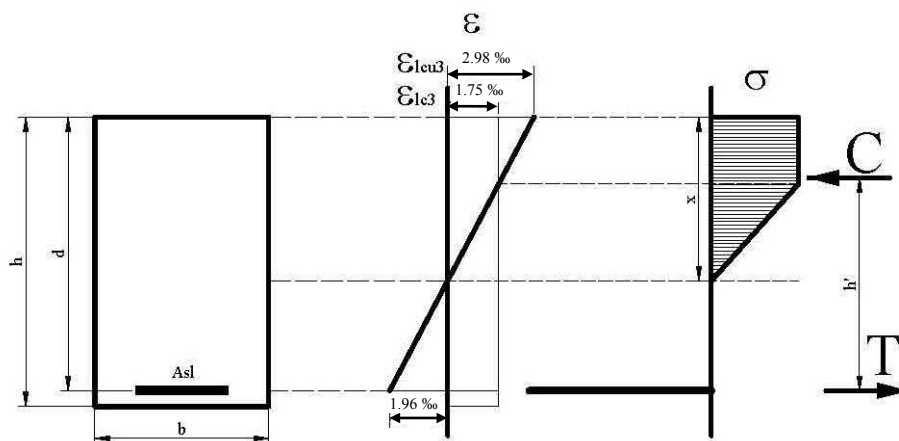


Fig.11.1. Deformation and tension diagram of r.c. section, build up with lightweight concrete ($f_{lck} = 35$ MPa, $\rho = 1650$ kg/m³), for collapse condition in which maximum resisting bending moment is reached with reinforcement at elastic design limit.

