

Example: Idea → Mathematical Model (Epidemic Spread)

Demonstration of the full workflow on a concrete idea.

Phase 1 — Parse

Idea (user): "A new contagious disease appears in a city of 1M people. What happens?"

- **System:** human population of the city, compartmentalized by disease status
- **State:** counts of Susceptible / Infectious / Recovered individuals over time
- **Inputs:** initial infections, contact patterns
- **Goal:** prediction — peak size and timing, total infected, does it die out?
- **Horizon:** months, daily resolution

Phase 2 — Decompose

Sub-problem	Nature
Transmission on contact	interaction → but mass-action aggregation valid for large city ⇒ treat as
Recovery process	flow
Population turnover (births/deaths)	flow (negligible on months horizon)

Coupling: transmission consumes susceptibles, feeds infectious; recovery drains infectious.

Phase 3 — Parameters

Symbol	Name	Unit	Exo/Endo	Range	Source	Sensitivity	
β	transmission rate	1/day	exo	0.2–0.5	lit.	high	Excluded:
γ	recovery rate	1/day	exo	0.05–0.2	lit.	high	
$I(0)$	initial infected	persons	exo	1–50	est.	medium	
$S(t), I(t), R(t)$	compartment counts	persons	endo	≥ 0	derived	—	

seasonal forcing (horizon $\ll$ 1 yr), spatial structure (first pass), demographics (months horizon).
Derived quantity: $R_0 = \beta/\gamma$ — threshold at $R_0 = 1$.

Phase 4 — Assumptions

#	Assumption	Type	Violation consequence
1	Homogeneous mixing [R]	Structural	Overestimates early growth if network is clustered
2	Constant β, γ in time [S]	Parametric	Wrong peak timing if behavior/seasonality shifts rates
3	No imports/exports [R]	Boundary	Misses re-seeding waves
4	Permanent immunity after recovery [S]	Structural	Multi-wave dynamics lost if immunity wanes

Assumptions 2 and 4 are [S] → included in sensitivity sweep.

Phase 5 — Perspectives

Deterministic (primary)

$$\frac{dS}{dt} = -\beta \frac{SI}{N}, \quad \frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I, \quad \frac{dR}{dt} = \gamma I$$

Insight: sharp threshold — epidemic iff $R_0 > 1$; final size equation predicts total attack rate independent of details. Blind spot: no chance of early stochastic fade-out.

Stochastic (validation)

Continuous-time Markov chain with same rates, Gillespie simulation. Insight: with a single index case $I(0)=1$, extinction before major outbreak has probability $\approx 1/R_0$ even when $R_0 > 1$ (for larger seeds it falls roughly as $(1/R_0)^{I(0)}$) — invisible to ODEs. Blind spot: expensive, gives distributions not clean thresholds.

Optimization (secondary)

Policy question layered on top: choose closure level $c \in [0,1]$ reducing $\beta \rightarrow \beta(1-c)$, minimize economic cost $k \cdot c^2$ + medical cost $m \cdot I_{\text{peak}}$. Insight: reveals acceptable intervention intensity trade-off. Blind spot: assumes cost curves are known.

Agent-based (rejected)

Rejected: heterogeneity and network structure deliberately excluded in first pass; ABM adds cost without answering THIS question better. Would become relevant if assumption 1 fails validation against data.

Phase 6 — Comparison & Recommendation

Criterion	Det	Stoch	Opt	ABM	Recom-
Fidelity (this question)	4	4	3	4	
Data needs	low	low	med	high	
Compute cost	tiny	low	tiny	high	
Answers goal question	yes peak/timing	yes fade-out prob	policy layer	no overkill	
mendation: deterministic SIR primary + stochastic CTMC validation. Code below implements both.					

Phase 7 — Implementation

See `skills/axiomize/tools/validate.py` usage:

```
python skills/axiomize/tools/validate.py --model sir --beta 0.3 --gamma 0.1 --I0 10 --N 100000
```

Outputs: peak height/timing, final size vs theoretical prediction (consistency check), R_0 , sensitivity sweep over $\beta \in [0.2, 0.5]$.

Phase 8 — Falsifiability

Model dies if observed data show: (a) multiple waves without behavior change (assumption 4 broken), (b) early exponential growth far from $\beta SI/N$ prediction (mixing assumption broken), (c)

sustained endemic plateau (waning immunity).

Confidence Ledger

Claim	Type	Basis
$R_0 > 1 \implies$ epidemic; final-size equation	established	standard SIR theory
Stochastic fade-out $\approx 1/R_0$ for single index case	established	branching-process result
β, γ ranges and constant rates	assumption	lit. ranges; [S] on constancy
Permanent immunity	assumption	[S], sweep-covered
Announcement coverage p achievable	speculation	unvalidated policy claim