Sensitivity calculation of a radio neutrino detector

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1 Introduction

This note discusses how to properly calculate the sensitivity of a neutrino detector from MC simulations.

2 Calculation of effective volume/area etc.

We assume that simulations for discrete neutrino energies were performed. The following equations refer to one specific neutrino energy E_ν.

The effective volume is defined via

$$V_{\mathrm{eff}} = V/N \sum_{i \in \mathrm{triggered}} \omega_i \,, \tag{1}$$

where V is the simulation volume, N is the total number of simulated events. The sum runs over all triggered events and ω_i is the weight (i.e. the probability of a neutrino reaching the simulation volume) of event i.

The effective area is then given by

$$A_{\mathrm{eff}} = V_{\mathrm{eff}}/L_{\mathrm{int}}(E_\nu, \rho) \,, \tag{2}$$

where $L_{\mathrm{int}}(E_\nu, \rho)$ is the interaction length of neutrinos of energy E_ν in a medium with density ρ. The interaction length is given by

$$L_{\mathrm{int}} = \frac{M_N}{\sigma(E_\nu)\rho} \,, \tag{3}$$

according to [?] where M_N is the nucleon mass which we approximate with the mass of the proton m_p and ρ is the average density along the path of the neutrino, which we approximate with the density of deep ice of $\rho = 0.917\,\mathrm{g/cm^3}$. This should be a reasonable assumption as the firn is typically only a small fraction of the full simulation module. However, we can easily replace the a constant density by a the averaged density which will depend on the incoming direction and vertex position in the future.

For diffuse flux sensitivities it is often useful to define the "effective volume steradian" which is

$$V_{\mathrm{eff,sr}} = V_{\mathrm{eff}} \times \Delta\Omega \,, \tag{4}$$

where $\Delta\Omega$ is the solid angle of the simulation which is typically 4π.

The exposure is the time integrated effective ares

$$\epsilon = \int \mathrm{d}t A_{\mathrm{eff}} = T \times A_{\mathrm{eff}} \,, \tag{5}$$

where the last equality holds if the effective area is independent of time and T is the total integration time.

2.1 Historical definition

Typically, the water equivalent effective volume is quoted for an inter-experimental comparison. Thus, the effective volume is multiplied by the ratio of $\rho_{\mathrm{ice}}/\rho_{\mathrm{water}}$. Internally in the code, this definition is error prone, as the interaction length then needs to be calculated for water to convert to effective area. Therefore, we don't use it internally but we provide utility functions to convert to water-equivalent effective volume.

3 Diffuse limit

Assuming a non-observation in a certain neutrino energy interval ΔE a limit of the diffuse neutrino flux can be derived. The Feldman-Cousins 90% CL upper limit corresponds to the flux level that predicts 2.4 events in an interval ΔE. Typically we assume that the neutrino flux follows a $N_\nu \propto E_\nu^{-2}$ spectrum.

The number of events for a diffuse neutrino flux $F(E_\nu)$ is given by

$$N = \int \mathrm{d}E_\nu F(E_\nu)\epsilon(E_\nu)4\pi\,, \tag{6}$$

where $\epsilon(E_\nu)$ is the average exposure over the full 4π sky, i.e., the time integral of the averaged A_{eff} over all incoming directions. For $F(E_\nu) = kE^{-\gamma}$ we get

$$N = \int \mathrm{d}E_\nu kE^{-\gamma}\epsilon(E_\nu)4\pi \tag{7}$$

$$= \int \mathrm{d}\log_{10}E\;\ln(10)\,k\,E^{-\gamma+1}\,\epsilon(E_\nu)\,4\pi \tag{8}$$

The flux limit is then given by

$$k = \frac{2.4}{\int \mathrm{d}\log_{10}E\;\ln(10)\,E^{-\gamma+1}\,\epsilon(E_\nu)\,4\pi} \tag{9}$$

$$= \frac{2.4\,E^{\gamma-1}}{\Delta\log_{10}E\;\ln(10)\,V_{\mathrm{eff}}(E)/L_{\mathrm{int}}(E)\,T\,4\pi}\,. \tag{10}$$

For multiple stations T can be replaced by $T \times N_{\mathrm{stations}}$ assuming that there is no overlap in the detected events between stations.

4 Point source fluence limits

The number of events for a flux $F(E) = k\,E^{-\gamma}$ from a point source is given by

$$N = \int \mathrm{d}E\,F(E_\nu)\epsilon(E) \tag{11}$$

$$= \int \mathrm{d}E\,k\,E^{-\gamma}\epsilon(E) \tag{12}$$

$$= \int \mathrm{d}\log_{10}E\;\ln(10)\,k\,E^{-\gamma+1}\,\epsilon(E) \tag{13}$$

$$= \int \mathrm{d}\log_{10}E\;\ln(10)\,k\,E^{-\gamma+1} \tag{14}$$

The flux normalization k depends on γ and has units of [area^{-2} $\times$ energy$^{\gamma-1}$]. Thus, the flux normalization k always needs to be presented together with the spectral index γ, and the reference energy.