

1 Nomenclature

1.1 Indices and Sets

$g \in \mathcal{G}$	Set of thermal generators.
$g \in \mathcal{G}_{on}^0$	Set of thermal generators which are initially committed (on).
$g \in \mathcal{G}_{off}^0$	Set of thermal generators which are not initially committed (off).
$w \in \mathcal{W}$	Set of renewable generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \dots, T$, $T = \text{time_periods}$
$l \in \mathcal{L}_g$	Piecewise production cost intervals for thermal generator g : $1, \dots, L_g$.
$s \in \mathcal{S}_g$	Startup categories for thermal generator g , from hottest (1) to coldest (S_g): $1, \dots, S_g$.

1.2 System Parameters

$D(t)$	Load (demand) at time t (MW), demand .
$R(t)$	Spinning reserve at time t (MW), reserves .

1.3 Thermal Generator Parameters

CS_g^s	Startup cost in category s for generator g (\$), startup['cost'] .
CP_g^l	Cost of operating at piecewise generation point l for generator g (MW), piecewise_production['cost'] .
DT_g	Minimum down time for generator g (h), time_down_minimum .
DT_g^0	Number of time periods the unit has been off prior to the first time period for generator g , time_down_t0 .
\bar{P}_g	Maximum power output for generator g (MW), power_output_maximum .
\underline{P}_g	Minimum power output for generator g (MW), power_output_minimum .
P_g^0	Power output for generator g (MW) in the time period prior to $t=1$, power_output_t0 .
P_g^l	Power level for piecewise generation point l for generator g (MW); $P_g^1 = \underline{P}_g$ and $P_g^{L_g} = \bar{P}_g$, piecewise_production['mw'] .
RD_g	Ramp-down rate for generator g (MW/h), ramp_down_limit .
RU_g	Ramp-up rate for generator g (MW/h), ramp_up_limit .
SD_g	Shutdown capability for generator g (MW), ramp_shutdown_limit .
SU_g	Startup capability for generator g (MW), ramp_startup_limit .
TS_g^s	Time offline after which the startup category s becomes active (h), startup['lag'] .
UT_g	Minimum up time for generator g (h), time_up_minimum .
UT_g^0	Number of time periods the unit has been on prior to the first time period for generator g , time_up_t0 .
U_g^0	Initial on/off status for generator g , $U_g^0 = 1$ for $g \in \mathcal{G}_{on}^0$, $U_g^0 = 0$ for $g \in \mathcal{G}_{off}^0$, unit_on_t0 .
U_g	Must-run status for generator g , must_run .

1.4 Renewable Generator Parameters

$\bar{P}_w(t)$	Maximum renewable generation available from renewable generator w at time t (MW), power_output_maximum .
$\underline{P}_w(t)$	Minimum renewable generation available from renewable generator w at time t (MW), power_output_minimum .

1.5 Variables

$c_g(t)$	Cost of power produced above minimum for thermal generator g at time t (MW), $\in \mathbb{R}$.
$p_g(t)$	Power above minimum for thermal generator g at time t (MW), ≥ 0 .
$p_w(t)$	Renewable generation used from renewable generator w at time t (MW), ≥ 0 .
$r_g(t)$	Spinning reserves provided by thermal generator g at time t (MW), ≥ 0 .
$u_g(t)$	Commitment status of thermal generator g at time t , $\in \{0, 1\}$.

$v_g(t)$	Startup status of thermal generator g at time t , $\in \{0, 1\}$.
$w_g(t)$	Shutdown status of thermal generator g at time t , $\in \{0, 1\}$.
$\delta_g^s(t)$	Startup in category s for thermal generator g at time t , $\in \{0, 1\}$.
$\lambda_g^l(t)$	Fraction of power from piecewise generation point l for generator g at time t (MW), $\in [0, 1]$.

2 Model Description

Below we describe the unit commitment model given by [1], with the piecewise production cost description from [2]. The unit commitment problem can then be formulated as:

$$\min \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(c_g(t) + CP_g^1 u_g(t) + \sum_{s=1}^{S_g} (CS_g^s \delta_g^s(t)) \right) \quad (1)$$

subject to:

$$\sum_{g \in \mathcal{G}} (p_g(t) + \underline{P}_g u_g(t)) + \sum_{w \in \mathcal{W}} p_w(t) = D(t) \quad \forall t \in \mathcal{T} \quad (2)$$

$$\sum_{g \in \mathcal{G}} r_g(t) \geq R(t) \quad \forall t \in \mathcal{T} \quad (3)$$

$$\sum_{t=1}^{\min\{UT_g - UT_g^0, T\}} (u_g(t) - 1) = 0 \quad \forall g \in \mathcal{G}_{on}^0 \quad (4)$$

$$\sum_{t=1}^{\min\{DT_g - DT_g^0, T\}} u_g(t) = 0 \quad \forall g \in \mathcal{G}_{off}^0 \quad (5)$$

$$u_g(1) - U_g^0 = v_g(1) - w_g(1) \quad \forall g \in \mathcal{G} \quad (6)$$

$$\sum_{s=1}^{S_g-1} \sum_{t=\max\{1, TS_g^{s+1} - DT_g^0 + 1\}}^{\min\{TS_g^{s+1} - 1, T\}} \delta_g^s(t) = 0 \quad \forall g \in \mathcal{G} \quad (7)$$

$$p_g(1) + r_g(1) - U_g^0 (P_g^0 - \underline{P}_g) \leq RU_g \quad \forall g \in \mathcal{G} \quad (8)$$

$$U_g^0 (P_g^0 - \underline{P}_g) - p_g(1) \leq RD_g \quad \forall g \in \mathcal{G} \quad (9)$$

$$U_g^0 (P_g^0 - \underline{P}_g) \leq (\bar{P}_g - \underline{P}_g) U_g^0 - \max\{(\bar{P}_g - SD_g), 0\} w_g(1) \quad \forall g \in \mathcal{G} \quad (10)$$

$$u_g(t) \geq U_g \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (11)$$

$$u_g(t) - u_g(t-1) = v_g(t) - w_g(t) \quad \forall t \in \mathcal{T} \setminus \{1\}, \forall g \in \mathcal{G} \quad (12)$$

$$\sum_{i=t-\min\{UT_g, T\}+1}^t v_g(i) \leq u_g(t) \quad \forall t \in \{\min\{UT_g, T\}, \dots, T\}, \forall g \in \mathcal{G} \quad (13)$$

$$\sum_{i=t-\min\{DT_g, T\}+1}^t w_g(i) \leq 1 - u_g(t) \quad \forall t \in \{\min\{DT_g, T\}, \dots, T\}, \forall g \in \mathcal{G} \quad (14)$$

$$\delta_g^s(t) \leq \sum_{i=TS_g^s}^{TS_g^{s+1}-1} w_g(t-i) \quad \forall t \in \{TS_g^{s+1}, \dots, T\}, \forall s \in \mathcal{S}_g \setminus \{S_g\}, \forall g \in \mathcal{G} \quad (15)$$

$$v_g(t) = \sum_{s=1}^{S_g} \delta_g^s(t) \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (16)$$

$$p_g(t) + r_g(t) \leq (\bar{P}_g - \underline{P}_g)u_g(t) - \max\{\bar{P}_g - SU_g, 0\}v_g(t) \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (17)$$

$$p_g(t) + r_g(t) \leq (\bar{P}_g - \underline{P}_g)u_g(t) - \max\{\bar{P}_g - SD_g, 0\}w_g(t+1) \quad \forall t \in \mathcal{T} \setminus \{T\}, \forall g \in \mathcal{G} \quad (18)$$

$$p_g(t) + r_g(t) - p_g(t-1) \leq RU_g \quad \forall t \in \mathcal{T} \setminus \{1\}, \forall g \in \mathcal{G} \quad (19)$$

$$p_g(t-1) - p_g(t) \leq RD_g \quad \forall t \in \mathcal{T} \setminus \{1\}, \forall g \in \mathcal{G} \quad (20)$$

$$p_g(t) = \sum_{l \in \mathcal{L}_g} (P_g^l - P_g^1) \lambda_g^l(t) \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (21)$$

$$c_g(t) = \sum_{l \in \mathcal{L}_g} (CP_g^l - CP_g^1) \lambda_g^l(t) \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (22)$$

$$u_g(t) = \sum_{l \in \mathcal{L}_g} \lambda_g^l(t) \quad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \quad (23)$$

$$\underline{P}_w(t) \leq p_w(t) \leq \bar{P}_w(t) \quad \forall t \in \mathcal{T}, \forall w \in \mathcal{W} \quad (24)$$

Note that in constraints (4), (5), and (7), we use the convention that empty sums are 0.

References

- [1] MORALES-ESPAÑA, G., LATORRE, J. M., AND RAMOS, A. Tight and compact MILP formulation for the thermal unit commitment problem. *IEEE Transactions on Power Systems* 28, 4 (2013), 4897–4908.
- [2] SRIDHAR, S., LINDEROTH, J., AND LUEDTKE, J. Locally ideal formulations for piecewise linear functions with indicator variables. *Operations Research Letters* 41, 6 (2013), 627–632.