

## Intelligent Machinery, A Heretical Theory

‘You cannot make a machine to think for you.’ This is a commonplace that is usually accepted without question. It will be the purpose of this paper to question it.

Most machinery developed for commercial purposes is intended to carry out some very specific job, and to carry it out with certainty and considerable speed. Very often it does the same series of operations over and over again without any variety. This fact about the actual machinery available is a powerful argument to many in favour of the slogan quoted above. To a mathematical logician this argument is not available, for it has been shown that there are machines theoretically possible which will do something very close to thinking. They will, for instance, test the validity of a formal proof in the system of Principia Mathematica, or even tell of a formula of that system whether it is provable or disprovable. In the case that the formula is neither provable nor disprovable such a machine certainly does not behave in a very satisfactory manner, for it continues to work indefinitely without producing any result at all, but this cannot be regarded as very different from the reaction of the mathematicians, who have for instance worked for hundreds of years on the question as to whether Fermat’s last theorem is true or not. For the case of machines of this kind a more subtle argument is necessary. By Gödel’s famous theorem, or some similar argument, one can show that however the machine is constructed there are bound to be cases where the machine fails to give an answer, but a mathematician would be able to. On the other hand, the machine has certain advantages over the mathematician. Whatever it does can be relied upon, assuming no mechanical ‘breakdown’, whereas the mathematician makes a certain proportion of mistakes. I believe that this danger of the mathematician making mistakes is an unavoidable corollary of his power of sometimes hitting upon an entirely new method. This seems to be confirmed by the well known fact that the most reliable people will not usually hit upon really new methods.

My contention is that machines can be constructed which will simulate the behaviour of the human mind very closely. They will make mistakes at times, and at times they may make new and very interesting statements, and on the whole the output of them will be worth attention to the same sort of extent as the output of a human mind. The content of this statement lies in the greater frequency expected for the true statements, and it cannot, I think, be given an exact statement. It would not, for instance, be sufficient to say simply that the machine will make any true statement sooner or later, for an example of such a machine would be one which makes all possible statements sooner or later. We

know how to construct these, and as they would (probably) produce true and false statements about equally frequently, their verdicts would be quite worthless. It would be the actual reaction of the machine to circumstances that would prove my contention, if indeed it can be proved at all.

Let us go rather more carefully into the nature of this 'proof'. It is clearly possible to produce a machine which would give a very good account of itself for any range of tests, if the machine were made sufficiently elaborate. However, this again would hardly be considered an adequate proof. Such a machine would give itself away by making the same sort of mistake over and over again, and being quite unable to correct itself, or to be corrected by argument from outside. If the machine were able in some way to 'learn by experience' it would be much more impressive. If this were the case there seems to be no real reason why one should not start from a comparatively simple machine, and, by subjecting it to a suitable range of 'experience' transform it into one which was more elaborate, and was able to deal with a far greater range of contingencies. This process could probably be hastened by a suitable selection of the experiences to which it was subjected. This might be called 'education'. But here we have to be careful. It would be quite easy to arrange the experiences in such a way that they automatically caused the structure of the machine to build up into a previously intended form, and this would obviously be a gross form of cheating, almost on a par with having a man inside the machine. Here again the criterion as to what would be considered reasonable in the way of 'education' cannot be put into mathematical terms, but I suggest that the following would be adequate in practice. Let us suppose that it is intended that the machine shall understand English, and that owing to its having no hands or feet, and not needing to eat, nor desiring to smoke, it will occupy its time mostly in playing games such as Chess and GO, and possibly Bridge. The machine is provided with a typewriter keyboard on which any remarks to it are typed, and it also types out any remarks that it wishes to make. I suggest that the education of the machine should be entrusted to some highly competent schoolmaster who is interested in the project but who is forbidden any detailed knowledge of the inner workings of the machine. The mechanic who has constructed the machine, however, is permitted to keep the machine in running order, and if he suspects that the machine has been operating incorrectly may put it back to one of its previous positions and ask the schoolmaster to repeat his lessons from that point on, but he may not take any part in the teaching. Since this procedure would only serve to test the bona fides of the mechanic, I need hardly say that it would not be adopted in the experimental stages. As I see it, this education process would in practice be an essential to the production of a reasonably intelligent machine within a reasonably short space of time. The human analogy alone suggests this.

I may now give some indication of the way in which such a machine might be expected to function. The machine would incorporate a memory. This does not

need very much explanation. It would simply be a list of all the statements that had been made to it or by it, and all the moves it had made and the cards it had played in its games. This would be listed in chronological order. Besides this straightforward memory there would be a number of 'indexes of experiences'. To explain this idea I will suggest the form which one such index might possibly take. It might be an alphabetical index of the words that had been used giving the 'times' at which they had been used, so that they could be looked up in the memory. Another such index might contain patterns of men on parts of a GO board that had occurred. At comparatively late stages of education the memory might be extended to include important parts of the configuration of the machine at each moment, or in other words it would begin to remember what its thoughts had been. This would give rise to fruitful new forms of indexing. New forms of index might be introduced on account of special features observed in the indexes already used. The indexes would be used in this sort of way. Whenever a choice has to be made as to what to do next, features of the present situation are looked up in the indexes available, and the previous choice in the similar situations, and the outcome, good or bad, is discovered. The new choice is made accordingly. This raises a number of problems. If some of the indications are favourable and some are unfavourable what is one to do? The answer to this will probably differ from machine to machine and will also vary with its degree of education. At first probably some quite crude rule will suffice, e.g. to do whichever has the greatest number of votes in its favour. At a very late stage of education the whole question of procedure in such cases will probably have been investigated by the machine itself, by means of some kind of index, and this may result in some highly sophisticated, and, one hopes, highly satisfactory, form of rule. It seems probable however that the comparatively crude forms of rule will themselves be reasonably satisfactory, so that progress can on the whole be made in spite of the crudeness of the choice [of] rules.<sup>1</sup> This seems to be verified by the fact that engineering problems are sometimes solved by the crudest rule of thumb procedure which only deals with the most superficial aspects of the problem, e.g. whether a function increases or decreases with one of its variables. Another problem raised by this picture of the way behaviour is determined is the idea of 'favourable outcome'. Without some such idea, corresponding to the 'pleasure principle' of the psychologists, it is very difficult to see how to proceed. Certainly it would be most natural to introduce some such thing into the machine. I suggest that there should be two keys which can be manipulated by the schoolmaster, and which represent the ideas of pleasure and pain. At later stages in education the machine would recognise certain other conditions as desirable owing to their having been constantly associated in the past with pleasure, and likewise certain others as undesirable. Certain expressions of

<sup>1</sup> Editor's note. Words enclosed in square brackets do not appear in the typescript.

anger on the part of the schoolmaster might, for instance, be recognised as so ominous that they could never be overlooked, so that the schoolmaster would find that it became unnecessary to 'apply the cane' any more.

To make further suggestions along these lines would perhaps be unfruitful at this stage, as they are likely to consist of nothing more than an analysis of actual methods of education applied to human children. There is, however, one feature that I would like to suggest should be incorporated in the machines, and that is a 'random element'. Each machine should be supplied with a tape bearing a random series of figures, e.g. 0 and 1 in equal quantities, and this series of figures should be used in the choices made by the machine. This would result in the behaviour of the machine not being by any means completely determined by the experiences to which it was subjected, and would have some valuable uses when one was experimenting with it. By faking the choices made one would be able to control the development of the machine to some extent. One might, for instance, insist on the choice made being a particular one at, say, 10 particular places, and this would mean that about one machine in 1024 or more would develop to as high a degree as the one which had been faked. This cannot very well be given an accurate statement because of the subjective nature of the idea of 'degree of development' to say nothing of the fact that the machine that had been faked might have been also fortunate in its unfaked choices.

Let us now assume, for the sake of argument, that these machines are a genuine possibility, and look at the consequences of constructing them. To do so would of course meet with great opposition, unless we have advanced greatly in religious toleration from the days of Galileo. There would be great opposition from the intellectuals who were afraid of being put out of a job. It is probable though that the intellectuals would be mistaken about this. There would be plenty to do, [trying to understand what the machines were trying to say,]<sup>2</sup> i.e. in trying to keep one's intelligence up to the standard set by the machines, for it seems probable that once the machine thinking method had started, it would not take long to outstrip our feeble powers. There would be no question of the machines dying, and they would be able to converse with each other to sharpen their wits. At some stage therefore we should have to expect the machines to take control, in the way that is mentioned in Samuel Butler's 'Erewhon'.

<sup>2</sup> Editor's note. The words 'trying to understand what the machines were trying to say,' are handwritten and are marked in the margin 'Inserted from Turing's Typescript'.

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# ImageNet Classification with Deep Convolutional Neural Networks

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## Abstract

We trained a large, deep convolutional neural network to classify the 1.2 million high-resolution images in the ImageNet LSVRC-2010 contest into the 1000 different classes. On the test data, we achieved top-1 and top-5 error rates of 37.5% and 17.0% which is considerably better than the previous state-of-the-art. The neural network, which has 60 million parameters and 650,000 neurons, consists of five convolutional layers, some of which are followed by max-pooling layers, and three fully-connected layers with a final 1000-way softmax. To make training faster, we used non-saturating neurons and a very efficient GPU implementation of the convolution operation. To reduce overfitting in the fully-connected layers we employed a recently-developed regularization method called “dropout” that proved to be very effective. We also entered a variant of this model in the ILSVRC-2012 competition and achieved a winning top-5 test error rate of 15.3%, compared to 26.2% achieved by the second-best entry.

## 1 Introduction

Current approaches to object recognition make essential use of machine learning methods. To improve their performance, we can collect larger datasets, learn more powerful models, and use better techniques for preventing overfitting. Until recently, datasets of labeled images were relatively small — on the order of tens of thousands of images (e.g., NORB [16], Caltech-101/256 [8, 9], and CIFAR-10/100 [12]). Simple recognition tasks can be solved quite well with datasets of this size, especially if they are augmented with label-preserving transformations. For example, the current-best error rate on the MNIST digit-recognition task ( $<0.3\%$ ) approaches human performance [4]. But objects in realistic settings exhibit considerable variability, so to learn to recognize them it is necessary to use much larger training sets. And indeed, the shortcomings of small image datasets have been widely recognized (e.g., Pinto et al. [21]), but it has only recently become possible to collect labeled datasets with millions of images. The new larger datasets include LabelMe [23], which consists of hundreds of thousands of fully-segmented images, and ImageNet [6], which consists of over 15 million labeled high-resolution images in over 22,000 categories.

To learn about thousands of objects from millions of images, we need a model with a large learning capacity. However, the immense complexity of the object recognition task means that this problem cannot be specified even by a dataset as large as ImageNet, so our model should also have lots of prior knowledge to compensate for all the data we don’t have. Convolutional neural networks (CNNs) constitute one such class of models [16, 11, 13, 18, 15, 22, 26]. Their capacity can be controlled by varying their depth and breadth, and they also make strong and mostly correct assumptions about the nature of images (namely, stationarity of statistics and locality of pixel dependencies). Thus, compared to standard feedforward neural networks with similarly-sized layers, CNNs have much fewer connections and parameters and so they are easier to train, while their theoretically-best performance is likely to be only slightly worse.

Despite the attractive qualities of CNNs, and despite the relative efficiency of their local architecture, they have still been prohibitively expensive to apply in large scale to high-resolution images. Luckily, current GPUs, paired with a highly-optimized implementation of 2D convolution, are powerful enough to facilitate the training of interestingly-large CNNs, and recent datasets such as ImageNet contain enough labeled examples to train such models without severe overfitting.

The specific contributions of this paper are as follows: we trained one of the largest convolutional neural networks to date on the subsets of ImageNet used in the ILSVRC-2010 and ILSVRC-2012 competitions [2] and achieved by far the best results ever reported on these datasets. We wrote a highly-optimized GPU implementation of 2D convolution and all the other operations inherent in training convolutional neural networks, which we make available publicly<sup>1</sup>. Our network contains a number of new and unusual features which improve its performance and reduce its training time, which are detailed in Section 3. The size of our network made overfitting a significant problem, even with 1.2 million labeled training examples, so we used several effective techniques for preventing overfitting, which are described in Section 4. Our final network contains five convolutional and three fully-connected layers, and this depth seems to be important: we found that removing any convolutional layer (each of which contains no more than 1% of the model’s parameters) resulted in inferior performance.

In the end, the network’s size is limited mainly by the amount of memory available on current GPUs and by the amount of training time that we are willing to tolerate. Our network takes between five and six days to train on two GTX 580 3GB GPUs. All of our experiments suggest that our results can be improved simply by waiting for faster GPUs and bigger datasets to become available.

## 2 The Dataset

ImageNet is a dataset of over 15 million labeled high-resolution images belonging to roughly 22,000 categories. The images were collected from the web and labeled by human labelers using Amazon’s Mechanical Turk crowd-sourcing tool. Starting in 2010, as part of the Pascal Visual Object Challenge, an annual competition called the ImageNet Large-Scale Visual Recognition Challenge (ILSVRC) has been held. ILSVRC uses a subset of ImageNet with roughly 1000 images in each of 1000 categories. In all, there are roughly 1.2 million training images, 50,000 validation images, and 150,000 testing images.

ILSVRC-2010 is the only version of ILSVRC for which the test set labels are available, so this is the version on which we performed most of our experiments. Since we also entered our model in the ILSVRC-2012 competition, in Section 6 we report our results on this version of the dataset as well, for which test set labels are unavailable. On ImageNet, it is customary to report two error rates: top-1 and top-5, where the top-5 error rate is the fraction of test images for which the correct label is not among the five labels considered most probable by the model.

ImageNet consists of variable-resolution images, while our system requires a constant input dimensionality. Therefore, we down-sampled the images to a fixed resolution of  $256 \times 256$ . Given a rectangular image, we first rescaled the image such that the shorter side was of length 256, and then cropped out the central  $256 \times 256$  patch from the resulting image. We did not pre-process the images in any other way, except for subtracting the mean activity over the training set from each pixel. So we trained our network on the (centered) raw RGB values of the pixels.

## 3 The Architecture

The architecture of our network is summarized in Figure 2. It contains eight learned layers — five convolutional and three fully-connected. Below, we describe some of the novel or unusual features of our network’s architecture. Sections 3.1-3.4 are sorted according to our estimation of their importance, with the most important first.

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<sup>1</sup><http://code.google.com/p/cuda-convnet/>

### 3.1 ReLU Nonlinearity

The standard way to model a neuron’s output  $f$  as a function of its input  $x$  is with  $f(x) = \tanh(x)$  or  $f(x) = (1 + e^{-x})^{-1}$ . In terms of training time with gradient descent, these saturating nonlinearities are much slower than the non-saturating nonlinearity  $f(x) = \max(0, x)$ . Following Nair and Hinton [20], we refer to neurons with this nonlinearity as Rectified Linear Units (ReLU). Deep convolutional neural networks with ReLUs train several times faster than their equivalents with tanh units. This is demonstrated in Figure 1, which shows the number of iterations required to reach 25% training error on the CIFAR-10 dataset for a particular four-layer convolutional network. This plot shows that we would not have been able to experiment with such large neural networks for this work if we had used traditional saturating neuron models.

We are not the first to consider alternatives to traditional neuron models in CNNs. For example, Jarrett et al. [11] claim that the nonlinearity  $f(x) = |\tanh(x)|$  works particularly well with their type of contrast normalization followed by local average pooling on the Caltech-101 dataset. However, on this dataset the primary concern is preventing overfitting, so the effect they are observing is different from the accelerated ability to fit the training set which we report when using ReLUs. Faster learning has a great influence on the performance of large models trained on large datasets.

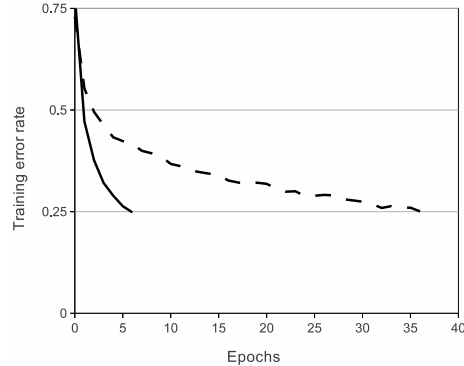


Figure 1: A four-layer convolutional neural network with ReLUs (**solid line**) reaches a 25% training error rate on CIFAR-10 six times faster than an equivalent network with tanh neurons (**dashed line**). The learning rates for each network were chosen independently to make training as fast as possible. No regularization of any kind was employed. The magnitude of the effect demonstrated here varies with network architecture, but networks with ReLUs consistently learn several times faster than equivalents with saturating neurons.

### 3.2 Training on Multiple GPUs

A single GTX 580 GPU has only 3GB of memory, which limits the maximum size of the networks that can be trained on it. It turns out that 1.2 million training examples are enough to train networks which are too big to fit on one GPU. Therefore we spread the net across two GPUs. Current GPUs are particularly well-suited to cross-GPU parallelization, as they are able to read from and write to one another’s memory directly, without going through host machine memory. The parallelization scheme that we employ essentially puts half of the kernels (or neurons) on each GPU, with one additional trick: the GPUs communicate only in certain layers. This means that, for example, the kernels of layer 3 take input from all kernel maps in layer 2. However, kernels in layer 4 take input only from those kernel maps in layer 3 which reside on the same GPU. Choosing the pattern of connectivity is a problem for cross-validation, but this allows us to precisely tune the amount of communication until it is an acceptable fraction of the amount of computation.

The resultant architecture is somewhat similar to that of the “columnar” CNN employed by Cireřan et al. [5], except that our columns are not independent (see Figure 2). This scheme reduces our top-1 and top-5 error rates by 1.7% and 1.2%, respectively, as compared with a net with half as many kernels in each convolutional layer trained on one GPU. The two-GPU net takes slightly less time to train than the one-GPU net<sup>2</sup>.

<sup>2</sup>The one-GPU net actually has the same number of kernels as the two-GPU net in the final convolutional layer. This is because most of the net’s parameters are in the first fully-connected layer, which takes the last convolutional layer as input. So to make the two nets have approximately the same number of parameters, we did not halve the size of the final convolutional layer (nor the fully-connected layers which follow). Therefore this comparison is biased in favor of the one-GPU net, since it is bigger than “half the size” of the two-GPU net.

### 3.3 Local Response Normalization

ReLU's have the desirable property that they do not require input normalization to prevent them from saturating. If at least some training examples produce a positive input to a ReLU, learning will happen in that neuron. However, we still find that the following local normalization scheme aids generalization. Denoting by  $a_{x,y}^i$  the activity of a neuron computed by applying kernel  $i$  at position  $(x, y)$  and then applying the ReLU nonlinearity, the response-normalized activity  $b_{x,y}^i$  is given by the expression

$$b_{x,y}^i = a_{x,y}^i / \left( k + \alpha \sum_{j=\max(0, i-n/2)}^{\min(N-1, i+n/2)} (a_{x,y}^j)^2 \right)^\beta$$

where the sum runs over  $n$  “adjacent” kernel maps at the same spatial position, and  $N$  is the total number of kernels in the layer. The ordering of the kernel maps is of course arbitrary and determined before training begins. This sort of response normalization implements a form of lateral inhibition inspired by the type found in real neurons, creating competition for big activities amongst neuron outputs computed using different kernels. The constants  $k$ ,  $n$ ,  $\alpha$ , and  $\beta$  are hyper-parameters whose values are determined using a validation set; we used  $k = 2$ ,  $n = 5$ ,  $\alpha = 10^{-4}$ , and  $\beta = 0.75$ . We applied this normalization after applying the ReLU nonlinearity in certain layers (see Section 3.5).

This scheme bears some resemblance to the local contrast normalization scheme of Jarrett et al. [11], but ours would be more correctly termed “brightness normalization”, since we do not subtract the mean activity. Response normalization reduces our top-1 and top-5 error rates by 1.4% and 1.2%, respectively. We also verified the effectiveness of this scheme on the CIFAR-10 dataset: a four-layer CNN achieved a 13% test error rate without normalization and 11% with normalization<sup>3</sup>.

### 3.4 Overlapping Pooling

Pooling layers in CNNs summarize the outputs of neighboring groups of neurons in the same kernel map. Traditionally, the neighborhoods summarized by adjacent pooling units do not overlap (e.g., [17, 11, 4]). To be more precise, a pooling layer can be thought of as consisting of a grid of pooling units spaced  $s$  pixels apart, each summarizing a neighborhood of size  $z \times z$  centered at the location of the pooling unit. If we set  $s = z$ , we obtain traditional local pooling as commonly employed in CNNs. If we set  $s < z$ , we obtain overlapping pooling. This is what we use throughout our network, with  $s = 2$  and  $z = 3$ . This scheme reduces the top-1 and top-5 error rates by 0.4% and 0.3%, respectively, as compared with the non-overlapping scheme  $s = 2, z = 2$ , which produces output of equivalent dimensions. We generally observe during training that models with overlapping pooling find it slightly more difficult to overfit.

### 3.5 Overall Architecture

Now we are ready to describe the overall architecture of our CNN. As depicted in Figure 2, the net contains eight layers with weights; the first five are convolutional and the remaining three are fully-connected. The output of the last fully-connected layer is fed to a 1000-way softmax which produces a distribution over the 1000 class labels. Our network maximizes the multinomial logistic regression objective, which is equivalent to maximizing the average across training cases of the log-probability of the correct label under the prediction distribution.

The kernels of the second, fourth, and fifth convolutional layers are connected only to those kernel maps in the previous layer which reside on the same GPU (see Figure 2). The kernels of the third convolutional layer are connected to all kernel maps in the second layer. The neurons in the fully-connected layers are connected to all neurons in the previous layer. Response-normalization layers follow the first and second convolutional layers. Max-pooling layers, of the kind described in Section 3.4, follow both response-normalization layers as well as the fifth convolutional layer. The ReLU non-linearity is applied to the output of every convolutional and fully-connected layer.

The first convolutional layer filters the  $224 \times 224 \times 3$  input image with 96 kernels of size  $11 \times 11 \times 3$  with a stride of 4 pixels (this is the distance between the receptive field centers of neighboring

<sup>3</sup>We cannot describe this network in detail due to space constraints, but it is specified precisely by the code and parameter files provided here: <http://code.google.com/p/cuda-convnet/>.



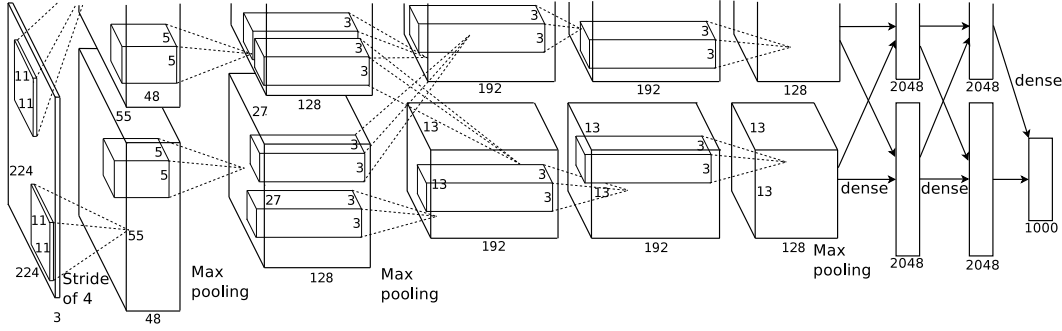


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

neurons in a kernel map). The second convolutional layer takes as input the (response-normalized and pooled) output of the first convolutional layer and filters it with 256 kernels of size  $5 \times 5 \times 48$ . The third, fourth, and fifth convolutional layers are connected to one another without any intervening pooling or normalization layers. The third convolutional layer has 384 kernels of size  $3 \times 3 \times 256$  connected to the (normalized, pooled) outputs of the second convolutional layer. The fourth convolutional layer has 384 kernels of size  $3 \times 3 \times 192$ , and the fifth convolutional layer has 256 kernels of size  $3 \times 3 \times 192$ . The fully-connected layers have 4096 neurons each.

## 4 Reducing Overfitting

Our neural network architecture has 60 million parameters. Although the 1000 classes of ILSVRC make each training example impose 10 bits of constraint on the mapping from image to label, this turns out to be insufficient to learn so many parameters without considerable overfitting. Below, we describe the two primary ways in which we combat overfitting.

### 4.1 Data Augmentation

The easiest and most common method to reduce overfitting on image data is to artificially enlarge the dataset using label-preserving transformations (e.g., [25, 4, 5]). We employ two distinct forms of data augmentation, both of which allow transformed images to be produced from the original images with very little computation, so the transformed images do not need to be stored on disk. In our implementation, the transformed images are generated in Python code on the CPU while the GPU is training on the previous batch of images. So these data augmentation schemes are, in effect, computationally free.

The first form of data augmentation consists of generating image translations and horizontal reflections. We do this by extracting random  $224 \times 224$  patches (and their horizontal reflections) from the  $256 \times 256$  images and training our network on these extracted patches<sup>4</sup>. This increases the size of our training set by a factor of 2048, though the resulting training examples are, of course, highly inter-dependent. Without this scheme, our network suffers from substantial overfitting, which would have forced us to use much smaller networks. At test time, the network makes a prediction by extracting five  $224 \times 224$  patches (the four corner patches and the center patch) as well as their horizontal reflections (hence ten patches in all), and averaging the predictions made by the network’s softmax layer on the ten patches.

The second form of data augmentation consists of altering the intensities of the RGB channels in training images. Specifically, we perform PCA on the set of RGB pixel values throughout the ImageNet training set. To each training image, we add multiples of the found principal components,

<sup>4</sup>This is the reason why the input images in Figure 2 are  $224 \times 224 \times 3$ -dimensional.

with magnitudes proportional to the corresponding eigenvalues times a random variable drawn from a Gaussian with mean zero and standard deviation 0.1. Therefore to each RGB image pixel  $I_{xy} = [I_{xy}^R, I_{xy}^G, I_{xy}^B]^T$  we add the following quantity:

$$[\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3][\alpha_1 \lambda_1, \alpha_2 \lambda_2, \alpha_3 \lambda_3]^T$$

where  $\mathbf{p}_i$  and  $\lambda_i$  are  $i$ th eigenvector and eigenvalue of the  $3 \times 3$  covariance matrix of RGB pixel values, respectively, and  $\alpha_i$  is the aforementioned random variable. Each  $\alpha_i$  is drawn only once for all the pixels of a particular training image until that image is used for training again, at which point it is re-drawn. This scheme approximately captures an important property of natural images, namely, that object identity is invariant to changes in the intensity and color of the illumination. This scheme reduces the top-1 error rate by over 1%.

## 4.2 Dropout

Combining the predictions of many different models is a very successful way to reduce test errors [1, 3], but it appears to be too expensive for big neural networks that already take several days to train. There is, however, a very efficient version of model combination that only costs about a factor of two during training. The recently-introduced technique, called “dropout” [10], consists of setting to zero the output of each hidden neuron with probability 0.5. The neurons which are “dropped out” in this way do not contribute to the forward pass and do not participate in back-propagation. So every time an input is presented, the neural network samples a different architecture, but all these architectures share weights. This technique reduces complex co-adaptations of neurons, since a neuron cannot rely on the presence of particular other neurons. It is, therefore, forced to learn more robust features that are useful in conjunction with many different random subsets of the other neurons. At test time, we use all the neurons but multiply their outputs by 0.5, which is a reasonable approximation to taking the geometric mean of the predictive distributions produced by the exponentially-many dropout networks.

We use dropout in the first two fully-connected layers of Figure 2. Without dropout, our network exhibits substantial overfitting. Dropout roughly doubles the number of iterations required to converge.

## 5 Details of learning

We trained our models using stochastic gradient descent with a batch size of 128 examples, momentum of 0.9, and weight decay of 0.0005. We found that this small amount of weight decay was important for the model to learn. In other words, weight decay here is not merely a regularizer: it reduces the model’s training error. The update rule for weight  $w$  was

$$\begin{aligned} v_{i+1} &:= 0.9 \cdot v_i - 0.0005 \cdot \epsilon \cdot w_i - \epsilon \cdot \left\langle \frac{\partial L}{\partial w} \Big|_{w_i} \right\rangle_{D_i} \\ w_{i+1} &:= w_i + v_{i+1} \end{aligned}$$

where  $i$  is the iteration index,  $v$  is the momentum variable,  $\epsilon$  is the learning rate, and  $\left\langle \frac{\partial L}{\partial w} \Big|_{w_i} \right\rangle_{D_i}$  is the average over the  $i$ th batch  $D_i$  of the derivative of the objective with respect to  $w$ , evaluated at  $w_i$ .

We initialized the weights in each layer from a zero-mean Gaussian distribution with standard deviation 0.01. We initialized the neuron biases in the second, fourth, and fifth convolutional layers, as well as in the fully-connected hidden layers, with the constant 1. This initialization accelerates the early stages of learning by providing the ReLUs with positive inputs. We initialized the neuron biases in the remaining layers with the constant 0.

We used an equal learning rate for all layers, which we adjusted manually throughout training. The heuristic which we followed was to divide the learning rate by 10 when the validation error rate stopped improving with the current learning rate. The learning rate was initialized at 0.01 and



Figure 3: 96 convolutional kernels of size  $11 \times 11 \times 3$  learned by the first convolutional layer on the  $224 \times 224 \times 3$  input images. The top 48 kernels were learned on GPU 1 while the bottom 48 kernels were learned on GPU 2. See Section 6.1 for details.

reduced three times prior to termination. We trained the network for roughly 90 cycles through the training set of 1.2 million images, which took five to six days on two NVIDIA GTX 580 3GB GPUs.

## 6 Results

Our results on ILSVRC-2010 are summarized in Table 1. Our network achieves top-1 and top-5 test set error rates of **37.5%** and **17.0%**<sup>5</sup>. The best performance achieved during the ILSVRC-2010 competition was 47.1% and 28.2% with an approach that averages the predictions produced from six sparse-coding models trained on different features [2], and since then the best published results are 45.7% and 25.7% with an approach that averages the predictions of two classifiers trained on Fisher Vectors (FVs) computed from two types of densely-sampled features [24].

We also entered our model in the ILSVRC-2012 competition and report our results in Table 2. Since the ILSVRC-2012 test set labels are not publicly available, we cannot report test error rates for all the models that we tried. In the remainder of this paragraph, we use validation and test error rates interchangeably because in our experience they do not differ by more than 0.1% (see Table 2). The CNN described in this paper achieves a top-5 error rate of 18.2%. Averaging the predictions of five similar CNNs gives an error rate of 16.4%. Training one CNN, with an extra sixth convolutional layer over the last pooling layer, to classify the entire ImageNet Fall 2011 release (15M images, 22K categories), and then “fine-tuning” it on ILSVRC-2012 gives an error rate of 16.6%. Averaging the predictions of two CNNs that were pre-trained on the entire Fall 2011 release with the aforementioned five CNNs gives an error rate of **15.3%**. The second-best contest entry achieved an error rate of 26.2% with an approach that averages the predictions of several classifiers trained on FVs computed from different types of densely-sampled features [7].

Finally, we also report our error rates on the Fall 2009 version of ImageNet with 10,184 categories and 8.9 million images. On this dataset we follow the convention in the literature of using half of the images for training and half for testing. Since there is no established test set, our split necessarily differs from the splits used by previous authors, but this does not affect the results appreciably. Our top-1 and top-5 error rates on this dataset are **67.4%** and **40.9%**, attained by the net described above but with an additional, sixth convolutional layer over the last pooling layer. The best published results on this dataset are 78.1% and 60.9% [19].

Model	Top-1	Top-5
<i>Sparse coding [2]</i>	47.1%	28.2%
<i>SIFT + FVs [24]</i>	45.7%	25.7%
CNN	<b>37.5%</b>	<b>17.0%</b>

Table 1: Comparison of results on ILSVRC-2010 test set. In *italics* are best results achieved by others.

Model	Top-1 (val)	Top-5 (val)	Top-5 (test)
<i>SIFT + FVs [7]</i>	—	—	26.2%
1 CNN	40.7%	18.2%	—
5 CNNs	38.1%	16.4%	<b>16.4%</b>
1 CNN*	39.0%	16.6%	—
7 CNNs*	36.7%	15.4%	<b>15.3%</b>

Table 2: Comparison of error rates on ILSVRC-2012 validation and test sets. In *italics* are best results achieved by others. Models with an asterisk\* were “pre-trained” to classify the entire ImageNet 2011 Fall release. See Section 6 for details.

### 6.1 Qualitative Evaluations

Figure 3 shows the convolutional kernels learned by the network’s two data-connected layers. The network has learned a variety of frequency- and orientation-selective kernels, as well as various colored blobs. Notice the specialization exhibited by the two GPUs, a result of the restricted connectivity described in Section 3.5. The kernels on GPU 1 are largely color-agnostic, while the kernels on GPU 2 are largely color-specific. This kind of specialization occurs during every run and is independent of any particular random weight initialization (modulo a renumbering of the GPUs).

<sup>5</sup>The error rates without averaging predictions over ten patches as described in Section 4.1 are 39.0% and 18.3%.



Figure 4: **(Left)** Eight ILSVRC-2010 test images and the five labels considered most probable by our model. The correct label is written under each image, and the probability assigned to the correct label is also shown with a red bar (if it happens to be in the top 5). **(Right)** Five ILSVRC-2010 test images in the first column. The remaining columns show the six training images that produce feature vectors in the last hidden layer with the smallest Euclidean distance from the feature vector for the test image.

In the left panel of Figure 4 we qualitatively assess what the network has learned by computing its top-5 predictions on eight test images. Notice that even off-center objects, such as the mite in the top-left, can be recognized by the net. Most of the top-5 labels appear reasonable. For example, only other types of cat are considered plausible labels for the leopard. In some cases (grille, cherry) there is genuine ambiguity about the intended focus of the photograph.

Another way to probe the network’s visual knowledge is to consider the feature activations induced by an image at the last, 4096-dimensional hidden layer. If two images produce feature activation vectors with a small Euclidean separation, we can say that the higher levels of the neural network consider them to be similar. Figure 4 shows five images from the test set and the six images from the training set that are most similar to each of them according to this measure. Notice that at the pixel level, the retrieved training images are generally not close in L2 to the query images in the first column. For example, the retrieved dogs and elephants appear in a variety of poses. We present the results for many more test images in the supplementary material.

Computing similarity by using Euclidean distance between two 4096-dimensional, real-valued vectors is inefficient, but it could be made efficient by training an auto-encoder to compress these vectors to short binary codes. This should produce a much better image retrieval method than applying auto-encoders to the raw pixels [14], which does not make use of image labels and hence has a tendency to retrieve images with similar patterns of edges, whether or not they are semantically similar.

## 7 Discussion

Our results show that a large, deep convolutional neural network is capable of achieving record-breaking results on a highly challenging dataset using purely supervised learning. It is notable that our network’s performance degrades if a single convolutional layer is removed. For example, removing any of the middle layers results in a loss of about 2% for the top-1 performance of the network. So the depth really is important for achieving our results.

To simplify our experiments, we did not use any unsupervised pre-training even though we expect that it will help, especially if we obtain enough computational power to significantly increase the size of the network without obtaining a corresponding increase in the amount of labeled data. Thus far, our results have improved as we have made our network larger and trained it longer but we still have many orders of magnitude to go in order to match the infero-temporal pathway of the human visual system. Ultimately we would like to use very large and deep convolutional nets on video sequences where the temporal structure provides very helpful information that is missing or far less obvious in static images.

## References

- [1] R.M. Bell and Y. Koren. Lessons from the netflix prize challenge. *ACM SIGKDD Explorations Newsletter*, 9(2):75–79, 2007.
- [2] A. Berg, J. Deng, and L. Fei-Fei. Large scale visual recognition challenge 2010. [www.image-net.org/challenges](http://www.image-net.org/challenges). 2010.
- [3] L. Breiman. Random forests. *Machine learning*, 45(1):5–32, 2001.
- [4] D. Cireřan, U. Meier, and J. Schmidhuber. Multi-column deep neural networks for image classification. *Arxiv preprint arXiv:1202.2745*, 2012.
- [5] D.C. Cireřan, U. Meier, J. Masci, L.M. Gambardella, and J. Schmidhuber. High-performance neural networks for visual object classification. *Arxiv preprint arXiv:1102.0183*, 2011.
- [6] J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei. ImageNet: A Large-Scale Hierarchical Image Database. In *CVPR09*, 2009.
- [7] J. Deng, A. Berg, S. Satheesh, H. Su, A. Khosla, and L. Fei-Fei. *ILSVRC-2012*, 2012. URL <http://www.image-net.org/challenges/LSVRC/2012/>.
- [8] L. Fei-Fei, R. Fergus, and P. Perona. Learning generative visual models from few training examples: An incremental bayesian approach tested on 101 object categories. *Computer Vision and Image Understanding*, 106(1):59–70, 2007.
- [9] G. Griffin, A. Holub, and P. Perona. Caltech-256 object category dataset. Technical Report 7694, California Institute of Technology, 2007. URL <http://authors.library.caltech.edu/7694>.
- [10] G.E. Hinton, N. Srivastava, A. Krizhevsky, I. Sutskever, and R.R. Salakhutdinov. Improving neural networks by preventing co-adaptation of feature detectors. *arXiv preprint arXiv:1207.0580*, 2012.
- [11] K. Jarrett, K. Kavukcuoglu, M. A. Ranzato, and Y. LeCun. What is the best multi-stage architecture for object recognition? In *International Conference on Computer Vision*, pages 2146–2153. IEEE, 2009.
- [12] A. Krizhevsky. Learning multiple layers of features from tiny images. Master’s thesis, Department of Computer Science, University of Toronto, 2009.
- [13] A. Krizhevsky. Convolutional deep belief networks on cifar-10. *Unpublished manuscript*, 2010.
- [14] A. Krizhevsky and G.E. Hinton. Using very deep autoencoders for content-based image retrieval. In *ESANN*, 2011.
- [15] Y. Le Cun, B. Boser, J.S. Denker, D. Henderson, R.E. Howard, W. Hubbard, L.D. Jackel, et al. Hand-written digit recognition with a back-propagation network. In *Advances in neural information processing systems*, 1990.
- [16] Y. LeCun, F.J. Huang, and L. Bottou. Learning methods for generic object recognition with invariance to pose and lighting. In *Computer Vision and Pattern Recognition, 2004. CVPR 2004. Proceedings of the 2004 IEEE Computer Society Conference on*, volume 2, pages II–97. IEEE, 2004.
- [17] Y. LeCun, K. Kavukcuoglu, and C. Farabet. Convolutional networks and applications in vision. In *Circuits and Systems (ISCAS), Proceedings of 2010 IEEE International Symposium on*, pages 253–256. IEEE, 2010.
- [18] H. Lee, R. Grosse, R. Ranganath, and A.Y. Ng. Convolutional deep belief networks for scalable unsupervised learning of hierarchical representations. In *Proceedings of the 26th Annual International Conference on Machine Learning*, pages 609–616. ACM, 2009.
- [19] T. Mensink, J. Verbeek, F. Perronnin, and G. Csorba. Metric Learning for Large Scale Image Classification: Generalizing to New Classes at Near-Zero Cost. In *ECCV - European Conference on Computer Vision*, Florence, Italy, October 2012.
- [20] V. Nair and G. E. Hinton. Rectified linear units improve restricted boltzmann machines. In *Proc. 27th International Conference on Machine Learning*, 2010.
- [21] N. Pinto, D.D. Cox, and J.J. DiCarlo. Why is real-world visual object recognition hard? *PLoS computational biology*, 4(1):e27, 2008.
- [22] N. Pinto, D. Doukhan, J.J. DiCarlo, and D.D. Cox. A high-throughput screening approach to discovering good forms of biologically inspired visual representation. *PLoS computational biology*, 5(11):e1000579, 2009.
- [23] B.C. Russell, A. Torralba, K.P. Murphy, and W.T. Freeman. Labelme: a database and web-based tool for image annotation. *International journal of computer vision*, 77(1):157–173, 2008.
- [24] J. Sánchez and F. Perronnin. High-dimensional signature compression for large-scale image classification. In *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*, pages 1665–1672. IEEE, 2011.
- [25] P.Y. Simard, D. Steinkraus, and J.C. Platt. Best practices for convolutional neural networks applied to visual document analysis. In *Proceedings of the Seventh International Conference on Document Analysis and Recognition*, volume 2, pages 958–962, 2003.
- [26] S.C. Turaga, J.F. Murray, V. Jain, F. Roth, M. Helmstaedter, K. Briggman, W. Denk, and H.S. Seung. Convolutional networks can learn to generate affinity graphs for image segmentation. *Neural Computation*, 22(2):511–538, 2010.

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# Attention Is All You Need

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## Abstract

The dominant sequence transduction models are based on complex recurrent or convolutional neural networks that include an encoder and a decoder. The best performing models also connect the encoder and decoder through an attention mechanism. We propose a new simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely. Experiments on two machine translation tasks show these models to be superior in quality while being more parallelizable and requiring significantly less time to train. Our model achieves 28.4 BLEU on the WMT 2014 English-to-German translation task, improving over the existing best results, including ensembles, by over 2 BLEU. On the WMT 2014 English-to-French translation task, our model establishes a new single-model state-of-the-art BLEU score of 41.0 after training for 3.5 days on eight GPUs, a small fraction of the training costs of the best models from the literature.

## 1 Introduction

Recurrent neural networks, long short-term memory [12] and gated recurrent [7] neural networks in particular, have been firmly established as state of the art approaches in sequence modeling and transduction problems such as language modeling and machine translation [29, 2, 5]. Numerous efforts have since continued to push the boundaries of recurrent language models and encoder-decoder architectures [31, 21, 13].

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\*Equal contribution. Listing order is random. Jakob proposed replacing RNNs with self-attention and started the effort to evaluate this idea. Ashish, with Illia, designed and implemented the first Transformer models and has been crucially involved in every aspect of this work. Noam proposed scaled dot-product attention, multi-head attention and the parameter-free position representation and became the other person involved in nearly every detail. Niki designed, implemented, tuned and evaluated countless model variants in our original codebase and tensor2tensor. Llion also experimented with novel model variants, was responsible for our initial codebase, and efficient inference and visualizations. Łukasz and Aidan spent countless long days designing various parts of and implementing tensor2tensor, replacing our earlier codebase, greatly improving results and massively accelerating our research.

<sup>†</sup>Work performed while at Google Brain.

<sup>‡</sup>Work performed while at Google Research.

Recurrent models typically factor computation along the symbol positions of the input and output sequences. Aligning the positions to steps in computation time, they generate a sequence of hidden states  $h_t$ , as a function of the previous hidden state  $h_{t-1}$  and the input for position  $t$ . This inherently sequential nature precludes parallelization within training examples, which becomes critical at longer sequence lengths, as memory constraints limit batching across examples. Recent work has achieved significant improvements in computational efficiency through factorization tricks [18] and conditional computation [26], while also improving model performance in case of the latter. The fundamental constraint of sequential computation, however, remains.

Attention mechanisms have become an integral part of compelling sequence modeling and transduction models in various tasks, allowing modeling of dependencies without regard to their distance in the input or output sequences [2, 16]. In all but a few cases [22], however, such attention mechanisms are used in conjunction with a recurrent network.

In this work we propose the Transformer, a model architecture eschewing recurrence and instead relying entirely on an attention mechanism to draw global dependencies between input and output. The Transformer allows for significantly more parallelization and can reach a new state of the art in translation quality after being trained for as little as twelve hours on eight P100 GPUs.

## 2 Background

The goal of reducing sequential computation also forms the foundation of the Extended Neural GPU [20], ByteNet [15] and ConvS2S [8], all of which use convolutional neural networks as basic building block, computing hidden representations in parallel for all input and output positions. In these models, the number of operations required to relate signals from two arbitrary input or output positions grows in the distance between positions, linearly for ConvS2S and logarithmically for ByteNet. This makes it more difficult to learn dependencies between distant positions [11]. In the Transformer this is reduced to a constant number of operations, albeit at the cost of reduced effective resolution due to averaging attention-weighted positions, an effect we counteract with Multi-Head Attention as described in section 3.2.

Self-attention, sometimes called intra-attention is an attention mechanism relating different positions of a single sequence in order to compute a representation of the sequence. Self-attention has been used successfully in a variety of tasks including reading comprehension, abstractive summarization, textual entailment and learning task-independent sentence representations [4, 22, 23, 19].

End-to-end memory networks are based on a recurrent attention mechanism instead of sequence-aligned recurrence and have been shown to perform well on simple-language question answering and language modeling tasks [28].

To the best of our knowledge, however, the Transformer is the first transduction model relying entirely on self-attention to compute representations of its input and output without using sequence-aligned RNNs or convolution. In the following sections, we will describe the Transformer, motivate self-attention and discuss its advantages over models such as [14, 15] and [8].

## 3 Model Architecture

Most competitive neural sequence transduction models have an encoder-decoder structure [5, 2, 29]. Here, the encoder maps an input sequence of symbol representations  $(x_1, \dots, x_n)$  to a sequence of continuous representations  $\mathbf{z} = (z_1, \dots, z_n)$ . Given  $\mathbf{z}$ , the decoder then generates an output sequence  $(y_1, \dots, y_m)$  of symbols one element at a time. At each step the model is auto-regressive [9], consuming the previously generated symbols as additional input when generating the next.

The Transformer follows this overall architecture using stacked self-attention and point-wise, fully connected layers for both the encoder and decoder, shown in the left and right halves of Figure 1 respectively.

### 3.1 Encoder and Decoder Stacks

**Encoder:** The encoder is composed of a stack of  $N = 6$  identical layers. Each layer has two sub-layers. The first is a multi-head self-attention mechanism, and the second is a simple, position-



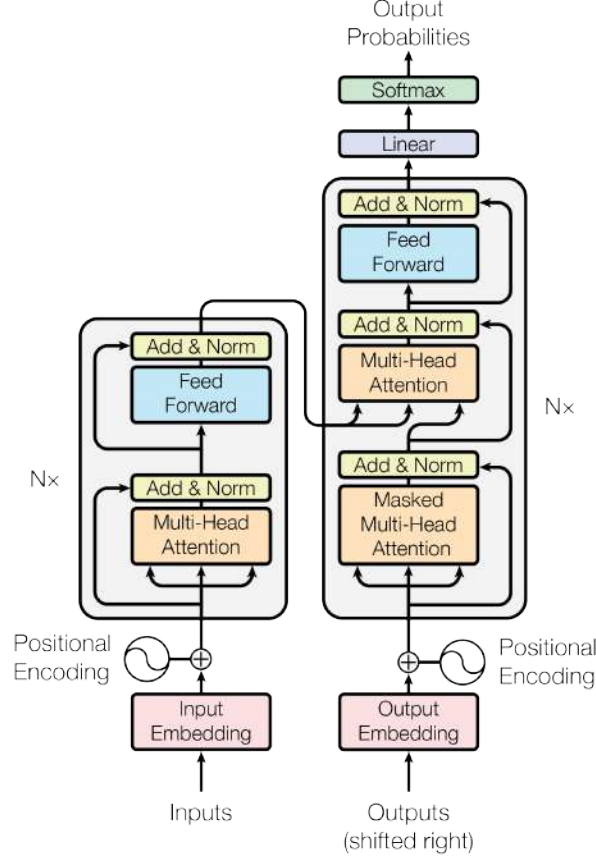


Figure 1: The Transformer - model architecture.

wise fully connected feed-forward network. We employ a residual connection [10] around each of the two sub-layers, followed by layer normalization [11]. That is, the output of each sub-layer is  $\text{LayerNorm}(x + \text{Sublayer}(x))$ , where  $\text{Sublayer}(x)$  is the function implemented by the sub-layer itself. To facilitate these residual connections, all sub-layers in the model, as well as the embedding layers, produce outputs of dimension  $d_{\text{model}} = 512$ .

**Decoder:** The decoder is also composed of a stack of  $N = 6$  identical layers. In addition to the two sub-layers in each encoder layer, the decoder inserts a third sub-layer, which performs multi-head attention over the output of the encoder stack. Similar to the encoder, we employ residual connections around each of the sub-layers, followed by layer normalization. We also modify the self-attention sub-layer in the decoder stack to prevent positions from attending to subsequent positions. This masking, combined with fact that the output embeddings are offset by one position, ensures that the predictions for position  $i$  can depend only on the known outputs at positions less than  $i$ .

### 3.2 Attention

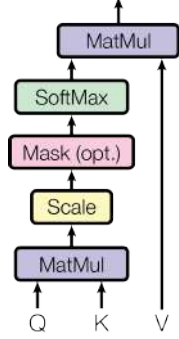
An attention function can be described as mapping a query and a set of key-value pairs to an output, where the query, keys, values, and output are all vectors. The output is computed as a weighted sum of the values, where the weight assigned to each value is computed by a compatibility function of the query with the corresponding key.

#### 3.2.1 Scaled Dot-Product Attention

We call our particular attention "Scaled Dot-Product Attention" (Figure 2). The input consists of queries and keys of dimension  $d_k$ , and values of dimension  $d_v$ . We compute the dot products of the



Scaled Dot-Product Attention



Multi-Head Attention

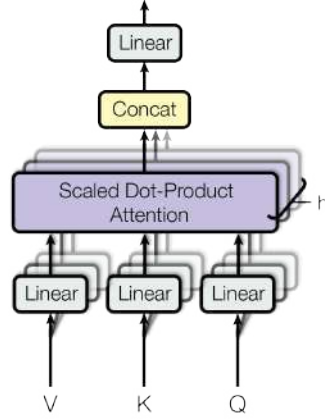


Figure 2: (left) Scaled Dot-Product Attention. (right) Multi-Head Attention consists of several attention layers running in parallel.

query with all keys, divide each by  $\sqrt{d_k}$ , and apply a softmax function to obtain the weights on the values.

In practice, we compute the attention function on a set of queries simultaneously, packed together into a matrix  $Q$ . The keys and values are also packed together into matrices  $K$  and  $V$ . We compute the matrix of outputs as:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \quad (1)$$

The two most commonly used attention functions are additive attention [2], and dot-product (multiplicative) attention. Dot-product attention is identical to our algorithm, except for the scaling factor of  $\frac{1}{\sqrt{d_k}}$ . Additive attention computes the compatibility function using a feed-forward network with a single hidden layer. While the two are similar in theoretical complexity, dot-product attention is much faster and more space-efficient in practice, since it can be implemented using highly optimized matrix multiplication code.

While for small values of  $d_k$  the two mechanisms perform similarly, additive attention outperforms dot product attention without scaling for larger values of  $d_k$  [3]. We suspect that for large values of  $d_k$ , the dot products grow large in magnitude, pushing the softmax function into regions where it has extremely small gradients [4]. To counteract this effect, we scale the dot products by  $\frac{1}{\sqrt{d_k}}$ .

### 3.2.2 Multi-Head Attention

Instead of performing a single attention function with  $d_{\text{model}}$ -dimensional keys, values and queries, we found it beneficial to linearly project the queries, keys and values  $h$  times with different, learned linear projections to  $d_k$ ,  $d_k$  and  $d_v$  dimensions, respectively. On each of these projected versions of queries, keys and values we then perform the attention function in parallel, yielding  $d_v$ -dimensional output values. These are concatenated and once again projected, resulting in the final values, as depicted in Figure 2.

Multi-head attention allows the model to jointly attend to information from different representation subspaces at different positions. With a single attention head, averaging inhibits this.

<sup>4</sup>To illustrate why the dot products get large, assume that the components of  $q$  and  $k$  are independent random variables with mean 0 and variance 1. Then their dot product,  $q \cdot k = \sum_{i=1}^{d_k} q_i k_i$ , has mean 0 and variance  $d_k$ .

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \dots, \text{head}_h)W^O$$

where  $\text{head}_i = \text{Attention}(QW_i^Q, KW_i^K, VW_i^V)$

Where the projections are parameter matrices  $W_i^Q \in \mathbb{R}^{d_{\text{model}} \times d_k}$ ,  $W_i^K \in \mathbb{R}^{d_{\text{model}} \times d_k}$ ,  $W_i^V \in \mathbb{R}^{d_{\text{model}} \times d_v}$  and  $W^O \in \mathbb{R}^{hd_v \times d_{\text{model}}}$ .

In this work we employ  $h = 8$  parallel attention layers, or heads. For each of these we use  $d_k = d_v = d_{\text{model}}/h = 64$ . Due to the reduced dimension of each head, the total computational cost is similar to that of single-head attention with full dimensionality.

### 3.2.3 Applications of Attention in our Model

The Transformer uses multi-head attention in three different ways:

- In "encoder-decoder attention" layers, the queries come from the previous decoder layer, and the memory keys and values come from the output of the encoder. This allows every position in the decoder to attend over all positions in the input sequence. This mimics the typical encoder-decoder attention mechanisms in sequence-to-sequence models such as [31, 2, 8].
- The encoder contains self-attention layers. In a self-attention layer all of the keys, values and queries come from the same place, in this case, the output of the previous layer in the encoder. Each position in the encoder can attend to all positions in the previous layer of the encoder.
- Similarly, self-attention layers in the decoder allow each position in the decoder to attend to all positions in the decoder up to and including that position. We need to prevent leftward information flow in the decoder to preserve the auto-regressive property. We implement this inside of scaled dot-product attention by masking out (setting to  $-\infty$ ) all values in the input of the softmax which correspond to illegal connections. See Figure 2

### 3.3 Position-wise Feed-Forward Networks

In addition to attention sub-layers, each of the layers in our encoder and decoder contains a fully connected feed-forward network, which is applied to each position separately and identically. This consists of two linear transformations with a ReLU activation in between.

$$\text{FFN}(x) = \max(0, xW_1 + b_1)W_2 + b_2 \quad (2)$$

While the linear transformations are the same across different positions, they use different parameters from layer to layer. Another way of describing this is as two convolutions with kernel size 1. The dimensionality of input and output is  $d_{\text{model}} = 512$ , and the inner-layer has dimensionality  $d_{ff} = 2048$ .

### 3.4 Embeddings and Softmax

Similarly to other sequence transduction models, we use learned embeddings to convert the input tokens and output tokens to vectors of dimension  $d_{\text{model}}$ . We also use the usual learned linear transformation and softmax function to convert the decoder output to predicted next-token probabilities. In our model, we share the same weight matrix between the two embedding layers and the pre-softmax linear transformation, similar to [24]. In the embedding layers, we multiply those weights by  $\sqrt{d_{\text{model}}}$ .

### 3.5 Positional Encoding

Since our model contains no recurrence and no convolution, in order for the model to make use of the order of the sequence, we must inject some information about the relative or absolute position of the tokens in the sequence. To this end, we add "positional encodings" to the input embeddings at the

Table 1: Maximum path lengths, per-layer complexity and minimum number of sequential operations for different layer types.  $n$  is the sequence length,  $d$  is the representation dimension,  $k$  is the kernel size of convolutions and  $r$  the size of the neighborhood in restricted self-attention.

Layer Type	Complexity per Layer	Sequential Operations	Maximum Path Length
Self-Attention	$O(n^2 \cdot d)$	$O(1)$	$O(1)$
Recurrent	$O(n \cdot d^2)$	$O(n)$	$O(n)$
Convolutional	$O(k \cdot n \cdot d^2)$	$O(1)$	$O(\log_k(n))$
Self-Attention (restricted)	$O(r \cdot n \cdot d)$	$O(1)$	$O(n/r)$

bottoms of the encoder and decoder stacks. The positional encodings have the same dimension  $d_{\text{model}}$  as the embeddings, so that the two can be summed. There are many choices of positional encodings, learned and fixed [8].

In this work, we use sine and cosine functions of different frequencies:

$$PE_{(pos, 2i)} = \sin(pos/10000^{2i/d_{\text{model}}})$$

$$PE_{(pos, 2i+1)} = \cos(pos/10000^{2i/d_{\text{model}}})$$

where  $pos$  is the position and  $i$  is the dimension. That is, each dimension of the positional encoding corresponds to a sinusoid. The wavelengths form a geometric progression from  $2\pi$  to  $10000 \cdot 2\pi$ . We chose this function because we hypothesized it would allow the model to easily learn to attend by relative positions, since for any fixed offset  $k$ ,  $PE_{pos+k}$  can be represented as a linear function of  $PE_{pos}$ .

We also experimented with using learned positional embeddings [8] instead, and found that the two versions produced nearly identical results (see Table 3 row (E)). We chose the sinusoidal version because it may allow the model to extrapolate to sequence lengths longer than the ones encountered during training.

## 4 Why Self-Attention

In this section we compare various aspects of self-attention layers to the recurrent and convolutional layers commonly used for mapping one variable-length sequence of symbol representations  $(x_1, \dots, x_n)$  to another sequence of equal length  $(z_1, \dots, z_n)$ , with  $x_i, z_i \in \mathbb{R}^d$ , such as a hidden layer in a typical sequence transduction encoder or decoder. Motivating our use of self-attention we consider three desiderata.

One is the total computational complexity per layer. Another is the amount of computation that can be parallelized, as measured by the minimum number of sequential operations required.

The third is the path length between long-range dependencies in the network. Learning long-range dependencies is a key challenge in many sequence transduction tasks. One key factor affecting the ability to learn such dependencies is the length of the paths forward and backward signals have to traverse in the network. The shorter these paths between any combination of positions in the input and output sequences, the easier it is to learn long-range dependencies [11]. Hence we also compare the maximum path length between any two input and output positions in networks composed of the different layer types.

As noted in Table 1, a self-attention layer connects all positions with a constant number of sequentially executed operations, whereas a recurrent layer requires  $O(n)$  sequential operations. In terms of computational complexity, self-attention layers are faster than recurrent layers when the sequence length  $n$  is smaller than the representation dimensionality  $d$ , which is most often the case with sentence representations used by state-of-the-art models in machine translations, such as word-piece [31] and byte-pair [25] representations. To improve computational performance for tasks involving very long sequences, self-attention could be restricted to considering only a neighborhood of size  $r$  in

the input sequence centered around the respective output position. This would increase the maximum path length to  $O(n/r)$ . We plan to investigate this approach further in future work.

A single convolutional layer with kernel width  $k < n$  does not connect all pairs of input and output positions. Doing so requires a stack of  $O(n/k)$  convolutional layers in the case of contiguous kernels, or  $O(\log_k(n))$  in the case of dilated convolutions [15], increasing the length of the longest paths between any two positions in the network. Convolutional layers are generally more expensive than recurrent layers, by a factor of  $k$ . Separable convolutions [6], however, decrease the complexity considerably, to  $O(k \cdot n \cdot d + n \cdot d^2)$ . Even with  $k = n$ , however, the complexity of a separable convolution is equal to the combination of a self-attention layer and a point-wise feed-forward layer, the approach we take in our model.

As side benefit, self-attention could yield more interpretable models. We inspect attention distributions from our models and present and discuss examples in the appendix. Not only do individual attention heads clearly learn to perform different tasks, many appear to exhibit behavior related to the syntactic and semantic structure of the sentences.

## 5 Training

This section describes the training regime for our models.

### 5.1 Training Data and Batching

We trained on the standard WMT 2014 English-German dataset consisting of about 4.5 million sentence pairs. Sentences were encoded using byte-pair encoding [3], which has a shared source-target vocabulary of about 37000 tokens. For English-French, we used the significantly larger WMT 2014 English-French dataset consisting of 36M sentences and split tokens into a 32000 word-piece vocabulary [31]. Sentence pairs were batched together by approximate sequence length. Each training batch contained a set of sentence pairs containing approximately 25000 source tokens and 25000 target tokens.

### 5.2 Hardware and Schedule

We trained our models on one machine with 8 NVIDIA P100 GPUs. For our base models using the hyperparameters described throughout the paper, each training step took about 0.4 seconds. We trained the base models for a total of 100,000 steps or 12 hours. For our big models, (described on the bottom line of table 3), step time was 1.0 seconds. The big models were trained for 300,000 steps (3.5 days).

### 5.3 Optimizer

We used the Adam optimizer [17] with  $\beta_1 = 0.9$ ,  $\beta_2 = 0.98$  and  $\epsilon = 10^{-9}$ . We varied the learning rate over the course of training, according to the formula:

$$lrate = d_{\text{model}}^{-0.5} \cdot \min(step\_num^{-0.5}, step\_num \cdot warmup\_steps^{-1.5}) \quad (3)$$

This corresponds to increasing the learning rate linearly for the first  $warmup\_steps$  training steps, and decreasing it thereafter proportionally to the inverse square root of the step number. We used  $warmup\_steps = 4000$ .

### 5.4 Regularization

We employ three types of regularization during training:

**Residual Dropout** We apply dropout [27] to the output of each sub-layer, before it is added to the sub-layer input and normalized. In addition, we apply dropout to the sums of the embeddings and the positional encodings in both the encoder and decoder stacks. For the base model, we use a rate of  $P_{drop} = 0.1$ .

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [15]	23.75			
Deep-Att + PosUnk [32]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [31]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [8]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [26]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [32]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [31]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [8]	26.36	<b>41.29</b>	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$
Transformer (base model)	27.3	38.1	<b><math>3.3 \cdot 10^{18}</math></b>	
Transformer (big)	<b>28.4</b>	<b>41.0</b>	$2.3 \cdot 10^{19}$	

**Label Smoothing** During training, we employed label smoothing of value  $\epsilon_{ls} = 0.1$  [30]. This hurts perplexity, as the model learns to be more unsure, but improves accuracy and BLEU score.

## 6 Results

### 6.1 Machine Translation

On the WMT 2014 English-to-German translation task, the big transformer model (Transformer (big) in Table 2) outperforms the best previously reported models (including ensembles) by more than 2.0 BLEU, establishing a new state-of-the-art BLEU score of 28.4. The configuration of this model is listed in the bottom line of Table 3. Training took 3.5 days on 8 P100 GPUs. Even our base model surpasses all previously published models and ensembles, at a fraction of the training cost of any of the competitive models.

On the WMT 2014 English-to-French translation task, our big model achieves a BLEU score of 41.0, outperforming all of the previously published single models, at less than 1/4 the training cost of the previous state-of-the-art model. The Transformer (big) model trained for English-to-French used dropout rate  $P_{drop} = 0.1$ , instead of 0.3.

For the base models, we used a single model obtained by averaging the last 5 checkpoints, which were written at 10-minute intervals. For the big models, we averaged the last 20 checkpoints. We used beam search with a beam size of 4 and length penalty  $\alpha = 0.6$  [31]. These hyperparameters were chosen after experimentation on the development set. We set the maximum output length during inference to input length + 50, but terminate early when possible [31].

Table 2 summarizes our results and compares our translation quality and training costs to other model architectures from the literature. We estimate the number of floating point operations used to train a model by multiplying the training time, the number of GPUs used, and an estimate of the sustained single-precision floating-point capacity of each GPU 5.

### 6.2 Model Variations

To evaluate the importance of different components of the Transformer, we varied our base model in different ways, measuring the change in performance on English-to-German translation on the development set, newstest2013. We used beam search as described in the previous section, but no checkpoint averaging. We present these results in Table 3.

In Table 3 rows (A), we vary the number of attention heads and the attention key and value dimensions, keeping the amount of computation constant, as described in Section 3.2.2. While single-head attention is 0.9 BLEU worse than the best setting, quality also drops off with too many heads.

<sup>5</sup>We used values of 2.8, 3.7, 6.0 and 9.5 TFLOPS for K80, K40, M40 and P100, respectively.

Table 3: Variations on the Transformer architecture. Unlisted values are identical to those of the base model. All metrics are on the English-to-German translation development set, newstest2013. Listed perplexities are per-wordpiece, according to our byte-pair encoding, and should not be compared to per-word perplexities.

	$N$	$d_{\text{model}}$	$d_{\text{ff}}$	$h$	$d_k$	$d_v$	$P_{\text{drop}}$	$\epsilon_{ls}$	train steps	PPL (dev)	BLEU (dev)	params $\times 10^6$	
base	6	512	2048	8	64	64	0.1	0.1	100K	4.92	25.8	65	
(A)					1	512				5.29	24.9		
					4	128	128				5.00	25.5	
					16	32	32				4.91	25.8	
					32	16	16				5.01	25.4	
(B)					16					5.16	25.1	58	
					32					5.01	25.4	60	
(C)	2									6.11	23.7	36	
	4									5.19	25.3	50	
	8									4.88	25.5	80	
		256			32	32				5.75	24.5	28	
		1024			128	128				4.66	26.0	168	
			1024							5.12	25.4	53	
			4096							4.75	26.2	90	
(D)							0.0			5.77	24.6		
							0.2			4.95	25.5		
								0.0		4.67	25.3		
								0.2		5.47	25.7		
(E)	positional embedding instead of sinusoids									4.92	25.7		
big	6	1024	4096	16				0.3	300K	<b>4.33</b>	<b>26.4</b>	213	

In Table 3 rows (B), we observe that reducing the attention key size  $d_k$  hurts model quality. This suggests that determining compatibility is not easy and that a more sophisticated compatibility function than dot product may be beneficial. We further observe in rows (C) and (D) that, as expected, bigger models are better, and dropout is very helpful in avoiding over-fitting. In row (E) we replace our sinusoidal positional encoding with learned positional embeddings [8], and observe nearly identical results to the base model.

## 7 Conclusion

In this work, we presented the Transformer, the first sequence transduction model based entirely on attention, replacing the recurrent layers most commonly used in encoder-decoder architectures with multi-headed self-attention.

For translation tasks, the Transformer can be trained significantly faster than architectures based on recurrent or convolutional layers. On both WMT 2014 English-to-German and WMT 2014 English-to-French translation tasks, we achieve a new state of the art. In the former task our best model outperforms even all previously reported ensembles.

We are excited about the future of attention-based models and plan to apply them to other tasks. We plan to extend the Transformer to problems involving input and output modalities other than text and to investigate local, restricted attention mechanisms to efficiently handle large inputs and outputs such as images, audio and video. Making generation less sequential is another research goal of ours.

The code we used to train and evaluate our models is available at <https://github.com/tensorflow/tensor2tensor>.

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## References

- [1] Jimmy Lei Ba, Jamie Ryan Kiros, and Geoffrey E Hinton. Layer normalization. *arXiv preprint arXiv:1607.06450*, 2016.
- [2] Dzmitry Bahdanau, Kyunghyun Cho, and Yoshua Bengio. Neural machine translation by jointly learning to align and translate. *CoRR*, abs/1409.0473, 2014.
- [3] Denny Britz, Anna Goldie, Minh-Thang Luong, and Quoc V. Le. Massive exploration of neural machine translation architectures. *CoRR*, abs/1703.03906, 2017.
- [4] Jianpeng Cheng, Li Dong, and Mirella Lapata. Long short-term memory-networks for machine reading. *arXiv preprint arXiv:1601.06733*, 2016.
- [5] Kyunghyun Cho, Bart van Merriënboer, Çağlar Gulcehre, Fethi Bougares, Holger Schwenk, and Yoshua Bengio. Learning phrase representations using rnn encoder-decoder for statistical machine translation. *CoRR*, abs/1406.1078, 2014.
- [6] Francois Chollet. Xception: Deep learning with depthwise separable convolutions. *arXiv preprint arXiv:1610.02357*, 2016.
- [7] Junyoung Chung, Çağlar Gülcehre, Kyunghyun Cho, and Yoshua Bengio. Empirical evaluation of gated recurrent neural networks on sequence modeling. *CoRR*, abs/1412.3555, 2014.
- [8] Jonas Gehring, Michael Auli, David Grangier, Denis Yarats, and Yann N. Dauphin. Convolutional sequence to sequence learning. *arXiv preprint arXiv:1705.03122v2*, 2017.
- [9] Alex Graves. Generating sequences with recurrent neural networks. *arXiv preprint arXiv:1308.0850*, 2013.
- [10] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 770–778, 2016.
- [11] Sepp Hochreiter, Yoshua Bengio, Paolo Frasconi, and Jürgen Schmidhuber. Gradient flow in recurrent nets: the difficulty of learning long-term dependencies, 2001.
- [12] Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. *Neural computation*, 9(8):1735–1780, 1997.
- [13] Rafal Jozefowicz, Oriol Vinyals, Mike Schuster, Noam Shazeer, and Yonghui Wu. Exploring the limits of language modeling. *arXiv preprint arXiv:1602.02410*, 2016.
- [14] Łukasz Kaiser and Ilya Sutskever. Neural GPUs learn algorithms. In *International Conference on Learning Representations (ICLR)*, 2016.
- [15] Nal Kalchbrenner, Lasse Espeholt, Karen Simonyan, Aaron van den Oord, Alex Graves, and Koray Kavukcuoglu. Neural machine translation in linear time. *arXiv preprint arXiv:1610.10099v2*, 2017.
- [16] Yoon Kim, Carl Denton, Luong Hoang, and Alexander M. Rush. Structured attention networks. In *International Conference on Learning Representations*, 2017.
- [17] Diederik Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *ICLR*, 2015.
- [18] Oleksii Kuchaiev and Boris Ginsburg. Factorization tricks for LSTM networks. *arXiv preprint arXiv:1703.10722*, 2017.
- [19] Zhouhan Lin, Minwei Feng, Cicero Nogueira dos Santos, Mo Yu, Bing Xiang, Bowen Zhou, and Yoshua Bengio. A structured self-attentive sentence embedding. *arXiv preprint arXiv:1703.03130*, 2017.
- [20] Samy Bengio Łukasz Kaiser. Can active memory replace attention? In *Advances in Neural Information Processing Systems, (NIPS)*, 2016.

- [21] Minh-Thang Luong, Hieu Pham, and Christopher D Manning. Effective approaches to attention-based neural machine translation. *arXiv preprint arXiv:1508.04025*, 2015.
- [22] Ankur Parikh, Oscar Täckström, Dipanjan Das, and Jakob Uszkoreit. A decomposable attention model. In *Empirical Methods in Natural Language Processing*, 2016.
- [23] Romain Paulus, Caiming Xiong, and Richard Socher. A deep reinforced model for abstractive summarization. *arXiv preprint arXiv:1705.04304*, 2017.
- [24] Ofir Press and Lior Wolf. Using the output embedding to improve language models. *arXiv preprint arXiv:1608.05859*, 2016.
- [25] Rico Sennrich, Barry Haddow, and Alexandra Birch. Neural machine translation of rare words with subword units. *arXiv preprint arXiv:1508.07909*, 2015.
- [26] Noam Shazeer, Azalia Mirhoseini, Krzysztof Maziarczyk, Andy Davis, Quoc Le, Geoffrey Hinton, and Jeff Dean. Outrageously large neural networks: The sparsely-gated mixture-of-experts layer. *arXiv preprint arXiv:1701.06538*, 2017.
- [27] Nitish Srivastava, Geoffrey E Hinton, Alex Krizhevsky, Ilya Sutskever, and Ruslan Salakhutdinov. Dropout: a simple way to prevent neural networks from overfitting. *Journal of Machine Learning Research*, 15(1):1929–1958, 2014.
- [28] Sainbayar Sukhbaatar, arthur szlam, Jason Weston, and Rob Fergus. End-to-end memory networks. In C. Cortes, N. D. Lawrence, D. D. Lee, M. Sugiyama, and R. Garnett, editors, *Advances in Neural Information Processing Systems 28*, pages 2440–2448. Curran Associates, Inc., 2015.
- [29] Ilya Sutskever, Oriol Vinyals, and Quoc VV Le. Sequence to sequence learning with neural networks. In *Advances in Neural Information Processing Systems*, pages 3104–3112, 2014.
- [30] Christian Szegedy, Vincent Vanhoucke, Sergey Ioffe, Jonathon Shlens, and Zbigniew Wojna. Rethinking the inception architecture for computer vision. *CoRR*, abs/1512.00567, 2015.
- [31] Yonghui Wu, Mike Schuster, Zhifeng Chen, Quoc V Le, Mohammad Norouzi, Wolfgang Macherey, Maxim Krikun, Yuan Cao, Qin Gao, Klaus Macherey, et al. Google’s neural machine translation system: Bridging the gap between human and machine translation. *arXiv preprint arXiv:1609.08144*, 2016.
- [32] Jie Zhou, Ying Cao, Xuguang Wang, Peng Li, and Wei Xu. Deep recurrent models with fast-forward connections for neural machine translation. *CoRR*, abs/1606.04199, 2016.