$$X = \frac{x}{\lambda}, Y = \frac{y}{d}, F = \frac{f}{c}, G = \frac{g}{c\delta}, t = \frac{ct^*}{\lambda}, P = \frac{\rho d^2}{\mu c \lambda}, \bar{h} = \frac{H}{d},$$

$$\phi = \frac{C - C_\infty}{C_1 - C_\infty}, \theta = \frac{T - T_\infty}{T_h - T_\infty}, \tau_{XX} = \frac{d\tau_{xx}}{\mu c}, \tau_{XY} = \frac{d\tau_{xy}}{\mu c \lambda}, \tau_{YY} = \frac{d\tau_{yy}}{\mu c}. \quad (8)$$

On utilizing the non-dimensional transformations, the governing equations can be expressed as,

$$\frac{dF}{dX} + \frac{dG}{dY} = 0, \quad (9)$$

$$\frac{dP}{dX} = \frac{d\tau_{XY}}{dY} - Ha^2 F + Gr \sin\alpha\ \theta, \quad (10)$$

$$\frac{dP}{dY} = 0, \quad (11)$$

$$0 = \frac{d^2\theta}{dY^2} + Pr\ Ec\left(\frac{dF}{dY}\right)^2 + Pr\ Nb\frac{d\phi}{dY}\frac{d\theta}{dY} + Pr\ Nt\left(\frac{d\theta}{dY}\right)^2 + (Ha)^2 \Pr\ EcF^2, \quad (12)$$

$$0 = \frac{d^2\phi}{dY^2} + \frac{Nt}{Nb}\left(\frac{d^2\theta}{dY^2}\right). \quad (13)$$

The dimensionless boundary conditions are:

$$\frac{dF}{dY} = 0, \frac{d\theta}{dY} = 0, \frac{d\phi}{dY} = 0, at Y = 0, \quad (14)$$

$$F = 0, \theta = 1, \phi = 1, at Y = \bar{h}. \quad (15)$$

where, $Re = \frac{\rho dc}{\mu}$ denotes the Reynolds number, $Pr = \frac{c_p \mu}{k}$ denotes the Prandtl number, $Ec = \frac{c^2}{c_p(T_h - T_a)}$ denotes the Eckert number, $Br = Ec * Pr$ denotes the Brinkmann number, $Gr = \frac{\rho \beta g_r (T_h - T_a) d^2}{c\mu}$ denotes the Grashof number, $Nt = \frac{\rho D_T (T_h - T_a)}{T_a \mu}$ denotes the thermophoresis parameter, $Ha = \sqrt{\frac{\sigma}{\mu}} B_0 d$ denotes the Hartmann number and $Nb = \frac{\rho D_B (C_1 - C_\infty)}{\mu}$ denotes the denotes the Brownian motion parameter.

The simplification process of the governing equations is done based on the commonly used long wavelength and low Reynolds number assumptions in lubrication theory, which have already been extensively used in studies involving peristaltic flows. With such considerations, inertia effects are no longer significant compared to viscosity effects, and the pressure gradient will be primarily along the axial direction while the velocity gradients will be primarily along the transverse direction. As a result, the governing partial differential equations will be significantly simplified by considering only the dominant terms. In addition, owing to the long-wavelength assumption, the flow variables will no longer depend on the axial direction, except for the pressure term, hence resulting in a system of non-linear ordinary differential equations in the transverse direction.

The expression of the velocity components relative to the stream function (ψ) is given below:
$F = \frac{\partial\psi}{\partial Y}, G = -\frac{\partial\psi}{\partial X}$.

The transformed equation for the Ree-Eyring fluid model is given by:

$$\tau_{XY} = (1 + \gamma)\frac{dF}{dY}$$

here, γ represents the pseudo-plasticity co-efficient.

The dimensionless form of the symmetric wavy wall geometry is obtained by:

$$\bar{h} = 1 + \sum\nolimits_{i=1}^{m} \Delta_i \sin(2\pi i(X - t))$$

where, $\Delta_i = \frac{\xi_i}{d}$ for $i = 1, 2$ displays the amplitude ratio.

The dimensionless quantities characterizing momentum, heat, and mass transfer-namely skin friction coefficient (Cf), Nusselt number (Nu) and Sherwood number (Sh) are computed using Equation 10, (Equation 11), and (Equation 12). These are evaluated at the upper wall and expressed as (Vaidya et al. 2024):

$$Cf = \left.\frac{\partial F}{\partial Y}\frac{\partial \bar{h}}{\partial X}\right|_{Y=\bar{h}},$$

$$Nu = \left.\frac{\partial \theta}{\partial Y}\frac{\partial \bar{h}}{\partial X}\right|_{Y=\bar{h}},$$

$$Sh = \left.\frac{\partial \phi}{\partial Y}\frac{\partial \bar{h}}{\partial X}\right|_{Y=\bar{h}}.$$

In the analysis of peristaltic flow, the pumping mechanism plays a crucial role in that many physical parameters show strong sensitivity to the volumetric flow rate (Q), and the behavior can change noticeably around a critical value of Q. For the case of one wavelength and with constant pressure difference, the volumetric flow rate of the Ree-Eyring fluid is given by

$$Q = \int_0^{\bar{h}} F\ dY.$$