Quantum Polyspectra

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Uncompromising and Universal Evaluation of Quantum Measurements

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Quantum polyspectra are **a completely general and uncompromising approach to the evaluation of continuous quantum measurements** including

- Spin noise spectroscopy
- Quantum transport
- Circuit quantum electrodynamics

Quantum polyspectra are **directly calculated from any detector output** $z(t)$ including

- Gaussian-dominated noise, photon shot noise
- Telegraph noise, quantum jumps
- Stochastic click-events, photomultiplier output

Analytic quantum polyspectra follow rigorously from the stochastic master equation [1]. **Automatic fitting of analytic to measured spectra yields quantities** like

- Tunneling times
- Precession frequencies
- Coupling-tensors

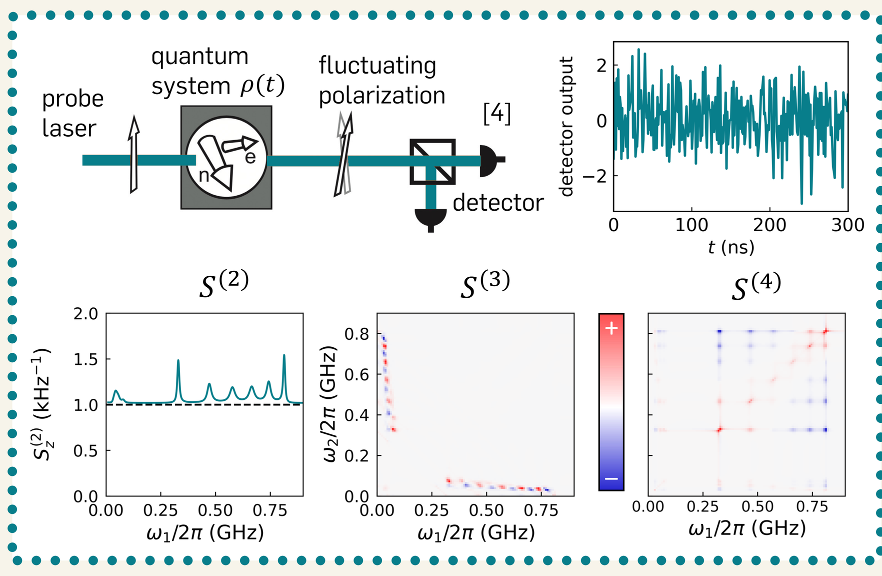
Outstanding features

The quantum polyspectra approach can handle

- Environmental damping
- Measurement backaction (Zeno effect) and arbitrary measurement strength
- Coherent quantum dynamics
- Stochastic in- and out-tunneling
- Additional detector noise
- Simultaneous measurement of non-commuting observables
- Incorporation of temperatures
- Completely automatic analysis of arbitrary measurement traces
- Covers all limiting case of weak spin noise measurements, strong measurements resulting in quantum jumps, and single photon sampling

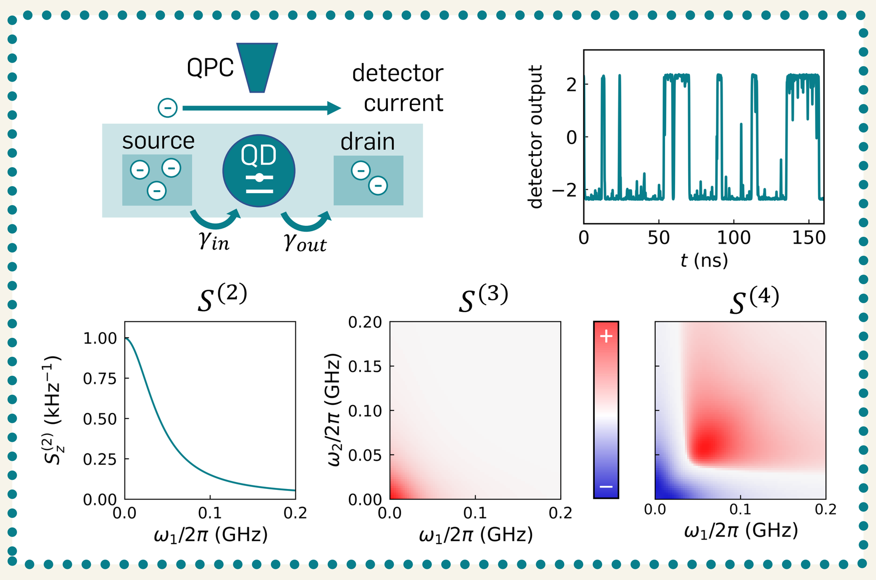
Three *Limiting Cases –* ***One*** *Theory*

Spin Noise Measurement [2]



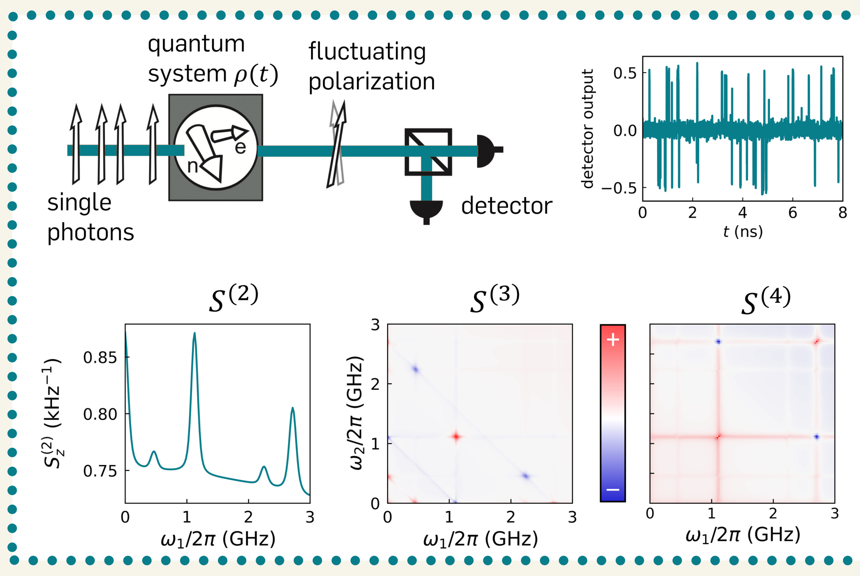
- Electron precesses in a superposition of six frequencies (5/2 nucleus in a magnetic field)
- **Weak measurement:** measurement of electron orientation dominated by Gaussian noise
- **Goal:** understanding precession dynamics
- The six frequencies can be seen in $S^{(2)}$
- $S^{(4)}$ shows correlations between frequencies
- Neighboring frequencies are positively correlated

Quantum Transport [3]



- Monitoring the charge state of a quantum dot
- **Strong measurement:** quantum state undergoes telegraph like switching (quantum jumps)
- **Goal:** determining the tunnelling rates γ_{in} and γ_{out}
- The $S^{(2)}$ is not sufficient to determine the tunnelling rates
- The $S^{(2)}$ and $S^{(3)}$ contain enough information to determine the tunnelling rates

Single Photon Measurement [4]



- Random-time sampling of a system
- **Ultra-weak measurement:** photon interacts with a quantum system for a short period
- **Goal:** reconstruction of the precession dynamics
- All higher-order correlations are visible

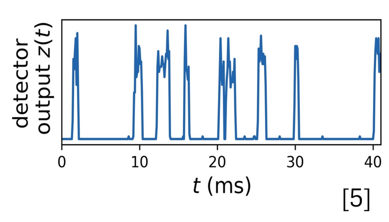
Calculate your own spectra with **SignalSnap**
github.com/MarkusSifft

SCAN ME

Evaluation of Quantum Measurements

Data

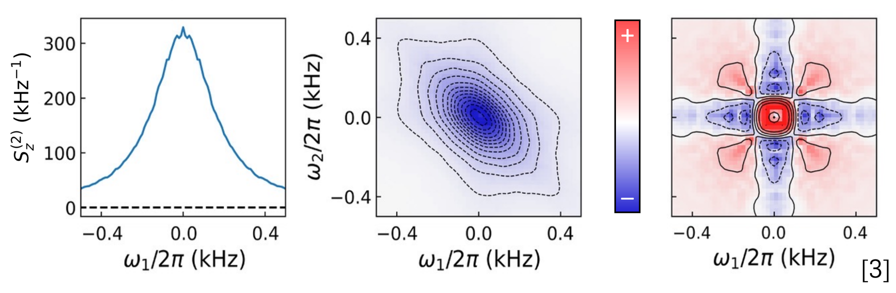
Detector Output z(t)



Fourier coefficients a_ω (see box right)

Measured Quantum Polyspectra

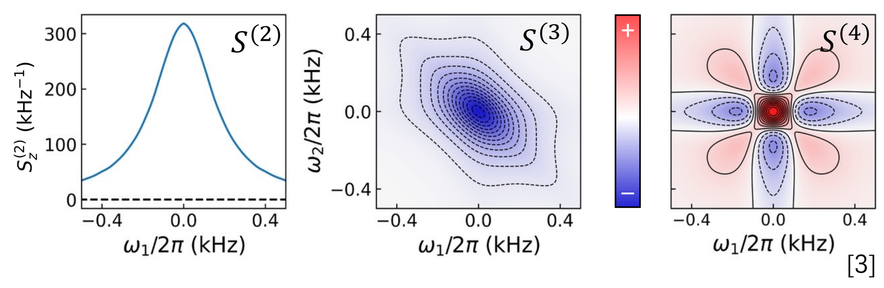
$S^{(2)} \propto \langle a_\omega a_\omega^* \rangle + \dots$ $S^{(3)} \propto \langle a_{\omega_1} a_{\omega_2} a^*_{\omega_1+\omega_2} \rangle + \dots$ $S^{(4)} \propto \langle a_{\omega_1} a^*_{\omega_1} a_{\omega_2} a^*_{\omega_2} \rangle + \dots$



The stochastic master equation allows for the simulation of a detector output by numerical integration

Fit

Analytic Quantum Polyspectra



Model

Stochastic Master Equation

$$d\rho = \mathcal{L}[\beta](\rho)dt + \beta\mathcal{S}[A](\rho)dW$$

$$z(t) = \beta^2 \mathrm{Tr}\left[\rho(t)(A + A^\dagger)/2\right] + \frac{1}{2}\beta\Gamma(t)$$

Analytical or numerical evaluation of quantum polyspectra formula (see bottom right)

Power Spectrum

The usual powerspectrum $S^{(2)}(\omega)$ is given by the expectation value $\langle a_\omega a_\omega^* \rangle$ and thus by the average intensity of $z(t)$ at frequency ω.

Bispectrum

Shows contributions if two frequencies are phase correlated with the sum of the frequencies and is sensitive to time-inversion (while $S^{(2)}$ is not)

Trispectrum

The two dimensional cut of the trispectrum (shown here) can be interpreted as a frequency-dependent intensity-intensity correlation.

From Data to Polyspectra

The detector output $z(t)$ is discretized and divided into time frames $z^{(n)}$ of length N

$$z_j^{(n)} = z(jT/N + nT),$$

The nth-order polyspectra $S_z^{(n)}$ is proportional to the nth-order cumulant C_n of the Fourier coefficients $a_k^{(n)}$ of the signal times window function

$$S_z^{(2)}(\omega_k) \propto C_2(a_k, a_k^*)$$

$$S_z^{(3)}(\omega_k, \omega_l) \propto C_3(a_k, a_l, a_{k+l}^*)$$

$$S_z^{(4)}(\omega_k, \omega_l) \propto C_4(a_k, a_k^*, a_l, a_l^*)$$

For infinitely many frames the cumulants C_n can be calculated as

$$C_2(x, y) = \langle yx \rangle - \langle y \rangle \langle x \rangle$$

$$C_3(x, y, z) = \langle zyx \rangle - \langle yx \rangle \langle z \rangle - \langle zx \rangle \langle y \rangle - \langle zy \rangle \langle x \rangle + 2\langle z \rangle \langle y \rangle \langle x \rangle$$

Since any measurement trace will be finite so-called cumulant estimator have to be used [6]

$$c_2(x, y) = \frac{m}{m-1}(\overline{xy} - \bar{x}\bar{y})$$

$$c_3(x, y, z) = \frac{m^2}{(m-1)(m-2)}\overline{(x-\bar{x})(y-\bar{y})(z-\bar{z})}$$

SME to Analytic Quantum Polyspectra

Stochastic Master Equation

Any quantum measurement of an observable A can be simulated with the SME [1]

$$d\rho = \frac{i}{\hbar}[\rho, H]dt + \sum_i c_i \mathcal{D}[c](\rho)dt + \beta^2\mathcal{D}[A](\rho)dt + \beta\mathcal{S}[A](\rho)dW$$

with the measurement strength β, damping terms

$$\mathcal{D}[c](\rho) = c\rho c^\dagger - \left(c^\dagger c\rho + \rho c^\dagger c\right)/2,$$

and backaction term

$$\mathcal{S}[c](\rho) = c\rho + \rho c^\dagger - \mathrm{Tr}\left[\left(c + c^\dagger\right)\rho\right]\rho.$$

The resulting detector output is

$$z(t) = \beta^2 \mathrm{Tr}\left[\rho(t)(A + A^\dagger)/2\right] + \tfrac{1}{2}\beta\Gamma(t),$$

where $\Gamma(t) = \dot{W}(t)$ is white noise.

Moments of the Detector Output

With the definition of

- the system propagator $\mathcal{G} = e^{\mathcal{L}t}\Theta(t)$
- the measurement superoperator $\mathcal{A}(x) = (Ax + xA^\dagger)/2$
- and the steady state ρ_0.

$$M_2(z(t_1), z(t_2)) = \langle z(t_1) z(t_2) \rangle = \beta^4 \sum_{prm.\, t_j} \mathrm{Tr}[\mathcal{A}\mathcal{G}(t_2 - t_1)\mathcal{A}\rho_0]$$

$$M_3(z(t_1), z(t_2), z(t_3)) = \langle z(t_1) z(t_2) z(t_3) \rangle = \beta^6 \sum_{prm.\, t_j} \mathrm{Tr}[\mathcal{A}\mathcal{G}(t_3 - t_2)\mathcal{A}\mathcal{G}(t_2 - t_1)\mathcal{A}\rho_0]$$

Fourth order expressions can be found in [2].

Cumulants

Polyspectra are defined via the cumulant of $z(\omega)$ (see box right). Compact expression can be found by rewriting

- $\mathcal{G}'(\tau) = \mathcal{G}(\tau) - \mathcal{G}(\infty)\Theta(\tau)$
- $\mathcal{A}'(x) = \mathcal{A}(x) - \mathrm{Tr}(\mathcal{A}\rho_0)x$

$$C_2(z(t_1), z(t_2)) = \beta^4 \sum_{prm.\, t_j} \mathrm{Tr}[\mathcal{A}'\mathcal{G}'(t_2 - t_1)\mathcal{A}'\rho_0]$$

$$C_3(z(t_1), z(t_2), z(t_3)) = \beta^6 \sum_{prm.\, t_j} \mathrm{Tr}[\mathcal{A}'\mathcal{G}'(t_3 - t_2)\mathcal{A}'\mathcal{G}'(t_2 - t_1)\mathcal{A}'\rho_0]$$

Fourth order expressions can be found in [2].

Quantum Polyspectra

A general definition of Polyspectra was given by Brillinger [7]

$$C_n(z(\omega_1), \dots, z(\omega_n)) = 2\pi\delta(\omega_1 + \dots + \omega_n) S_z^{(n)}(\omega_1, \dots, \omega_{n-1})$$

Expressions for the quantum polyspectra can, therefore, be found by Fourier transforming of the cumulant expressions [4]

$$S_z^{(2)}(\omega) = \beta^4(\mathrm{Tr}[\mathcal{A}'\mathcal{G}'(\omega)\mathcal{A}'\rho_0] + \mathrm{Tr}[\mathcal{A}'\mathcal{G}'(-\omega)\mathcal{A}'\rho_0]) + \beta^2/4$$

$$S_3(\omega_1, \omega_2, \omega_3 = -\omega_1 - \omega_2) = \beta^6 \sum_{prm.\, \omega_1, \omega_2, \omega_3} \mathrm{Tr}[\mathcal{A}'\mathcal{G}'(\omega_3)\mathcal{A}'\mathcal{G}'(\omega_3 + \omega_2)\mathcal{A}'\rho_0]$$

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[4] M. Sifft and D. Hägele, Continuous Quantum Measurements via Random-Time Sampling, arXiv:2109.05862 (2021)
[5] A. Kurzmann, P. Stegmann, J. Kerski, R. Schott, A. Ludwig, A. D. Wieck, J. König, A. Lorke, and M. Geller, Optical detection of single-electron tunneling into a semiconductor quantum dot, Phys. Rev. Lett. 122, 247403 (2019)
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