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# Coupled Exponentials & Logarithms

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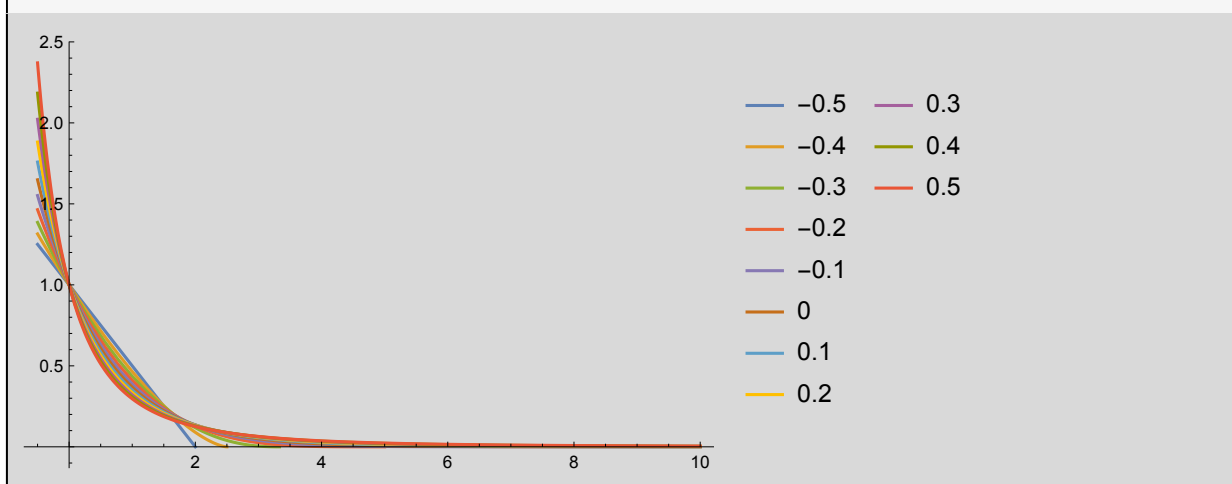
## Graphic of Coupled Exponential

Graph shows curves from linear ( $\kappa = -0.5$ ) to exponential ( $\kappa = 0$ )

`In[ ]:=`

```
CouplingValues = {-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5};  
Plot[CoupledExponential[x, #]^-1 & /@ CouplingValues // Evaluate,  
{x, -0.5, 10}, PlotLegends -> CouplingValues, PlotRange -> Full]
```

`Out[ ]:=`



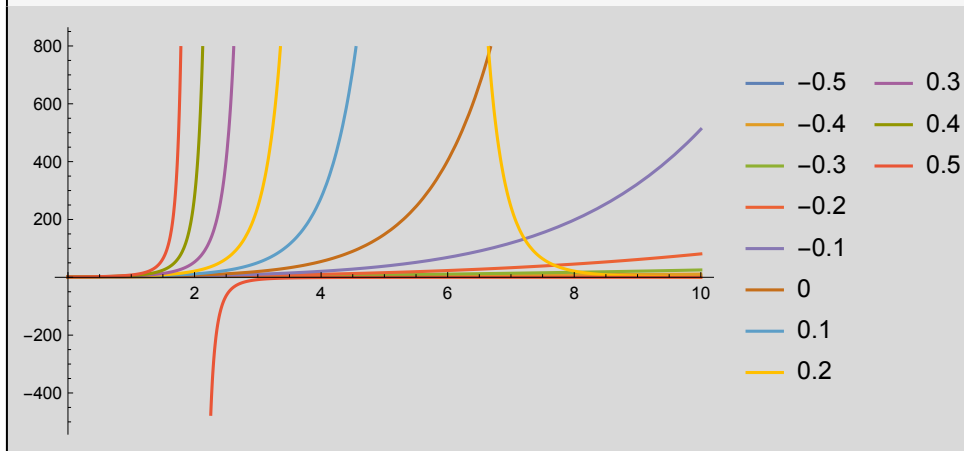
The curves are produced by the Coupled Exponential Function

$$(1 + \kappa x)^{-\frac{1+\kappa}{\kappa}}$$

In[ ]:=

```
CouplingValues = {-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5};
Plot[CoupledExponential[-x, #]^-1 & /@ CouplingValues // Evaluate,
{x, 0, 10}, PlotLegends -> CouplingValues]
```

Out[ ]:=



The curves are produced by the Coupled Exponential Function

$$(1 - \kappa x)^{\frac{1+\kappa}{-\kappa}}$$

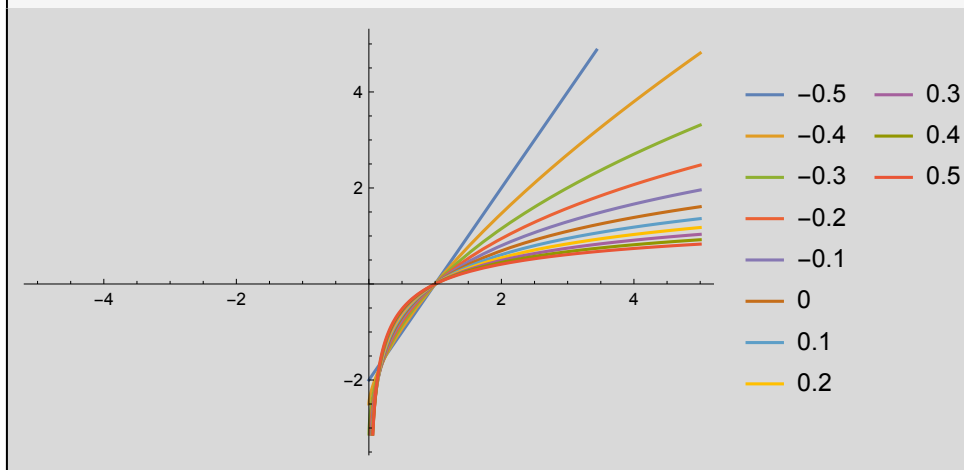
## Graphic of Coupled Logarithm

Graph shows curves from linear to logarithmic

In[ ]:=

```
CouplingValues = {-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5};
Quiet[Plot[
-CoupledLogarithm[x^-1, #] & /@ CouplingValues // Evaluate, {x, -5, 5},
PlotLegends -> CouplingValues],
{Power::infy}]
```

Out[ ]:=



The curves are produced by the Coupled Logarithmic Function

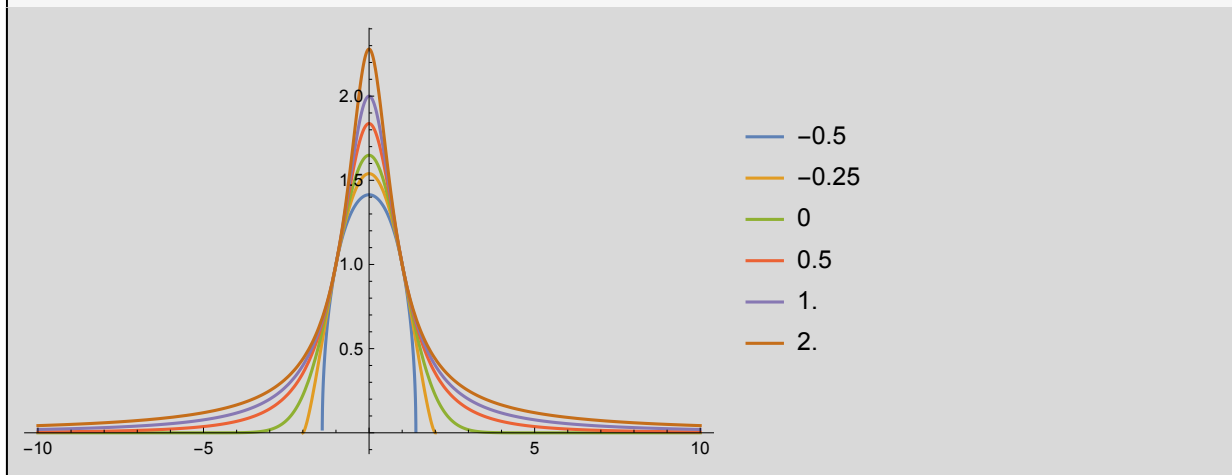
$$\frac{1}{-\kappa} \left( x^{\frac{-\kappa}{1+\kappa}} - 1 \right)$$

## Coupled Normal Distribution

In[ ]:=

```
CouplingValues = {-0.5, -0.25, 0, 0.5, 1.0, 2.0};  
Quiet[Plot[  
  PDF[CoupledNormalDistribution[0, 1, #], {x}] /  
    PDF[CoupledNormalDistribution[0, 1, #], {1}] & /@  
  CouplingValues // Evaluate, {x, -10, 10},  
  PlotLegends -> CouplingValues,  
  PlotRange -> Full],  
{Power::infy}]
```

Out[ ]:=

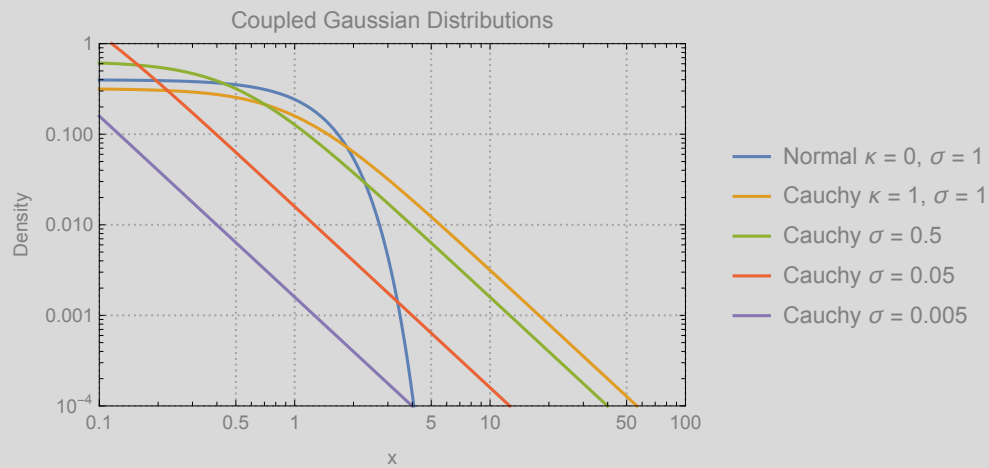


## Coupled Gaussian is Scale-Free as $\sigma \rightarrow 0$

In[ ]:=

```
Parameters = {{1, 1, 0.5, 0.05, 0.005}, {0, 1, 1, 1, 1}};
Quiet[LogLogPlot[MapThread[
  PDF[CoupledNormalDistribution[0, #1, #2], {x}] &, Parameters] // Evaluate,
{x, 0.1, 100},
PlotLegends → {"Normal  $\kappa = 0, \sigma = 1$ ",
  "Cauchy  $\kappa = 1, \sigma = 1$ ", "Cauchy  $\sigma = 0.5$ ",
  "Cauchy  $\sigma = 0.05$ ", "Cauchy  $\sigma = 0.005$ "},
LabelStyle → Directive[Gray, Smaller],
PlotRange → {{0.1, 100}, {10-4, 1}},
PlotTheme → {"Detailed"},
FrameLabel → {"x", "Density"},
PlotLabel → "Coupled Gaussian Distributions"],
{Power::infy}]
```

Out[ ]:=



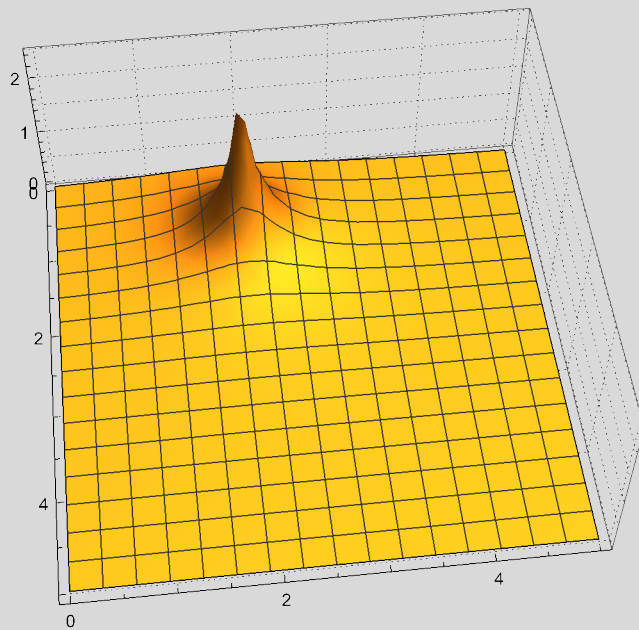
# Multivariate Coupled Distribution

## Multivariate Coupled Exponential

In[228]:=

```
Plot3D[PDF[MultivariateCoupledDistribution[{1, 2}, {{1, 0}, {0, 1}}, 2, 1],  
  {x, y}],  
  {x, 0, 5}, {y, 0, 5},  
  PlotLegends → None,  
  PlotTheme → "Detailed",  
  PlotRange → Full  
]
```

Out[228]=



## Multivariate Coupled Gaussian

In[230]:=

```
Plot3D[  
  PDF[MultivariateCoupledDistribution[{1, 2}, {{1, -0.01}, {0.01, 1}}, 0.01, 2],  
  {x, y}],  
  {x, -5, 5}, {y, -5, 5},  
  PlotLegends → None,  
  PlotTheme → "Detailed",  
  PlotRange → Full  
]
```

Out[230]=

