



## Outline

- ▶ Start from algorithms close to PG: TRPO and ACKTR
- ▶ Three aspects distinguish TRPO:
  - ▶ Surrogate return objective
  - ▶ Natural policy gradient
  - ▶ Conjugate gradient approach
- ▶ Differences in ACKTR:
  - ▶ Approximate second order gradient descent (Hessian)
  - ▶ Using Kronecker Factored Approximated Curvature

## Surrogate return objective

- ▶ The standard policy gradient algorithm for stochastic policies is:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_t[\nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \hat{A}_{\phi}^{\pi_{\theta}}]$$

- ▶ This gradient is obtained from differentiating

$$Loss^{PG}(\theta) = \mathbb{E}_t[\log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \hat{A}_{\phi}^{\pi_{\theta}}]$$

- ▶ But we obtain the same gradient from differentiating

$$Loss^{IS}(\theta) = \mathbb{E}_t\left[\frac{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\pi_{\theta_{old}}(\mathbf{a}_t | \mathbf{s}_t)} \hat{A}_{\phi}^{\pi_{\theta}}\right]$$

where  $\pi_{\theta_{old}}$  is the policy at the previous iteration

- ▶ Because  $\nabla_{\theta} \log f(\theta) |_{\theta_{old}} = \frac{\nabla_{\theta} f(\theta) |_{\theta_{old}}}{f(\theta_{old})} = \nabla_{\theta} \left( \frac{f(\theta)}{f(\theta_{old})} \right) |_{\theta_{old}}$

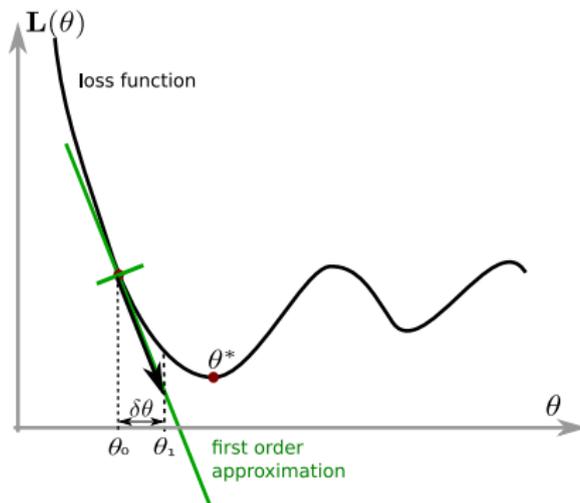
- ▶ Another view based on importance sampling
- ▶ See John Schulmann's Deep RL bootcamp lecture #5

<https://www.youtube.com/watch?v=SQtOI9jsrJ0>

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## Trust region

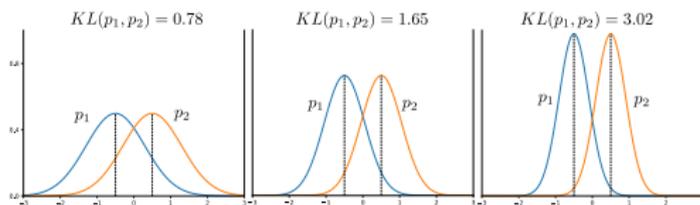


- ▶ The gradient of a function is only accurate close to the point where it is calculated
- ▶  $\nabla_{\theta} J(\theta)$  is only accurate close to the current policy  $\pi_{\theta}$
- ▶ Thus, when updating,  $\pi_{\theta}$  must not move too far away from a “trust region” around  $\pi_{\theta_{old}}$



Kakade, S. & Langford, J. (2002) Approximately optimal approximate reinforcement learning. In *ICML*, volume 2, pages 267–274

## Natural Policy Gradient



- ▶ One way to constrain two stochastic policies to stay close is constraining their KL divergence
- ▶ The KL divergence is smaller when the variance is larger
- ▶ Under fixed KL constraint, it is easier to move the mean further away when the variance is large
- ▶ Thus the mean policy converges first, then the variance is reduced
- ▶ Ensures a large enough amount of exploration noise
- ▶ Other properties presented in the Pierrot et al. (2018) paper



Sham M. Kakade. A natural policy gradient. In *Advances in neural information processing systems*, pp. 1531–1538, 2002



Pierrot, T., Perrin, N., & Sigaud, O. (2018) First-order and second-order variants of the gradient descent: a unified framework. *arXiv preprint arXiv:1810.08102*



## Trust Region Policy Optimization

- ▶ Theory: monotonous improvement towards the optimal policy  
(Assumptions do not hold in practice)
- ▶ To ensure small steps, TRPO uses a natural gradient update instead of standard gradient
- ▶ Minimize Kullback-Leibler divergence to previous policy



$$\max_{\theta} \mathbb{E}_t \left[ \frac{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\pi_{\theta_{old}}(\mathbf{a}_t | \mathbf{s}_t)} A_{\phi}^{\pi_{\theta_{old}}}(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$\text{subject to } \mathbb{E}_t [KL(\pi_{\theta_{old}}(\cdot | \mathbf{s}) || \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t))] \leq \delta$$

- ▶ In TRPO, optimization performed using a conjugate gradient method to avoid approximating the Fisher Information matrix



Schulman, J., Levine, S., Moritz, P., Jordan, M. I., & Abbeel, P. (2015) Trust Region Policy Optimization. *CoRR*, [abs/1502.05477](https://arxiv.org/abs/1502.05477)



## Advantage estimation

- ▶ To get  $\hat{A}_\phi^{\pi_\theta}$ , an empirical estimate of  $V^{\pi_\theta}(s)$  is needed
- ▶ TRPO uses a MC estimate approach through regression, but constrains it (as for the policy):

$$\min_{\phi} \sum_{n=0}^N \|V_{\phi}^{\pi_\theta}(s_n) - V^{\pi_\theta}(s_n)\|^2$$

subject to  $\frac{1}{N} \sum_{n=0}^N \frac{\|V_{\phi}^{\pi_\theta}(s_n) - V_{\phi_{old}}^{\pi_\theta}(s_n)\|^2}{2\sigma^2} \leq \epsilon$

- ▶ Equivalent to a mean KL divergence constraint between  $V_{\phi}^{\pi_\theta}$  and  $V_{\phi_{old}}^{\pi_\theta}$

## Properties

- ▶ Moves slowly away from current policy
- ▶ Key: use of line search to deal with the gradient step size
- ▶ More stable than DDPG, performs well in practice, but less sample efficient
- ▶ Conjugate gradient approach not provided in standard tensor gradient libraries, thus not much used
- ▶ Greater impact of PPO
- ▶ Related work: NAC, REPS

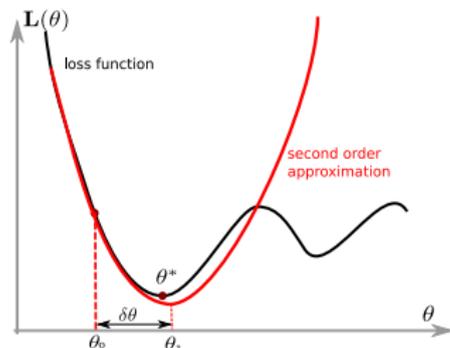
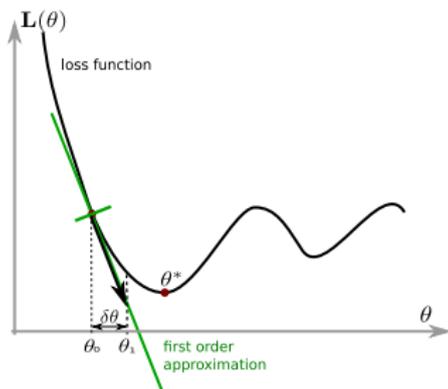


Jan Peters and Stefan Schaal. Natural actor-critic. *Neurocomputing*, 71 (7-9):1180–1190, 2008



Jan Peters, Katharina Mülling, and Yasemin Altun. Relative entropy policy search. In *AAAI*, pp. 1607–1612. Atlanta, 2010

## First order versus second order derivative



- ▶ In first order methods, need to define a step size
- ▶ Second order methods provide a more accurate approximation
- ▶ They also provide a true minimum, when the Hessian matrix is symmetric positive-definite (SPD)
- ▶ In both cases, the derivative is very local
- ▶ The trust region constraint applies too

## ACKTR

- ▶ K-FAC: Kronecker Factored Approximated Curvature: efficient estimate of the gradient
- ▶ Using block diagonal estimations of the Hessian matrix, to do better than first order
- ▶ ACKTR: TRPO with K-FAC natural gradient calculation
- ▶ But closer to actor-critic updates (see PPO)
- ▶ The per-update cost of ACKTR is only 10% to 25% higher than SGD
- ▶ Improves sample efficiency
- ▶ Not much excitement: less robust gradient approximation?
- ▶ Next lesson: PPO



Yuhui Wu, Elman Mansimov, Shun Liao, Roger Grosse, and Jimmy Ba (2017) Scalable trust-region method for deep reinforcement learning using Kronecker-factored approximation. *arXiv preprint arXiv:1708.05144*



Any question?



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Approximately optimal approximate reinforcement learning.  
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First-order and second-order variants of the gradient descent: a unified framework.  
*arXiv preprint arXiv:1810.08102*, 2018.



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