CURVES 2015: SnapPy worksheet 2 Thursday, June 25, 2015.

For links for SnapPy, SageMath, and other needed topics see: http://dunfield.info/curve.

1. Programming and searching.

   (a) Find all manifolds in the `OrientableClosedCensus` which have a 2- or 3-fold covering space with $b_1 > 0$.

   (b) The smallest volume manifold in the `OrientableClosedCensus` with $b_1 > 0$ is $m160(3,1)$ which has volume ≈ 3.1663333212. Find a closed manifold with $b_1 > 0$ with smaller volume by searching through 0 surgeries on the knots in `HTLinkExteriors`.

   (c) Express the first manifold M that you found in (b) as a Dehn filling on a 1-cusped manifold in the `OrientableCuspedCensus`.

   (d) The reason that the closed manifold M in part (c) is not in the `OrientableClosedCensus` is that its shortest geodesic has length smaller than the cutoff that was chosen by Hodgson and Weeks when they created this census. Determine this cutoff by finding length of the shortest geodesic for the all the manifolds in the `OrientableClosedCensus`.

2. Recall that two 3-manifolds are commensurable if they have a common finite cover.

   (a) In the `OrientableClosedCensus`, find several pairs of 1-cusped manifolds that appear to have the same volume. (You don't have to find all such pairs, though that can certainly be done.)

   (b) For each pair, increase your confidence that the volumes are the same by using the `ManifoldHP` type, which works with quad-double precision.

   (c) For each pair, try to either (a) find a common finite cover, or (b) show they are incommensurable by looking at the cusp density.

3. SageMath and SnapPy are friends.

   (a) Find the first nontrivial knot in `HTLinkExteriors` which has the same Alexander polynomial as the unknot.

   (b) Let M be the knot exterior you found in part (a). Via Matthias's new verify module in SnapPy, use interval arithmetic to rigorously prove that this manifold is indeed hyperbolic. Consequently, the corresponding knot is not the unknot.

   (c) Find exact expressions for the tetrahedra shapes of M, which live in some number field.

   (d) Check that the number field found in (c) is the same as the trace field of M.

4. SageMath and SnapPy are friends, part II.

   (a) Find many cusped hyperbolic 3-manifolds in the various censuses whose trace fields are $\mathbb{Q}(i)$. Hint: Computing trace fields is really expensive, so either (a) try with very small parameters or (b) numerically recognize elements of $\mathbb{Q}(i)$ that have small denominators.

   (b) Amongst the your examples in (a), find one with a non-integral trace. This manifold will not be arithmetic and will also contain a closed incompressible surface.